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A modified test for detecting influential decision-making units in data envelopment analysis

Irmak ACARLAR*, Sinem Tuğba ŞAHİN TEKİN† and H. Hasan ÖRKCÜ‡ §

Abstract

In data analyses based on a deterministic or stochastic approach, using pre-study is very important to identify observations that are not suitable to data in general. Among such observations, those that have a high tendency to change results negatively are called influential observations. In this paper, we propose a new method to identify influential observations in Data Envelopment Analysis (DEA). Our method is a modified version of the one proposed by Pastor et al. [12]. Both methods are compared by using two well-known data sets and the outcomes are discussed. A comparative analysis indicates that our method is an effective alternative to the Pastor et al. [12] method to identify influential observations in DEA.

Keywords: Data Envelopment Analysis, influential observations, super efficiency, likelihood ratio test, statistical hypothesis test.

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1. Introduction

Data envelopment analysis (DEA) is a mathematical programming approach to measure the relative efficiency of decision-making units (DMUs). First, Charnes et al. [5] introduced a well-known model (CCR). Then, Banker et al. [3] extended it from a constant returns-to-scale to a variable returns-to-scale situation (BCC). The efficiency is defined as a ratio, which is determined by dividing the weighted sum of outputs by the weighted sum of inputs. DEA has been extensively used to compare the efficiencies of non-profit and for-profit organizations such as schools, hospitals, shops, bank branches, and other environments in which there are relatively homogeneous DMUs [11, 14, 18].

*Gazi University, Graduate School of Natural and Applied Sciences, 06500 Teknikokullar, Ankara, TURKEY Email: irmakacarlar@gazi.edu.tr

†Gazi University, Faculty of Sciences, Department of Statistics, 06500 Teknikokullar, Ankara, TURKEY Email: sinemsahin@gazi.edu.tr

‡Gazi University, Faculty of Sciences, Department of Statistics, 06500 Teknikokullar, Ankara, TURKEY Email: hhorkcu@gazi.edu.tr

§Corresponding Author.

In some cases, there may be indifferent DMUs in data sets. These DMUs can be defined as outliers or influential observations under some circumstances. An outlier is an observation that is not consistent with the rest of the data in terms of a relevant variable. This observation also influences the values of mean and variance adversely. In addition, influential observations are different from outliers to some extent, so influential observations influence the results of the analysis including multiple variables. The importance of identifying influential observations was first emphasized by Cook [8] in multiple linear regression analysis. He also proposed a measure based on the Mahalanobis distance to identify these observations. The diagnostic procedure, based on the case deletion approach, was proposed by Cook [8]. It is a practical method to observe the differentiations on ordinary least squares estimations for each observation in a data set. Similar methods exist in Belsley et al. [4], Cook and Weisberg [9], and Chatterjee and Hadi [7].

The case deletion approach is divided into two forms: single-case deletion and multiple-case deletions. In the single-case deletion approach, the p^{th} observation is excluded from the data set and the results are obtained from the remaining data. Subsequently, the results calculated from the remaining data are compared to the results calculated by using all the data. After the effect of the p^{th} observation is examined, it is decided whether the observation is influential or not. While this procedure is applied for each observation repeatedly, detecting influential observations in data is the aim. In contrast, the multiple-case deletion approach is a generalized version of single-case deletion for multiple observations. In this approach, a subset with size k $(1 < k < n/2)$ is excluded from the data set with size n. Afterwards, the results calculated from the remaining data are compared to the results calculated by using all the data. Eventually, observations in the subset are examined to decide whether they are influential or not.

The idea of identifying influential observations using the case deletion method is improved for several statistical analyses. Similarly, the influential DMUs can be detected by employing the case deletion approach in DEA. The main principle underlying the identification of influential observations in DEA is similar to linear regression analysis. Indeed, an influential DMU is an efficient one, which extends the production possibility set according to its own coordinate. Therefore, it may cause several problems, which are as follows:
(1) The influential DMU may cause another DMU (which is near the efficiency frontier) to be inefficient. When the influential DMU is eliminated, this second DMU can be efficient. (2) The influential DMU may cause a decrease in the super efficiency scores for some efficient DMUs. (3) The influential DMU may cause a decrease in the efficiency scores for some inefficient DMUs.

In particular, the second and third failures are significant, because one of the main objectives of DEA is to identify the efficient DMUs and to give suggestions on improving the efficiency of inefficient DMUs. In addition, let consider the case mentioned in the third failure. As known that an inefficient DMU refers at least one efficient DMU. Therefore, if the relevant efficient DMU is influential, an inefficient DMU, which takes as references the efficient DMU, effects negatively. Clearly these influential DMUs may cause wrong suggestions in improving the efficiency of inefficient DMUs.

The reasons for emerging influential DMUs in the DEA are as follows:
The input or output value for any DMU may be recorded incorrectly in terms of measurement units. There might be an extreme DMU in the data set.

In these cases, the influential DMU is first identified. If there is a recording failure, it should be fixed and, as another possibility, if there is an extreme DMU, it will be deleted from the data.

Wilson [15] utilized the AP statistics based on the case deletion approach for linear regression in identifying influential DMUs in DEA. AP statistics was proposed by Andrews and Pregibon [1]. However, the disadvantage of this method is its impracticality when the numbers of variables and observations are high. Nevertheless, this disadvantage can be overcome by using computer software. Another method that employs the case deletion method was suggested by Wilson [16]. This method gives priority to efficient DMUs for future investigations and is intended to identify influential DMUs.

In addition, a statistical test was proposed by Pastor et al. [12] to identify influential DMUs. First, the efficiency score of DMU_o is obtained using all of the DMUs in the data set with this method. Subsequently, the p^{th} efficient DMU is taken out of the dataset to obtain the efficiency score of the DMU_o. This method is related to the ratio between these two efficiency scores. In cases where these ratios are larger or smaller than a critical point, a binary variable is defined. A statistical analysis is run on the sum of these variables' values for the p^{th} efficient DMU. Because the p^{th} efficient DMU is decided to be influential, the likelihood ratio test with the α significance level is utilized.

