Research Article

Examining Middle School Mathematics Teacher Candidates' Algebraic Habits of Mind in the Context of Problem Solving*

Emine Nur ÜNVEREN BİLGİÇ[1], **Ziya ARGÜN**[2]

Abstract

The aim of this research is to examine the algebraic mind habits in the context of problem solving of middle mathematics teacher candidates within the pedagogical field. The study which was dominated by qualitative paradigm was done in the pattern of case study. With this aim, data were gathered from 30 teacher candidates via "Algebraic Habits of Mind Worksheet" and interviews. The data were analyzed in the light of the components of theoretical title of algebraic habits of mind and according to the stages of descriptive analysis. The teacher candidates made solutions based on memorizations without writing what is given and wanted; however they clearly wrote what is given and wanted in the last two problems. While this seems to be a form of rules that represent direct functions in the questions seen as exercises; it causes them to use the thinking / reverse thinking step more actively when they are perceived as problems. At the interviews, it is seen that the fourth grade teacher candidates are more detailed about the construction on their students' knowledge than the first grade teacher candidates and that the first grade only focuses on solving.

Keywords: algebraic habits, mathematics education, problem solving, teacher education

1. INTRODUCTION

Students focus on solving fixed problem situations by following rules or formulas which they learned in the past and by applying them. This situation causes students to perceive that mathematics is a science which can be studied by only using specific rules on special conditions (Cuoco, Goldenberg & Mark, 1996). It is true that mathematics involve high level abstract thinking skills. However, this abstractness is not a structure which is always impossible to understand or only mathematicians can understand. As long as specific thinking skills are developed in learners, desire and power to do mathematics can be gained in them.

Thinking ability is one of humankind's basic traits. Besides equipping the individual with basic information regarding arithmetic, algebra and geometry; main purpose of mathematics education is to direct them to think and be aware to be consistent in judgements and results. Considering these features of mathematics, we can speak of a thinking that is special to mathematics (mathematical thinking). Mathematical thinking is described as a dynamic process which broadens our understanding and allows us to think further (Mason, Burton & Stacey, 1998). Harel (2007) defined internalization of thinking methods as mind habit. The point that needs to be highlighted here is that rather than exercising or solving routine problems; mind habits are strategies and approaches produced in dealing with problem situations. Individuals' thinking methods are quite important in explaining a mind habit (Harel, 2007). Mind habits are usually described as cognitive habits which allow students to improve their general heuristic repertoires and approaches that can be applied to problems they face in many different situations (Cuoco et al., 1996). In other words, mind habits are strategies which individuals personally prefer when dealing with a problem situation and tendencies which they display in their

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[1]*PhD, Sakarya University, eunveren@sakarya.edu.tr , Turkey*

[2] *Prof. Dr.,Gazi University, zyargun@gmail.com , Turkey*

Corresponding Author e-mail adress: eunveren@sakarya.edu.tr

applications. Cuoco, Goldenberg and Mark (1996) treated mind habits in two types; as general mind habits and mind habits specific to discipline. General mind habits include the most basic skills such as thinking, researching, realizing patterns and relationships, making definitions, discovering, hypothesizing and visualizing.

Mathematical habits of the mind is defined as possessing the ability to reason in different situations via intellectual activities and by considering the methods used by those who engage in the science of mathematics (Mark, Cuoco, Golderberg & Sword, 2010). Individuals' mathematical habits of mind differ according to their learning levels (Cuoco, Goldenberg, & Mark, 2010; Goldenberg, Shteingol & Feurzeig, 2003). For higher education mathematics, this situation can be described as carrying out thought experiments, finding, stating and explaining patterns, creating and using representations, generalizing examples, dogmatizing this generalization and revealing the mathematic by making sense of it (Cuoco et al., 2010).

Algebra which is in mathematical thinking skills is a language that explains mathematical thoughts. However, contrary to most languages, more than one representation can be used to explain one mathematical thought. For this reason, algebraic thinking is not that easy to explain in a simple way (Driscoll, 1999). Explaining algebraic thinking is to do with how you look at algebra. For instance, some mathematicians try to explain algebraic thinking by focusing on its abstract features which makes it different from arithmetic. With this aim, Langrall and Swafford (1997) defined algebraic thinking as "the ability to carry out processes in an unknown amount as if the quantity is known on the contrary of arithmetic reasoning that requires a known amount of processing" (cited in Driscoll, 1999). Some mathematicians explain algebraic thinking as the capacity of representing quantitative situations, considering that the concept of "function" plays an important role in algebra (Greenes & Findell, 1998; Trybulski, 2007). Another group of mathematicians focused on problem solving in algebra and became interested in individuals' modelling of the problem situation in the process of problem solving (Hebert & Brown, 1997; Kaf, 2007). Algebraic thinking skill is also carried out and improved through some mind habits.

