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Ranking decision making units with the integration of the multi-dimensional scaling algorithm into PCA-DEA

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Abstract

Data envelopment analysis (DEA) has being used commonly in a variety of fields since it was developed, and its development continues through interacting with other techniques. Since the method can be applied to multiple inputs and outputs, it interacts with multivariate statistical methods. Principle component analysis (PCA) is a multivariate analysis method used to destroy the independence structure between variables or to reduce the number of dimensions. In literature, PCA and DEA are compared for ranking decision making units. Then, PCA-DEA procedure was modified. In this study, the multi-dimensional scaling (MDS) algorithm, which is one of the commonly used methods in multivariate statistics, is integrated to the PCA-DEA method to rank the decision making units (DMUs). According to Spearman rank correlation, the proposed method gives a higher correlation with super efficiency compared to other methods.

Keywords: Data envelopment analysis, ranking problem, multivariate statistics, principle component analysis, multi-dimensional scaling.

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1. Introduction

DEA, non parametric method of evaluating relative efficiencies for groups of similar units in point of view of the produced product and service, was introduced by Charnes et al. [1]. The summary of the main characteristics of DEA method are to be able to identify the sources and the level of inefficiency for each Decision Making Unit (DMU) and their evaluated efficiencies are relative efficiencies since the level of efficiency of each DMU is obtained with respect to the other units, and making no assumptions on the variables.

DEA was first proposed by Charnes et. al. [1] (Charnes, Cooper, Rhodes (CCR) model) and then extended by Banker et. al. [2] (Banker, Charnes and Cooper (BCC) model). These methods are called classical models and they can not be used in ranking efficient units. Andersen and Petersen [3] provided ranking of efficient units through improving these methods. The basic idea in this model is to compare the analyzed decision making unit with the linear combinations of all the other decision making units. Decision making unit that has the highest super efficiency score occurs at the first place. The other decision making units are ranked in descending order according to their super efficiency scores. Additionally, a lot of different approaches about ranking problem could be seen in DEA literature as Sexton et. al. [4], Podinovski and Athanassopoulos [5], Meza and Lins [6], Sun and Lu [7], Jahanshahloo et al. [8, 9], Alirezaee and Afsharian [10], Orkcu and Bal [11], Hosseinzadeh et al. [12], Wu et al. [13], Bal et al. [14, 15], Lam [16], Cooper et al. [17], and Wang et al. [18, 19].

DEA has being used commonly in variety of fields since it was developed and its development continues through interacting with other techniques. Since the method can be applied to multiple inputs and outputs, it interacts with multivariate statistical methods. Sinuany-Stern and Friedman [20] proposed a method for ranking of DMUs which is a combination of DEA and discriminant analysis of ratios (DR/DEA approach). DEA is also combined with canonical correlation analysis by Friedman et al. [21]. Principle component analysis (PCA) is a multivariate analysis method used to destroy independence structure between variables or to reduce number of dimensions [22]. Moreover, it can be used for ranking units. Zhu [23] compared DEA and PCA for ranking decision making units. Premacandra [24] extended this approach by incorporating other important features of ranking that Zhu has not considered. Besides, Rossi and Tomas [25] and Azadeh et al. [26] have shown that the distance matrix also could be used to rank the DMUs.

In this study, the multi-dimensional scaling algorithm, a statistical technique to visualize dissimilarity data, is integrated to Zhu's PCA-DEA method for the benchmarking of DMUs. To see the proposed method performance based on MDS algorithm, and to compare with the other approaches, the rankings of methods are compared with super efficiency according to Spearman rank correlations. The proposed method gives a higher correlation with super efficiency compared to Zhu's and Premachandra's algorithms.

The rest of this paper is organized as follows. Section 2 presents the general information of DEA overview and super efficiency concept. Zhu's PCA-DEA procedure and Premachandra's extension for Zhu's PCA are introduced in Section 3. Section 4 represents the integration of MDS algorithm to Zhu's PCA-DEA. Section 5 demonstrates the methods with real data set which reflect the socio-economic performance of Turkish cities. The simulation studies are considered in Section 6. Conclusions are given in Section 7.

