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Improved ratio type estimators of population mean based on median of a study variable and an auxiliary variable

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Abstract

This paper deals with efficient ratio type estimators for estimating finite population mean under simple random sampling scheme by using the knowledge of known median of a study and an auxiliary variable. Expressions for the bias and mean squared error of the proposed ratio type estimators are derived up to first order of approximation. It is found that our proposed estimators perform better as compared to the traditional ratio estimator, regression estimator, Subramani and Kumarapandiyan [23], Subramani and Prabavathy [24] and Yadav et al. [28] estimators. In addition, theoretical findings are verified with the help of real data sets.

Keywords: Auxiliary variables, Bias, Efficiency, Median, Mean squared error, Ratio estimators, RRMSE.

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1. Introduction

To use the additional information provided by an auxiliary or subsidiary variables enhances the precision of the ratio, product and regression estimators. When correlation between study variable and auxiliary variable is positively (high) then ratio estimator proposed by Cochran [5] is used. On the other hand, product estimator suggested by Robson [15] and rediscovered by Murthy [13] is preferably used when correlation is negatively (high). A lot of work has been done in the area of survey sample for the estimation of finite population mean using information on an auxiliary variable. Several authors including Sisodia and Dwivedi [22], Prasad [14], Upadhyaya and Singh [25], Singh and Tailor [19], Kadilar and Cingi [6] and [7], Singh et al. [20] and [21], Koyuncu and Kadilar [8] and [9], Yan and Tian [29], Singh and Solanki [17] and [18], Yadav and Kadilar [27], Kumar [10], Abid et al. [1], [2] and [3] have developed various estimators or classes of estimators for improved estimation of population mean using an auxiliary information under different sampling schemes. Further, Subramani and Kumarapandiyan [23], Subramani and Prabavathy [24] and Yadav et al. [28] proposed ratio estimators to estimate population mean using linear combination of population mean and median of an auxiliary variable.

Consider a sample of size "n" drawn by simple random sampling without replacement (SRSWOR) from a population of size N with $n < N$. Let the values of Y and X for the ith unit denote the observations on the study variable and auxiliary variable, respectively. The notations used in this paper can be described as follows:

The population mean of the study variable and auxiliary variable are denoted by $\bar{Y} = N^{-1}\sum_{i=1}^{N} y_i$ and $\bar{X} = N^{-1}\sum_{i=1}^{N} x_i$ respectively, where, $\bar{y} = n^{-1}\sum_{i=1}^{n} y_i$ and $\bar{x} = n^{-1}\sum_{i=1}^{n} x_i$ be the sample mean of the study variable and auxiliary variable respectively, $Y_{0.5}$ is the population median of the study variable, $\hat{Y}_{0.5}$ is the sample median of the study variable, $X_{0.5}$ is the population median of the auxiliary variable, $\hat{X}_{0.5}$ is the sample median of the auxiliary variable, $S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$ is the population variance of the study variable, $S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$ is the population variance of the auxiliary variable, $S_{yx} = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})$ is the population covariance between y and x, $C_y^2 = (\bar{Y}^2)^{-1}S_y^2$ is the square of coefficient of variation of y, $C_x^2 = (\bar{X}^2)^{-1}S_x^2$ is the square of coefficient of variation of x, $\rho_{yx} = (S_y S_x)^{-1}S_{yx}$ is the correlation coefficient between y and x, $f = \frac{n}{N}$ is the sampling fraction, $R' = \frac{\bar{Y}}{Y_{0.5}}$ is the ratio of population mean to population median and $\eta = (\frac{1}{n} - \frac{1}{N})$ denote the finite population correction factor.

In order to find the bias and mean square error (MSE) of the existing and proposed estimators, we define the following relative error terms and their expectations.

Let $\zeta_o = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$, $\zeta_1 = \frac{\hat{Y}_{0.5} - Y_{0.5}}{Y_{0.5}}$ and $\zeta_2 = \frac{\bar{x} - \bar{X}}{\bar{X}}$ such that $E(\zeta_i = 0)$ for $i = 0, 1$ and 2 where $E(.)$ represents the mathematical expectation.

Let $E(\zeta_0^2) = \eta\frac{V(\bar{y})}{\bar{Y}^2}$, $E(\zeta_1^2) = \eta\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2}$, $E(\zeta_2^2) = \eta\frac{V(\bar{x})}{\bar{X}^2}$, $E(\zeta_0\zeta_1) = \eta\frac{Cov(\bar{y}, \hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}$ and $E(\zeta_0\zeta_2) = \eta\frac{Cov(\bar{y}, \bar{x})}{\bar{Y}\bar{X}}$.

Mean squared error or variance of the usual unbiased estimator $\hat{\bar{Y}}$ in simple random sampling without replacement (SRSWOR) is given as:

$$(1.1)\qquad MSE(\hat{\bar{Y}}) = V(\hat{\bar{Y}}) = \eta S_y^2$$

The usual ratio estimator proposed by Cochran [5] to estimate population mean $\bar{Y}$ of the study variable Y is defined by:

$$(1.2)\qquad \hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X}, \text{ where } \bar{x} \neq 0$$

The bias and MSE of the ratio estimator, $\hat{\bar{Y}}_R$, to first order of approximation, are given by

$$(1.3)\qquad Bias(\hat{\bar{Y}}_R) \cong \frac{\eta}{\bar{X}}\left(RS_x^2 - S_{yx}\right)$$

and

$$(1.4)\qquad MSE(\hat{\bar{Y}}_R) \cong \eta\left(S_y^2 + R^2S_x^2 - 2RS_{yx}\right), \text{ where } R = \frac{\bar{Y}}{\bar{X}}$$

The linear regression estimator suggested by Watson [26] to estimate the population mean $\bar{Y}$ of the study variable Y using one auxiliary variable is defined as,

$$(1.5)\qquad \hat{\bar{Y}}_{Reg} = \bar{y} + b(\bar{X} - \bar{x})$$

where $b = \frac{S_{yx}}{S_x^2}$ is the sample regression coefficient (assumed to be known) of Y on X. The MSE of the estimator $\hat{\bar{Y}}_{Reg}$ is given as,

$$(1.6)\qquad \hat{\bar{Y}}_{Reg} \cong \eta S_y^2\left(1 - \rho_{yx}^2\right)$$

Subramani and Kumarapandiyan [23] suggested modified ratio estimator for the estimation of population mean using the known value of median of an auxiliary variable as,

$$(1.7)\qquad \hat{\bar{Y}}_{SK} = \bar{y}\left(\frac{\bar{X} + X_{0.5}}{\bar{x} + X_{0.5}}\right)$$