Algebraic mind habits are steps which individuals prefer when faced with an algebraic situation. In literature, many researchers attempted to define algebraic mind habits (Bass, 2008; Cuoco et al., 1996; GorLGn, 2011; Jacobbe & Millman, 2009; Lim & Selden, 2009; Mark et al., 2010; Matsuura, Sword, Piecham, Stevens & Cuoco, 2013; Rolle, 2008). The point commonly emphasized by these researchers is that they define algebraic mind habits as actions performed by individuals who engage in mathematics in carrying out their processes. With this perspective, algebraic mind habits can be described as individuals transforming their existing mathematical information to habits via different ways of thinking. Driscoll (1999) examines algebraic mind habits under three components. These are thinking/reverse thinking, creating rules that represent functions and going from calculations to abstractions. Thinking/reverse thinking is quite a basic component for the other two algebraic mind habits. Driscoll (1999) describes thinking/reverse thinking as not a process of reaching a solution; but at the same time, a capacity to be able to go back to the given point in a problem situation with an answer, by controlling the entire problem situation. For example, as individuals engaging in algebra can solve $9x^2 - 16= 0$ directly, they can answer the question "What is the equation whose result is 4/3 and -4/3?" First stage of thinking/reverse thinking is to understand the problem. This skill can be described as what the student understands from the problem. At this stage, student reads the problem situation, comments and understands. An important component of understanding the problem is to define quantities and the relationship between them. Lastly, developing representation of these components with symbols, pictures, words, tables and algebraically. Another algebraic mind habit is creating rules that represent functions. This process of thinking is quite important in algebraic mind habit because with the help of this thinking, individual can define relationships and organize data. Herbest and Brown (1997) define algebraic thinking as using mathematical symbols and tool, applying and analyzing mathematical findings and displaying mathematical recycling with representations in order to discover terms and quantities in problem situations. With this definition, creating rules that represent functions can be described as searching for and defining relationship and reaching generalizations. Reaching abstractions from calculations can be described as the ability to think of the processes independent from numbers. For example, when teaching how to factorize, students may be asked to do some processing with the help of area model. Students can see the processing of which they do abstractions independent of the numbers in time. This is probably the most important one among the processes of algebraic mind habits.

Many researchers have emphasized that algebra is a critical issue for students and that algebraic thinking should be developed at an early age (Blanton & Kaput, 2003; Cai, 2004; Carraher & Schliemann, 2007; Kaput & Blanton, 2001; Kieran, 1996; Moses, 1995; NCTM, 2000; Schliemann, Carraher & Brizuela, 2007). However, in the literature, it is often stated that students have difficulties in algebra (Akgün, 2006; Bağdat & Anapa-Saban, 2014; Dede & Argün, 2003; Ersoy & Erbaş, 2005; Kaya & Keşan, 2014; Kaya, 2017; Yenilmez & Avcu, 2009; Özarslan, 2010; Soylu, 2008; Van Amerom, 2003).

In Secondary School Mathematics Class (5-8 th Grade) Teaching program (Turkish Ministry of National Education) [TMoNE], 2013), these application habits are stated in sub-title of "Reasoning" under "Mathematical Process Skills;"

- "Justifying accuracy and validity of the inferences,
- Making reasonable generalizations and inferences,
- Explaining and using mathematical patterns a
- Predicting the result of processing or measures by using strategies such as rounding up, grouping appropriate numbers, using first or last digits or self-developed strategies,
- Making predictions about measurements by considering a specific reference point" (TMoNE, 2013).

The necessity to teach these to students in mathematics teaching was stated. When deeply thought on, we can say that teaching algebraic mind habits to students is one of the aims of our teaching program. By teaching algebraic mind habits to students, we can develop their algebraic thinking and allow them to solve problems they face in different ways (Poindexter, 2011).

In the direction of this aim in Mathematics Teacher Qualifications (2008), for these performance indicators under the sub-qualifications of Developing Students' Problem Solving Skills and Developing Students' Reasoning Skills to be present in teachers, they need to be aware of the above-mentioned algebraic mind habits. These performance indicators are displayed below.

Developing Students' Problem Solving Skills

- Allows students to question the process of problem solving and confirm the results they reach.
- Guides students to develop and use different problem-solving strategies.

Developing Students' Reasoning Skills

- Makes practices towards developing mathematical reasoning skills.
- Allows students to use mathematical models, rules and relationships to explain their own thoughts.
- Regulates learning environments to develop students' prediction skills.
- Allows students to make inferences and generalizations by using reasoning skill.

Research Problem

It is very important to examine the existing habits of teachers and teacher candidates who are expected to develop algebraic mind habits in their students and and the effect of the mathematics teacher training program on the algebraic mind habits in a pedagogical sense, based on these expressions in Teacher Proficiency (2008) and Instructional Program (2013). With this aim, the problem of this study was stated as "How the algebraic mind habits of the teacher candidates who are studying in the Middle Education Mathematics Teaching Program are in the context of the pedagogical field?” In this context, these questions will be answered;

1) How are the algebraic mind habits of first grade teacher candidates in the pedagogical context at middle school mathematics teaching program?