2. DEA overwiev

In DEA there are many models which can be used to measure of efficiency, and these models are derived from the ratio models in which the ratio of weighted sum of outputs to the weighted sum of inputs [1]. In general terms, the efficiency of a particular unit can be defined as a ratio of the value of sum of outputs to the value of sum of inputs, where maximum efficiencies are restricted to 1; thus, the efficiency of a unit must be less than or equal to 1. It is assumed that there are n DMUs to be evaluated in terms of m inputs and s outputs. Let x_{ij} $(i = 1, ..., m)$ and y_{rj} $(r = 1, ..., s)$ represent the input and output values of DMU_j $(j = 1, ..., n)$, respectively. Here, v_i $(i = 1, ..., m)$ and u_r $(r = 1, ..., s)$ are the input and output weights assigned to i^{th} input and r^{th} output, respectively. DMU_o refers to the DMU under evaluation. The efficiency of DMU_o can be calculated as:

$$
\begin{aligned}
&\text{Max} \sum_{r=1}^{s} u_r y_{ro} \\
&\text{subject to} \\
&\sum_{i=1}^{m} v_i x_{io} = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad ; \quad j = 1, ..., n \\
&u_r \geq 0 \quad ; \quad r = 1, ..., s \\
&v_i \geq 0 \quad ; \quad i = 1, ..., m
\end{aligned} \tag{2.1}
$$

This linear programming problem is well known as CCR model [1], where j is is the DMU index, $j = 1, ..., n$; r is the output index, $r = 1, ..., s$; i is the input index, $i = 1, ..., m$; y_{rj} is the value of the r^{th} output for the j^{th} DMU, x_{ij} is the value of the i^{th} input for the j^{th} DMU, u_r is the weight given to the r^{th} output, v_i is the weight given to the i^{th} input. DMU_o is the under evaluation DMU. In this model, DMU_o is efficient if and only if objective function value is 1. Model (2.1) is known as multiplier model of input oriented CCR. The dual model of this multiplier model is know as envelopment model, and can be given as below:

$$
\begin{aligned}
&\text{Min } z_o = \theta_o \\
&\text{subject to} \\
&\theta_o x_{io} - \sum_{j=1}^{n} \lambda_j x_{ij} \geq 0 \quad ; \quad i = 1, ..., m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{ro} \quad ; \quad r = 1, ..., s \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \geq 0 \quad ; \quad j = 1, ..., n
\end{aligned} \tag{2.2}
$$

In DEA, variables need to be separated as input and output. The discrimination of variables as input and output is dependent on their effect on the unit. Retzlaff-Roberts [27] showed that it will be more accurate to use the concept of positive effective and negative effective variables instead of input and output variables. They proposed that variables whose increase provides the better evaluation of the unit are taken as positive effective; in contrast, variables whose decrease provides the better evaluation of the unit are taken as negative effective [27].

In DEA, DMUs are ranked according to efficiency scores obtained at the end of the analysis. DMU that has the highest efficiency score occurs at the first place while DMU that has the lowest efficiency score occurs at the last place. However, since efficiency score of all DMUs that are effective in DEA are assigned as '1', it is not possible to rank effective units between each other. DEA can be used only for ranking inefficient DMUs and in order to abolish this disadvantage various methods were developed [28]. The most commonly used method developed for ranking efficient decision making units is the super efficiency model proposed by Andersen and Petersen [3]. The basic idea in this model is to compare the analyzed decision making unit with the linear combinations of all the other decision making units. Decision making unit that has the highest super efficiency score occurs at the first place. The other decision making units are ranked in descending order according to their super efficiency scores.