The bias and MSE of the Subramani and Kumarapandiyan [23] estimator are given below,

$$(1.8)\qquad Bias(\hat{\bar{Y}}_{SK}) = \eta\bar{Y}\left(\theta^2C_x^2 - \theta\rho_{yx}C_yC_x\right)$$

$$(1.9)\qquad MSE(\hat{\bar{Y}}_{SK}) = \eta\bar{Y}^2\left(C_y^2 + \theta^2C_x^2 - 2\theta\rho_{yx}C_yC_x\right) \text{ where } \theta = \frac{\bar{X}}{\bar{X} + X_{0.5}}$$

Subramani and Prabavathy [24] are proposed following modified ratio estimators to estimate the population mean using the linear combination of population mean and median of an auxiliary variable,

$$(1.10)\qquad \hat{\bar{Y}}_{SP1} = \bar{y}\left(\frac{X_{0.5}Y_{0.5} + \bar{X}}{X_{0.5}\hat{Y}_{0.5} + \bar{X}}\right)$$

$$(1.11)\qquad \hat{\bar{Y}}_{SP2} = \bar{y}\left(\frac{\bar{X}Y_{0.5} + X_{0.5}}{\bar{X}\hat{Y}_{0.5} + X_{0.5}}\right)$$

The bias and MSE of the estimators $\hat{\bar{Y}}_{SP1}$ and $\hat{\bar{Y}}_{SP2}$ are given as,

$$(1.12)\qquad Bias(\hat{\bar{Y}}_{SPi}) = \eta\bar{Y}\left({\theta'_j}^2\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2} - \theta'_j\frac{Cov(\bar{y}, \hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}\right) \text{ where } i \text{ and } j = 1, 2$$

$$(1.13)\qquad MSE(\hat{\bar{Y}}_{SPi}) = \eta\left(S_y^2 + R'^2\theta_j'^2V(\hat{Y}_{0.5}) - 2R'\theta'_jCov(\bar{y}, \hat{Y}_{0.5})\right)$$

where $i\ and\ j = 1, 2$
and

$$\theta_1' = \frac{X_{0.5}Y_{0.5}}{X_{0.5}Y_{0.5} + \bar{X}}\ and\ \theta_2' = \frac{\bar{X}Y_{0.5}}{\bar{X}Y_{0.5} + X_{0.5}}.$$

Motivated by the estimators in Subramani and Prabavathy [24] and Prasad [14], the Yadav et al. [28] proposed some new improved ratio estimators based on median of an auxiliary variable. They showed that their proposed estimators perform more efficiently than the usual ratio estimator and the estimators proposed by Subramani and Prabavathy [24]. The Yadav et al. [28] proposed estimators are defined as,

$$(1.14)\qquad \hat{\bar{Y}}_{YD1} = k_1\bar{y}\left(\frac{X_{0.5}Y_{0.5} + \bar{X}}{X_{0.5}\hat{Y}_{0.5} + \bar{X}}\right)$$

$$(1.15)\qquad \hat{\bar{Y}}_{YD2} = k_2\bar{y}\left(\frac{\bar{X}Y_{0.5} + X_{0.5}}{\bar{X}\hat{Y}_{0.5} + X_{0.5}}\right)$$

where k_1 and k_2 are the suitable constants.
The expressions for bias and MSE of the Yadav et al. [28] estimators are given by,

$$(1.16)\qquad Bias(\hat{\bar{Y}}_{YDi}) \cong \bar{Y}\left((k_i - 1) + \eta\theta_i^2\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2} - \eta\theta_i\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}\right)$$

$$(1.17)\qquad MSE(\hat{\bar{Y}}_{YDi}) \cong \bar{Y}^2\left[(k_i - 1)^2 + \eta k_i^2[E(\zeta_o^2) + 3\theta_i^2E(\zeta_1^2) - 4\theta_iE(\zeta_o\zeta_1)]\right]$$
$$-2\bar{Y}^2\eta k_i\left[\theta_i^2E(\zeta_1^2) - \theta_iE(\zeta_o\zeta_1)\right]$$

The optimum value of k_i are,

$$k_i = \frac{1 + \eta\theta_i^2E(\zeta_1^2) - \eta\theta_iE(\zeta_0\zeta_1)}{1 + \eta E(\zeta_0^2) + 3\eta\theta i^2E(\zeta_1^2) - 4\eta\theta_iE(\zeta_0\zeta_1)}$$

$$k_i = \frac{1 + \eta\theta_i^2\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2} - \eta\theta_i\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}}{1 + \eta\frac{V(\bar{y})}{\bar{Y}^2} + 3\eta\theta_i^2\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2} - 4\eta\theta_i\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}}$$

$$k_i = \frac{A_i}{B_i}$$

where

$$A_i = 1 + \eta\theta_i^2\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2} - \eta\theta_i\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}$$

$$B_i = 1 + \eta\frac{V(\bar{y})}{\bar{Y}^2} + 3\eta\theta_i^2\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2} - 4\eta\theta_i\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}$$

After simplifying equation (1.17), the MSE of Yadav et al. [28] estimators can be written as,

$$(1.18)\qquad MSE(\hat{\bar{Y}}_{YDi}) \cong \bar{Y}^2\left(1 - \frac{A_i^2}{B_i}\right)\ \text{where}\ i = 1, 2$$

The remaining part of the paper is organized as follows: In section 2, the proposed ratio type estimators for estimating finite population mean using the known value of the median of a study variable and an auxiliary variable are defined. The conditions in which the proposed estimators perform better than the existing estimators are presented in section 3. In section 4, an empirical study is carried out to evaluate the performance of the proposed estimators. Finally, we close with summary conclusion in the last section.

