Konuralp Journal of Mathematics
Volume 4 No. 1 pp. 275–281 (2016) ©KJM

TRANSLATION SURFACES IN THE 3-DIMENSIONAL SIMPLY ISOTROPIC SPACE $\mathbb{I}_3^1$ SATISFYING $\Delta^{III}x_i = \lambda_i x_i$

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Abstract. In this paper, we classify translation surfaces in the three dimensional simply isotropic space $\mathbb{I}_3^1$ satisfying some algebraic equations in terms of the coordinate functions and the Laplacian operators with respect to the third fundamental form of the surface. We also give explicit forms of these surfaces.

1. Introduction

Let $\mathbf{x} : \mathbf{M} \to \mathbb{E}^m$ be an isometric immersion of a connected n-dimensional manifold in the m-dimensional Euclidean space $\mathbb{E}^m$. Denote by $\mathbf{H}$ and Δ the mean curvature and the Laplacian of $\mathbf{M}$ with respect to the Riemannian metric on $\mathbf{M}$ induced from that of $\mathbb{E}^m$, respectively. Takahashi ([19]) proved that the submanifolds in $\mathbb{E}^m$ satisfying $\Delta\mathbf{x} = \lambda\mathbf{x}$, that is, all coordinate functions are eigenfunctions of the Laplacian with the same eigenvalue $\lambda \in \mathbb{R}$ are either the minimal submanifolds of $\mathbb{E}^m$ or the minimal submanifolds of hypersphere $\mathbb{S}^{m-1}$ in $\mathbb{E}^m$.

As an extension of Takahashi theorem, Garay studied in [12] hypersurfaces in $\mathbb{E}^m$ whose coordinate functions are eigenfunctions of the Laplacian, but not necessarily associated to the same eigenvalue. He considered hypersurfaces in $\mathbb{E}^m$ satisfying the condition

$$\Delta\mathbf{x} = \mathbf{A}\mathbf{x}, \tag{1.1}$$

where $\mathbf{A} \in Mat\ (m, \mathbb{R})$ is an $m \times m$- diagonal matrix, and proved that such hypersurfaces are minimal ($\mathbf{H} = 0$) in $\mathbb{E}^m$ and open pieces of either round hyperspheres or generalized right spherical cylinders.

Related to this, Dillen, Pas and Verstraelen ([10]) investigated surfaces in $\mathbb{E}^3$ whose immersions satisfy the condition

$$\Delta\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}, \tag{1.2}$$

where $\mathbf{A} \in Mat\ (3, \mathbb{R})$ is a 3×3-real matrix and $\mathbf{B} \in \mathbb{R}^3$. In other words, each coordinate function is of 1-type in the sense of Chen ([9]). For the Lorentzian

Key words and phrases. Simply isotropic space, translation surfaces, Laplace operator.
Mathematics Subject Classification 2010: 53A35,53B30.

version of surfaces satisfying (1.2), Alias, Ferrandez and Lucas ([1]) proved that the only such surfaces are minimal surfaces and open pieces of Lorentz circular cylinders, hyperbolic cylinders, Lorentz hyperbolic cylinders, hyperbolic spaces or pseudo-spheres.

The notion of an isometric immersion $\mathbf{x}$ is naturally extended to smooth functions on submanifolds of Euclidean space or pseudo-Euclidean space. The most natural one of them is the Gauss map of the submanifold. In particular, if the submanifold is a hypersurface, the Gauss map can be identified with the unit normal vector field to it. Dillen, Pas and Verstraelen ([11]) studied surfaces of revolution in the 3-dimensional Euclidean space $\mathbb{E}^3$ such that its Gauss map $\mathbf{G}$ satisfies the condition