2) How are the algebraic mind habits of fourth grade teacher candidates in the pedagogical context at middle school mathematics teaching program?

2. METHOD

The purpose of the study was to demonstrate that mathematics teacher candidates are studying algebraic mind habits in the pedagogical context. With this aim, qualitative paradigm was followed in the study. The study was carried out in the pattern of case study. In internal case study under the title of case study, researcher deeply narrates the features in order to illuminate a situation (Johnson & Christensen, 2014). In this study which was carried out in the pattern of internal case study, the primary purpose of the researcher is to define the teacher candidates’ algebraic mind habits. For this, teacher candidates’ worksheets were examined in detail and their solutions were repeatedly analyzed. The interviews were conducted with the teacher candidates, who are determined by considering the solutions they had made on the worksheet, on evaluating their solutions in the pedagogical context.

2.1. Participants

The participants of the study were fifteen first grade and fifteen fourth grade middle school mathematics teacher candidates. A purposeful sampling method was used to identify the participants. Firstly, the investigator determined the properties of the relevant universe and then attempts to sample individuals with these properties (Christensen & Christensen, 2014). At this point, the researcher tried to determine what the algebraic mind habits of middle mathematics teacher candidates are in the context of the first and fourth grades. The selection of prospective teachers from the first and fourth grades in the study closely examines the effect of university education on algebraic mind habits.

2.2. Data Collection

Two different data gathering tools were used in the process of gathering data. One of these tools was “Algebraic Mind Habits Worksheet” which was prepared with two experts to define teacher candidates’ algebraic mind habits. Algebraic thinking is a way of thinking that includes necessary skills for mathematics such as reasoning, using representations, understanding variables, explaining the meaning of symbolic representations, working with models for developing mathematical ideas and making conversions between representations (Kaf, 2007). According to Hawker and Cowley (1997), this way of thinking includes an estimate that requires representation, structuring and generalized thinking of pattern and orders. In this context, the related literature (Cuoco et al., 1996; Nebraska-Lincoln University Report, 2006) was examined and two questions regarding displaying the general features, structures and generalizations of patterns and three questions regarding reasoning, using representations, understanding variables, explaining the meaning of symbolic representations, working with models for developing mathematical ideas and recycling among representations were placed in the worksheet. The honeycomb and the circle problems in the worksheet were problem situations which were most probably faced by the teacher candidates. The shopping problem was added to the worksheet as having less probability to be faced by teacher candidates compared to first two questions. The number problem was included as a problem situation in the worksheet to allow teacher candidates to display their proving skills. The aim here is to examine the strategies suggested by teacher candidates and their possible solutions to the problem situations in the context of algebraic mind habits.

Another data gathering tool was the interviews with teacher candidates determined by the answers given on the worksheet in an approach of guidance. The researcher comes to the interviews with guidance approach with a plan of discovering specific topics and asking the interviewee specific open-ended questions (Christensen & Christensen, 2014). In this context, the basic interview questions were given in the appendix.

2.3. Data Analysis

The data gathered from the worksheets were firstly numbered and then analyzed according to the components of the theoretical roof of "algebraic mind habits" and stages of descriptive analysis. The theoretical framework developed by Driscoll (1999) was used in order to draw a general framework for reaching and not reaching the generalization of teacher candidates in the data analysis from the research questions and the interviews. In this context, component developed for each algebraic mind habit (thinking/reverse thinking, creating rules that represent functions and moving from calculations to abstractions) were considered as themes and indicators of each components were considered as sub-theme and codes. Theme, sub-theme and codes were explained in Table 1. Miles and Huberman (1984) have used the principles of reducing data, choosing important parts of raw data, focusing on certain points, simplifying, summarizing and transforming the ideas to provide consensus between two researchers. Encoding consistency of individual generated categories is examined. Equation P = (Nax100) / (Na + Nd) (P: percent of maturity, Na: maturity, Nd: maturity) is used to calculate the compliance percentage (Türnüklü, 2000). As a result, the percentage of complaints was 72%. This value indicates that the study can be regarded as reliable. Afterwards data were read, organized and associated according to this frame. Lastly to explain the findings of the study, firstly the relationships between themes and sub-themes were visualized and then these relationships were presented with direct quotes from the participants and comments. In direct quotations, codes were used instead of the teacher candidates' names. For example, the first student in first grade was stated as FG1, the fifteenth student in first grade was stated as FG15, the first student in fourth (last) grade was stated as LG_1, the fifteenth student in last grade was stated as LG_{15}. The themes, sub-themes and codes covered in the frame drawn by Driscoll (1999) with descriptive analysis are given in Table 1.