2. Proposed Ratio Type Estimators

Motivated by the work of Subramani and Prabavathy [24] and Yadav et al. [28], we proposed the following ratio type estimators for estimating the population mean using the known value of the median of a study variable and an auxiliary variable.

$$\hat{\hat{Y}}_{P1} = t_1\bar{y}\left(\frac{X_{0.5}Y_{0.5}+\bar{X}}{X_{0.5}\hat{Y}_{0.5}+\bar{X}}\right)^{\frac{X_{0.5}Y_{0.5}}{X_{0.5}Y_{0.5}+\bar{X}}} \tag{2.1}$$

$$\hat{\hat{Y}}_{P2} = t_2\bar{y}\left(\frac{\bar{X}Y_{0.5}+X_{0.5}}{\bar{X}\hat{Y}_{0.5}+X_{0.5}}\right)^{\frac{\bar{X}Y_{0.5}}{\bar{X}Y_{0.5}+X_{0.5}}} \tag{2.2}$$

$$\hat{\hat{Y}}_{P3} = t_3\bar{y}\left(\frac{Y_{0.5}+1}{\hat{Y}_{0.5}+1}\right)^{\frac{Y_{0.5}}{Y_{0.5}+1}} \tag{2.3}$$

$$\hat{\hat{Y}}_{P4} = t_4\bar{y}\left(\frac{\bar{X}R'Y_{0.5}+X_{0.5}}{\bar{X}R'\hat{Y}_{0.5}+X_{0.5}}\right)^{\frac{\bar{X}R'Y_{0.5}}{\bar{X}R'Y_{0.5}+X_{0.5}}} \tag{2.4}$$

$$\hat{\hat{Y}}_{P5} = t_5\bar{y}\left(\frac{X_{0.5}Y_{0.5}+R'\bar{X}}{X_{0.5}\hat{Y}_{0.5}+R'\bar{X}}\right)^{\frac{X_{0.5}Y_{0.5}}{X_{0.5}Y_{0.5}+R'\bar{X}}} \tag{2.5}$$

$$\hat{\hat{Y}}_{P6} = t_6\bar{y}\left(\frac{Y_{0.5}+R'}{\hat{Y}_{0.5}+R'}\right)^{\frac{Y_{0.5}}{Y_{0.5}+R'}} \tag{2.6}$$

$$\hat{\hat{Y}}_{P7} = t_7\left(\bar{y}\left(\frac{X_{0.5}+Y_{0.5}}{X_{0.5}+\hat{Y}_{0.5}}\right)^{\frac{Y_{0.5}}{Y_{0.5}+X_{0.5}}} + b(Y_{0.5}-\hat{Y}_{0.5})\right) \tag{2.7}$$

$$\hat{\hat{Y}}_{P8} = t_8\left(\bar{y}\left(\frac{\bar{X}X_{0.5}+Y_{0.5}}{\bar{X}X_{0.5}+\hat{Y}_{0.5}}\right)^{\frac{Y_{0.5}}{Y_{0.5}+\bar{X}X_{0.5}}} + b(Y_{0.5}-\hat{Y}_{0.5})\right) \tag{2.8}$$

$$\hat{\hat{Y}}_{P9} = t_9\left(\bar{y}\left(\frac{\bar{X}+Y_{0.5}}{\bar{X}+\hat{Y}_{0.5}}\right)^{\frac{Y_{0.5}}{Y_{0.5}+\bar{X}}} + b(Y_{0.5}-\hat{Y}_{0.5})\right) \tag{2.9}$$

where $\left(\hat{\hat{Y}}_{Pi}, i = 1,2,3,...,9\right)$ and $(t_i, i = 1,2,3,...,9)$ are the unknown constants to be determined later.
where

$$\delta_1 = \frac{X_{0.5}Y_{0.5}}{X_{0.5}Y_{0.5}+\bar{X}}, \delta_2 = \frac{\bar{X}Y_{0.5}}{\bar{X}Y_{0.5}+X_{0.5}}, \delta_3 = \frac{Y_{0.5}}{Y_{0.5}+1}, \delta_4 = \frac{\bar{X}R'Y_{0.5}}{\bar{X}R'Y_{0.5}+X_{0.5}},$$
$$\delta_5 = \frac{X_{0.5}Y_{0.5}}{X_{0.5}Y_{0.5}+R'\bar{X}}, \delta_6 = \frac{Y_{0.5}}{Y_{0.5}+R'}, \delta_7 = \frac{Y_{0.5}}{Y_{0.5}+X_{0.5}}, \delta_8 = \frac{Y_{0.5}}{Y_{0.5}+\bar{X}X_{0.5}}$$
$$and\ \delta_9 = \frac{Y_{0.5}}{Y_{0.5}+\bar{X}}$$

After writing the proposed ratio type estimators $\left(\hat{\hat{Y}}_{Pi}, i = 1,2,3,4,5\ and\ 6\right)$ in terms of $\zeta_i' s$, we have obtained the following terms,

$$\hat{\hat{Y}}_{Pi} = t_i\bar{Y}(1+\zeta_0)(1+\delta_i\zeta_1)^{-\delta_i} \tag{2.10}$$

$$(2.11)\qquad \hat{\hat{Y}}_{Pi} = t_i\bar{Y}(1+\zeta_0)\left(1-\delta_i^2\zeta_1+\frac{1}{2}\delta_i^3(\delta_i+1)\zeta_1^2\right)$$

Expanding the right hand side of the equation (2.11) to the first degree of approximation and also subtracting $\bar{Y}$ from both sides of equation (2.11), we get:

$$(2.12)\qquad \hat{\hat{Y}}_{Pi}-\bar{Y} = \bar{Y}\left(t_i+t_i\zeta_0-t_i\delta_i^2\zeta_1+\frac{1}{2}t_i(\delta_i^4+\delta_i^3)\zeta_1^2-t_i\delta_i^2\zeta_0\zeta_1-1\right)$$

The bias of the proposed estimators, $\left(\hat{\hat{Y}}_{Pi}, i=1,2,3,4,5\ and\ 6\right)$ are defined as,

$$Bias(\hat{\hat{Y}}_{Pi}) \cong E(\hat{\hat{Y}}_{Pi}-\bar{Y})$$

So,

$$(2.13)\qquad Bias(\hat{\hat{Y}}_{Pi}) \cong \bar{Y}(t_i-1)+\eta\bar{Y}t_i\left[\frac{1}{2}(\delta_i^4+\delta_i^3)\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2}-\delta_i^2\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}\right]$$