Another diagnostic method was proposed by Ruiz and Sirvent [13], addressing the case of evaluating efficiency by employing both radial and non-radial DEA methods. The influence measures in their method can be easily computed by solving some linear programming problems that are modified versions of the DEA model, which is used to evaluate the efficiency [13].

Jahanshahloo et al. [10] presented a method for detecting influential observations in radial data envelopment analysis models. Since an efficient DMU has an influence on the efficiency of inefficient DMUs in this method, their influence measure is specified by using a half-line and a simple formulation. Moreover, Yang et al. [19] proposed a method based on the bootstrap approach to identify influential DMUs in deterministic nonparametric DEA. Furthermore, Witte and Marques [17] proposed an outlier detection procedure, applying a nonparametric model and accounting for undesired outputs and exogenous influences in the sample. To illustrate its capability, this method was applied in the Portuguese drinking water sector.

In this article, we propose a new method based on the likelihood ratio test to identify the influential DMUs in the DEA. The proposed method is a modified version of the one proposed by Pastor et al. [12]. Once the two applications are compared, then the outcomes are evaluated accordingly. The essential distinction between two methods is that our method also takes the influence on efficient DMUs into consideration, which is not included in the Pastor et al. [12] method.

The structure of this paper is as follows. In the second section, the method proposed by Pastor et al. [12] is introduced and a visual review is conducted on artificial data via a graph. In the third section, a new method is suggested based on the modified method of Pastor et al. [12]. In the fourth section, these two methods are compared by using two popular datasets that are frequently used in the literature. In the last section, the results are presented and discussed.

2. Pastor et al.'s method for detecting influential DMUs

An influential DMU is defined as one which affects the results of the DEA. One of the important effects on the results arises from the influential DMU's change in the production possibility set. As a result, the influential DMU extends the set to its own coordinate. Pastor et al. [12] dealt with two BCC models. The first model was the basic input-oriented BCC model given as:

$$
(2.1) \quad \begin{array}{l}
\text{Min } \theta_o^t \\
\text{subject to} \\
\theta_o^t x_{io} \geq \sum_{k=1}^{n} \lambda_k x_{ik} \quad ; \quad i = 1, ..., m \\
\sum_{k=1}^{n} \lambda_k x_{hk} \geq y_{ho} \quad ; \quad h = 1, ..., s \\
\sum_{k=1}^{n} \lambda_k = 1 \\
\lambda_k \geq 0 \quad ; \quad k = 1, ..., n
\end{array}
$$

where θ_o^t is efficiency score of DMU_o, λ_k is intensity of DMU_k, x_{io} and y_{ho} (all non-negative) are i^{th} input and h^{th} output value of DMU_o, respectively. Pastor et al. [12] called this model, given in model (2.1), the *total model*, since the efficiency score θ_o^t is obtained by using all data. They consider the second model as an input-oriented BCC model when one of the efficient DMU is excluded from the data. For an efficient DMU_p, this model is given as:

$$
(2.2) \quad \begin{array}{l}
\text{Min } \theta_{(p)o}^r \\
\text{subject to} \\
\theta_{(p)o}^r x_{io} \geq \sum_{k=1}^{n} \gamma_k x_{ik} \quad ; \quad i = 1, ..., m \quad ; k \neq p \\
\sum_{k=1}^{n} \gamma_k x_{hk} \geq y_{ho} \quad ; \quad h = 1, ..., s \quad ; k \neq p \\
\sum_{k=1}^{n} \gamma_k = 1 \quad ; k \neq p \\
\gamma_k \geq 0 \quad ; \quad k = 1, ..., n \quad ; k \neq p
\end{array}
$$

where $\theta_{(p)o}^r$ is efficiency score of DMU_o calculated disregarding DMU_p and γ_k is intensity of DMU_k for this model. Pastor et al. [12] called this model the *reduced model*, because the efficiency score $\theta_{(p)o}^r$ is obtained by omitting efficient DMU_p. The underlying idea to delete efficient DMU is that an influential DMU is also an efficient one due to the effects on the efficiency scores of other efficient DMUs.

The ratio of the efficiency score obtained from the model in (2.1) for DMU_o to the efficiency score obtained from the model in (2.2) be defined with the random variable [12]

$$
(2.3) \quad \Psi_{(p)o} = \frac{\theta_o^t}{\theta_{(p)o}^r}
$$

where $\Psi_{(p)o} \in (0, 1]$. As can be seen, if the efficiency score of DMU_o does not change when the DMU_p is erased from the data, then $\Psi_{(p)o} = 1$. Otherwise, if DMU_p has an effect on the inefficient DMU_o, the value of $\Psi_{(p)o}$ will be smaller than 1. To evaluate whether DMU_p is influential or not, let a cutoff value of $\bar{\Psi} = 0.95$ and a probability level $\pi_0 = 0.05$ be defined. If

$$
(2.4) \quad P(\Psi_p < \bar{\Psi}) > \pi_0
$$

DMU_p can be assessed as efficient [12]. Namely, if DMU_p is influential, the efficiency scores of inefficient DMUs will fall below $\bar{\Psi} \times 100\%$, this probability will be at least with π_0. With an inefficient DMU set of $\Theta = \{o \;/ \theta_o^t < 1 \;;\; o = 1, ..., n\}$; the binary variable in Eq. (2.5) is defined to observe the effect of DMU_p on inefficient DMUs [12].

$$
(2.5) \quad T_{(p)o} = \begin{cases} 1, & \text{if } \Psi_{(p)o} < \bar{\Psi} \;;\; o \in \Theta \\ 0, & \text{otherwise} \end{cases}
$$

where $T_{(p)o} \sim \text{Bernoulli}(\pi_0)$. For the π_0 parameter, a composite hypothesis is formed as such:

$$
(2.6) \quad \begin{array}{l}
H_0: \quad \pi \leq \pi_0 \\
H_1: \quad \pi > \pi_0
\end{array}
$$

In the case of rejecting the H_0 hypothesis presented in Eq. (2.6), the conclusion is "at α significance level, DMU_p affects the efficiency scores of $\%100 \times \pi_0$ of the inefficient DMUs and this effect is actualized by decreasing the scores of inefficient DMUs to $\%100 \times (1 - \bar{\Psi})$". This is a notable conclusion that means the DMU_p is influential. This hypothesis test is employed for all efficient DMUs. The likelihood ratio test is used to solve the problem in Eq. (2.6). Test statistics is defined as below [12]:

$$(2.7) \qquad T_{(p)} = \sum\nolimits_{o \in \Theta} T_{(p)o}$$

where, considering the number of elements for set Θ as c, $T_{(p)} \sim$ Binom(c,π_0). Therefore, the p-value is given as

$$(2.8) \qquad P(T \geq T_p) = 1 - F^B(T_p - 1)$$

where, the $F^B(.)$ is a cumulative distribution function of the binomial distribution.

