Investigation of Natural Frequency for Continuous Steel Bridges with Variable Cross-sections by using Finite Element Method

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Abstract- This paper mainly focuses on the natural frequencies of composite steel I-girder continuous-span bridges with straight haunched sections. The finite element analysis is performed to model dynamic behaviour of bridges by using CSIBridge package. Continuous-span bridges (two to six) with straight haunched section are considered. All the dimensions used for generating bridge models are designed according to AASHTO LRFD Standards (2014). Effect of various parameters such as span length, the depth ratios between haunched cross-section to mid-span cross-section (r), the length ratio between haunched section to span (α), span configuration and steel girder arrangement on natural frequencies are investigated by numerically generating one hundred fifty three bridge models. "r" and "α" values are set to be 0.5 - 2.0 and 0.1 - 0.5. The analysis results are given and discussed for the natural frequency of continuous-span composite steel bridges with straight haunched section.

Keywords Composite steel bridges, Natural frequency, Finite element analysis, Straight haunched, Non-prismatic cross-section

1. Introduction

Dynamic response has long been recognized as one of the significant factors affecting the service life and safety of bridge structures, and both analytical and experimental research has been performed [1]. Natural frequency of bridge structure is one of the most important parameter to determine dynamic response. To account for the dynamic effect of moving vehicles, static live load on bridges have been modified by a factor called "the dynamic load allowance" or "the impact factor" in bridge design specifications. The Ontario Highway Bridge Design Code (1983) (OHBDC) [2] is described to dynamic effect of moving vehicles as an equivalent static effect in terms of the natural frequency of the bridge structure. Australian Code (1992) (Austroads) [3] proposed similar dynamic load allowance to regard dynamic effect.

The AASHTO Standard Specifications (2002) [4] and AASHTO LRFD Specifications [5] limit the live load deflection depend on span length. Previous research by Roeder et al. [6] has shown that the justification for the current AASHTO live-load deflection limits are not clearly defined. Moreover, the bridge design specifications of other countries do not commonly employ deflection limits. Instead, vibration control is often achieved through a relationship between natural frequency, response acceleration and live-load deflection. In the Ontario Highway Bridge Design Code and Australian Code, live-load deflection limits are ensured by relationship with first flexural natural frequency for bridge structures. Therefore, it is important to demonstrate the influence of different variables on the natural frequency.

Frequently, beams are deepened by haunches near the supports to increase the support moment, which results in a considerable reduction of the span moment. Consequently, midspan depth can be reduced in order to obtain more clearance and/or less structural height [7]. Non-prismatic beam members are used commonly in bridge structures and less frequently in building structures [8]. It is especially true for continuous bridges. Despite many studies have been conducted to show influence of different variables on the natural frequency of continuous bridges with uniform cross-section, investigation for dynamic behaviour of bridges with variable cross-section is limited.

El-Mezaini et al. [7] discussed the general behaviour of non-prismatic members. The behaviour of non-prismatic members differs from that of prismatic ones due to the variation of cross section along the member and the discontinuity of the centroidal axis or its slope. Behaviour of non-prismatic members having T section was discussed by Balkaya [8]. However, the dynamic behaviour of composite continuous-span bridges having non-prismatic members was not considered. Gao et al. [9] studied on continuous concrete bridge with variable cross-section that has parabolic shape was used and proposed an empirical formulation to estimate fundamental frequency. Formulation includes two main parameters: "k" ratio defined as the ratio between side span and central span and "r" ratio defined as the height ratio between mid-span cross-section and the support cross-section but their studies focused on a narrow range of parameters.

The primary objective of this paper is to determine and demonstrate the influence of different variables on the dynamic behaviour of continuous bridges with straight haunched cross-section by finite element analysis (FEA) procedure using CSIBridge software [10]. Analysis results for sample bridges have been discussed.

2. Finite Element Model of Straight Haunched Continuous Bridge Structures

In this section, the procedure of FEA models with using CSIBridge [10] and assumptions are provided. Four-node shell element with six degrees of freedom at each node was used to model concrete slab as well as steel I girders. These elements consist of both membrane and plane-bending behaviour. For diaphragm members, two-node truss members were used as 'V' shape for all models to ensure the lateral stability. AASHTO specifications require full composite action between concrete slab and steel girder at the serviceability limit state [11]. Therefore, rigid-body behaviour was used to develop full composite action between steel I girder top flange and concrete slab so that corresponding nodes move together as a three-dimensional rigid body. Diaphragm members were connected to steel girders top and bottom flanges by hinged connection.

Material properties of bridge structure are listed in Table 1.

The finite element mesh was arranged to ensure good aspect ratio. To provide full composite action behaviour, steel I girder and concrete slab were divided into fine mesh that each node directly overlapped and connected as rigid-body. Typical3-D model mesh for continuous-span composite steel bridges with straight haunched section is shown in Fig.1.

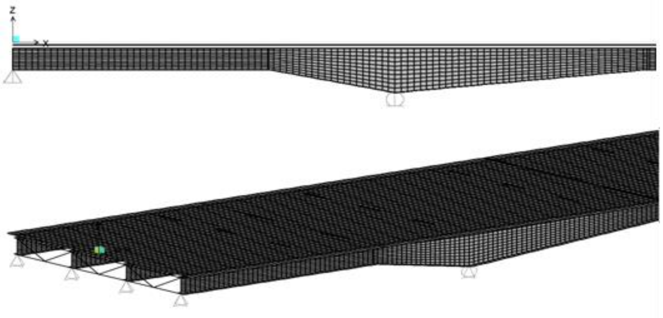


Fig.1. Typical 3-D finite element model mesh system for generated bridges

The supports were used as pin constraint that prevents three translational displacements at one support and roller constrain that prevents vertical and transverse displacement at the other supports rather than modelling piers and abutments. All constraints were placed along the bottom flanges at support locations.

Previous studies [11-12] show that parapet effect on vertical frequency of bridges maybe neglected. In these studies, maximum effect of parapet was found 4.8% between FEA model with and without parapet for Colquitz River Bridge in Canada. Therefore, parapets were neglected in this study.

General geometry of continuous bridges and notations are shown with using typical four-span bridge in Fig.2.Span lengths are selected as the same for all mid-spans (L_m) and are always bigger than side span lengths (L_s). The depth ratios between haunched cross-section to mid-span cross-section which is called 'r' and the length ratio between haunched section width to span which is called 'α' are shown in Fig. 2.