The MSE of the proposed estimators, $\left(\hat{\hat{Y}}_{Pi}, i=1,2,3,4,5\ and\ 6\right)$ are defined as,

$$MSE(\hat{\hat{Y}}_{Pi}) \cong E(\hat{\hat{Y}}_{Pi}-\bar{Y})^2$$

Hence,

$$(2.14)\qquad MSE(\hat{\hat{Y}}_{Pi}) \cong \bar{Y}^2\left[(t_i-1)^2+t_i^2\left(E(\zeta_0^2)+(2\delta_i^4+\delta_i^3)E(\zeta_1^2)-4\delta_i^2E(\zeta_0\zeta_1)\right)\right] - t_i\bar{Y}^2\left[(\delta_i^4+\delta_i^3)E(\zeta_1^2)-2\delta_i^2E(\zeta_0\zeta_1)\right]$$

To get the optimum value of t_i, we differentiate equation (2.14) with respect to t_i and equating it equal to zero, we get,

$$t_i = \frac{2+\left[(\delta_i^4+\delta_i^3)E(\zeta_1^2)-2\delta_i^2E(\zeta_0\zeta_1)\right]}{2+2\left[E(\zeta_0^2)+(2\delta_i^4+\delta_i^3)E(\zeta_1^2)-4\delta_i^2E(\zeta_0\zeta_1)\right]}$$

$$t_i = \frac{2+\eta\left((\delta_i^4+\delta_i^3)\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2}-2\delta_i^2\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}\right)}{2+2\eta\left(\frac{V(\bar{y})}{\bar{Y}^2}+(2\delta_i^4+\delta_i^3)\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2}-4\delta_i^2\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}\right)}$$

$$t_i = \frac{C_{1i}}{C_{2i}}$$

and

$$C_{1i} = 2+\eta\left((\delta_i^4+\delta_i^3)\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2}-2\delta_i^2\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}\right)$$

$$C_{2i} = 2+2\eta\left(\frac{V(\bar{y})}{\bar{Y}^2}+(2\delta_i^4+\delta_i^3)\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2}-4\delta_i^2\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}\right)$$

After putting the value of t_i in equation (2.14), we get,

$$MSE(\hat{\hat{Y}}_{Pi}) \cong \bar{Y}^2\left[\left(\frac{C_{1i}}{C_{2i}}-1\right)^2+\left(\frac{C_{1i}}{C_{2i}}\right)^2\left[E(\zeta_0^2)+(2\delta_i^4+\delta_i^3)E(\zeta_1^2)-4\delta_i^4E(\zeta_0\zeta_1)\right]\right]$$

$$(2.15)\qquad -\bar{Y}^2\left(\frac{C_{1i}}{C_{2i}}\right)\left[(\delta_i^4+\delta_i^3)E(\zeta_1^2)-2\delta_i^2E(\zeta_0\zeta_1)\right]$$

The minimum MSE of the proposed ratio type estimators is given as,

$$(2.16)\qquad MSE_{min}\left(\hat{\hat{Y}}_{Pi}\right) \cong \bar{Y}^2\left(1-\frac{C_{1i}^2}{2C_{2i}}\right)\ \text{ where } i=1,2,3,4,5 \text{ and } 6$$

Now, for the rest of the proposed ratio type estimators $\left(\hat{\hat{Y}}_{Pi}, i = 7, 8 \ and \ 9\right)$, can be written in terms of $\zeta_i' s$, as follows

$$\hat{\hat{Y}}_{Pi} = t_i \left[(\bar{Y} + \bar{Y}\zeta_0)(1 + \delta_i\zeta_1)^{-\delta_i} - bY_{0.5}\zeta_1\right]$$

or

$$\hat{\hat{Y}}_{Pi} = t_i \left[(\bar{Y} + \bar{Y}\zeta_0)\left(1 - \delta_i^2\zeta_1 + \frac{1}{2}\delta_i^3(\delta_i + 1)\zeta_1^2\right) - bY_{0.5}\zeta_1\right] \tag{2.17}$$

where $b = \dfrac{Cov(\bar{y}, \hat{Y}_{0.5})}{V(\hat{Y}_{0.5})}$

Subtracting $\bar{Y}$ from both sides of equation (2.17) and solve this term to first degree of approximation, we obtain,

$$\hat{\hat{Y}}_{Pi} - \bar{Y} = t_i\bar{Y} + t_i\bar{Y}\zeta_0 - t_i\bar{Y}\delta_i^2\zeta_1 - t_i bY_{0.5}\zeta_1 + \frac{1}{2}t_i\bar{Y}(\delta_i^4 + \delta_i^3)\zeta_1^2 - t_i\bar{Y}\delta_i^2\zeta_0\zeta_1 - \bar{Y} \tag{2.18}$$

The bias of the proposed estimators, $\left(\hat{\hat{Y}}_{Pi}, i = 7, 8 \ and \ 9\right)$, are defined as,

$$Bias(\hat{\hat{Y}}_{Pi}) \cong E(\hat{\hat{Y}}_{Pi} - \bar{Y})$$

So,

$$Bias(\hat{\hat{Y}}_{Pi}) \cong \bar{Y}(t_i - 1) + \eta\bar{Y}t_i\left[\frac{1}{2}(\delta_i^4 + \delta_i^3)\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2} - \delta_i^2\frac{Cov(\bar{y}, \hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}\right] \tag{2.19}$$

The MSE of the proposed estimators can be written as,

$$MSE(\hat{\hat{Y}}_{Pi}) \cong E(\hat{\hat{Y}}_{Pi} - \bar{Y})^2$$

Hence,

$$\begin{aligned} MSE(\hat{\hat{Y}}_{Pi}) \cong \ & (t_i\bar{Y} - \bar{Y})^2 + t_i^2\left[\bar{Y}^2E(\zeta_0^2) + \left(2\delta_i^4\bar{Y}^2 + b^2Y_{0.5}^2 + \delta_i^3\bar{Y}^2 + 2\delta_i^2 b\bar{Y}Y_{0.5}\right)E(\zeta_1^2)\right] \\ & - t_i^2\left[4\delta_i^2\bar{Y}^2 + 2b\bar{Y}Y_{0.5}\right]E(\zeta_0\zeta_1) - t_i\left[(\delta_i^4\bar{Y}^2 + \delta_i^3\bar{Y}^2)E(\zeta_1^2) - 2\delta_i^2\bar{Y}^2E(\zeta_0\zeta_1)\right] \end{aligned} \tag{2.20}$$