Table 1. Theme, sub-theme and codes descriptions

Themes	Sub-theme and Codes		
Thinking/reverse thinking	Understanding the problem • *Explaining what is given and wanted in the problem clearly (e.g; there is some kind of a bargain here and the common number is wanted?)* • *Taking steps based on memorization (e.g; based on past information, focusing on solving the problem without reading it)*	Understanding the quantities in the problem and the relationship between them • *Trial-and-error (e.g; making trials l giving values : quantities)* • *Counting (e.g; there's four difference between these numbers)*	Showing with symbols, pictures, words and tables • *Expressing with symbols (e.g; using variable)* • *Trying to tell with words, mimics and gestures (e.g; in this question, one is going up and the other is gong down (gestures), i mean, there has to be a common point but I don't know how to find it)* • *Showing with tables* • *Drawing*
Creating			

Rules that Represent Functions	Searching for relationship • *Trying familiar strategies (e.g; trying a relationship used in the past)* • *Acting with intuition or prediction (e.g; if I write a general statement like this it looks like it will stand for all*	Describing relationships • *Stating the strategies that were tried* • *Determining the invariables and writing them (e.g; while person A falls 25 units, person B increases 12,5 units)*	Reaching the rules • *Trying the defined statements for different stages of the problem (e.g; checking if the defined function gives the correct result at 6th step)* • *Confirming the solution by developing other solutions (e.g; this can also be solved on the table with the help of similarity in geometry as well as with the help of sequences)*
Going from calculations to abstractions	Ability to think of processings as independents from numbers • *Solving intuitively (not being aware of how you write the solution)* • *Extracting the numbers from processings done with the numbers in time and creating general structures (e.g; if you look at the number of circles around the shape, it is obvious that four points and a center point are stable. Then, it should be 4 times of steps we take and plus one)*	Producing shortcuts for the solution • *Trying to find a general statement* • *Emphasizing that you can intuitively find a shortcut (e.g; I think there will be a very short representation of it but I don't know how to show it)*	Producing an appropriate statement for the solution • *Designing a problem that corresponds the solution*

3. FINDINGS

The results gathered from this study which aims to determine middle school mathematics teacher candidates' algebraic mind habits were presented in accordance with the answers given to the problems.

Table 2. Descriptions of themes, sub-themes and categories in the context of problem situations

Theme, Sub-theme and Categories	Problem 1	Problem 2	Problem 3	Problem 4
Thinking/reverse thinking				
Understanding the problem				
• *Clearly stating what is given and wanted in the problem*	LG_1, LG_2, LG_5, LG_9, LG_{13}, LG_{14}	LG_1, LG_{11}, LG_{13}	LG_1, LG_2, LG_3, LG_4, LG_5, LG_6, LG_7, LG_8, LG_{10}, LG_{11}, LG_{12}, LG_{13}, LG_{14}	FG_1, FG_3, FG_6, FG_7, FG_{14}, LG_1 LG_2, LG_3, LG_6, LG_8, LG_{12}, LG_{13}, LG_{14}
• *Taking steps based on memorization*	FG_1,..., FG_{15}, LG_3, LG_6, LG_{12}, LG_{15}	FG_1,..., FG_{15}, LG_3, LG_5, LG_6, LG_9,	nonexistent	nonexistent

		LG_{15}		
Understanding the quantities in the problem and the relationship between them				
• *Trial-and-error*	LG_2 LG_9, LG_{14}	$LG_{1,}$ LG_{14}	nonexistent	nonexistent
• *Counting*	LG_3, $LG_{6,}$ LG_{10}, LG_{14}	$LG_{3,}$ LG_5, LG_6, LG_{13}, LG_{14}	nonexistent	nonexistent
Showing with symbols, pictures, words and tables				
• *Expressing with symbols*	FG_1,..., FG_{15}, LG_1, LG_2, LG_3, LG_5, LG_6, LG_9, LG_{12}, LG_{13}, LG_{14}, LG_{15}	FG_1,..., FG_{15}, LG_1, LG_2, LG_3, LG_5, LG_6, LG_9, LG_{11}, LG_{13}, LG_{14}, LG_{15}	LG_{14}	FG_1, FG_3, FG_6, FG_7, FG_{14}, LG_1, LG_2, LG_3, LG_6, LG_8, LG_{12}, LG_{13}, LG_{14}
• *Trying to express with verbal expressions and gestures and mimics*	nonexistent	nonexistent	LG_6	nonexistent
• *Showing with a table*	nonexistent	LG_1, LG_{13}	nonexistent	nonexistent
• *Drawing shapes*	LG_{14}	LG_{11}	LG_6, LG_{14}	nonexistent
Creating Rules which Respresent Functions				
Searching for Relationship				
• *Trying familiar strategies*	LG_2, LG_5	LG_{13}, LG_6, LG_9, FG_1,..., FG_{15}	nonexistent	LG_3, $LG_{8,}$ LG_{13}
• *Acting with intuition or prediction*	LG_{14}	LG_1, $LG_{11,}$ LG_{14}	LG_6, LG_7, LG_8, LG_{10}, $FG_{12,}$ LG_{14}	LG_1, LG_2, LG_4, LG_{14}
Defining relationships				
• *Writing used strategies as statements*	LG_2, LG_{14}	LG_1, LG_2, LG_3, LG_5, LG_6, LG_9, LG_{11}, $LG_{13,}$ LG_{14}	LG_6, LG_{14}	nonexistent
• *Determining invariables and writing them*	nonexistent	nonexistent	$LG_{3,}$ LG_6, LG_{14}	nonexistent
Reaching rules				