Table 1. Material properties of bridge structure

Components	Elastic Modulus [Mpa]	Shear Modulus [MPa]	Minimum Yield/Tensile Stress (F_y/F_u) [MPa]	Concrete Compressive Strength (F_c) [MPa]	Poisson Ratio	Thickness [m]
Concrete Slab (Shell)	33000	13750	-	30	0.2	0.2
Steel Girders (Shell)	210000	80769	355/510	-	0.3	-
Diaphragm Beams	210000	80769	355/510	-	0.3	-

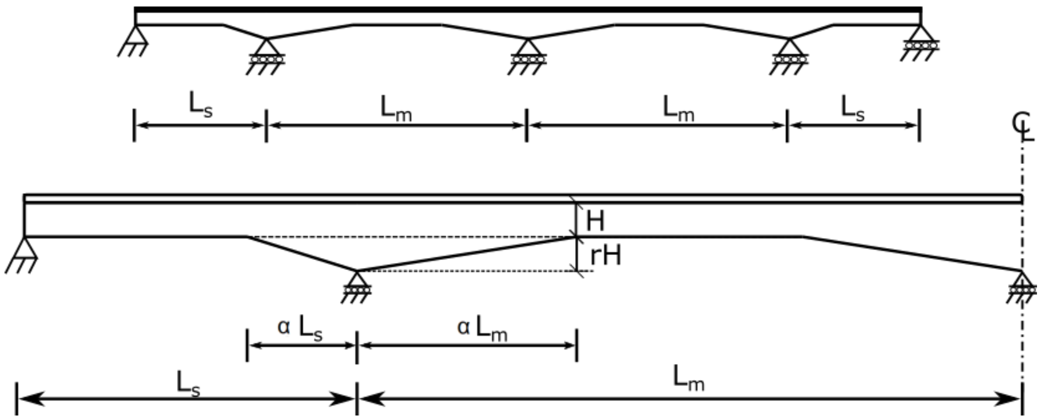


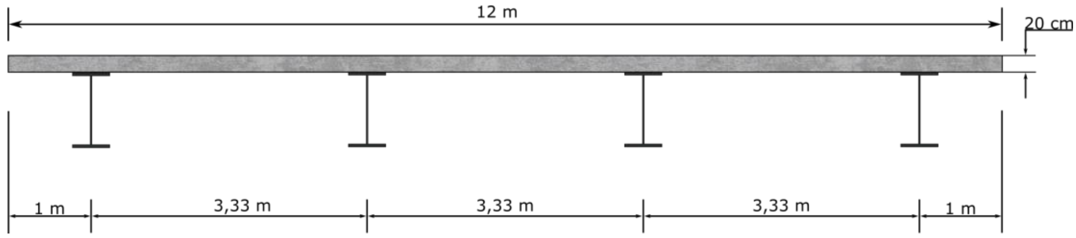
Fig.2. General geometry and notations of continuous-span bridges with straight haunched section

2.1 Variables

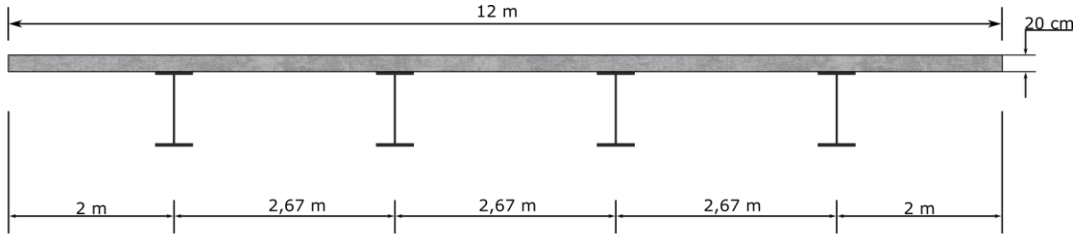
To investigate the effect of various parameters such as span arrangements, maximum span length, ɣ ratio (L_s/L_m), r and α ratios on natural frequency of continues-span bridges, total of 153 models are analysed. All dimensions used for generated bridge models are designed according to AASHTO LRFD Standards [8]. The dimensions of cross sections are determined with using 6.10.2.1.2-1, 6.10.2.2-1, 6.10.2.2-2, 6.10.2.2-3,6.10.2.2-4 limitations from AASHTO LRFD Standards; e.g. for four-span continuous bridges having maximum span length, r ratio and the ratio between steel I girder depth to span length are respectively equal to 30m, 2 and 0.03, the full depth of steel I girder is equal to 2.7m. According to limitations of AASHTO LRFD Standards, web depth of steel I girder (D), web thickness (t_w), flange thickness (t_f), full width of the flange (b_f) were respectively used as 2.63m, 0.0185m, 0.035m, 0.45mm. Concrete slab thickness (that also satisfy to AASHTO LRFD Standards, Section 9.7.1.1) and width are respectively used with same dimensions as 0.2m and 12m for all numerical models. Typical cross-section types are shown in Fig.3.

Cross-frames are designed as V-type. UNP380 steel profile was used as V-type cross-frames for all models. Colquitz River Bridge in Canada [12] that has almost same concrete slab width and thickness is used to design steel profile type of cross-frame. Despite arbitrary 25 ft. (7.62m) spacing limit for cross-frames and diaphragms was given by AASHTO Standard Design Specification, there isn't any limitation for spacing of cross-frames or diaphragms at AASHTO LRFD Standards. Therefore, 7.5m spacing of cross-frames is used so that the span lengths used for study can be equally divided and close to given spacing limit in [4]. Analysis cases are summarized in Table 2 for continuous-span bridges with straight haunched cross section.