Differentiating equation (2.20) with respect to t_i and then equating this equation equal to zero, we get values of t_i as,

$$t_i = \frac{2\bar{Y}^2 + (\delta_i^4\bar{Y}^2 + \delta_i^3\bar{Y}^2)E(\zeta_1^2) - 2\delta_i^2\bar{Y}^2E(\zeta_0\zeta_1)}{2\bar{Y}^2 + 2\left[\bar{Y}^2E(\zeta_0^2) + \left(2\delta_i^4\bar{Y}^2 + b^2Y_{0.5}^2 + \delta_i^3\bar{Y}^2 + 2\delta_i^2 b\bar{Y}Y_{0.5}\right)E(\zeta_1^2) - \left(4\delta_i^2\bar{Y}^2 + 2b\bar{Y}Y_{0.5}\right)E(\zeta_0\zeta_1)\right]}$$

$$t_i = \frac{2\bar{Y}^2 + \eta\left[(\delta_i^4 + \delta_i^3)R'^2V(\hat{Y}_{0.5}) - 2\delta_i^2R'Cov(\bar{y}, \hat{Y}_{0.5})\right]}{2\bar{Y}^2 + 2\eta\left[V(\bar{y}) + (2\delta_i^4R'^2 + b^2 + \delta_i^3R'^2 + 2\delta_i^2bR')V(\hat{Y}_{0.5}) - (4\delta_i^2R' + 2b)Cov(\bar{y}, \hat{Y}_{0.5})\right]}$$

$$t_i = \frac{C_{3i}}{C_{4i}}$$

and

$$C_{3i} = 2\bar{Y}^2 + \eta\left[(\delta_i^4 + \delta_i^3)R'^2V(\hat{Y}_{0.5}) - 2\delta_i^2R'Cov(\bar{y}, \hat{Y}_{0.5})\right]$$

$$C_{4i} = 2\bar{Y}^2 + 2\eta\left[V(\bar{y}) + \left(2\delta_i^4R'^2 + b^2 + \delta_i^3R'^2 + 2\delta_i^2bR'\right)V(\hat{Y}_{0.5}) - \left(4\delta_i^2R' + 2b\right)Cov(\bar{y}, \hat{Y}_{0.5})\right]$$

After substituting the value of t_i in equation (2.20), we get,

$$MSE(\hat{\bar{Y}}_{Pi}) \cong \left(\frac{C_{3i}}{C_{4i}}\bar{Y}-\bar{Y}\right)^2 +$$
$$+\left(\frac{C_{3i}}{C_{4i}}\right)^2\left[\bar{Y}^2E(\zeta_0^2)+\left(2\delta_i^4\bar{Y}^2+b^2Y_{0.5}^2+\delta_i^3\bar{Y}^2+2\delta_i^2b\bar{Y}Y_{0.5}\right)E(\zeta_1^2)\right]$$
$$-\left(\frac{C_{3i}}{C_{4i}}\right)^2\left[4\delta_i^2\bar{Y}^2+2b\bar{Y}Y_{0.5}\right]E(\zeta_0\zeta_1)-$$
$$(2.21)\qquad -\left(\frac{C_{3i}}{C_{4i}}\right)\left[(\delta_i^4\bar{Y}^2+\delta_i^3\bar{Y}^2)E(\zeta_1^2)-2\delta_i^2\bar{Y}^2E(\zeta_0\zeta_1)\right]$$

Thus, the minimum MSE of the proposed ratio type estimators is given as,

$$(2.22)\qquad MSE_{min}\left(\hat{\bar{Y}}_{Pi}\right) \cong \left[\bar{Y}^2-\frac{C_{3i}^2}{2C_{4i}}\right] \text{ where } i=7,8 \text{ and } 9$$

3. Efficiency comparison of proposed estimators with existing estimators

In this section, the conditions for which the proposed ratio type estimators based on the known value of the median will have minimum mean square error as compared to usual ratio estimator, the regression estimator, Subramani and Kumarapandiyan [23] estimator, Subramani and Prabavathy [24] estimators and Yadav et al. [28] estimators for estimating the finite population mean have been derived algebraically.

3.1. The usual unbiased estimator. We compare MSE of usual unbiased estimator with the MSE of the proposed ratio type estimators by using the expressions of (1.1) and (2.16) as follows,

$$MSE_{min}\left(\hat{\bar{Y}}_{Pi}\right) < MSE\left(\hat{\bar{Y}}\right)$$
$$\bar{Y}^2\left[1-\frac{C_{1i}^2}{2C_{2i}}\right] < \eta S_y^2$$
$$(3.1)\qquad \left[\eta C_y^2+\frac{C_{1i}^2}{2C_{2i}}\right] > 1 \text{ where } i=1,2,3,4,5 \text{ and } 6$$

and
By (1.1) and (2.22)

$$MSE_{min}\left(\hat{\bar{Y}}_{Pi}\right) < MSE\left(\hat{\bar{Y}}\right)$$
$$(3.2)\qquad \left[\eta C_y^2+\frac{C_{3i}^2}{2\bar{Y}^2C_{4i}}\right] > 1 \text{ where } i=7,8 \text{ and } 9$$

If the conditions given in equations (3.1) and (3.2) are satisfied, then the proposed ratio type estimators are more efficient than the usual unbiased estimator.

3.2. The usual ratio estimator. We compare MSE of usual ratio estimator with MSE of proposed ratio type estimators using expressions of (1.4) and (2.16) as

$$MSE_{min}\left(\hat{\bar{Y}}_{Pi}\right) < MSE\left(\hat{\bar{Y}}_R\right)$$
$$\bar{Y}^2\left[1-\frac{C_{1i}^2}{2C_{2i}}\right] < \eta\left(S_y^2+R^2S_x^2-2RS_{yx}\right)$$
$$(3.3)\qquad \eta\left[C_y^2+C_x^2-2\rho_{yx}C_yC_x\right]+\frac{C_{1i}^2}{2C_{2i}} > 1 \text{ where } i=1,2,3,4,5 \text{ and } 6$$

and
By (1.4) and (2.22)

$$MSE_{min}\left(\hat{\bar{Y}}_{Pi}\right) < MSE\left(\hat{\bar{Y}}_{R}\right)$$

$$(3.4) \qquad \eta\left[C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right] + \frac{C_{3i}^2}{2\bar{Y}^2C_{4i}} > 1 \text{ where } i = 7, 8 \text{ and } 9$$

If the above conditions are satisfied, then our proposed ratio type estimators perform more efficiently than the usual ratio estimator.