According to this method, the decision rule is: "If the p-value for the efficient DMU_p is greater than the α significance level, the DMU is influential".

To assess the method visually, artificial data are drawn (see the graph in Figure 1). Considering the convenience of the graph, one input and one output variable are counted. Furthermore, this dataset has 14 DMUs, four of which are efficient. In this case, $\Theta = \{E, F, G, H, I, J, K, L, M, N\}$ occurs. Thus it can be clearly observed that the maximum value for the T_p statistics, the statistically significant differentation number in inefficient DMUs scores, will be $c = 10$. In Figure 1, let DMU_C be removed from the data based

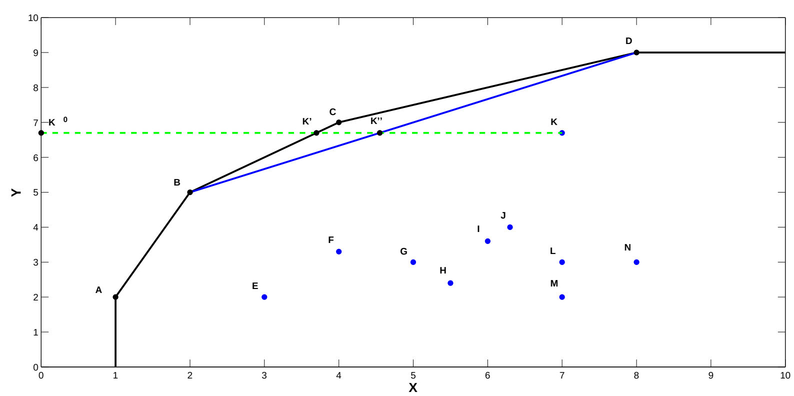

Figure 1. The graph is an example to analyze the method based on efficiency scores proposed by Pastor et al. [12].

on case deletion approach. After the efficient DMU is removed from data, the proposed method by Pastor et al. [12] is based on observing the differentations in the inefficient DMUs' efficiency scores. In Figure 1, DMU_K from the inefficient DMUs is taken to demonstrate the method of Pastor et al. [12]. By removing DMU_C from the data, the new efficiency frontier is shown by the $[BD]$ straight line on this DMU's adjacency. In addition to this, there are no shifts of the frontiers composed of other efficient DMUs. Therefore, as can be clearly seen from Figure 1, the production possibility set becomes smaller. When DMU_C maintains its position with regard to the data, the point on the efficiency frontier of K on the horizontal hypotetical line is K'. When DMU_C is removed from the data, the point on the efficiency frontier of K on the horizontal hypotetical line is K''. K^0 is the projection of point K on the vertical axis. When the data includes DMU_C, the effiecinecy score of DMU_K is $\theta^t_K=[K^0K']/[K^0K]$. In contrast, when DMU_C is excluded from the data, the efficiency score of DMU_K is $\theta^r_{(C)K}=[K^0K'']/[K^0K]$. At that point, $\theta^t_K \leq \theta^r_{(C)K}$ is clearly seen and $\psi_{(C)K} = [K^0K']/[K^0K'']$ the rate states the decline in DMU_K's efficiency score. Similarly, this review is also applied for the other inefficient DMUs. Next, considering 0.95 cutoff values after DMU_C is removed, the T_C shows the number of inefficient DMUs that have a decline in their efficiency scores. The T_C binomial variable is used to determine whether the influential DMU_C is efficient. T_C is a test statistic for the testing of the composite hypothesis given in Eq.(2.6). The likelihood ratio test is used to conduct this test. If the H_0 hypothesis is rejected, it means that this efficient DMU_C is influential.

3. A modified method for detecting influential DMUs

In this study we consider two models. The first one is called a *total super-efficiency* model that provides the super-efficiency scores by using all data. This model is given as follows:

$$
\begin{array}{ll}
 & \text{Min } \phi^t_o \\
 & \text{subject to} \\
(3.1) & \phi^t_o x_{io} \geq \sum_{k=1,k\neq o}^{n} \delta_k x_{ik} \quad ; \quad i=1,...,m, \\
 & \sum_{k=1,k\neq o}^{n} \delta_k x_{hk} \geq y_{ho} \quad ; \quad h=1,...,s, \\
 & \sum_{k=1,k\neq o}^{n} \delta_k = 1 \quad ; \\
 & \delta_k \geq 0 \quad ; \quad k=1,...,n \quad ; k\neq o
\end{array}
$$

where ϕ^t_o is the super-efficiency score of DMU_o and δ_k is the intensity of DMU_k.

The main idea in the super-efficiency model is to remove DMU_o from the weighted sum section in the model in Eq. (2.1) to obtain, as shown in Eq. (3.1). The super-efficiency scores of inefficient DMUs and their efficiency scores obtained from Eq. (2.1) are the same. If inefficient DMUs are to be put in an order, the use of their efficiency-scores or their super-efficiency scores yield the same order. Indeed, this case is not the same for efficient DMUs. While the efficiency-scores of efficient DMUs are equal to 1, their super-efficiency scores are greater than 1 and have different values from the efficiency scores. When super efficiency scores are compared to the efficiency scores, the advantage of super-efficiency scores is that they provide the degree of effectiveness of efficient DMUs.