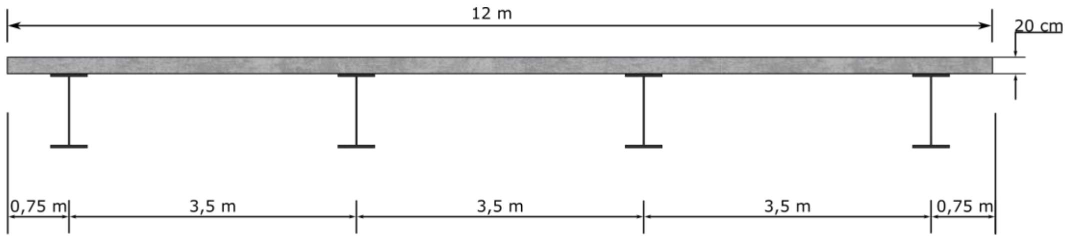
For Case A and B in Table 2; each L_m lengths are used according to given parameters. For Case C; total of 66 models are designed to investigate the relation between effect of r and α ratios on the natural frequency. Every r or α ratios are used as the other parameters given Table 2 are constant; e.g. while r ratios is equal 0.6 and the other parameters are constant as given, α ratios are used as 0.1, 0.2, 0.3, 0.33, 0.4 and 0.5. For Case D; each ɣ ratio is used with given L_m lengths for three, four, five and six-span continuos bridges. For Case E; five different cross-sections (Fig.3) are used according to given constant length and ratios in Table 2.



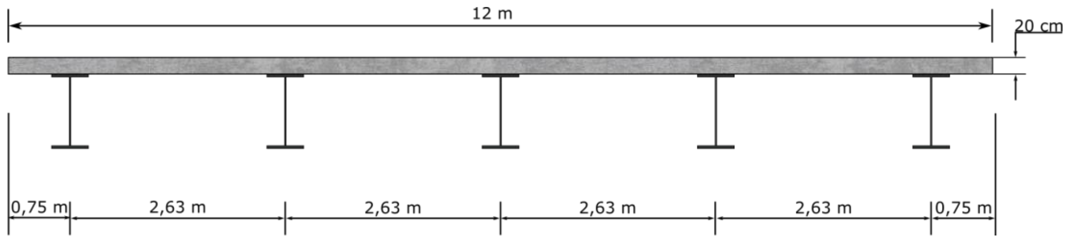
(a) Cross-Section-1 (CS-1)



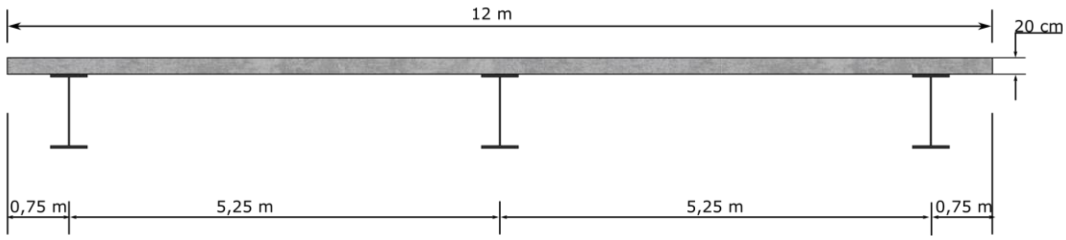
(b) Cross-Section-2 (CS-2)



(c) Cross-Section-3 (CS-3)



(d) Cross-Section-4 (CS-4)



(e) Cross-Section-5 (CS-5)

Fig.3. Cross sectional arrangements for mid-spans of bridges

Table 2. Analysis cases for straight haunched continuous bridges

Case	Span length, L_m (m)	H/L_m ratio	r ratio	α ratio	ɣ ratio	Cross Section Type	Span Configuration
A	15, 30, 45, 60, 75, 90	0,03	1	0,33	-	3	simple span
B	15, 30, 45, 60, 75, 90	0,03	1	0,33	1	3	two-span
C	30	0,03	0,5 0,6 0,7 0,8 0,9 1,00 1,20 1,40 1,60 1,80 2,00	0.1 0.2 0.3 0.33 0.4 0.5	0,5	3	four-span
D	15, 30, 45, 60, 75, 90	0,03	1	0,33	0,25 0,33 0,5 0,6 0,67 0,75 0,8 0,83 1	3	three-span four-span five-span six-span
E	30	0,03	1	0,33	0,5	1, 2, 3, 4, 5	four-span

3. Dynamic Analysis Results

The effects of r, α and ɣ ratios, maximum span length, cross-section configuration and span configuration on natural frequency are discussed and presented here.

3.1 Effect of r and α ratios:

To investigate the effects of r and α ratios on the natural frequency of steel I-girder continuous-span bridges with straight haunched sections, four-span continuous bridges are used with the span configuration as 15m-30m-30m-15m. The r and α ratios are respectively varied from 0.5 to 2.0. and 0.1 to 0.5 (Table 2) as shown in Fig. 4.

As “r” ratios increase, there is a clear increment mostly follows curve for the natural frequency while α ratios and all other variables are constant. As shown in Fig. 5, curves are converged to second-order equation. These increments of natural frequencies are valid for almost all α ratios but α is equal to 0.1. As α is equal to 0.1, there is a clear decreasing of natural frequencies after r = 1.0. While the r values increase from 1.0, natural frequencies decrease with second-order curve due to the relation between mass increment and natural frequency. Natural frequency can be more influenced by mass increment than stiffness increments as a result of haunched section which is generated in very narrow haunched length (Fig.5a). Additionally, the increment of frequency is more obvious for the case between α = 0.5 and r = 1.0 and 1.2 respectively. Increment of frequency rate is 3.35%. Differences of frequencies relative to given “r” and “α” values are given as Table 3.