3.3. The linear regression estimator. We compare MSE of linear regression estimator with MSE of proposed ratio type estimators by using expressions of (1.6) and (2.16) as follows:

$$MSE_{min}\left(\hat{\bar{Y}}_{Pi}\right) < MSE\left(\hat{\bar{Y}}_{Reg}\right)$$

$$\bar{Y}^2\left[1 - \frac{C_{1i}^2}{2C_{2i}}\right] < \eta S_y^2\left(1 - \rho_{yx}^2\right)$$

$$(3.5) \qquad \left[\eta C_y^2\left(1 - \rho_{yx}^2\right) + \frac{C_{1i}^2}{2C_{2i}}\right] > 1 \text{ where } i = 1, 2, 3, 4, 5 \text{ and } 6$$

and
By (1.6) and (2.22)

$$MSE_{min}\left(\hat{\bar{Y}}_{Pi}\right) < MSE\left(\hat{\bar{Y}}_{Reg}\right)$$

$$(3.6) \qquad \left[\eta C_y^2\left(1 - \rho_{yx}^2\right) + \frac{C_{3i}^2}{2\bar{Y}^2C_{4i}}\right] > 1 \text{ where } i = 7, 8 \text{ and } 9$$

The proposed estimators are more superior than the linear regression estimator, when the conditions given in equation (3.5) and (3.6) are satisfied.

3.4. Subramani and Kumarapandiyan [23] **proposed estimator.** We compare MSE value of the Subramani and Kumarapandiyan [23] proposed estimator with MSE value of the proposed ratio type estimators using expressions of (1.9) and (2.16) as follows:

$$MSE_{min}\left(\hat{\bar{Y}}_{Pi}\right) < MSE\left(\hat{\bar{Y}}_{SK}\right)$$

$$\bar{Y}^2\left[1 - \frac{C_{1i}^2}{2C_{2i}}\right] < \eta\bar{Y}^2\left(C_y^2 + \theta^2C_x^2 - 2\theta\rho_{yx}C_yC_x\right)$$

$$(3.7) \qquad \left[\eta\left(C_y^2 + \theta^2C_x^2 - 2\theta\rho_{yx}C_yC_x\right) + \frac{C_{1i}^2}{2C_{2i}}\right] > 1 \text{ where } i = 1, 2, 3, 4, 5 \text{ and } 6$$

where $\theta = \dfrac{\bar{X}}{\bar{X} + X_{0.5}}$
and
By (1.9) and (2.22)

$$MSE_{min}\left(\hat{\bar{Y}}_{Pi}\right) < MSE\left(\hat{\bar{Y}}_{SK}\right)$$

$$(3.8) \qquad \left[\eta\bar{Y}^2\left(C_y^2 + \theta^2C_x^2 - 2\theta\rho_{yx}C_yC_x\right) + \frac{C_{3i}^2}{2C_{4i}} - \bar{Y}^2\right] > 0 \text{ where } i = 7, 8 \text{ and } 9$$

where $\theta = \dfrac{\bar{X}}{\bar{X} + X_{0.5}}$
Our proposed estimators perform better as compared to the estimator proposed by Subramani and Kumarapandiyan [23], if the conditions given in equations (3.7) and (3.8) are fulfilled.

3.5. Subramani and Prabavathy [24] **suggested estimators.** We compare MSE of the Subramani and Prabavathy [24] estimators with MSE of proposed ratio type estimators by using the expressions of (1.13) and (2.16) as:

$$MSE_{min}\left(\hat{\hat{Y}}_{Pi}\right) < MSE\left(\hat{\hat{Y}}_{SPi}\right)$$

$$\bar{Y}^2\left[1-\frac{C_{1i}^2}{2C_{2i}}\right] < \eta\left(S_y^2 + R'^2\theta_j' 2V(\hat{Y}_{0.5}) - 2R'\theta_j' Cov(\bar{y},\hat{Y}_{0.5})\right)$$

where

$$\theta_1' = \frac{X_{0.5}Y_{0.5}}{X_{0.5}Y_{0.5}+\bar{X}} \; and \; \theta_2' = \frac{\bar{X}Y_{0.5}}{\bar{X}Y_{0.5}+X_{0.5}} \text{ for } j = 1,2$$

$$\eta\left[C_y^2 + \theta_j'^2\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2} - 2\theta_j'\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}\right] + \frac{C_{1i}^2}{2C_{2i}} > 1 \text{ where } i = 1,2,3,\ldots,6 \quad (3.9)$$

and
By(1.13) and (2.22)

$$MSE_{min}\left(\hat{\hat{Y}}_{Pi}\right) < MSE\left(\hat{\hat{Y}}_{SPi}\right)$$

$$\eta\left[C_y^2 + \theta_j'^2\frac{V(\hat{Y}_{0.5})}{Y_{0.5}^2} - 2\theta_j'\frac{Cov(\bar{y},\hat{Y}_{0.5})}{\bar{Y}Y_{0.5}}\right] + \frac{C_{3i}^2}{2\bar{Y}^2C_{4i}} > 1 \text{ where } i = 7,8 \text{ and } 9 \quad (3.10)$$

If the above two conditions are fulfilled, then the proposed ratio type estimators are more efficient as compared to the estimators suggested by Subramani and Prabavathy [24].

