Communications in Advanced Mathematical Sciences
Vol. I, No. 1, 35-38, 2018

Notes on UP-ideals in UP-algebras

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Abstract
The concept of UP-algebras was introduced and analyzed in 2017 by A. Iampan. In his article, he introduced the concept of UP-ideals in such algebras. In this article, we show that this concept can be determined in some other way than it was done in Iampan's article. In addition, we are more profoundly analyzing UP-ideals in UP-algebras and establish some of their additional properties.

Keywords: UP-algebra, UP-ideal, UP-homomorphism
2010 AMS: Primary 03G25

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Received: 3 July 2018, **Accepted:** 17 September 2018, **Available online:** 30 September 2018

1. Introduction

The basic concepts of UP-algebra are taken from the text [2]. The author in his article has introduced and analyzed the concepts of UP-algebra, UP-subalgebra and UP-ideals. In the article [9] the authors introduced the concept of UP-filter in UP algebras. This latter concept has a non-standard attitude towards the concept of UP-ideals. In our recently published article [7] and in the forthcoming article [8] we introduce and analyze the concept of proper UP-filters in UP-algebra. A number of authors investigated the reflections of the UP-substructures in UP-algebras within the fuzzy environment (See articles [1, 4, 5, 10, 9]). In the article [1], the authors determined the concept of a strong UP ideas. Since according to Theorem 2.1 in [1], a subset J in a UP-algebra A is a strong UP-ideal in A if and only if $J = A$, it seems to us that this concept will not be of interest in the further researching of properties of UP-algebras.

In this article, we are more profoundly analyze UP-ideals in UP-algebras and establish some of their additional properties. We show that the concept of UP-ideals in a UP-algebra can be determined in some other way than it was done in Iampan's article. Finally, we have shown the theorem that we can look at as the Second theorem on isomorphisms between UP-algebras.

2. Preliminaries

First, let us recall the definition of UP-algebra.

Definition 2.1 ([2], Definition 1.3)**.** *An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a* UP- algebra *if it satisfies the following axioms:*

(UP - 1): $(\forall x, y, z \in A)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,
(UP - 2): $(\forall x \in A)(0 \cdot x = x)$,
(UP - 3): $(\forall x \in A)(x \cdot 0 = 0)$,
(UP - 4): $(\forall x, y \in A)((x \cdot y = 0 \wedge y \cdot x = 0) \Longrightarrow x = y)$.

On a UP-algebra $(A,\cdot,0)$, we define a binary relation $'\leqslant'$ on A as follows:

$$(\forall x,y\in A)(x\leqslant y \iff x\cdot y=0).$$

Second, in the following we give definition of the concept of UP-ideals of UP-algebra.

Definition 2.2 ([2], Definition 2.1)**.** *Let A be a UP-algebra. A subset J of A is called a* UP-ideal *of A if it satisfies the following properties:*
(1) $0\in J$,
(2) $(\forall x,y,z\in A)((x\cdot(y\cdot z)\in J\wedge y\in J)\Longrightarrow x\cdot z\in J)$.

Third, let's remind ourselves of UP-homomorphisms between UP-algebras.

Definition 2.3 ([2], Definition 4.1)**.** *A mapping $f:A\longrightarrow B$ between two UP-algebras is a UP-homomorphism if the following holds*

$$(\forall x,y\in A)(f(x\cdot y)=f(x)\cdot f(y)).$$

3. Main results

For this talk, the recognizable feature of the UP-ideal is given in the following theorem. Although the statements (3) and (4) of this theorem are proved in the text [2], we show extraordinarily simpler evidence of these claims here than was done in Iampan's article.

Theorem 3.1. *Let A be a UP-algebra and J a UP-ideal of A. Then*
(3) $(\forall x,y\in A)((x\cdot y\in J\wedge x\in J)\Longrightarrow y\in J)$,
(4) $(\forall x,y\in A)(y\in J\Longrightarrow x\cdot y\in J)$.

Proof. If we put $x=0$, $y=x$ and $z=y$ in (2) we got (3) taking into account (UP - 2).
If we put $z=y$ in (2) we have $(x\cdot(y\cdot y)\in J\wedge y\in J)\Longrightarrow x\cdot y\in J$. Thus $0=x\cdot 0\in J$ and $y\in J$ implies $x\cdot y\in J$. So, the property (4) is proven taking into account (1) of Theorem 1.7 in [2], (1) and (UP - 3). □

The following property of UP-ideal follows immediately from (3):

Corollary 3.2. *Let J be a UP-ideal of UP-algebra A. Then*
(5) $(\forall x,y\in A)((x\leqslant y\wedge x\in J)\Longrightarrow y\in J)$.

Let us show (3) $\wedge$ (4) implies (1) $\wedge$ (2) if we assume $J\neq\emptyset$.