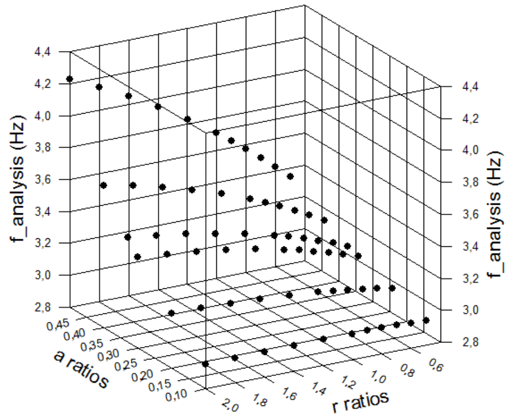
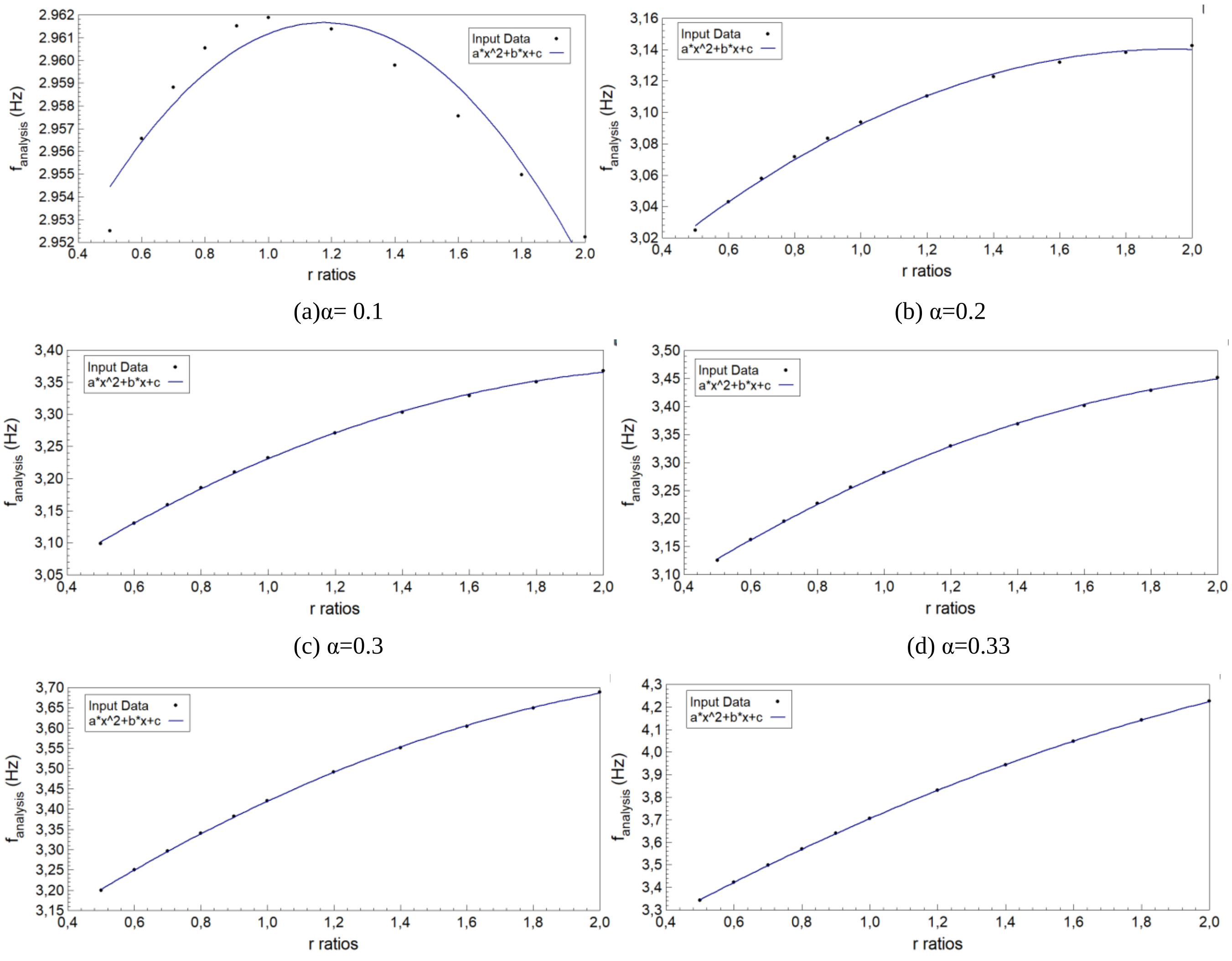


Fig.4. Natural frequencies versus “r” and “α” ratios



(a)α= 0.1

(b) α=0.2

(c) α=0.3

(d) α=0.33

(e) α=0.4

(f) α=0.5

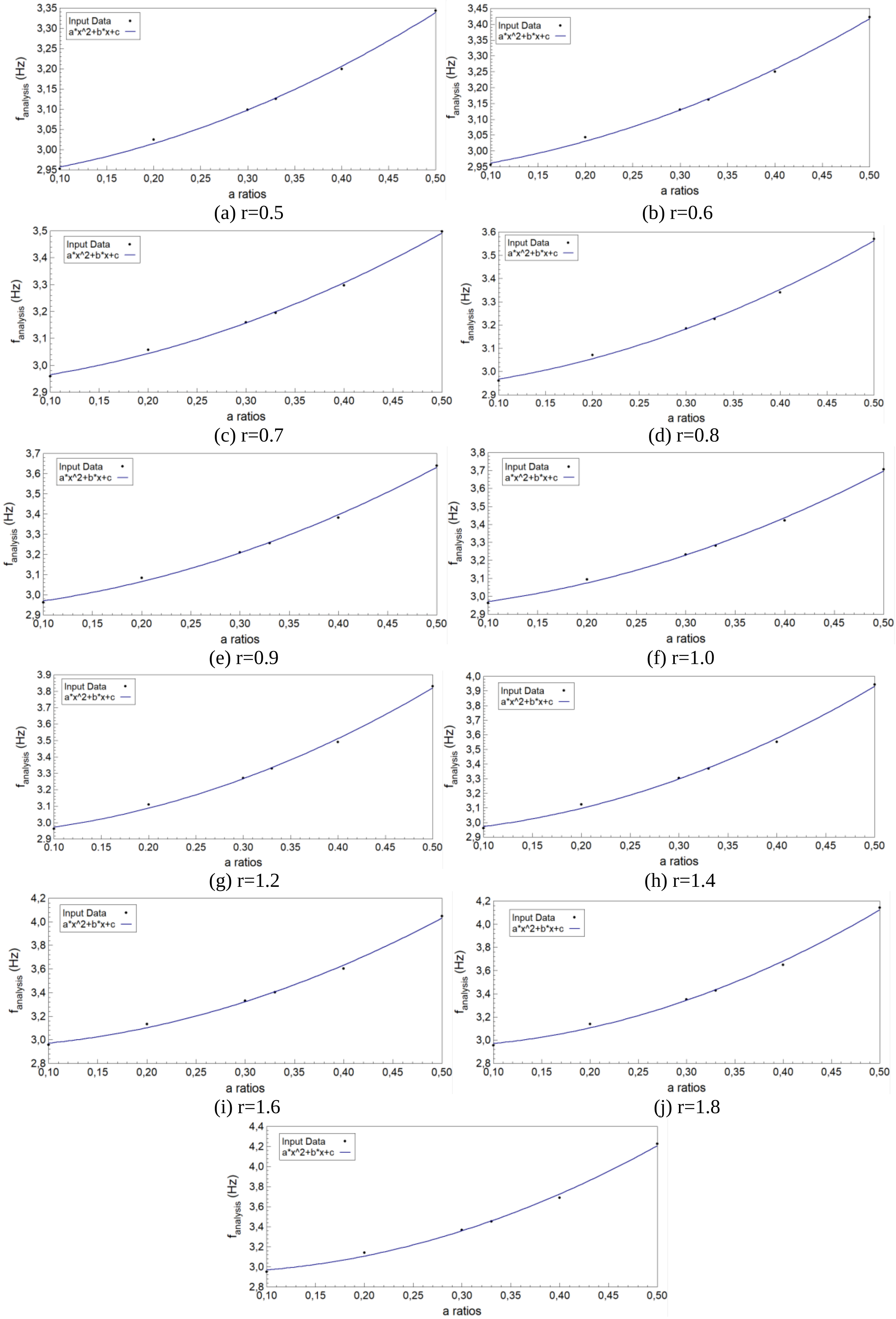
Fig.5. Natural frequencies versus “r” ratios corresponding to constant “α” for four-span straight haunched continuous bridges