Let the second model be called a *reduced super-efficiency model* that provides the super-efficiency scores by deleting efficient DMU_p. This model is given as follows:

$$
\begin{array}{ll}
 & \text{Min } \phi^t_{(p)o} \\
 & \text{subject to} \\
(3.2) & \phi^r_{(p)o} x_{io} \geq \sum_{k=1,k\neq o,k\neq p}^{n} \beta_k x_{ik} \quad ; \quad i=1,...,m, \\
 & \sum_{k=1,k\neq o,k\neq p}^{n} \beta_k x_{hk} \geq y_{ho} \quad ; \quad h=1,...,s, \\
 & \sum_{k=1,k\neq o,k\neq p}^{n} \beta_k = 1 \quad ; \\
 & \beta_k \geq 0 \quad ; \quad k=1,...,n \quad ; k\neq o, \ k\neq p
\end{array}
$$

where $\phi^r_{(p)o}$ is the super-efficiency score of DMU_o disregarding the efficient DMU_p, and β_k is the intensity of DMU_k.

For DMU_o, let the super-efficiency score obtained from all the data be ϕ^t_o and the super-efficiency score obtained from after removing the efficient DMU_p be $\phi^r_{(p)o}$. In this case, the ratio of the two super-efficiency scores obtained for DMU_o is defined as:

$$(3.3) \qquad \tau_{(p)o} = \frac{\phi_o^t}{\phi_{(p)o}^r}$$

since the super-efficiency scores are $\phi_o^t \leq \phi_{(p)o}^r$, so $0 < \tau_{(p)o} \leq 1$. Eq. (3.3) is thought to be $\phi_o^t = \phi_{(p)o}^r \tau_{(p)o}$. Consequently, when the influential DMU is removed from the data set, the value of the ϕ_o^t score will increase the multiplier $(1 - \tau_{(p)o})/\tau_{(p)o}$.

Let the $\tau_{(p)o}$ ratio close to 1, namely, when the efficient DMU_p is deleted from the data, the $\phi_{(p)o}^r$ score obtained for DMU_o will change very little when compared with the ϕ_o^t score. Along with this, if the $\tau_{(p)o}$ is not close to 1, the differentiation between ϕ_o^t and $\phi_{(p)o}^r$ is noteworthy. Therefore, small values for the $\tau_{(p)o}$ indicate the possibility that DMU_p can be an influential observation.

To determine whether DMU_p is influential or not and address a high number of efficiency and super-efficiency scores, it is practical to form a statistical approach. For this purpose, let the cutoff value be $0 < \tau^* < 1$ for the change in scores as the tolerance value τ^*. Moreover, let the probability value be π_0, defined as $0 < \pi_0 < 1$. Thus, the statistical approach can be constructed based on the inequality $P(\tau_{(p)o} < \tau^*) > \pi_0$. τ^* and π_0 are external parameters. These can be taken as $\tau^* = 0.95$ and $\pi_0 = 0.05$ [12].

$$(3.4) \qquad \begin{array}{ll} H_0: & \pi \leq \pi_0 \\ H_1: & \pi > \pi_0 \end{array}$$

Let the composite hypothesis in Eq. (3.4) be defined to identify influential observations. In the method suggested by Pastor et al. [12], this hypothesis was defined as $\Theta = \{o \mid \theta_o^t < 1, o = 1, ..., n\}$. Here, different from the set Θ, let the set created by all observations without DMU_p be defined as $\Phi = \{o \mid o = 1, ..., n; o \neq p\}$. Furthermore, the binary variable in Eq. (3.5) for any $o \in \Phi$ is defined as below to obtain the test statistics.

$$(3.5) \qquad S_{(p)o} = \{ \begin{array}{ll} 1, & \tau_{(p)o} < \tau^* \\ 0, & \text{otherwise} \end{array}$$

in this case, the test statistics are defined as such:

$$(3.6) \qquad S_{(p)} = \sum_{o \in \Phi}^{n} S_{(p)o} \sim \mathrm{Binom}(n-1, \pi_o).$$

It can be clearly seen $T_{(p)} \leq S_{(p)}$, because $\Theta \subset \Phi$. The p-value is defined as below:

$$(3.7) \qquad p\text{-value} = P(S \geq S_{(p)}) = 1 - F^B[S_{(p)} - 1].$$

In Eq. (3.7), $F^B[.]$ is the cumulative distribution function of the $\mathrm{Binom}(n-1, \pi_o)$ distribution. In this situation, the decision rule is defined as follows: If the p-value$< \alpha$ for DMU_p, it is influential. To evaluate the method visually in this study, the artificial data in Figure 1 is reconsidered. Because the proposed method depends on the changes in the super-efficiency scores, the graphic regarding this example is redrawn in Figure 2. In Figure 2, the case deletion method is utilized as in Figure 1. Accordingly, after removing from the data set the condition of DMU_C, the results are re-examined. The only change occurring on the frontier when DMU_C is removed from the data is that [BD] straight line becomes a part of the new frontier. As a result of this, there is a decrease in the production possibility set in Figure 2 as in Figure 1.

Regarding the knowledge that efficiency scores and super-efficiency scores of inefficient DMUs are equal to one another, when inefficient DMUs are considered in the proposed method, this is the same as the method of Pastor et al. [12]. Despite this, our method, in comparison with the method of Pastor et al. [12], is slightly different. Namely, when an efficient DMU is excluded from the data, differentiations of results from remaining

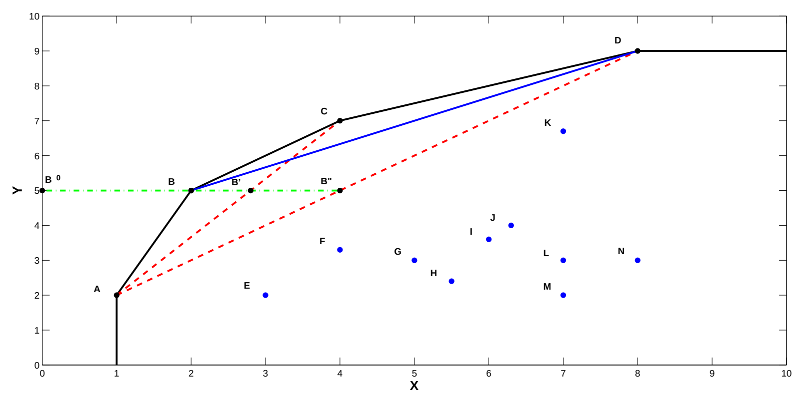

Figure 2. The graph is an example to analyze our proposed method based on super-efficiency score.

efficient DMUs are also examined in our procedure. This is the most important advantage of the method. Since this differentiation cannot be observed from the efficient DMUs efficiency scores, the change in the super-efficiency scores forms the basis of the suggested method. Therefore, in Figure 2, when DMU_C is removed from the data set, the change in the super-efficiency score of the efficient DMU_B is shown. The geometrical presentations from the aspect of inefficient DMUs are in Figure 1.