• *Trying defined statements for different stages of the problem*	LG_{14}	LG_2, LG_{11}, LG_{14}	nonexistent	nonexistent
• *Confirming the solution by developing different solutions*	LG_{14}	LG_{14}	LG_{14}	nonexistent
Going from Calculations to Abstractions				
Ability to think of processings as independents from numbers				
• *Solving intuitively*	FG_1,..., $FG_{15,}$ LG_1, LG_9, LG_{12}, LG_{13}, LG_{15}	FG_1,..., $FG_{15,}$ LG_{15}	nonexistent	nonexistent
• *Extracting the numbers from processings done with the numbers in time and creating general structures*	LG_2, LG_3, LG_6, LG_{14}	LG_1, LG_2, LG_{11}, LG_{13}, LG_{14}	nonexistent	nonexistent
Creating Shortcuts for Solution				
• *Trying to find a general statement*	LG_{14}	LG_{14}	LG_{14}	nonexistent
• *Emphazing intuitively that there could be a shortcut*	nonexistent	nonexistent	nonexistent	nonexistent
Producing an appropriate statement for the solution				nonexistent
• *Designing a problem which corresponds with the solution*	nonexistent	nonexistent	nonexistent	nonexistent

When teacher candidates' solutions are examined, it is seen that most of the teacher candidates were able to reach generalizations for the problem situation in the first two questions. However this was different in third and fourth problems. While there was not a first grader teacher candidate who was able to generalize in the third question; some of the fourth graders were able to generalize. In the interviews, FG_3 stated:

"*…But in high school mathematics for example, while solving a polynomial question, we just apply the formula and not think about the reason behind it. I did not question this. We had to learn mathematics for the exam. I mean, we had to memorize. And we were always criticizing while preparing for the exam. When we come here, there is an imposition that we should understand the logic behind it. I mean we are always in a chaos… And also, it is easier to understand the shape but in* *third and fourth questions, you can understand the shape more easily than the verbal part. But for example (for the 3rd and 4th questions) when I read this, I should firstly understand it from all these words, I mean I should find a logic and LG_{14} it according to the logic but it is difficult compared to the first two questions. If I were to understand, at least I would have written a correlation, a function in the third question. I was used to the first two questions but I could not understand the other questions because I was not familiar.. Obviously, before I tell you how, I focus on my own sense. I think I can transfer it after I understand the story..*"

None of the teacher candidates were able to reach generalizations in the fourth question. LG_{14} comments on the reason they could not generalize and the importance of reaching generalizations as a mathematics teacher saying:

"*Now if I were someone who did not study at university, probably I would be able to solve the third question. I mean I could solve one and two, but 'probably' could solve the third. I don't think I could solve the fourth question.*
Also we have numerical data in the first three questions. We have direct, clear quantities. We donn't have it in the fourth questions. Fourth is more like algebraic. I mean, we can express with symbols. So the solution is difficult because it is a bit abstract. I mean, the way of thinking is different. For instance the reason I was able to solve the third question, is solely because I have gained a different perspective at university. Otherwise I would have just solved with sequences and leave it. But to be honest, I would not bother to think how I could get it to a simpler level. I think the teaching methods and mathematics field knowledge needs to be mixed together more. I doubt that my friends here can solve the third question. As teachers, we should be able to solve these questions and make them understandable for students."

From the statements of teacher candidates, it is understood that their past learnings and numerical representations (numbers, shapes) helped them understand the first two questions but that they found the other two questions less familiar and higher level. For this reason, they stated that they could reach generalizations in the first two questions by understanding them and could not generalize and catch "a correlation, a function" between the quantities because they had difficulty in understanding the problem. Although the teacher candidates were quite successful at reaching generalization in the first and second problem situations, very few of them noted what was given and wanted in the problem. In the first two problem situations, the teacher candidates reached generalizations via past solution approaches, taking steps based on memorization and without any processing. However, while the number of teacher candidates who were able to generalize in the third and fourth questions was fewer, teacher candidates took more notes of what was given and wanted in these problems. None of the teacher candidates took steps based on memorizations in the last two problem situations. LG_{11}, says about this situation,

"*I have always been like this. I don't wait while solving something I know. But if I am less familiar to the question, I mean if I see it for the first time, I write everything in the question. I suppose it has to do with my middle school teacher. S/he always solved it taught us like this. And now I continue to do the same thing in questions I don't know. I can even solve questions I don't think I can solve by writing. But it didn't work on the last question (laughs)*." From these statements, noting what is given and wanted is an algebraic mind habit of LG_{11} gained in the past.