Table 3. Differences of natural frequencies relative to given r ratios corresponding α ratios

No	r ratio	α ratio	Freq.	D %*	No	r ratio	α ratio	Freq.	D %*	No	r ratio	α ratio	Freq.	D %*
1	0.5	0.1	2.953	**0.137**	23	0.5	0.3	3.099	1.028	45	0.5	0.4	3.199	1.580
2	0.6		2.957	0.076	24	0.6		3.130	0.916	46	0.6		3.250	1.440
3	0.7		2.959	0.050	25	0.7		3.159	0.844	47	0.7		3.297	1.344
4	0.8		2.96	0.041	26	0.8		3.186	0.765	48	0.8		3.341	1.240
5	0.9		2.962	0.012	27	0.9		3.210	0.692	49	0.9		3.382	1.145
6	1.0		2.962	0.033	28	1.0		3.232	**1.197**	50	1.0		3.421	**2.044**
7	1.2		2.961	0.038	29	1.2		3.271	0.977	51	1.2		3.491	1.740
8	1.4		2.960	0.075	30	1.4		3.303	0.795	52	1.4		3.552	1.481
9	1.6		2.958	0.087	31	1.6		3.329	0.643	53	1.6		3.604	1.258
10	1.8		2.955	0.093	32	1.8		3.351	0.516	54	1.8		3.650	1.067
11	2.0		2.952	-	33	2.0		3.368	-	55	2.0		3.689	-
12	0.5	0.2	3.025	**0.590**	34	0.5	0.33	3.125	1.178	56	0.5	0.5	3.343	2.372
13	0.6		3.043	0.497	35	0.6		3.162	1.058	57	0.6		3.422	2.202
14	0.7		3.058	0.445	36	0.7		3.195	0.981	58	0.7		3.498	2.078
15	0.8		3.072	0.386	37	0.8		3.227	0.895	59	0.8		3.570	1.948
16	0.9		3.083	0.334	38	0.9		3.256	0.816	60	0.9		3.640	1.827
17	1.0		3.094	0.539	39	1.0		3.282	**1.428**	61	1.0		3.706	**3.355**
18	1.2		3.110	0.398	40	1.2		3.329	1.184	62	1.2		3.831	2.961
19	1.4		3.123	0.288	41	1.4		3.369	0.979	63	1.4		3.944	2.617
20	1.6		3.132	0.202	42	1.6		3.402	0.806	64	1.6		4.047	2.316
21	1.8		3.138	0.135	43	1.8		3.429	0.661	65	1.8		4.141	2.051
22	2.0		3.142	-	44	2.0		3.452	-	66	2.0		4.226	-

**Difference relative to r ratio; D % = $[(f(x_{i+1})-f(x_i))/ f(x_{i+1})]*100$*
Peak values are shown as bold

While "r" ratios and all other parameters are constant, the effect of "α" ratio on the natural frequency is investigated as shown Fig. 6. As α values increase, the natural frequencies of bridges also increase with curves which converge to second-order equation. The increment of natural frequency is more obvious for the case between r =2.0 and α= 0.4 and 0.5 respectively. Increment of frequency rate is 14.57 %. The differences of frequencies between cases are shown in Table 4.



(a) r=0.5 (b) r=0.6

(c) r=0.7 (d) r=0.8

(e) r=0.9 (f) r=1.0

(g) r=1.2 (h) r=1.4

(i) r=1.6 (j) r=1.8

(k) r=2.0

Fig.6. Natural frequencies versus "α" ratios
for four-span straight haunched continuous bridges

Table 4. Differences of natural frequencies relative to given α ratios corresponding r ratios