Super efficiency model for under evaluation decision making unit DMU_o is defined as follows:

$$
\begin{aligned}
&\text{Max} \sum_{r=1}^{s} u_r y_{ro} \\
&\text{subject to} \\
&\sum_{i=1}^{m} v_i x_{io} = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad ; \quad j = 1, ..., n; \quad ; j \neq o \\
&u_r \geq 0 \quad ; \quad r = 1, ..., s \\
&v_i \geq 0 \quad ; \quad i = 1, ..., m
\end{aligned} \tag{2.3}
$$

where o denotes the under evaluation decision making unit and $j \neq o$ means removing the analyzed decision making unit from the constraint group, this is the basic idea of super efficiency model.

3. A method based on principal component analysis for ranking decision making units

PCA is a statistical method that converts correlated number of p variables into the uncorrelated number of k variables which are linear combinations of the original variables provide $p \geq k$. The covariance or correlation matrices structures are used to find these linear combinations. Σ is the covariance matrix, and ρ is the correlation matrix of random vector $X' = [X'_1 \; X'_2 \; . \; . \; . \; X'_p]$. $\lambda_1 \geq \lambda_2 \geq \; . \; . \; . \; \geq \lambda_p$ are the eigen values and $l_1, l_2, \; . \; . \; . \; l_p$ are ortagonal eigen vectors of correlation matrix [29]. Linear combinations of variables can be calculated as $PC_i = l_i X$, $(i = 1, \; . \; . \; . \; , p)$. Explanation ratio of total variance of k^{th} principal component is described as $\frac{\lambda_k}{\lambda_1 + \lambda_2 + \; . \; . \; . \; + \lambda_p}$ [22, 29].

According to Zhu's [23] approach, the ratio of weighted sum of output to weighted sum of input is used as a variable in PCA to provide correspondence of DEA and PCA methods [29]. Thus, for each DMU_j $(j = 1, \; . \; . \; . \; , n)$.

$$
d_{ir}^{j} = \frac{y_{rj}}{x_{ij}} \; ; \; (i = 1, ..., m \; ; \; r = 1, ..., s) \tag{3.1}
$$

ratios will be the new variables which are used in PCA to evaluate an alternative approach to DEA , thus, the bigger the d_{ir}^{j}, the better the performance of DMU_j in terms of the r^{th} output and the i^{th} input [29].

Let $d_k^j = d_{ir}^j$, with, e.g., $k = 1$ corresponds to $i = 1, r = 1$ and $k = 2$ corresponds to $i = 1, r = 2$, etc., where $k = 1, \; . \; . \; . \; p$; $p = m \times s$. $n \times p$ data matrix composed by d_k^j is defined as follows [29]:

$$
D = (\underline{d_1}, \; . \; . \; . \; , \underline{d_p})_{n \times p} \tag{3.2}
$$

where each row represents p individual ratios of d_k^j for each DMU and each column represents a spesific output/input ratio. That is, $d_k = (d_k^1, \; . \; . \; . \; , d_k^n)_{1 \times n}$, $(k = 1, \; . \; . \; . \; , p)$ [29].

The general concept of PCA approach is given step by step in [22, 29]. Seven basic steps can be mentioned briefly as below similar to [22, 29]. For data matrix, PCA is processed as follows:

Step 1: Correlation matrix of sample, R, is computed.

Step 2: Eigen value and Eigen vectors of correlation matrix of sample are computed.
Step 3: Principal components are computed.
Step 4: The first m principal components are selected.
Step 5: The sings of weights of the principle components are determinated.
Step 6: Matrix D is standardaized to use the principal components' scores in ranking.
Step 7: Principal components scores are computed to rank the DMUs according to values of scores.

Zhu [23] applied the above algorithm to rank DMUs. The PCA ranking procedure used by Zhu is based on ratios of individual inputs and outputs. If a particular DMU has a large value for the relevant ratio, it can be expected that the DMU will perform better in terms of the relevant input and output and obtain a higher rank. Therefore, Zhu attempted to rank DMUs using PCA. Premachandra [24] extended this approach by modifying the PCA-DEA procedure as follows:

Matrix D is modified D' by adding another variable $((m+n+1)^{th}$ variable) whose elements for each DMU are equivalent to the sum of the elements in the first $(m \times s)$ columns of the matrix D; it is supposed to take into account the overall performance of each DMU with respect to all variables.