$$\Delta \mathbf{G} = \mathbf{A}\mathbf{G}, \tag{1.3}$$

where $\mathbf{A} \in Mat\ (3, \mathbb{R})$. Baikoussis and Verstraelen ([4]) studied the helicoidal surfaces in $\mathbb{E}^3$. Choi ([7]) completely classified the surfaces of revolution satisfying the condition (1.3) in the 3- dimensional Minkowski space $\mathbb{E}^3_1$. The authors ([8, 20]) classified surfaces of revolution satisfying (1.2) and (1.3) in the 3-dimensional Minkowski space and pseudo-Galilean space. The authors ([21]) classified the translation surfaces in the 3-dimensional Galilean space under the condition $\Delta \mathbf{x}^i = \lambda^i \mathbf{x}^i$, where $\lambda^i \in \mathbb{R}$.The authors ([6]) classified translation surfaces in the 3-dimensional space satisfying $\Delta^{\mathbf{III}} \mathbf{r}_i = \mu_i \mathbf{r}_i$. The authors ([13]) classified surfaces of revolution in the 3-dimensional Minkowski space satisfying $\Delta^{\mathbf{III}} \mathbf{r} = \mathbf{A}\mathbf{r}$, where $\mathbf{A} \in Mat\ (3, \mathbb{R})$. The authors ([14]) classified the translation surfaces in the 3-dimensional simply isotropic space under the condition $\Delta^j \mathbf{x}_i = \lambda_i \mathbf{x}_i$, where $\lambda_i \in \mathbb{R}$ and $j = \mathbf{I}, \mathbf{II}$,

The main purpose of this paper is to complete classification of translation surfaces in the three dimensional simply isotropic space $\mathbb{I}^1_3$ in terms of the position vector field and the Laplacian operator.

2. Preliminaries

A simply isotropic space $\mathbb{I}^1_3$ is a Cayley–Klein space defined from the three dimensional projective space $\mathcal{P}(\mathbb{R}^3)$ with the absolute figure which is an ordered triple (w, f_1, f_2), where w is a plane in $\mathcal{P}(\mathbb{R}^3)$ and f_1, f_2 are two complex-conjugate straight lines in w. The homogeneous coordinates in $\mathcal{P}(\mathbb{R}^3)$ are introduced in such a way that the absolute plane w is given by $x_0 = 0$ and the absolute lines f_1, f_2 by $x_0 = x_1 + ix_2 = 0$, $x_0 = x_1 - ix_2 = 0$. The intersection point $\mathbb{F}(0:0:0:1)$ of these two lines is called the absolute point. The group of motions of the simply isotropic space is a six-parameter group given in the affine coordinates $x = \frac{x_1}{x_0}, y = \frac{x_2}{x_0}, z = \frac{x_3}{x_0}$ by

$$\begin{aligned} \overline{x} &= a + x\cos\theta - y\sin\theta \\ \overline{y} &= b + x\sin\theta + y\cos\theta \\ \overline{z} &= c + c_1 x + c_2 y + z, \end{aligned} \tag{2.1}$$

where $a, b, c, c_1, c_2, \theta \in \mathbb{R}$. Such affine transformations are called isotropic congruence transformations or i-motions [15, 17].

Isotropic geometry has different types of lines and planes with respect to the absolute figure. A line is called non-isotropic (resp. completely isotropic) if its point at infinity does not coincide (coincides) with the point $\mathbb{F}$. A plane is called non-isotropic (resp. isotropic) if its line at infinity does not contain $\mathbb{F}$. Completely isotropic lines and isotropic planes in this affine model appear as vertical, i.e., parallel to the z-axis. Finally, the metric of the simply isotropic space $\mathbb{I}_3^1$ is given by

$$ds^2 = dx^2 + dy^2.$$

A surface $\mathbf{M}$ immersed in $\mathbb{I}_3^1$ is called admissible if it has no isotropic tangent planes. For such a surface, the coefficients E, F, G of its first fundamental form are calculated with respect to the induced metric and the coefficients L, M, N of the second fundamental form, with respect to the normal vector field of a surface which is always completely isotropic. The (isotropic) Gaussian and mean curvature are defined by

$$\mathbf{K} = k_1 k_2 = \frac{LN - M^2}{EG - F^2}, \quad 2\mathbf{H} = k_1 + k_2 = \frac{EN - 2FM + GL}{EG - F^2}, \tag{2.2}$$

where k_1, k_2 are principal curvatures, i.e., extrema of the normal curvature determined by the normal section (in completely isotropic direction) of a surface. Since $EG - F^2 > 0$, for the function in the denominator we often put $W^2 = EG - F^2$.The surface $\mathbf{M}$ is said to be isotropic flat (resp. isotropic minimal) if $\mathbf{K}$ (resp. $\mathbf{H}$) vanishes [17].