No	r ratio	α ratio	Freq.	D %*	No	r ratio	α ratio	Freq.	D %*	No	r ratio	α ratio	Freq.	D %*
1		0.1	2.953	2.455	25		0.1	2.962	4.117	49		0.1	2.958	5.891
2		0.2	3.025	2.433	26		0.2	3.083	4.110	50		0.2	3.132	6.307
3	0.5	0.3	3.099	0.858	27	0.9	0.3	3.210	1.418	51	1.6	0.3	3.329	2.168
4		0.33	3.125	2.368	28		0.33	3.256	3.889	52		0.33	3.402	5.961
5		0.4	3.199	**4.497**	29		0.4	3.382	**7.618**	53		0.4	3.604	**12.296**
6		0.5	3.343	-	30		0.5	3.640	-	54		0.5	4.047	
7		0.1	2.957	2.918	31		0.1	2.962	4.452	55		0.1	2.955	6.197
8		0.2	3.043	2.879	32		0.2	3.094	4.481	56		0.2	3.138	6.775
9	0.6	0.3	3.130	1.008	33	1.0	0.3	3.232	1.543	57	1.8	0.3	3.351	2.334
10		0.33	3.162	2.775	34		0.33	3.282	4.228	58		0.33	3.429	6.437
11		0.4	3.250	**5.312**	35		0.4	3.421	**8.344**	59		0.4	3.650	**13.469**
12		0.5	3.422	-	36		0.5	3.706	-	60		0.5	4.141	-
13		0.1	2.959	3.351	37		0.1	2.961	5.050	61		0.1	2.952	6.440
14		0.2	3.058	3.308	38		0.2	3.110	5.165	62		0.2	3.142	7.181
15	0.7	0.3	3.159	1.151	39	1.2	0.3	3.271	1.774	63	2.0	0.3	3.368	2.481
16		0.33	3.195	3.164	40		0.33	3.329	4.861	64		0.33	3.452	6.866
17		0.4	3.297	**6.103**	41		0.4	3.491	**9.736**	65		0.4	3.689	**14.574**
18		0.5	3.498	-	42		0.5	3.831	-	66		0.5	4.226	-
19		0.1	2.960	3.759	43		0.1	2.960	5,507					
20		0.2	3.072	3.718	44		0.2	3.123	5.773					
21	0.8	0.3	3.186	1.288	45	1.4	0.3	3.303	1.982					
22		0.33	3.227	3.535	46		0.33	3.369	5.437					
23		0.4	3.341	**6.871**	47		0.4	3.552	**11.053**					
24		0.5	3.570	-	48		0.5	3.944	-					

**Difference relative to r ratio; D % = [(f(xi+1)-f(xi))/ f(xi+1)] *100*
Peak values are shown as bold

3.2 Effect of Maximum Span Length:

The effects of maximum span lengths on natural frequencies of steel I-girder continuous-span bridges with straight haunched sections are investigated. For this purpose; single, two, three, four, five and six continuous-span bridges which have α and r ratios respectively equal to 0.33 and 1, are considered. Maximum span lengths of those bridges have been selected as 15, 30 ,45, 60, 75 and 90 m. As shown in Fig. 7 and Fig. 8, while maximum span lengths (L_{max}) increase, frequencies of continuous bridges clearly decrease. Span configuration's effect on natural frequency decreases with increment of span length. In other words, as maximum span lengths values increase, the natural frequencies of bridges that have different span configuration tend to close each other. This is also observed by Barth [10]. In general, frequencies of continuous bridges having same maximum span length decrease with increment of span number. However, Fig. 8 shows that frequency of three span bridges have higher natural frequencies than the others.

Especially three-span continuous bridges show dispersion on frequencies even though bridges have same maximum span lengths. This dispersion can be result of side span's effect on natural frequency. As side spans are generated in wide range that is from 0.25 portion of middle span to 1.0, middle span is considered as maximum span length for three span continuous bridges and middle span's dynamic contribution decreases with different range of side spans.

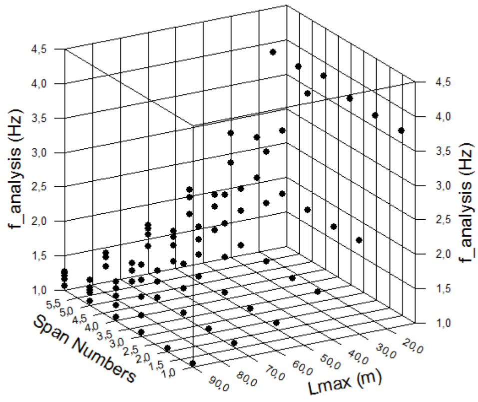


Fig.7. Natural Frequencies versus span number and L_{max}

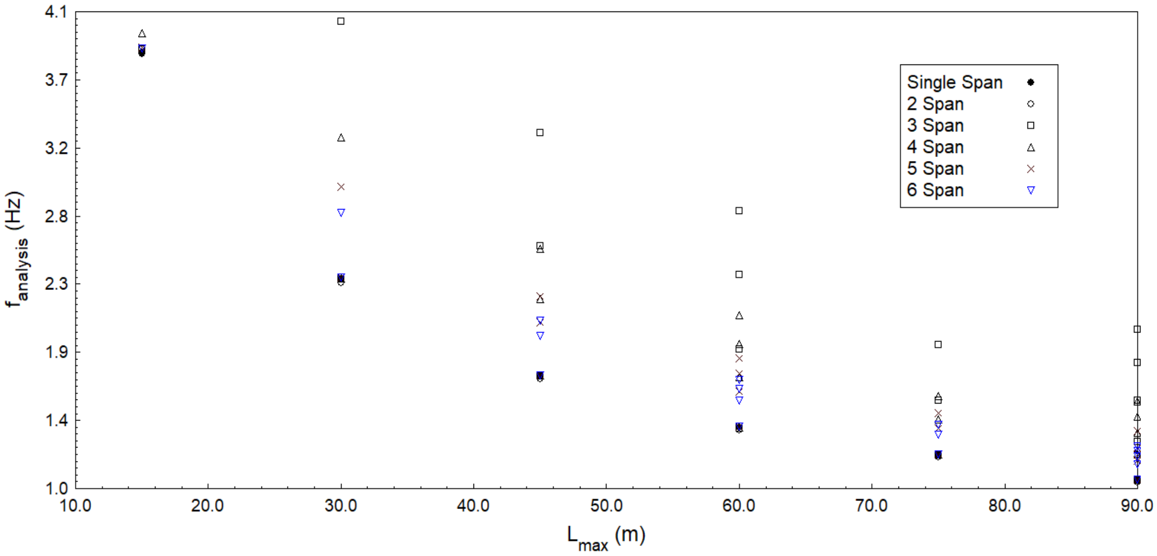
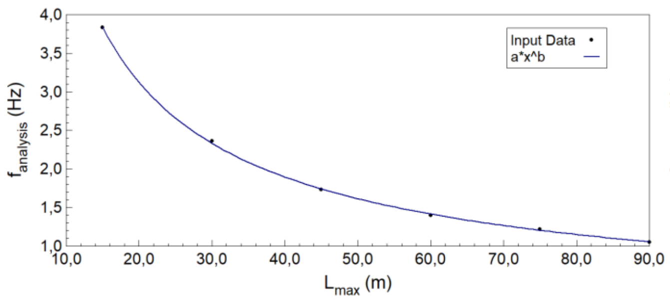
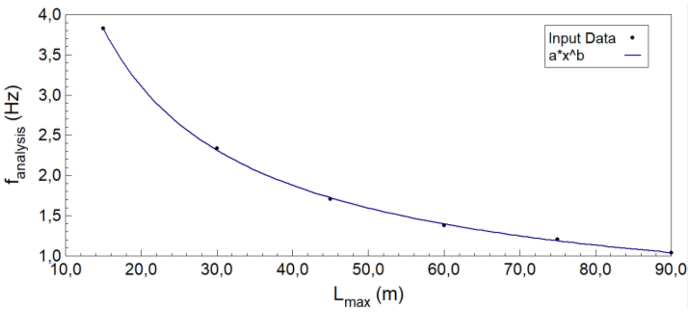