When ranking, it is important that the performance of each DMU be evaluated relative to other DMUs in the sample. As a second step, in order to incorporate this feature into the PCA, it is suggested that D'' be obtained by dividing all the elements in each column of the matrix D' by its column minimum. Therefore, each element in any column k of the matrix D'' would be indicate how good each DMU is with respect to the i^{th} input and r^{th} output when compared to worst DMU with respect to the same variables. Then, PCA is performed on the matrix D'' in the usual manner [24].

4. MDS algorithm

The PCA procedure uses the covariance matrix. We applied a distance matrix instead of a covariance matrix in *Step* 2, which was modified according to the multi-dimensional scaling algorithm. Rossi and Tomas [25] and Azadeh et al. [26] also used the distance matrix to rank the DMUs in their studies.

The distance matrix as used in MDS was integrated into the PCA-DEA algorithm, so the new algorithm was established in order to be integrated through the PCA procedure in the Zhu's study. The algorithm based on MDS is given as follows:

Step 1: The distance matrix between DMUs, is computed (*this is different from the matrix in Step 6 in the PCA procedure*).
Step 2: Generate the matrix E, whose elements are: $e_{ij} = -\frac{1}{2} d_{ij}$.
Step 3: Matrix F is obtained by removing the row and column averages and adding the overall average of E to each element of matrix E.
Step 4: Remove the column averages from each column element of matrix F and divide by column standard deviation, so each column of F is standardised as F_{sd} [22].
Step 5: The principle component scores are obtained by the PCA procedure by using the matrix F_{sd} in Step 2 of Zhu's PCA-DEA procedure.

This algoritm is different from Zhu's and Premachandra's procedure. Their algorithm take into account correlation matrix and distance matrix, respectively. Both correlation matrix and distance matrix between variables are calculated. In our approach, distance matrix between decision making units is integrated into the algoritm. The three approaches Zhu's, Premachandra's, and proposed methods are for comparison purposes in

Section 5 and Section 6. Section 5 contains a real world data application related to socio-economic performance of Turkish cities. In Section 6, the simulation study is applied with nine levels of n, and three levels for each pair of (m, r) to see the cases of these algorithms.

5. A numerical application

We illustrate the new algorithm based on MDS by applying it to the real world data of the 81 Turkish cities. The variables which reflect the socio-economic performance in Turkey were chosen when determining the process of input and output variables. The set extracted from [30] characterizes each city by 10 outputs and 3 inputs, as illustrated in Table 1.

Table 1. Output and input variables

Outputs	Inputs
y_1: The number of working people in agriculture sector	x_1:Infant mortality population
y_2: The number of working people in industry sector	x_2:Municipal expenditures
y_3: The number of working people in trade sector	x_3:Public investment amount
y_4: The number of paid workers	
y_5: The number of employers	
y_6: The literate population	
y_7: Number of beds in hospitals	
y_8: Gross domestic product	
y_9: Number of bank branches	
y_{10}: Total electric consumption amount	

Zhu's, Premachandra's and the proposed algorithm were evaluated using a real data set relative to the ranking of 81 Turkish cities. Also, super efficiency rankings were obtained to compare these methods with super efficiency according to Spearman rank correlations. To evaluate the performance of the algorithm based on MDS versus the other algorithms mentioned in this study, the ranking scores and Spearman rank correlations between three methods and super efficiency are given in Table 2 and Table 3, respectively.

As seen in Table 2 and Table 3, the rankings and correlation results for the three algorithms are investigated. The correlation between ranking of proposed algorithm and super efficiency ranking based on 81 Turkish cities is 0.7148 and it is significant for $\alpha = 0.05$.