Teacher candidates used solutions and statements in given problem situations in the first two questions to indicate that they understand. In this context, it is beneficial to share solutions and statements of LG_{11} and LG_{14} who used trial-and-error and counting. LG_{11} drew the shape below shown in Figure 1 for the circle problem. Additionally, s/he clearly stated the steps by saying, "*I thought there would be a LGt in the center. I thought if we wanted to draw a circle around it, it would pass from four points. In a sense, I tried. Then, I looked for the other steps too. So if we say, x is the number of circles, I wrote the general term as 4x-3*."

In addition to this, s/he told that s/he could show it on a geometry board.

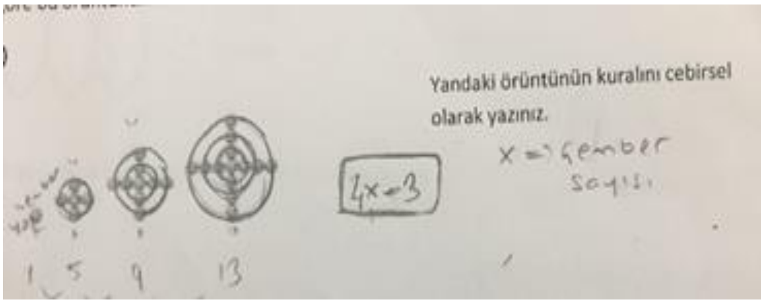

Figure1. Solution of LG_{11} regarding the Circle Problem

You can see this problem, from Figure 1, Appendixes I. The participant (LG_{14}) say x fort he number of circles. And tried to find a rule. LG_{14} who developed different ways of solution for the first problem situation said about her/his solutions

"*I did the solutions like this: I solved the first problem in four ways. First, I tried to write from the first step by breaking it down and using the amount of increase and reach a general formula by following these steps. Second, I did not break down the first step. I kept the amount of increase in the first step and wrote the second, third and fourth steps and tried to reach the general term like that. I used generally known shortcuts in the third way. I used amount of increase and the method to gain the first term, I mean the shortcut, the memorization method. In the fourth way, I tried to draw it and count it differently. I mean, I said in the fourth way, for example everyone has a different way of counting. For example some count one by one, some others may group them. I thought I would use grouping here. Like this, and I wanted to count differently. How can I count in a more practical and faster way? Which ones are the mutual blocks? For example, let's draw a hexagon like this. (pause, drawing the shape) Oh, okay let's go from the solution. For example, there is a mutual corner for two polygons. For example, one more makes two mutual corners. There can be more different solutions. For example in sequences or counting one by one like I said or according to how you break it down. My purpose here was… to show the common ones. I searched for a correlation. In fifth way, I wrote another way here I've just seen it. Here, except for the first one, because the others are common, I counted the shape I used in the fourth way and continued. I focused on shapes rather than corners*." LG_{14} was the only teacher candidate who confirmed the solution with multiple solution strategies. S/he explains it by saying "*A teacher should never be limited to one solution. Mathematics is a sea. But everyone looks at that sea from a different place. Yes, the sea we see is the same. But we have to know from which point the students looks at it. For this reason, we should produce multiple solutions…*" Solution of LG_{14} is shared in Figure2.

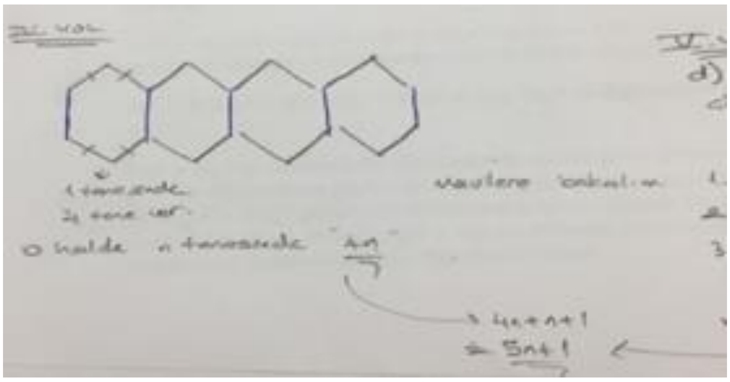

Figure2. Solution of LG_{14} regarding the Honeycomb Problem

It was found that teacher candidates preferred to show with symbols in all of the problem situations and used very little tables, shapes or gestures and mimics. You can see this problem, from Figure 2, Appendixes I. The participant (LG_{14}) paint two sides blue. Then count the black sides for n hexagon. At the end of his or her processes added the blue sides. LG_{14} drew attention to this point and said, “ *the third question, I thought the third questions was a high school question. Because there is something here…. Err… sequence. Was it alternate that always decreases at a certain amount? I can’t remember. That is here. Then I said, how can I do it without using that formula? I mean if the student does not know the sequence, how can I do it? and I said can we put it a graphic? And I drew linear functions. I increased one while decreasing the other. I thought the two would definitely cross at one point at this linear function. I tried to find that point. And err, it came directly. The result, I mean. I did it by creating familiarity. I used geometry. Actually it is a mathematical question but I used geometry. Another solution is the one with sequences that came first in my mind. I even increased one side while decreasing the other and showed that n should go infinitely for the two results to be equal to each other*.”