3.6. Yadav et al. [28] **proposed estimators.** We compare MSE of the Yadav et al. [28] estimators with MSE of proposed ratio type estimators using expressions of (1.18) and (2.16) as follows:

$$MSE_{min}\left(\hat{\hat{Y}}_{Pi}\right) < MSE\left(\hat{\hat{Y}}_{YDi}\right)$$

$$\bar{Y}^2\left[1-\frac{C_{1i}^2}{2C_{2i}}\right] < \bar{Y}^2\left(1-\frac{A_i^2}{B_i}\right)$$

$$\frac{C_{1i}^2}{2C_{2i}} - \frac{A_i^2}{B_i} > 0 \text{ where } i = 1,2,3,4,5 \; and \; 6 \quad (3.11)$$

and
By(1.18) and (2.22)

$$MSE_{min}\left(\hat{\hat{Y}}_{Pi}\right) < MSE\left(\hat{\hat{Y}}_{YDi}\right)$$

$$\frac{C_{3i}^2}{2\bar{Y}^2C_{4i}} - \frac{A_i^2}{B_i} > 0 \text{ where } i = 7,8 \; and \; 9 \quad (3.12)$$

If the conditions mentioned in equations (3.11) and (3.12) are fulfilled, then our suggested estimators perform better as compared to the estimators proposed by Yadav et al. [28].

In general, we can say that, our proposed ratio type estimators are more efficient as compared to the existing estimators consider in this study when all the derived conditions are satisfied.

4. Numerical Illustration

In this section, the performance of the proposed ratio type estimators and the existing ratio estimators is evaluated by using two natural populations. The population 1 and 2 are taken from Mukhopadhyay [11]. The characteristics of the two populations are given below in Tables 1 and 2, respectively. In Table 3, the values of MSEs of the existing and

Table 1. Characteristics of population 1.

$N = 20$	$X_{0.5} = 407.50$	$Y_{0.5} = 40.50$
$n = 03$	$V(\hat{Y}_{0.5}) = 26.1307$	$Cov(\bar{y}, \hat{Y}_{0.5}) = 21.0918$
$\bar{Y} = 41.50$	$V(\bar{y}) = 27.1254$	$Cov(\bar{y}, \bar{x}) = 182.7425$
$\bar{X} = 441.95$	$V(\bar{x}) = 2894.3089$	$\rho_{yx} = 0.6522$

Table 2. Characteristics of population 2.

$N = 20$	$X_{0.5} = 407.50$	$Y_{0.5} = 40.50$
$n = 05$	$V(\hat{Y}_{0.5}) = 10.8348$	$Cov(\bar{y}, \hat{Y}_{0.5}) = 9.0665$
$\bar{Y} = 41.50$	$V(\bar{y}) = 14.3605$	$Cov(\bar{y}, \bar{x}) = 96.7461$
$\bar{X} = 441.95$	$V(\bar{x}) = 1532.2812$	$\rho_{yx} = 0.6522$

Table 3. The values of the MSEs of the existing and proposed estimators.

Existing and Proposed estimators	**Population 1**	**Population 2**
$\hat{\bar{Y}}$	7.6855	2.1540
$\hat{\bar{Y}}_R$	5.1923	1.4553
$\hat{\bar{Y}}_{Reg}$	4.4163	1.2378
$\hat{\bar{Y}}_{SK}$	4.5836	1.2846
$\hat{\bar{Y}}_{SP1}$	3.1314	1.0582
$\hat{\bar{Y}}_{SP2}$	3.1425	1.0603
$\hat{\bar{Y}}_{YD1}$	3.1195	1.0572
$\hat{\bar{Y}}_{YD2}$	3.1304	1.0593
$\hat{\bar{Y}}_{P1}$	3.0520	1.0447
$\hat{\bar{Y}}_{P2}$	3.0707	1.0482
$\hat{\bar{Y}}_{P3}$	3.0616	1.0465
$\hat{\bar{Y}}_{P4}$	3.0733	1.0487
$\hat{\bar{Y}}_{P5}$	3.0490	1.0442
$\hat{\bar{Y}}_{P6}$	3.0587	1.0460
$\hat{\bar{Y}}_{P7}$	2.8578	1.0156
$\hat{\bar{Y}}_{P8}$	**2.8571**	**1.0154**
$\hat{\bar{Y}}_{P9}$	2.8576	1.0155

proposed estimators are computed by using the MSE formulas which are given in section

1 and 2, respectively. From an analysis of Table 3, several interesting observations can be made:

- **•:** The existing modified ratio estimators proposed by Subramani and Kumarapandiyan [23], Subramani and Prabavathy [24] and Yadav et al. [28] have the smaller MSE values as compared to usual unbiased estimator, the usual ratio estimator and the linear regression estimator.
- **•:** It can be seen that the proposed estimators have smaller values of MSE as compared to the usual unbiased estimator, the ratio estimator, the linear regression estimator, Subramani and Kumarapandiyan [23] estimator, Subramani and Prabavathy [24] estimators and Yadav et al. [28] estimators which indicates that the proposed estimators are more efficient as compared to the existing estimators consider in this study.
- **•:** It is observed that the proposed ratio type estimator, $\hat{\bar{Y}}_{P8}$ has a smaller MSE value i.e. (2.8571 and 1.0154) as compared to all the proposed ratio type estimators and existing estimator for two real populations consider in this study.
- **•:** It is to be also noted that the first two proposed estimators i,e. $\hat{\bar{Y}}_{P1}$ and $\hat{\bar{Y}}_{P2}$ produce similar results as compared to the Yadav et al. [28] estimators when the value of $\delta_i = 1$. where i =1 and 2.

Table 4. PREs of proposed estimators with respect to competing estimators for Population 1.