When DMU_C is in the data set, $\phi^t_B = [B^0B']/[B^0B]$, and $\phi^r_{(C)B} = [B^0B'']/[B^0B]$ when DMU_C is removed from the data set. Consequently, $\tau_(C)B = [B^0B']/[B^0B'']$. Naturally, $\phi^t_B \leq \phi^r_{(C)B}$ requires $\tau_{(C)B} \leq 1$. The $\tau_{(C)B}$ rate shows the decline in the super-efficiency score of the efficient DMU_B when DMU_C is removed from the data set.

Similarly, $\tau_(C)o$ the ratios are obtained from the remaining $n-1$ DMUs without DMU_C. Considering 0.95 cutoff value in this method, $S_{(C)}$ statistics show how many $n-1$ DMUs' super-efficiency scores have changed. If it is proven statistically that the probability of these changes is at least 0.05 under a certain error, DMU_C is called an influential DMU (as can be seen in Eq. (3.4)). The likelihood ratio test is utilized to test the composite hypothesis that is given in Eq. (3.4).

For our proposed method, to evaluate the effectiveness among efficient DMUs, an artificial data set, including two influential DMUs, is taken into consideration and then data structures are analysed from graphs and super-efficiency scores. The artificial set with super-efficiency scores of DMUs in the data are presented in Table 1.

Table 1. The artificial data and super-efficiency scores

DMU	A	B	C	D	E	F	G	H	I
X	1	1.25	3	4.5	3.5	4	4.5	4.25	5
Y	1	3	6	6.5	4,5	5.5	5	6.25	5.25
ϕ^t_o	1.25	1.44	1.34	1.07	0.61	0.68	0.54	0.88	0.51

The graphical demonstration of obtaining super-efficiency scores for DMU_A and DMU_B is presented in Figure 3 and Figure 4.

As can be seen in Figure 3 and Figure 4, while DMUs A and B are getting a wider production possibility set, other DMUs are getting smaller super-efficiency scores. The

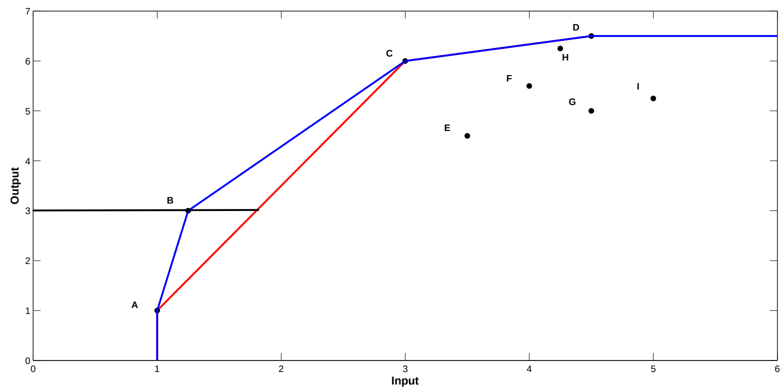

Figure 3. Calculation of the super-efficiency score for DMU_B (for full data).

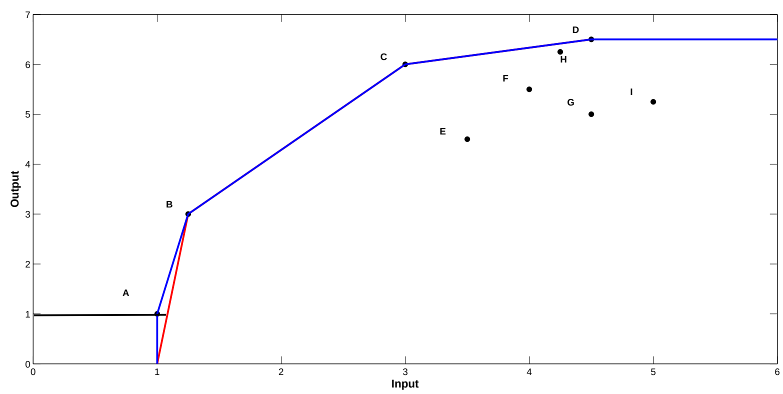

Figure 4. Calculation of the super efficiency score for DMU_A (for full data).

Table 2. Super-efficiency scores when DMU_A is deleted from data

DMU	A	B	C	D	E	F	G	H	I
ϕ_o^t	1.25	1.44	1.34	1.07	0.61	0.68	0.54	0.88	0.51
$\phi_{(A)o}^r$	-	2.4	1.34	1.26	0.61	0.68	0.54	0.88	0.51

probability of these DMUs being influential is high. Therefore, the case deletion approach is utilized. First, consider the situation after removing DMU_A. Figure 5 demonstrates the related graph and Table 2 presents the results.

As can be seen in Figure 5, the decrease can be observed when DMU_A is excluded from the data, accordingly the improvement in super-efficiency scores can be observed in B and D from other efficient DMUs, as shown in Figure 2. Because inefficient DMUs do not refer to DMU_A, their scores do not change and DMU_A cannot be determined as influential according to Pastor et al. [12].