6. Simulation study

In the preceding section, the results obtained undoubtedly applied to one sample. In this section, the computational investigation considers randomly generated instances with nine levels of n, and three levels for each pair of (m, r). For each combination of n, (m, r), and 1000 random instances are generated. The fixed and variable inputs are aggregated to produce the outputs through the following production technology: $\ln Y_i = \ln \beta + \sum_{j=1}^{n} \alpha_j \ln x_j - u_i + v_i$ where $Y_i; i = 1, \ldots, s$ denotes each output; $X_j; j = 1, \ldots, m$ denotes each variable input; β is the fixed input, u_i represents a normally distributed random disturbance for each output $u_i \sim N(\mu, \sigma)$, and v_i represents the truncated normal disturbance denoting inefficiency $v_i \sim N^+(\mu, \sigma)$ [31]. Spearman rank correlation coefficients were calculated for each trial between the super efficiency method and Zhu's, Premachandra's, and the proposed algorithms, respectively. To evaluate the

Table 2. Rankings of 81 Turkish Cities according to Super Efficiency (SE), Zhu's PCA-DEA, Premachandra algorithm (PA) and Proposed Algorithm

No	Cities (DMUs)	SE	Zhu	PA	Proposed	No	Cities (DMUs)	SE	Zhu	PA	Proposed
1	Adana	10	5	6	8	42	Konya	5	24	23	11
2	Adiyaman	40	58	59	52	43	Kutahya	57	39	40	37
3	Afyon	66	43	44	46	44	Malatya	55	47	46	42
4	Agri	13	67	70	40	45	Manisa	8	3	4	3
5	Amasya	78	46	49	58	46	K.Maras	41	55	54	47
6	Ankara	4	6	5	10	47	Mardin	2	7	11	4
7	Antalya	6	10	9	6	48	Mugla	45	32	28	35
8	Artvin	12	18	18	16	49	Mus	31	81	81	77
9	Aydin	58	30	29	28	50	Nevsehir	71	57	55	65
10	Balikesir	25	22	21	9	51	Nigde	70	59	58	63
11	Bilecik	62	41	41	53	52	Ordu	39	52	53	44
12	Bingol	67	78	79	76	53	Rize	32	36	33	30
13	Bitlis	52	66	68	59	54	Sakarya	69	37	38	39
14	Bolu	3	2	2	2	55	Samsun	76	56	56	61
15	Burdur	23	16	13	14	56	Siirt	7	26	34	17
16	Bursa	26	27	26	38	57	Sinop	81	74	73	81
17	Canakkale	50	35	30	34	58	Sivas	63	70	69	69
18	Cankiri	27	28	31	24	59	Tekirdag	19	17	15	21
19	Corum	74	64	61	68	60	Tokat	29	19	25	19
20	Denizli	44	23	22	25	61	Trabzon	33	38	39	32
21	Diyarbakir	80	77	76	80	62	Tunceli	34	69	64	74
22	Edirne	54	25	20	22	63	S.Urfa	22	71	71	55
23	Elazig	35	33	36	29	64	Usak	17	9	12	12
24	Erzincan	56	68	65	67	65	Van	28	61	63	43
25	Erzurum	61	51	57	62	66	Yozgat	43	63	67	56
26	Eskisehir	51	21	19	33	67	Zonguldak	68	34	32	41
27	Gaziantep	9	11	14	15	68	Aksaray	65	42	45	45
28	Giresun	49	54	52	50	69	Bayburt	73	79	78	78
29	Gumushane	24	72	72	70	70	Karaman	72	50	50	57
30	Hakkari	64	76	77	75	71	Kirikkale	53	48	42	60
31	Hatay	46	14	17	7	72	Batman	38	44	48	36
32	Isparta	11	29	27	27	73	Sirnak	14	60	62	49
33	Mersin	75	45	43	51	74	Bartin	59	62	60	64
34	Istanbul	1	1	1	1	75	Ardahan	47	73	75	72
35	Izmir	21	4	3	5	76	Igdir	42	75	74	73
36	Kars	79	80	80	79	77	Yalova	15	12	10	23
37	Kastamonu	48	49	47	48	78	Karabuk	20	15	16	26
38	Kayseri	36	8	7	18	79	Kilis	37	31	35	31
39	Kirklareli	30	13	8	13	80	Osmaniye	16	20	24	20
40	Kirsehir	60	53	51	54	81	Duzce	77	65	66	71
41	Kocaeli	18	40	37	66						