It is well known in terms of local coordinates $\{u, v\}$ of $\mathbf{M}$ the Laplacian operator $\Delta^{\mathbf{III}}$ of the third fundamental form on $\mathbf{M}$ is defined by ([13])

$$\Delta^{\mathbf{III}}\mathbf{x} = -\frac{\sqrt{EG - F^2}}{LN - M^2}\left[\begin{array}{c} \frac{\partial}{\partial u}\left(\frac{Z\mathbf{x}_u - Y\mathbf{x}_v}{(LN-M^2)\sqrt{EG-F^2}}\right) - \\ \frac{\partial}{\partial v}\left(\frac{Y\mathbf{x}_u - X\mathbf{x}_v}{(LN-M^2)\sqrt{EG-F^2}}\right) \end{array}\right], \tag{2.3}$$

where

$$\begin{aligned} X &= EM^2 - 2FLM + GL^2, \\ Y &= EMN - FLN + GLM - FM^2, \\ Z &= GM^2 - 2FNM + EN^2. \end{aligned}$$

3. Translation Surfaces in $\mathbb{I}_3^1$

In order to describe the isotropic analogues of translation surfaces of constant curvatures, we consider translation surfaces obtained by translating two planar curves. The local surface parametrization is given by

$$\mathbf{x}(u, v) = \alpha(u) + \beta(v). \tag{3.1}$$

Since there are, with respect to the absolute figure, different types of planes in $\mathbb{I}_3^1$, there are in total three different possibilities for planes that contain translated curves: the translated curves can be curves in isotropic planes (which can be chosen, by means of isotropic motions, as $y = 0$, resp. $x = 0$); or one curve is in a non-isotropic plane ($z = 0$) and one curve in an isotropic plane ($y = 0$); or both curves are curves in non-isotropic perpendicular planes ($y - z = \pi$, resp. $y + z = \pi$). Therefore, the obtained translation surfaces allow the following parametrizations:

Type 1: The surface $\mathbf{M}$ is parametrized by

$$\mathbf{x}(u, v) = (u, v, f(u) + g(v))\,, \tag{3.2}$$

and the translated curves are $\alpha(u) = (u, 0, f(u))$, $\beta(v) = (0, v, g(v))\,.$

Type 2: The surface $\mathbf{M}$ is parametrized by

$$\mathbf{x}(u, v) = (u, f(u) + g(v), v)\,, \tag{3.3}$$

and the translated curves are $\alpha(u) = (u, f(u), 0)$, $\beta(v) = (0, g(v), v)$. In order to obtain admissible surfaces, $g'(v) \neq 0$ is assumed (i.e. $g(v) \neq$const.).

Type 3: The surface $\mathbf{M}$ is parametrized by

$$\mathbf{x}(u,v) = \frac{1}{2}\left(f(u) + g(v), u - v + \pi, u + v\right), \tag{3.3}$$

and the translated curves are

$$\alpha(u) = \frac{1}{2}\left(f(u), u + \frac{\pi}{2}, u - \frac{\pi}{2}\right), \beta(v) = \left(g(v), \frac{\pi}{2} - v, \frac{\pi}{2} + v\right).$$

In order to obtain admissible surfaces, $f'(u) + g'(v) \neq 0$ is assumed (i.e. $f'(u) \neq -g'(v) = a$ =constant.) ([17]).

In this paper, we will investigate the surface Type 1.

4. Translation Surfaces Satisfying $\Delta^{\mathbf{III}}\mathbf{x}_i = \lambda_i \mathbf{x}_i$

In this section, we classify translation surface in $\mathbb{I}_3^1$ satisfying the equation

$$\Delta^{\mathbf{III}}\mathbf{x}_i = \lambda_i \mathbf{x}_i, \tag{4.1}$$

where $\lambda_i \in \mathbb{R}$, $i=1,2,3$ and

$$\Delta^{\mathbf{III}}\mathbf{x} = \left(\Delta^{\mathbf{III}}\mathbf{x}_1, \Delta^{\mathbf{III}}\mathbf{x}_2, \Delta^{\mathbf{III}}\mathbf{x}_3\right),$$

where

$$\mathbf{x}_1 = u,\ \mathbf{x}_2 = v,\ \mathbf{x}_3 = f(u) + g(v).$$

For the translation surface is given by (3.2), the coefficients of the first and second fundamental form are

$$E = 1, F = 0, G = 1, \tag{4.2}$$

$$L = f'', M = 0, N = g'', \tag{4.3}$$

respectively. The Gaussian curvature $\mathbf{K}$ and the mean curvature $\mathbf{H}$ are

$$\mathbf{K} = f''(u)g''(v), \quad \mathbf{H} = \frac{f''(u) + g''(v)}{2}, \tag{4.4}$$

respectively.