Theorem 3.3. *Let J be a non empty subset of UP-algebra for which the formulas* (3) *and* (4) *are valid. Then J is a UP-ideal in A.*

Proof. Let us suppose that for a nonempty subset J in A formulas (3) and (4) are valid.
Since the set J is nonempty, there exists at least one element y in J. If we put $x=y$ in (4), we get $y\cdot y\in J$. Then $0\in J$ considering the statement (1) of the Theorem 1.7 in [2].
Let $x,y,z\in A$ be arbitrary elements such that $x\cdot(y\cdot z)\in J$ and $y\in J$. From $y\in J$ follows $x\cdot y\in J$ by (4). On the other hand, (UP-1) gives us $y\cdot z\leqslant(x\cdot y)\cdot(x\cdot z)$. If we assume that $\neg(yz\in J)$, we would have $(\neg(yz\in J)\wedge x\in J)\Longrightarrow\neg(x\cdot(y\cdot z)\in J)$ by the contraposition of (3). This is in a contradiction with the first hypothesis. Therefore, it must be $yz\in J$. Thus, from $y\cdot z\in J$ and $y\cdot z\leqslant(x\cdot y)\cdot(x\cdot z)$, we get $(x\cdot y)\cdot(x\cdot z)\in J$ by (5). Now, from last and $z\cdot y\in J$ we have $x\cdot z\in J$ by (3). So, (2) is proven. □

In addition to the previous criterion, we have the following possibility to check whether to nonempty subset J of a UP-algebra A is a UP-ideal in A or not.

Theorem 3.4. *A subset J of a UP-algebra A such that $0\in J$ is a UP-ideal in A if and only if the following holds*
(6) $(\forall x,y,z\in A)((\neg(x\cdot z\in J)\wedge y\in J)\Longrightarrow\neg(x\cdot(y\cdot z)\in J))$.

Proof. Let J be a UP-ideal in UP-algebra A. Suppose $\neg(x\cdot z\in J)$ and $y\in J$ hold. If $x\cdot(y\cdot z)\in J$, we would have $x\cdot z\in J$, which is contradictory to the first hypothesis. So it has to be $\neg(x\cdot(y\cdot z)\in J)$. Therefore, (6) is proven.
Opposite, let (6) be holds. Suppose that hypothesis in the formula (2) are valid. If it were $\neg(x\cdot z\in J)$, then it would have $\neg(x\cdot(y\cdot z)\in J$ by (6) in contradiction with $x\cdot(y\cdot z)\in J$. So it has to be $x\cdot z\in J$. Therefore, (3) is proven. □

Corollary 3.5. *Let J be a UP-ideal in a UP-algebra A. Then*
(7) $(\forall x, y \in A)((x \in J \wedge \neg(y \in J) \implies \neg(x \cdot y \in J))$.

Proof. Is we put $x = 0$, $y = x$ and $z = y$ in (6) we obtain (7). $\square$

Theorem 3.6. *A subset J of a UP-algebra A such that $0 \in J$ is a UP-ideal in A if and only if the following holds*
(8) $(\forall x, y, z \in A)((\neg(x \cdot z \in J) \wedge x \cdot (y \cdot z) \in J) \implies \neg(y \in J))$.

Proof. Let J be a UP-ideal in UP-algebra A. Suppose $\neg(x \cdot z \in J)$ and $x \cdot (y \cdot z) \in J$ hold. If $y \in J$, we would have $x \cdot z \in J$, which is contradictory to the first hypothesis. So it has to be $\neg(y \in J)$. Therefore, (8) is proven.

Opposite, let (8) be holds. Suppose that hypothesis in the formula (2) are valid. If it were $\neg(x \cdot z \in J)$, then it would have $\neg(y \in J)$ by (8) in contradiction with $y \in J$. So it has to be $x \cdot z \in J$. Therefore, (3) is proven. $\square$

Corollary 3.7. *Let J be a UP-ideal in a UP-algebra A. Then*
(9) $(\forall x, y \in A)((x \cdot y \in J \wedge \neg(y \in J) \implies \neg(x \in J))$.

Proof. Is we put $x = 0$, $y = x$ and $z = y$ in (8) we obtain (9). $\square$

One part of the following theorem is proved in [2] (Theorem 2.6). We repeat this proof as it has some useful consequences.

Theorem 3.8. *The family $\mathfrak{J}_A$ of all UP-ideals in a UP-algebra A forms the completely lattice.*

Proof. Let $\{J_i\}_{i \in I}$ be a family of UP-ideals in a UP-algebra A.

(a) Obviously, the following is true $0 \in \bigcup_{i \in I} J_i$ and $0 \in \bigcap_{i \in I} J_i$.