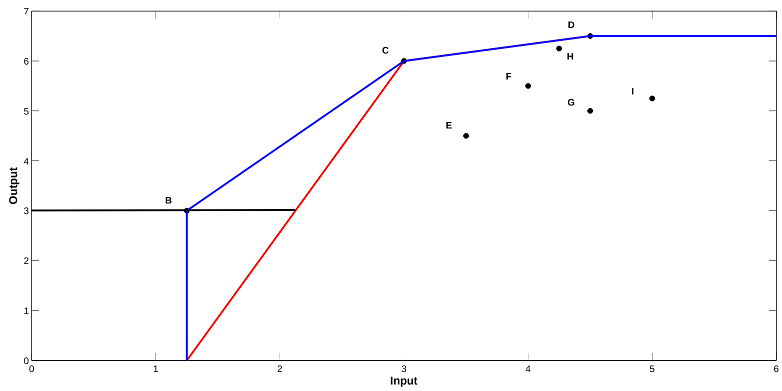

Figure 5. Calculation of the super-efficiency score for DMU_B (without DMU_A).

Second, consider the situation after removing DMU_B. Figure 6 demonstrates the related graph and Table 3 presents the results.

Table 3. Super-efficiency scores when DMU_B is deleted from data

DMU	A	B	C	D	E	F	G	H	I
ϕ_o^t	1.25	1.44	1.34	1.07	0.61	0.68	0.54	0.88	0.51
$\phi_{(B)o}^r$	3	-	1.36	1	0.69	0.7	0.58	0.88	0.54

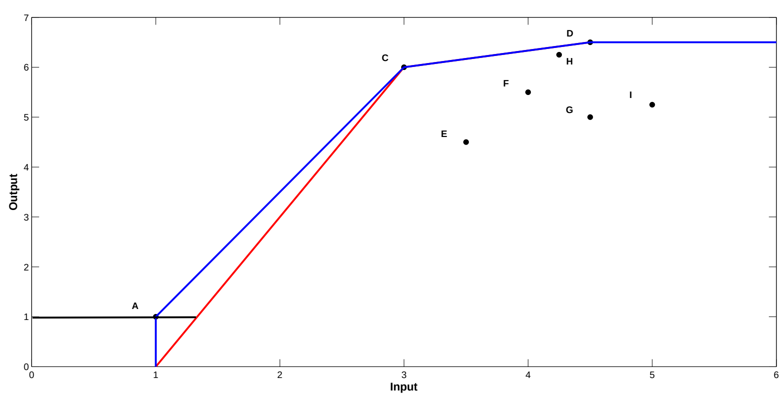

Figure 6. Calculation of the super-efficiency score for DMU_A (without DMU_B).

As can be seen in Figure 6, after deleting DMU_B from the data set, the production possibility set has decreased. According to the results in Table 4, the improvement can be observed in super-efficiency scores in A and C from efficient DMU and in E, F, G, and I from inefficient DMUs. Both examples indicate that influential A and B DMUs cannot mask their effects on one another so these examples illustrate that efficient DMUs do not interact with each other for our proposed method.

4. Application

To compare the proposed test of Pastor et al. [12], the data in Charnes et al. [6] and the data in Bal et al. [2] are used.

4.1. Application 1. Pastor et al. [12] adopted the data in Charnes et al. [6], comprising $n = 70$ DMUs, to assess their methods. Therefore, we have used the same data to compare our proposed method with the method of Pastor et al. [12]. As 27 DMUs are efficient, in Charnes et al. [6] data, it is obvious that the remaining 43 inefficient DMUs have equal super-efficiency and efficiency scores to one another. For the Pastor et al. [12] method, efficiency scores of inefficient DMUs are taken into consideration and only change in these efficiency scores are interpreted. In addition to this, the results obtained using our proposed method is interpreted with respect to the changes in the super-efficiency scores of both efficient and inefficient DMUs.

The values of $\tau^* = 0.95$ and $\pi = 0.05$ from Pastor et al. [12] are used. This means that "*if the efficient* DMU_p *affects the super-efficiency scores more than 5% of other* $n-1$ *DMUs besides itself, the super-efficiency scores' values for these DMUs reduce their efficiency less than 95%*". According to this rule, the composite hypotheses are established in Eq. (3.4) for each of the 27 efficient DMUs and statistical tests are conducted for each of them. At the end of the testing procedure, the $S_{(p)}$ values and p-values are obtained (see Table 4). Furthermore, the results of the method of Pastor et al. [12] are presented in the same table for the same data.

When the results in Table 4 are examined, the differences between the $T(p)$ and $S(p)$ values from efficient DMUs are clearly seen. To examine these differences, first let us address how DMU_{59} and $T_{(59)} = 0$ are obtained. Namely, when the efficient DMU_{59} is omitted from the data, all efficiency scores for inefficient DMUs are not less than 95% of their own values, so the p-value is obtained as 1. Since the significance level in the decision rule is $\alpha = 0.05$ and the p-value is greater than α, DMU_{59} is not influential in accordance with the method of Pastor et al. [12]. In addition, $S_{(59)} = 0$ is interpreted as follows: when DMU_{59} is removed from the data, 3 super-efficiency scores of efficient DMUs decrease to less than 95% of their own values. In contrast, the p-value obtained from our proposed method for DMU_{59} is 0.1505. Therefore, when $\alpha = 0.05$ and p-value$> \alpha$, H_0 cannot be rejected, and according to our proposed method, it is concluded that DMU_{59} is not influential. Hence, DMU_{59} cannot be detected as influential while using both methods, so to assess the difference between the two methods, it is necessary to use a more extreme unit.

When the results obtained for DMU_{44} in Table 4 are observed, it can be seen that $T_{(44)} = 3$. Namely, significant changes occur in the efficiency-score of 3 inefficient DMUs when DMU_{44} is removed from the data. The relevant p-value is calculated as 0.1505. Since $\alpha = 0.05$ and the p-value is greater than α, this efficient DMU is not influential according to Pastor et al. [12]. However, it is emphasized in the method of Pastor et al. [12] that DMU_{44} can be influential. Considering the fundamental statistical decision rule, it is concluded that DMU_{44} is not influential, because in their method the obtained p-value$> \alpha$ and accordingly the H_0 cannot be rejected. When the results obtained from our method are observed, it is found that $S_{(44)} = 7$. In other words, significant changes to the super-efficiency score of 7 out of the remaining 69 arise by removing DMU_{44} from the

Table 4. Super-efficiency scores and statistical results for the data in Charnes et al. [6].