Suppose that the surface has non zero Gaussian curvature, so $f''(u)g''(v) \neq 0, \forall u \in I$. By a straightforward computation, the Laplacian operator on $\mathbf{M}$ with the help of (4.2),(4.3) and (2.3) turns out to be

$$\Delta^{\mathbf{III}}\mathbf{x} = \left(\frac{f'''}{f''^3}, \frac{g'''}{g''^3}, \frac{-f''^3 g''^2 - f''^2 g''^3 + f' g''^3 f''' + g' f''^3 g'''}{f''^3 g''^3}\right). \tag{4.5}$$

Equation (4.1) by means of (4.5) gives rise to the following system of ordinary differential equations

$$\left(\frac{f'''}{f''^3}\right) = \lambda_1 u, \tag{4.6}$$

$$\left(\frac{g'''}{g''^3}\right) = \lambda_2 v, \tag{4.7}$$

$$\frac{-f''^3 g''^2 - f''^2 g''^3 + f' g''^3 f''' + g' f''^3 g'''}{f''^3 g''^3} = \lambda_3 \left(f(u) + g(v)\right), \tag{4.8}$$

where λ_1, λ_2 and $\lambda_3 \in \mathbb{R}$.This means that $\mathbf{M}$ is at most of 3- types. Combining equations (4.6), (4.7) and (4.8), we have

$$-\frac{1}{f''} + \lambda_1 u f' - \lambda_3 f = \frac{1}{g''} - \lambda_2 v g' + \lambda_3 g. \tag{4.9}$$

Here u and v are independent variables, so each side of (4.9) is equal to constant, call it a. Hence, we have

$$-\frac{1}{f''} + \lambda_1 u f' - \lambda_3 f = a = \frac{1}{g''} - \lambda_2 v g' + \lambda_3 g. \tag{4.10}$$

If we choose $a = 0$, then we get

$$-\frac{1}{f''} + \lambda_1 u f' - \lambda_3 f = 0 = \frac{1}{g''} - \lambda_2 v g' + \lambda_3 g, \tag{4.11}$$

where $c_i, \lambda_i \in \mathbb{R}$. If we solve (4.10) and (4.11), there are no any suitable solutions. If we differentiate both sides of (4.10) respect to u and v, respectively, we get the following

$$\begin{aligned} f''' + \lambda_1 u f''^3 + f' f''^2 (\lambda_1 - \lambda_3) &= 0, \\ g''' + \lambda_2 v g''^3 + g' g''^2 (\lambda_2 - \lambda_3) &= 0. \end{aligned} \tag{4.12}$$

We discuss four cases according to constants $\lambda_1, \lambda_2, \lambda_3$.

Case 1: Let $\lambda_1 = \lambda_2 = \lambda_3 = \lambda \neq 0$, from (4.12), we obtain

$$\begin{aligned} f'''(u) + \lambda u f''^3(u) &= 0, \\ g'''(v) + \lambda v g''^3(u) &= 0, \end{aligned} \tag{4.13}$$

and their general solutions are

$$\begin{aligned} f(u) &= c_1 + u c_2 + \frac{\sqrt{\lambda u^2 - 2c_3}}{\lambda} \pm \frac{u \ln\left(\lambda u + \sqrt{\lambda\left(\lambda u^2 - 2c_3\right)}\right)}{\sqrt{\lambda}}, \\ g(v) &= c_4 + v c_5 + \frac{\sqrt{\lambda v^2 - 2c_6}}{\lambda} \pm \frac{v \ln\left(\lambda v + \sqrt{\lambda\left(\lambda v^2 - 2c_6\right)}\right)}{\sqrt{\lambda}}, \end{aligned} \tag{4.14}$$

where $\lambda, c_i \in \mathbb{R}$. In this case, $\mathbf{M}$ is parametrized by

$$\mathbf{x}(u,v) = \begin{pmatrix} u, \\ v, \\ \left(c_1 + u c_2 + \frac{\sqrt{\lambda u^2 - 2c_3}}{\lambda} \pm \frac{u \ln\left(\lambda u + \sqrt{\lambda(\lambda u^2 - 2c_3)}\right)}{\sqrt{\lambda}}\right) \\ + \left(c_4 + v c_5 + \frac{\sqrt{\lambda v^2 - 2c_6}}{\lambda} \pm \frac{v \ln\left(\lambda v + \sqrt{\lambda(\lambda v^2 - 2c_6)}\right)}{\sqrt{\lambda}}\right) \end{pmatrix}. \tag{4.15}$$