(b) Let $x, y, z \in A$ arbitrary elements such that $x \cdot (y \cdot z) \in \bigcap_{i \in I} J_i$ and $y \in \bigcap_{i \in I} J_i$. Thus $x \cdot (y \cdot z) \in J_i$ and $y \in J_i$ for any $i \in I$. Then $x \cdot z \in J_i$ since J_i is a UP-ideal in A. Therefore, $x \cdot z \in \bigcap_{i \in I} J_i$. So, $\bigcap_{i \in I} J_i$ is a UP-ideal in A.

(c) Let $\mathfrak{X}$ be the family of all UP-filters of UP-algebra A contained the union $\bigcup_{i \in I} J_i$. The $\bigcap \mathfrak{X}$ is a UP-ideal in A by the first part of this proof.

(d) If we put $\sqcap_{i \in I} J_i = \bigcap_{i \in I} J_i$ and $\sqcup_{i \in I} J_i = \bigcap \mathfrak{X}$, then $(\mathfrak{J}, \sqcap, \sqcup)$ is a completely lattice. $\square$

Corollary 3.9. *Let X be an arbitrary subset of UP-algebra A. Then there is the minimal UP-ideal $\langle X \rangle$ containing the set X. Specifically, for every element x in A there is the minimal UP-ideal in A that contains x.*

Proof. Let $F = \{J : J \text{ is a UP-ideal contains} X\}$. Then $\bigcap F$ is the minimal UP-ideal in A contains the subset X. Specifically, for $X = \{x\}$ we get the second part of this claim. $\square$

Let $f : A \longrightarrow B$ be a UP-homomorphism between two UP-algebras. In [2] it has been shown (Theorem 4.5) that $Ker f$ is an UP-ideal and that $f(A)$ is an UP-subalgebra of algebra A. Without major difficulties, it can be proved that if J is an UP-ideal in UP-algebra A and $' \sim '$ the congruence on A determined by the ideal J ([2], Proposition 3.5), then $A/J \equiv A/\sim \; = \; \{[x]_J : x \in A\}$ is also UP-algebra with the internal operation $' \cdot '$ defined by

$$(\forall x, y \in A)([x]_J \cdot [y]_J = [x \cdot y]_J)$$

and the fixed element J. The following claims is proven by direct verification. Furthermore, without major difficulties, it can be shown ([8], Theorem 3.5) that if $f : A \longrightarrow B$ is a UP-homomorphism between UP-algebras, then there is an UP-isomorphism $g : A/Ker f \longrightarrow f(A)$ such that $f = g \circ \pi$, where π is the natural UP-epimorphism. We can look at this as the First theorem on isomorphisms between UP-algebras. In addition, if J and K are UP-ideals in UP-algebra A such that J is contained in K, then K/J is a UP-ideal in UP-algebra A/J ([8], Theorem 3.6). For more details, see articles [3, 6].

Now we can express the following theorem, so called the second isomorphism theorem. The proof of this theorem differs significantly from the proof of the analogous theorem in the article [6].

Theorem 3.10. *Let J and K be UP-ideals of a UP-algebra A such that $J \subseteq K$. Then K/J is a UP-ideal of UP-algebra A/J and the following holds*

$$(A/J)/(K/J) \cong A/K.$$

Proof. Since $K/J = \{[x]_J : x \in K\}$ is a UP-ideal in UP-algebra $A/J = \{[x]_J : x \in A\}$, then the factor-set $(A/J)/(K/J) = \{[[x]_J]_{K/J}] : [x]_J \in A/J\}$ can be correctly determined as the factor-algebra of the UP-algebra A/J by the UP-ideal K/J. If we define mapping $\varphi : (A/J)/(K/J) \longrightarrow A/K$ by the following way $\varphi([[x]_J]_{K/J} = [x]_K$ without major difficulties we can verify that this mapping is a UP isomorphism. Since it is obvious that φ is a UP-endomorphism, it is sufficient to check that φ is a UP-monomorphism. Let $[x]_K = \varphi([[x]_J]_{K/J})$ and $[y]_K = \varphi([[y]_J]_{K/J})$ be arbitrary elements of A/K such that $[x]_K = [y]_K$. This means $x \cdot y \in K$ and $y \cdot x \in K$. Thus $[x \cdot y]_J \in K/J$ and $[y \cdot x]_J \in K/J$. Then $[x]_J \cdot [y]_J \in K/J$ and $[y]_J \cdot [x]_J \in K/J$. Last equality means $[[x]_J]_{K/J} = [[y]_J]_{K/J}$. So, the UP-homomorphism φ is a UP-monomorphism. $\square$

4. Conclusion

In this article analyzing the axioms by which the concept of UP-ideals in UP-algebras was determined we offered some possibilities for introduction of UP-ideals on different ways. This analysis enables us to gain a more serious insight into the properties of ideals in these algebraic. Finally, we have shown the Theorem that we can look at as the Second theorem on isomorphisms between UP-algebras.

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