Proposed Estimators	**Existing Estimators**							
	$\hat{\bar{Y}}$	$\hat{\bar{Y}}_R$	$\hat{\bar{Y}}_{Reg}$	$\hat{\bar{Y}}_{SK}$	$\hat{\bar{Y}}_{SP1}$	$\hat{\bar{Y}}_{SP2}$	$\hat{\bar{Y}}_{YD1}$	$\hat{\bar{Y}}_{YD2}$
$\hat{\bar{Y}}_{P1}$	251.8185	170.1278	144.7018	150.1835	102.6016	102.9653	102.2117	102.5688
$\hat{\bar{Y}}_{P2}$	250.2850	169.0917	143.8206	149.2689	101.9767	102.3382	101.5892	101.9442
$\hat{\bar{Y}}_{P3}$	251.0289	169.5943	144.2481	149.7126	102.2799	102.6424	101.8912	102.2472
$\hat{\bar{Y}}_{P4}$	250.0732	168.9487	143.6990	149.1426	101.8905	102.2517	101.5033	101.8579
$\hat{\bar{Y}}_{P5}$	252.0663	170.2952	144.8442	150.3313	102.7025	103.0667	102.3122	102.6697
$\hat{\bar{Y}}_{P6}$	251.2669	169.7551	144.3849	149.8545	102.3768	102.7397	101.9878	102.3441
$\hat{\bar{Y}}_{P7}$	268.9306	181.6887	154.5350	160.3891	109.5738	109.9622	109.1574	109.5388
$\hat{\bar{Y}}_{P8}$	268.9965	181.7332	154.5728	160.4284	109.6006	109.9891	109.1841	109.5656
$\hat{\bar{Y}}_{P9}$	268.9495	181.7014	154.5458	160.4003	109.5815	109.9699	109.1650	109.5465

To show the dominance of the proposed ratio type estimators over the existing estimators used in this study, we have also found the percent relative efficiencies (PREs) for population 1 and 2. The percentage relative efficiencies (PREs) of the proposed ratio type estimators (p) with respect to the existing estimators (e) is computed as

$$PRE(e,p) = \frac{MSE(e)}{MSE(p)} * 100 \quad (4.1)$$

and are given in Tables 4 and 5.
From Tables 4 and 5, it can be observed that PREs of the proposed ratio type estimators with regards to the existing estimators consider in this study are much higher, which shows that they are more efficient for population 1 and 2. To get more insight in this study, we have also find the relative root mean square error (RRMSE) which is a very

Table 5. PREs of proposed estimators with respect to competing estimators for Population 2.

Proposed Estimators	**Existing Estimators**							
	$\hat{\bar{Y}}$	$\hat{\bar{Y}}_R$	$\hat{\bar{Y}}_{Reg}$	$\hat{\bar{Y}}_{SK}$	$\hat{\bar{Y}}_{SP1}$	$\hat{\bar{Y}}_{SP2}$	$\hat{\bar{Y}}_{YD1}$	$\hat{\bar{Y}}_{YD2}$
$\hat{\bar{Y}}_{P1}$	206.1836	139.3031	118.4838	122.9635	101.2922	101.4933	101.1965	101.3975
$\hat{\bar{Y}}_{P2}$	205.4951	138.8380	118.0882	122.5529	100.9540	101.1544	100.8586	101.0590
$\hat{\bar{Y}}_{P3}$	205.8290	139.0635	118.2800	122.7520	101.1180	101.3187	101.0225	101.2231
$\hat{\bar{Y}}_{P4}$	205.3972	138.7718	118.0318	122.4945	100.9059	101.1061	100.8105	101.0108
$\hat{\bar{Y}}_{P5}$	206.2823	139.3699	118.5405	123.0224	101.3407	101.5419	101.2450	101.4461
$\hat{\bar{Y}}_{P6}$	205.9273	139.1300	118.3365	122.8107	101.1663	101.3671	101.0707	101.2715
$\hat{\bar{Y}}_{P7}$	212.0914	143.2946	121.8787	126.4868	104.1946	104.4013	104.0961	104.3029
$\hat{\bar{Y}}_{P8}$	212.1331	143.3228	121.9027	126.5117	104.2151	104.4219	104.1166	104.3234
$\hat{\bar{Y}}_{P9}$	212.1123	143.3087	121.8907	126.4993	104.2048	104.4116	104.1064	104.3131

Table 6. RRMSE values of the existing and proposed estimators.

Existing and Proposed estimators	**Population 1**	**Population 2**
$\hat{\bar{Y}}$	0.0668	0.0353
$\hat{\bar{Y}}_R$	0.0549	0.0290
$\hat{\bar{Y}}_{Reg}$	0.0506	0.0268
$\hat{\bar{Y}}_{SK}$	0.0515	0.0273
$\hat{\bar{Y}}_{SP1}$	0.0426	0.0247
$\hat{\bar{Y}}_{SP2}$	0.0427	0.0248
$\hat{\bar{Y}}_{YD1}$	0.0425	0.0247
$\hat{\bar{Y}}_{YD2}$	0.0426	0.0248
$\hat{\bar{Y}}_{P1}$	0.0420	0.0246
$\hat{\bar{Y}}_{P2}$	0.0422	0.0246
$\hat{\bar{Y}}_{P3}$	0.0421	0.0246
$\hat{\bar{Y}}_{P4}$	0.0422	0.0246
$\hat{\bar{Y}}_{P5}$	0.0420	0.0246
$\hat{\bar{Y}}_{P6}$	0.0421	0.0246
$\hat{\bar{Y}}_{P7}$	0.0407	0.0242
$\hat{\bar{Y}}_{P8}$	0.0407	0.0242
$\hat{\bar{Y}}_{P9}$	0.0407	0.0242

common measure to compare the precision of the estimators (cf. Silva and Skinner [16], Yan and Tian [29], Munoz et al. [12] and Alvarez et al. [4]). The RRMSEs of the existing ratio estimators and the proposed ratio type estimators are calculated by using the following formula.

$$RRMSE = \frac{\sqrt{MSE(\hat{\phi})}}{\phi} \tag{4.2}$$

where mean square error $MSE(\hat{\phi})$ is given by

$$MSE(\hat{\phi}) = \frac{1}{n}\sum_{i=1}^{n}(\hat{\phi} - \phi)^2 \tag{4.3}$$

where $\hat{\phi}$ is the estimate of ϕ on the ith sample.
The results of RRMSEs are shown in Table 6. From Table 6, it is observed that the proposed estimators perform more efficiently as to the all the existing estimators consider in this study.

5. Conclusions

In sample survey, the availability of auxiliary information enhances the efficiency of the estimators. In this study, we have proposed several ratio type estimators using known value of population median by using the information on the study variable and the auxiliary variable. It is observed that the mean squared errors of the suggested estimators based on the knowledge of the median are smaller than those for the existing ratio estimators consider in this study for the two known populations considered for the numerical study. Also, it is observed that the proposed estimators are more efficient than the existing estimators in terms of percentage relative efficiencies and relative root mean square error. Hence, we strongly recommend the use of our proposed ratio type estimators over the existing ratio estimators consider in this study for the practical consideration.

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