Table 3. Spearman's rank correlations between three methods and super efficiency

Methods	Zhu	Premachandra	Proposed Method
Super Efficiency	0.6205	0.6005	0.7148

average of the correlation coefficients for each algorithm, the sum of the correlation coefficients in 1000 trials was divided by 1000. The averages of Spearman rank correlations for the algorithms are summarized in Tables 4-6 and in Figures 1-3, which indicate the number of DMUs, n, as the x axis and the averages of correlation coefficients in 1000 trials as the y axis.

As seen in Table 4 and Figure 1, for the case of $m = 2, r = 2$, proposed and Premachandra algorithms have higher correlations with super efficiency compared to Zhu's algorithm. As the number of n (DMUs) increases, the differences in the averages between the proposed and Premachandra's algorithms tend to decrease. Also, it can be seen that the proposed and Premachandra's algoritms have the highest correlations in all trials.

Table 4. Spearman's rank correlations averages for the methods, $m = 2, r = 2$)

n	Zhu	Premachandra	Proposed
20	0.8534	0.9062	0.9027
30	0.8650	0.9031	0.8991
40	0.8660	0.9052	0.9063
50	0.8825	0.9134	0.9139
60	0.8847	0.9196	0.9204
70	0.8967	0.9215	0.9208
80	0.8971	0.9210	0.9218
90	0.8941	0.9243	0.9244
100	0.8999	0.9242	0.9285

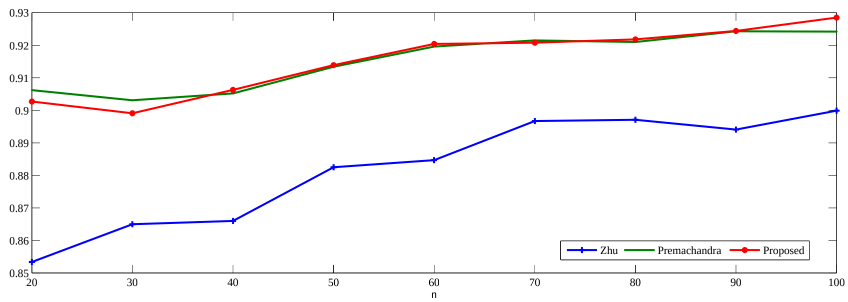

Figure 1. Average Spearman rank correlations for methods $m = 2, r = 2$.

Table 5. Spearman's rank correlations averages for the methods, $m = 2, r = 3$)

n	Zhu	Premachandra	Proposed
20	0.8011	0.8911	0.8824
30	0.8315	0.8991	0.9015
40	0.8460	0.9068	0.9094
50	0.8594	0.9090	0.9129
60	0.8693	0.9149	0.9100
70	0.8998	0.9160	0.9110
80	0.8726	0.9161	0.9141
90	0.8730	0.9162	0.9143
100	0.8801	0.9168	0.9144

In Table 5 and Figure 2, there are simulation results according to $m = 2$ and $r = 3$. Similarly, the proposed and Premachandra's algorithms are higher than Zhu's algorithm in respect to the Spearman rank correlations. Furthermore, the proposed algorithm has high correlation in all algorithms in all cases. Briefly, the proposed and Premachandra's algoritms have the highest correlations in all trials.