DMU	$T_{(p)}$	$P(T > T_{(p)})$	$S_{(p)}$	$P(S > S_{(p)})$
5	0	1	1	0.7497
11	0	1	0	1
12	0	1	0	1
15	0	1	1	0.7497
17	0	1	0	1
18	0	1	0	1
20	0	1	0	1
21	0	1	0	1
22	0	1	0	1
24	0	1	1	0.7497
27	0	1	0	1
32	0	1	0	1
35	0	1	0	1
38	0	1	0	1
44	3	0.1505	7	0.0003 *
45	0	1	1	0.7497
47	0	1	1	0.7497
48	0	1	0	1
49	0	1	0	1
52	3	0.1505	6	0.0019 *
54	0	1	0	1
56	0	1	1	0.7497
58	0	1	1	0.7497
59	0	1	3	0.1505
62	0	1	2	0.3939
68	0	1	0	1
69	1	0.7497	3	0.1505

* The values smaller than 0.05, and the DMUs that gives these values, are influential.

data. The calculated p-value for this case is 0.0003, and H_0 is rejected due to p-value$< \alpha$. Therefore, we reach the result that the efficient DMU_{44} is also an influential DMU.

A similar condition is valid for DMU_{52} as well. In the proposed test, it is found that $S_{(52)} = 6$. Furthermore, this efficient DMU is influential because the p-value obtained for DMU_{52} is smaller than α $(0.0019 < 0.05)$ in our method. According to Pastor et al. [12], the p-value for DMU_{52} is 0.1505 and it is clearly shown that the p-value is greater than α, so DMU_{52} cannot be determined as influential. Although Pastor et al. [12] describe DMU_{44} and DMU_{52} as influential in their proposed method, in accordance with the fundamental statistical decision rule, these DMUs cannot be determined as influential. From the results of our proposed test method, it can be clearly and statistically concluded that DMU_{44} and DMU_{52} are influential.

The detailed results showing the change in the super-efficiency scores of the remaining DMUs after removing DMU_{44} and DMU_{52} from the data via the case deletion approach are presented in Table 5.

In Table 5, the values that are smaller than 0.95 for $\tau_{(44)o}$ and $\tau_{(52)o}$ are shown in bold. The corresponding values of the DMUs, highlighted in Table 5, are the units whose super-efficiency scores have significantly changed. By removing DMU_{44} from the data, a significant change occurs in the super-efficiency scores of the inefficient DMUs 8, 16, 39, and 43, and the efficient DMUs 35, 59, and 68. Similarly, if DMU_{52} is considered, removing DMU_{52} from the data causes a significant change in the super-efficiency scores of inefficient DMUs 1, 10, 40, 50, and 57, and the efficient DMU_{11}. Depending on these results, the $S_{(p)}$ statistics value is calculated, and it is found that these two efficient DMUs are influential due to the statistical procedure. We pay attention not only to the changes in the results of the inefficient DMUs but also to the changes in the results of the efficient DMUs. This is a notable difference between our proposed method and the method proposed by Pastor et al. [12].

4.2. Application 2. For the second application, the economic data of the OECD countries from Bal et al. [2] are examined. The data includes thirty OECD countries, and it has five output and three input variables given as: y_1 national income per capita (USA dollars, 2006), y_2 human development index (life expectancy from birth, 2006); y_3 education index (2006); y_4 contribution rate to the labour force of the women population (2006); y_5 health expenditure per capita (USA dollars, 2005); x_1 unemployment ratio (2006); x_2 rate of inflation (2005); and x_3 infant mortality rate (2005). For the data, scores are obtained from the method proposed by Pastor et al. [12] and the proposed method in our study is presented in Table 6.

The results in Table 6 demonstrate the efficient 14 countries: Australia, Canada, Denmark, Finland, Iceland, Ireland, Japan, Luxembourg, New Zealand, Norway, Sweden, Switzerland, England, and the USA. Naturally, influential DMUs are detected among these countries. When the significance level is 0.05, according to the method of Pastor et al. [12], Iceland, Norway, and Switzerland are identified as influential DMUs.

In contrast, by using our proposed method, Australia, Denmark, Finland, Japan, Luxembourg, Sweden, and England are identified as influential DMUs in addition to the three countries mentioned above. Consequently, our proposed method identifies 7 more efficient DMUs as influential that cannot be identified by the method of Pastor et al. [12]. The USA, which is one of the DMUs that is extreme in respect to its super-efficiency score, is not identified as influential in either of two methods, despite having a very high score. Therefore, DMUs with very high super-efficiency scores may not be influential.

5. Conclusion

In this study, a new modified method is proposed to identify the influential DMUs in DEA. There are already some useful methods presented in the literature to identify influential DMUs in DEA. One of them is proposed by Pastor et al. [12] and is based on a statistical test. Our method based on the super-efficiency scores is a modified version of the Pastor et al. [12] method. Because the efficiency scores of the efficient DMUs are fixed as 1, the method of Pastor et al. [12] only considers the changes in the scores of inefficient DMUs. Unlike Pastor et al [12], we consider the changes in the scores of both the efficient and inefficient DMUs using our proposed method. Therefore, this is the most important advantage of our method in comparison with that of Pastor et al. [12]. This also provides an advantage in decreasing the p-value and identifying the influential DMUs in comparison with the method of Pastor et al. [12]. The results of two applications in the fourth section clearly show that our method has significantly more advantages.