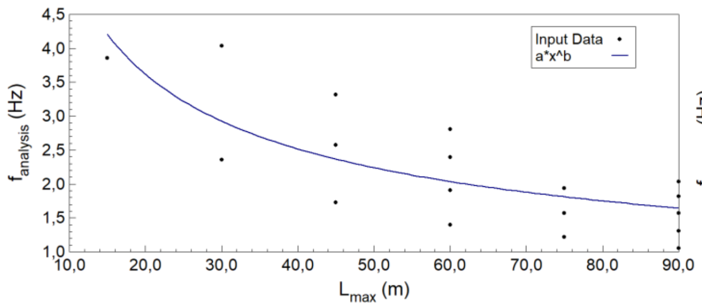
Fig.8. Distribution of natural frequencies versus span numbers and maximum span lengths



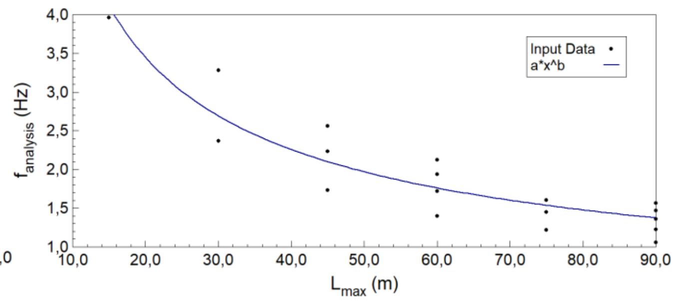
(a) Simple Span



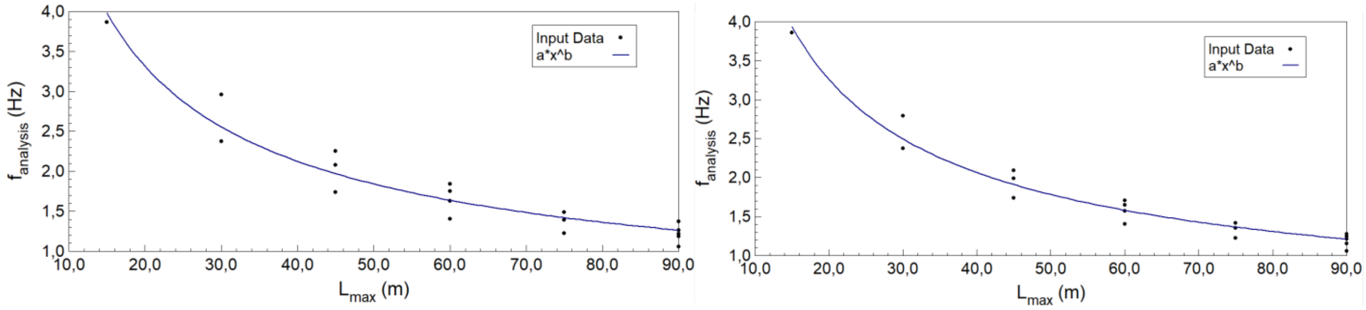
(b) 2 Span



(c) 3 Span



(d) 4 Span



(e) 5 Span (f) 6 Span

Fig.9. Natural frequencies versus maximum span lengths for different span numbers of straight haunched continuous bridges

According to Fig. 9, as maximum span lengths increase, the natural frequencies decrease with following curve. For almost all curves, ‘$a.x^b$’ curve gives the best fitting. Curve fitting parameters a and b, are given in Table 5 for different span arrangements

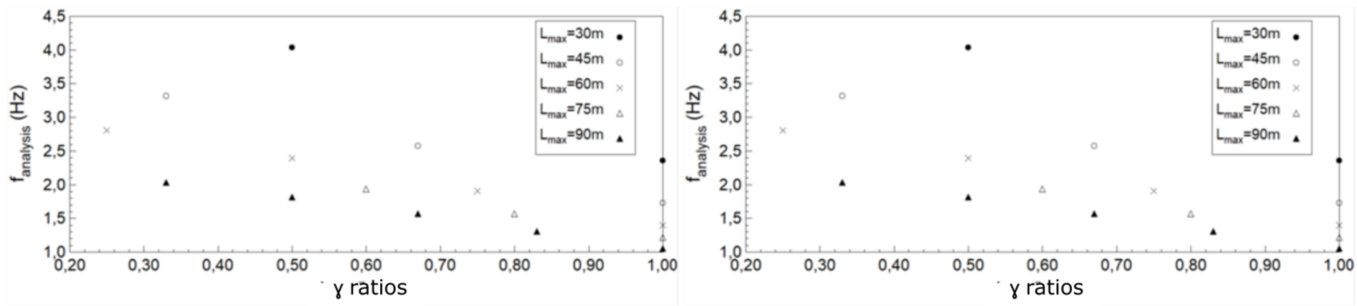
Table 5 Curve fitting parameters

a	b	Span Arrangement
26.947	-0.720	single-span
27.377	-0.726	two-span
17.336	-0.523	three-span
21.494	-0.611	four-span
22.557	-0.640	five-span
23.381	-0.658	six-span