Case 2: Let $\lambda_1 = \lambda_2 = 0$, $\lambda_3 \neq 0$, from (4.12), we obtain

$$\begin{aligned} f'''(u) - \lambda_3 f'(u) f''^2(u) &= 0, \\ g'''(v) - \lambda_3 g'(v) g''^2(v) &= 0. \end{aligned} \tag{4.16}$$

Differential equation (4.16) admits the particular solutions

$$\begin{aligned} f(u) &= c_1 \text{ or } f(u) = c_1 u + c_2, \\ g(v) &= c_2 \text{ or } g(v) = c_3 v + c_4, \end{aligned} \tag{4.17}$$

where $c_i \in \mathbb{R}$. Using the solutions (4.17) give rise a contradiction with our assumption saying that the solution must be non-degenerate second fundamental form. In the other cases, there are no any suitable solutions

Case 3: Let $\lambda_1 \neq 0, \lambda_2 \neq 0$, $\lambda_3 = 0$, from (4.12), we obtain

$$f'''(u) + \lambda_1 u f''^3(u) + \lambda_1 f'(u) f''^2(u) = 0, \tag{4.18}$$

$$g'''(v) + \lambda_2 v g''^3(v) + \lambda_2 g'(v) g''^2(v) = 0.$$

Differential equation (4.18) admits the particular solutions

$$f(u) = c_1 \text{ or } f(u) = c_1 u + c_2, \tag{4.19}$$

$$g(v) = c_2 \text{ or } g(v) = c_3 v + c_4,$$

where $c_i \in \mathbb{R}$. Using the solutions (4.19) give rise a contradiction with our assumption saying that the solution must be non-degenerate second fundamental form. In the other cases, there are no any suitable solutions.

Case 4: Let $\lambda_1 = \lambda_2 = 0 = \lambda_3 = 0$, from (4.12), we obtain

$$f'''(u) = 0, \tag{4.20}$$

$$g'''(v) = 0,$$

and their general solutions are

$$f(u) = c_1 u^2 + c_2 u + c_3, \tag{4.21}$$

$$g(v) = c_4 v^2 + c_5 v + c_6,$$

where $c_i \in \mathbb{R}$. In this case, $\mathbf{M}$ is parametrized by

$$\mathbf{x}(u,v) = \left(u, v, \left(c_1 u^2 + c_2 u + c_3\right) + \left(c_4 v^2 + c_5 v + c_6\right)\right). \tag{4.22}$$

Definition 4.1. A surface of in the three dimensional simple isotropic space is said to be **III**-harmonic if it satisfies the condition $\Delta^{\mathbf{III}}\mathbf{x} = \mathbf{0}$.

Theorem 4.1. *Let* $\mathbf{M}$ *be a translation surface given by (3.2) in the three dimensional simply isotropic space* $\mathbb{I}_3^1$. *If* $\mathbf{M}$ *is* **III***-harmonic, then it is congruent to an open part of the following surface*

$$\mathbf{x}(u,v) = \left(u, v, \left(c_1 u^2 + c_2 u + c_3\right) + \left(c_4 v^2 + c_5 v + c_6\right)\right).$$

where $c_i \in \mathbb{R}$.

Theorem 4.2. *(Classification)Let* $\mathbf{M}$ *be a translation surface with non-degenerate second fundamental form given by (3.2) in the three dimensional simply isotropic space* $\mathbb{I}_3^1$. *The surfaces* $\mathbf{M}$ *satisfies the condition* $\Delta^{\mathbf{III}}\mathbf{x}_i = \lambda_i \mathbf{x}_i$, *where* $\lambda_i \in \mathbb{R}$, *then it is congruent to an open part of the fallowing surface*

$$\mathbf{x}(u,v) = \left(\begin{array}{c} u, \\ v, \\ \left(c_1 + u c_2 + \frac{\sqrt{\lambda u^2 - 2c_3}}{\lambda} \pm \frac{u \ln\left(\lambda u + \sqrt{\lambda(\lambda u^2 - 2c_3)}\right)}{\sqrt{\lambda}}\right) \\ + \left(c_4 + v c_5 + \frac{\sqrt{\lambda v^2 - 2c_6}}{\lambda} \pm \frac{v \ln\left(\lambda v + \sqrt{\lambda(\lambda v^2 - 2c_6)}\right)}{\sqrt{\lambda}}\right) \end{array} \right),$$

where $\lambda, c_i \in \mathbb{R}$.