Table 5. The detailed results of our proposed method for DMU_{44} and DMU_{52}

DMU	ϕ_o^t	$\phi_{(44)o}^r$	$\tau_{(44)o}$	$\phi_{(52)o}^r$	$\tau_{(52)o}$
1	0.9621	0.9621	1	1.0359	**0.9288**
2	0.9010	0.9315	0.9673	0.9010	1
3	0.9348	0.9606	0.9732	0.9791	0.9548
4	0.9016	0.9016	1	0.9021	0.9995
5	1.0504	1.0504	1	1.0504	1
6	0.9099	0.9153	0.9941	0.9100	0.9999
7	0.8914	0.8914	1	0.8929	0.9984
8	0.9050	1.0220	**0.8855**	0.9050	1
9	0.8585	0.8657	0.9917	0.8858	0.9692
10	0.9408	0.9750	0.9648	0.9998	**0.9409**
11	1.0577	1.0577	1	1.1136	**0.9498**
12	1.0487	1.0586	0.9906	1.0895	0.9625
13	0.8623	0.8658	0.9960	0.8630	0.9992
14	0.9897	0.9897	1	0.9897	1
15	1.2868	1.2868	1	1.2868	1
16	0.9501	1.1777	**0.8068**	0.9501	1
17	1.2360	1.2360	1	1.2360	1
18	1.0393	1.0393	1	1.0453	0.9943
19	0.9526	0.9842	0.9678	0.9577	0.9947
20	1.1421	1.1472	0.9956	1.1421	1
21	1.1122	1.1123	0.9999	1.1122	1
22	1.0158	1.0188	0.9971	1.0263	0.9898
23	0.9748	1.0257	0.9504	0.9771	0.9977
24	1.1055	1.1055	1	1.1055	1
25	0.9787	0.9829	0.9957	0.9864	0.9922
26	0.9425	0.9577	0.9842	0.9425	1
27	1.0630	1.0814	0.9830	1.0639	0.9992
28	0.9903	0.9903	1	0.9903	1
29	0.8833	0.8833	1	0.8833	1
30	0.8934	0.9033	0.9890	0.8948	0.9984
31	0.8369	0.8369	1	0.8369	1
32	1.0615	1.0615	1	1.0615	1
33	0.9521	0.9868	0.9649	0.9578	0.9941
34	0.8590	0.8818	0.9742	0.8642	0.9940
35	1.0299	1.2051	**0.8546**	1.0299	1
36	0.7929	0.7929	1	0.8012	0.9897
37	0.8393	0.8459	0.9922	0.8599	0.9760
38	1.1455	1.1455	1	1.1455	1
39	0.9415	1.0048	**0.9370**	0.9415	1
40	0.9498	0.9498	1	0.9999	**0.9499**
41	0.9523	0.9575	0.9946	0.9526	0.9997
42	0.9531	0.9531	1	0.9531	1
43	0.8647	0.9259	**0.9339**	0.8696	0.9945
44	2.0816	-	-	2.0816	1
45	1.0120	1.0120	1	1.0120	1
46	0.9129	0.9300	0.9815	0.9277	0.9841
47	1.1089	1.1089	1	1.1089	1
48	1.3018	1.3018	1	1.3018	1
49	1.0690	1.0929	0.9782	1.0690	1
50	0.9587	0.9587	1	1.0316	**0.9293**
51	0.9199	0.9199	1	0.9199	1
52	1.1863	1.1863	1	-	-
53	0.8696	0.8734	0.9957	0.8707	0.9987
54	1.2186	1.2186	1	1.266	0.9626
55	0.9994	1.0423	0.9588	1.0089	0.9905
56	1.0921	1.0921	1	1.0921	1
57	0.9269	0.9269	1	0.9994	**0.9275**
58	1.3514	1.3514	1	1.3514	1
59	1.6130	2.0750	**0.7773**	1.6427	0.9819
60	0.9804	0.9804	1	0.9838	0.9966
61	0.8927	0.8927	1	0.8964	0.9959
62	1.5541	1.5541	1	1.5541	1
63	0.9634	0.9634	1	0.9937	0.9695
64	0.9303	0.9335	0.9966	0.9563	0.9729
65	0.9754	0.9754	1	0.9754	1
66	0.9356	0.9356	1	0.9501	0.9848
67	0.9462	0.9476	0.9985	0.9714	0.9740
68	1.1897	1.4481	**0.8216**	1.1897	1
69	1.6448	1.6448	1	1.6448	1
70	0.9640	0.9640	1	0.9671	0.9968
-	-	-	$S_{44} = 7$	-	$S_{52} = 6$

Table 6. Super-efficiency scores and statistical results for the data in Bal et al. [2]

Countries	ϕ_o^t	$T_{(p)}$	$P(T \geq T_{(p)})$	$S_{(p)}$	$P(S \geq S_{(p)})$
Australia	1.7843	1	0.5123	7	0.0000*
Austria	0.6707	-	-	-	-
Belgium	0.7347	-	-	-	-
Canada	1.1804	0	1	1	0.5123
Chec. Rep.	0.6000	-	-	-	-
Denmark	1.2578	0	1	3	0.0301*
Finland	41.5337	0	1	3	0.0301*
France	0.8732	-	-	-	-
Germany	0.6000	-	-	-	-
Greece	0.6000	-	-	-	-
Hungary	0.4281	-	-	-	-
Iceland	16.1047	4	0.0042*	9	0.0000*
Ireland	1.5254	0	1	2	0.1530
Italy	0.5253	-	-	-	-
Japan	8.7437	0	1	3	0.0301*
S. Korea	0.8006	-	-	-	-
Luxembourg	19.5169	0	1	3	0.0301*
Mexico	0.6392	-	-	-	-
Netherland	0.7863	-	-	-	-
N.Zealand	1.7610	0	1	2	0.1530*
Norway	10.4978	5	0.0004*	12	0.0000*
Poland	0.4971	-	-	-	-
Portugal	0.5000	-	-	-	-
Slovak. Rep.	0.3750	-	-	-	-
Spain	0.8540	-	-	-	-
Sweden	1.2573	2	0.1530	6	0.0000*
Switzerland	10.6916	8	0.0000*	11	0.0000*
Turkey	0.2263	-	-	-	-
England	1.1550	2	0.1530	7	0.0000*
USA	16.7049	0	1	1	0.5123

* The values smaller than 0.05 and the DMUs that give these values are influential.

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