4. DISCUSSION and CONCLUSION

Summarizing the algebraic mind habits of the teacher candidates; it is seen that there are three main titles of gathering a function, observing under what conditions this function works and does not and lastly, reaching a generalization. This situation corresponds to the below structure developed by Driscoll (1999).

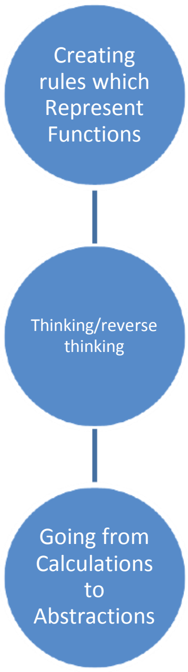

Figure 3. Functions and correlations Processing and Structures

The results showed that the teacher candidates could not make generalizations at the desired level under the title of algebraic mind habits, within the scope of thinking/reverse thinking, creating rules that represent functions and going from calculations to abstractions and in the context of the sub-qualifications of Teacher Qualifications (2008) and Developing Students’ Problem-Solving Skills and Developing Students Reasoning Skills. The teacher candidates were more successful at generalizing the problem situations with which they were familiar (Honeycomb Problem, Circle Problem). Especially the first grader teacher candidates could not display any mind habits in the last two problem situations while they took memorized steps to solve the first two questions without any processing. Teacher candidates displayed similar algebraic mind habits in similar problem situations. On the other hand, algebraic mind habits of the fourth grader teacher candidates regarding the first two questions varied. It was found that they could not generalize the problems they faced for the first time (shopping problem and number problem) and most of them got stuck on the step of thinking and could not move to stage Creating rules which Represent Functions Thinking/reverse thinking Going from Calculations to Abstractions of creating rules that represent functions. This situation; suggests that teacher candidates gain and develop algebraic mind habits in the context of problem solving throughout their university education. However, none of the teacher candidates could produce a problem that would fit the solution. From Table 2 on the algebraic habits of teacher candidates on the basis of all these expressions, it is understood from the following table that fourth grade teacher candidates are approaching a genuine thinking habit away from memorization-based steps in *reaching a generalization* which is the ultimate goal of algebra. First-rate teacher candidates wrote direct results without any action on the first two questions, but fourth grade teacher candidates have developed different solutions and tested them. It is also seen that the solutions of interviews with the first grade teacher candidates are similar, while the solutions of the fourth grade teacher candidates are diversified. In addition, according to the first two questions, it is seen that the fourth grade teacher candidates are more successful in the problems where encounter probabilities are lower. This can be explained as the reason why algebraic mind habits are diversified and enlarged in accordance with the education they receive. For this purpose, quantitative studies may be proposed to examine the existence of such a situation for larger groups.

Theme, Sub-theme and Categories	Problem 1	Problem 2	Problem 3	Problem 4
Was able to reach generalization	FG_1,..., FG_{15}, LG_1, LG_2, LG_3, LG_5, LG_6, LG_9, LG_{12}, LG_{13}, LG_{14}, LG_{15}	FG_1,..., FG_{15}, LG_1, LG_2, LG_3, LG_5, LG_6, LG_9, LG_{11}, LG_{13}, LG_{14}, LG_{15}	LG_1, LG_8, LG_{10}, LG_{11}, LG_{12}, LG_{13}, LG_{14}	Nonexistent
Was not able to reach generalization	LG_4, LG_7, LG_8, LG_{10}, LG_{11}	LG_4, LG_7, LG_8, LG_{10}, LG_{12}	FG_1,…, FG_{11}, FG_{13}, FG_{14}, FG_{15}, LG_2, LG_3, LG_4, LG_5, LG_6, LG_7, LG_9, LG_{15}	FG_1,FG_2, FG_3, FG_4, FG_5, FG_6, FG_7, FG_8, FG_9, FG_{10}, FG_{11}, FG_{12}, FG_{13}, FG_{14}, FG_{15}, LG_1, LG_2, LG_3, LG_4, LG_5, LG_6, LG_7, LG_8, LG_9, LG_{10}, LG_{11}, LG_{12}, LG_{13}, LG_{14}, LG_{15}

The teacher candidates made solutions based on memorizations without writing what is given and wanted; however they clearly wrote what is given and wanted in the last two problems. While this seems to be a form of rules that represent direct functions in the questions seen as exercises; it causes them to use the thinking / reverse thinking step more actively when they are perceived as problems. This situation brings forward the issue of examining their existing algebraic mind habits for different problems. For this reason, making participants deal with a various amount of problems may be suggested for future research on determining algebraic mind habits.