References

[1] L. J. Alias, A. Ferrandez and P. Lucas, Surfaces in the 3-dimensional Lorentz-Minkowski space satisfying $\Delta\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}$, Pacific J. Math. 156 (1992), 201–208.

[2] M.E.Aydin, Classification results on surfaces in the isotropic 3-space, http://arxiv.org/pdf/1601.03190.pdf

[3] M.E.Aydin, A generalization of translation surfaces with constant curvature in the isotropic space, J. Geom, DOI 10.1007/s00022-015-0292-0

[4] C.Baikoussis and L. Verstraelen, On the Gauss map of helicoidal surfaces,Rend. Sem. Math. Messina Ser. II 2(16) (1993), 31–42.

[5] M.Bekkar, Surfaces of Revolution in the 3-Dimensional Lorentz-Minkowski Space Satisfying $\Delta\mathbf{x}^i = \lambda^i\mathbf{x}^i$,Int. J. Contemp. Math. Sciences, Vol. 3, 2008, no. 24, 1173 - 1185

[6] M.Bekkar, B. Senoussi, Translation surfaces in the 3-dimensional space satisfying $\Delta^{III}r_i = \mu_i r_i$, J. Geom. 103 (2012), 367–374

[7] S.M.Choi, On the Gauss map of surfaces of revolution in a 3-dimensional Minkowski space, Tsukuba J. Math. 19 (1995), 351–367.

[8] S.M.Choi, Y. H.Kim and D.W.Yoon, Some classification of surfaces of revolution in Minkowski 3-space, J. Geom. 104 (2013), 85–106

[9] B.Y. Chen, A report on submanifold of finite type, Soochow J. Math. 22 (1996),117–337.

[10] F. Dillen, J. Pas and L. Vertraelen, On surfaces of finite type in Euclidean 3-space,Kodai Math. J. 13 (1990), 10–21.

[11] F. Dillen, J. Pas and L. Vertraelen, On the Gauss map of surfaces of revolution,Bull. Inst. Math. Acad. Sinica 18 (1990), 239–246.

[12] O. J. Garay, An extension of Takahashi's theorem, Geom. Dedicata 34 (1990), 105–112.

[13] G. Kaimakamis, B. Papantoniou, K. Petoumenos, Surfaces of revolution in the 3-dimensional Lorentz-Minkowski space satisfying $\Delta^{\mathbf{III}}\mathbf{r} = \mathbf{A}\mathbf{r}$, Bull.Greek Math. Soc. 50 (2005), 75-90.

[14] M.K.Karacan, D.W.Yonn and B.Bukcu, Translation surfaces in the three dimensional simply isotropic space $\mathbb{I}_3^1$,International Journal of Geometric Methods in Modern Physics, accepted.

[15] H. Sachs, Isotrope Geometrie des Raumes, Vieweg Verlag, Braunschweig, 1990.

[16] B.Senoussi, M. Bekkar, Helicoidal surfaces with $\Delta^J\mathbf{r} = \mathbf{A}\mathbf{r}$ in 3-dimensional Euclidean space,Stud. Univ. Babes-Bolyai Math. 60(2015), No. 3, 437-448

[17] Z.M. Sipus, Translation Surfaces of constant curvatures in a simply isotropic space,Period Math. Hung. (2014) 68:160–175.

[18] K. Strubecker, Differentialgeometrie des isotropen Raumes III, Flachentheorie, Math. Zeitsch.48 (1942), 369-427.

[19] T. Takahashi, Minimal immersions of Riemannian manifolds, J. Math. Soc. Japan,18 (1966), 380–385.

[20] D.W.Yoon, Surfaces of Revolution in the three dimensional pseudo-galilean space,Glasnik Matematicki,Vol. 48 (68) (2013), 415 – 428.

[21] D.W.Yoon, Some Classification of Translation Surfaces in Galilean 3-Space, Int. Journal of Math. Analysis, 6(28) 2012,1355 - 1361.

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