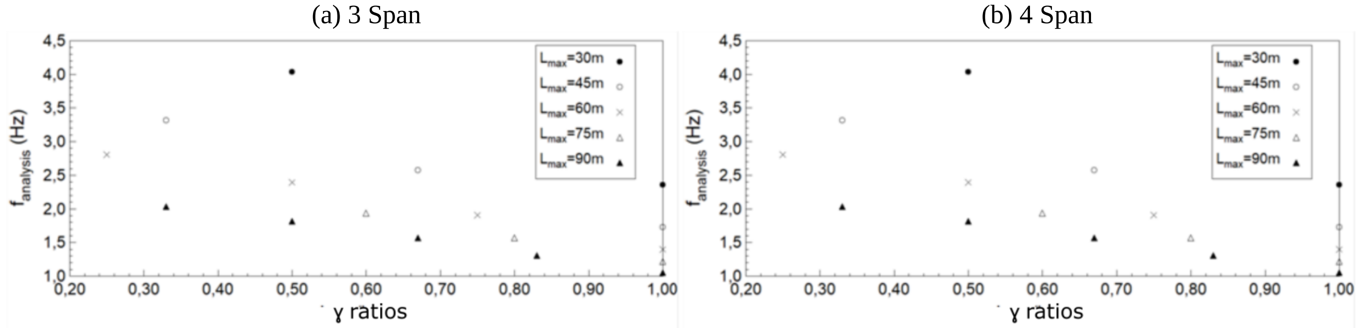
3.3 *Effect of ɣ Ratio:*

The effect of ɣ ratio on natural frequencies of steel I-girder continuous-span bridges with straight haunched sections is investigated. To determine effect of ɣ ratios on the natural frequencies; three, four, five and six continuous-span bridges are generated with using ɣ ratios between 0.25 to 1.0. For all considered bridge models, α and r ratios are respectively taken as 0.33 and 1.

Additionally, the relation between ɣ ratio and natural frequency of bridge is shown Fig. 10 considering maximum span lengths for all types of span numbers. According to Fig. 10, it seems that natural frequency of bridge decreases while ɣ ratio increases with constant maximum span length of bridge. Typically, decreasing of natural frequencies follow to straight line.



(a) 3 Span (b) 4 Span



(c) 5 Span (d) 6 Span

Fig.10. Natural Frequencies versus ɣ ratios

3.4 Effect of Cross-Section Arrangement:

To investigate cross-sectional effect on the natural frequency, five different steel bridges with variable cross-section are generated. For all generated bridges; α and r ratios are respectively taken as 0.33 and 1.0 as shown in Table 2, Case E. 15m-30m-30m-15m span arrangement is used. Concrete slab thickness and width are respectively used with same dimensions as 0.2 m and 12 m in all numerically modelled continuous bridges.

The natural frequencies of CS-1, CS-2, CS-3, CS-4 and CS-5 respectively equal to 3.289, 3.280, 3.282, 3.510 and 2.963.

CS-1, CS-2 and CS-3 have equal steel girder. Girder spacing and diaphragm lengths of CS-2 are smaller than CS-1 and CS-3. The natural frequencies of CS-1, CS-2 and CS-3 are slightly changed with changing steel girder space. Maximum difference of natural frequencies is obtained as 0.26 % between CS-1 and CS-2. With decreasing diaphragm lengths and therefore decreasing mass can affect to this small increment on the natural frequency.

CS-3, CS-4 and CS-5 which have same distance to concrete edge, are used to obtain effect of girder number on the natural frequency. Vertical rigidity of CS-4 should be more than the other models and accordingly, CS-4 has biggest frequency value, as CS-5's frequency is the smallest. The maximum difference of the natural frequency is obtained as 18.5 % between CS-5 having 3 steel I-girders and CS-4 having 5 steel I-girders.

3.5 Effect of Support Condition:

From two-span to six-span continuous bridges are used.to investigate support condition on the natural frequency of continuous-span bridge. The pin constrains that prevent three translational displacements are used at edge supports for continuous bridges. Roller constrains that prevent vertical and transverse displacements are used at the middle supports. General geometry is shown on typical four-span continuous bridge with straight haunched section in Fig.11.

For all models include two, three, four, five and six-span continuous bridges are analysed with two different support conditions (Fig. 2 and Fig. 11). The natural frequencies of bridges having two pin constrains at edge supports are smaller than the natural frequencies of bridges having one pin constrains at starting support. The maximum difference is observed as 15 % in all analysed cases of continuous bridges with different span arrangements.

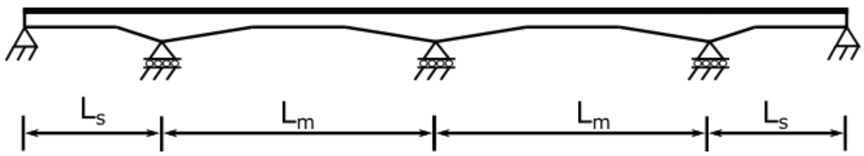


Fig.11. Geometry of continuous-span bridges with two pin constrains at starting and ending supports

4. Summary and Conclusions

This paper investigates the effect of various parameters on the natural frequency of continuous-span composite steel I-girder bridges with straight haunched section. 153 numerical models are analysed by using 3-D finite element models by using CSIBridge. The following conclusions can be drawn based on the results of the analyses:

1. While other parameters are constant; as r values increases, the natural frequency of continuous bridge increases. Likewise, the natural frequencies of continuous bridges increase with increment of α values. As given Table 3, the maximum difference is found as 3.35 % under the condition which α values are constant and r values are variable. Otherwise, the maximum difference is found as 14.57 %under the condition which r values are constant and α values are variable as shown Table 4.
2. Maximum span length of bridge has important effect on the natural frequencies of continuous-span bridges. In general, the natural frequency of bridge decreases with following curve while maximum span length increases. For all curves, '$a.x^b$' expression gives best fitting. Depend on results, a and b values has been produced corresponding span arrangement.
3. Effect of the ratio between middle span length and side span length (ɣ) on the natural frequency is also important factor. Natural frequencies of bridges that have same span number, linearly decrease with increasing ɣ ratios.
4. The natural frequencies of bridges can be changed by using different steel girder numbers and girder spacing. The steel girder numbers are more efficient on the natural frequency than steel girder spacing. The difference is obtained as 0.26 %due to effect of steel girder spacing with the same number of steel I-girders while the difference is obtained up to 18.5 %due to the effect of steel girder numbers decreasing from 5 steel I-girders to 3 steel I-girders.
5. Finally it is observed that the edge supports conditions of continuous bridges have also effect on the natural frequency. As ending support condition is changed from roller to pinned support, difference is obtained up to 15 % in studied cases with different span arrangement.

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