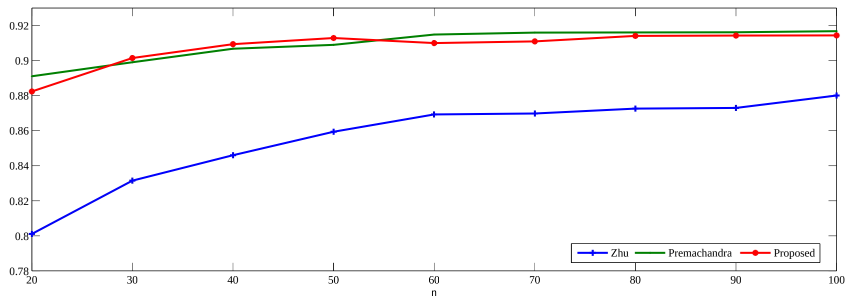

Figure 2. Average Spearman rank correlations for methods $m = 2, r = 2$.

Table 6. Spearman's rank correlations averages for the methods, $m = 3, r = 2$)

n	Zhu	Premachandra	Proposed
20	0.7799	0.8739	0.8711
30	0.8307	0.8942	0.8975
40	0.8389	0.8999	0.9048
50	0.8572	0.9125	0.9114
60	0.8649	0.9113	0.9104
70	0.8679	0.9109	0.9127
80	0.8657	0.9111	0.9127
90	0.8693	0.9111	0.9140
100	0.8799	0.9148	0.9164

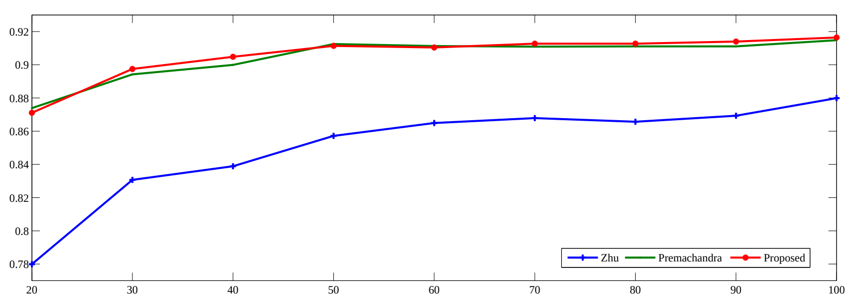

Figure 3. Average Spearman rank correlations for methods $m = 2, r = 2$.

When $m = 3$ and $r = 2$, similar to other (m, r) sets, the proposed and Premachandra's algorithms are higher than Zhu's algorithm in respect to the Spearman rank correlations. The proposed and Premachandra's algoritms have the highest correlations in all trials, as seen in Table 6 and Figure 3.

The real world and simulation data set results show that the ranking scores of the proposed algoritm are generally closer than the other algorithms' rankings to the super efficiency rankings. According to the real data set and simulations studies, especially when the number of DMUs is high, the highest correlations with the super efficiency method belong to the algorithm based on MDS. Both in the study of socio-economic performance of Turkish cities and in the simulation studies in respect to different levels of n and pair of (m, r), the proposed algorithm represents good performance for the ranking of DMUs according to Spearman rank correlations with super efficiency.

7. Conlusion

DEA measures the relative efficiency of DMUs with common inputs and outputs. DEA is not only used to determine efficient and non-efficient DMUs, but is also used to rank DMUs. Since the method can be applied to multiple inputs and outputs, it also interacts with multivariate statistical methods. In this study, the distance matrix, which is modified according to the multi-dimensional scaling algorithm, is integrated into Zhu's PCA-DEA to rank the DMUs. And the proposed algorithm gives good performance in terms of ranking. According to the real data set application and simulation studies, the proposed algorithm has high correlation averages with the super efficiency method according to Spearman rank correlations. According to the real data set application and simulation studies, it is observed that there is an obviously significant difference in favour of the proposed method in terms of the correlation averages with the super efficiency method according to Spearman rank correlations compared with the Zhu method for all considered input-output cases and n combinations. In addition to, there is a slightly difference in the favour of the proposed method for correlations as compared to the Premachandra's method some considered input-output cases and sample size.

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