It was also seen that one of the teacher candidates said that he or she had wrote the given and the asked all problem situations. He or she also siad that this stuation could be for his or her midlle school teacher. This situation can be covered in more detail in the framework of algebraic mind habits.

In addition, students who have been studied many times in the literature can examine algebraic operations and interpretations within the framework of algebraic mind habits(Çelik & Güneş, 2013, Geller & Chard, 2011, Gökkurt, Şahin & Soylu, 2016, Yıldız, Çiftçi, Akar & Sezer, 2015).

Performance indicators of examining the accuracy of the solutions in problem situation and developing different problem solving strategies that are a part of Teacher Qualifications (2008) are expected to be improved in the teacher candidates. Only one of the fourth grader teacher candidate could evaluate regarding confirming the solution which is under the title of algebraic mind habits. None of the other teacher candidates could develop a different strategy or way of confirming the solution. Additionally, some of the fourth grader teacher candidates felt the lack of this situation but only one of them could produce different strategies. This situation stands as an obstacle to be overcome for the candidates who are going to be teachers in the future.

At the interviews, it is seen that the fourth grade teacher candidates are more detailed about the construction on their stunets' knowledge than the first grade teacher candidates and that the first grade only focuses on solving. This leads to the conclusion that teacher candidates are aware of the importance of their components and their components in the development of their students even though they are not under the name of algebraic mind habits during the training they receive during the teacher training program.

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6. APPENDIX

Appendix1. Algebraic Habits of Mind Worksheet

Name- Surname:

Grade:

Phone Number:

Algebraic Habits of Mind Worksheet (10/04/2017)

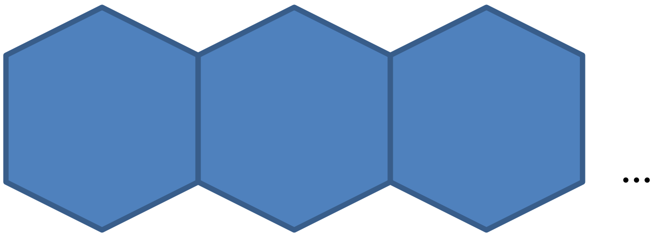
What are your solution strategies for the problem situations given below?

How would you comment on the possible solutions in the context of mathematics education?

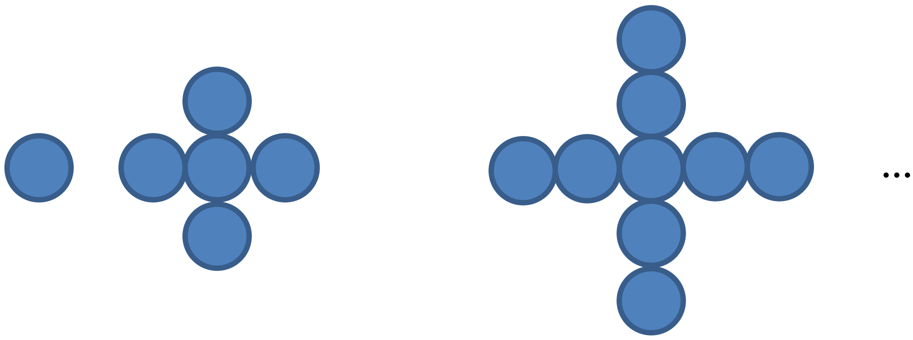
Create a new problem situation from your solutions.

Question1 (Honeycomb Problem)

Omer wants to create a honeycomb model made of hexagons by using sticks. You see on the figure below how many sticks are needed for each honeycomb. Write the rule of this pattern algebraically.



Question2. (Circle Problem)



Write the rule of the pattern above algebraically.

Question3. (Shopping Problem)

Person A wants to sell a product to person B for 100 kuruş. Person B says s/he will only give 75 kuruş for this product. At the end of the bargains, person A goes down to 75 kuruş and person B goes up to 62,5 kuruş. While the bargain continues, both persons give the average of the number they last say. Write the algebraic rule of this situation.

Question4 (Number Problem)

Write algebraically that the multiplication of two numbers that can be written as the total of two perfect squares can be written as the total of two perfect squares.

Appendix2. Interview Form

Interview Form

This interview is going to be carried out in accordance with your answers on the "Algebraic Habits of Mind Worksheet." I would like to record this interview which will be carried out with this purpose. Do you confirm?

1) Can you explain your process of solving the problem?
2) Is there a general strategy you developed in solving this problem? Did you learn this strategy somewhere or develop it yourself? (Why did you use this strategy?)
3) Can you evaluate the impact of your education life on the strategies you prefer in solving problems?
4) Could you evaluate the possible impact of these habits in your teaching life? Please evaluate this situation for your students.

Thank you.