

A Novel Multi-Criteria Decision Making Method based on The Ranking Values of Interval Type-2 Fuzzy Sets: An Application of a Manager Selection for a Telecommunication Company

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Abstract: Type-2 fuzzy sets (T2FSs), characterized by a fuzzy membership function, are much useful tool for representing the decision knowledge in the decision making process. Interval Type-2 fuzzy sets (IT2FSs) are the most commonly used T2FSs. In this study, a method based upon ranking values of IT2FSs is used to tackle multi-criteria decision making (MCDM) problems. First, some basic concepts and arithmetic operations for IT2FSs are presented. Then, three kinds of fuzzy ranking methods, proposed by [1], based on arithmetic average (AA), geometric average (GA) and harmonic average (HA) operators to compute the ranking values of IT2FSs are applied. Finally, the outcomes of MCDM methods based on the ranking values of IT2FSs are obtained and also compared with the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method based on Type-1 fuzzy sets (T1FSs) for a numerical example.

Aralık Tip-2 Bulanık Kümelerin Sıralanmasına dayalı Yeni Bir Çok Kriterli Karar Verme Yöntemi: Bir Telekomünikasyon Şirketi için Yönetici Seçimi Uygulaması

Anahtar Kelimeler

Çok kriterli karar verme,
Aralık Tip-2 bulanık kümeler,
Bulanık sıralama yöntemi

Özet: Bulanık üyelik fonksiyonuna sahip olan Tip-2 bulanık kümeler, karar verme aşamasında karar matrisinin belirlenmesinde çok kullanışlı bir araçtır. Aralık Tip-2 bulanık kümeler, en yaygın olarak kullanılan Tip-2 bulanık kümelerdir. Bu çalışmada, çok kriterli karar verme (ÇKKV) problemlerini çözmek için aralık Tip-2 bulanık kümelerin sıralanmasına dayalı bir yöntem kullanıldı. İlk olarak, aralık Tip-2 bulanık kümeler için bazı temel kavramlar ve aritmetik işlemleri tanıtıldı. Daha sonra, aralık Tip-2 bulanık kümelerin sıralama değerlerini hesaplamak için [1] tarafından önerilen, aritmetik ortalama (AO), geometrik ortalama (GO) ve harmonik ortalama (HO) işlemlerine dayalı 3 çeşit bulanık sıralama yöntemi uygulandı. Son olarak, aralık Tip-2 bulanık kümelerin sıralanmasına dayalı ÇKKV yönteminin sonuçları elde edildi ve sayısal bir örnek ile Tip-1 bulanık kümelere dayalı ideal çözüme yakınlığa göre tercihlerin sıralanması (TOPSIS) yöntemi ile karşılaştırıldı.

1. Introduction

The object of multi-criteria decision-making (MCDM) is to select a top option from a group of alternatives by evaluating multiple criteria. So far, many approaches have been presented for MCDM problems with Type-1 fuzzy sets (T1FSs). For instance, [2] introduced a method to handle MCDM problems utilizing the similarity measure of fuzzy sets. [3] presented an extension of the TOPSIS method for group decision making under fuzzy environment. [4] proposed a method to handle MCDM problems adapted from the fuzzy preference information. [5]

introduced a method for modelling the notion of risk in human decision processes. [6] proposed an application of TOPSIS with assessment aircraft training in a fuzzy setting. [7] introduced a fuzzy optimization method for MCDM. [8] proposed an approach for MCDM accompany incomplete information. [9] presented a dynamic MCDM model by using the grey number evaluations.

However, fuzzy MCDM method based upon T1FSs, proposed by [10], are unsuccessful to deal with high complexity and vagueness. For that reason, Type-2 fuzzy sets (T2FSS), which can be considered as an

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extension of T1FSs, were presented by [11]. Recently, some methods using the T2FSs have been proposed for MCDM problem. For example, [12] proposed a new method for fuzzy multiple criteria group decision making (FMCGDM) based upon the arithmetic operations of Interval Type-2 fuzzy sets (IT2FSs). [13] proposed a method for FMCGDM by utilizing ranking values and the arithmetic operations of IT2FSs. [14] presented a FMCGDM by using the IT2 TOPSIS method. [15] presented a FMCGDM based on ranking IT2FSs. [4] presented multi-attribute group decision making models for IT2F environment. [1] introduced a new method to deal MCDM problems with merged ranking values in IT2F environment. [16] introduced a merged method of Analytical Hierarchy Process and TOPSIS method based on IT2FSs.

In this study, we use a method based on ranking values of IT2FSs to handle MCDM problems. In Section 2, we introduce some basic concepts and arithmetic operations for IT2FSs and three kinds of fuzzy ranking methods proposed by Qin and Liu (2015) for calculating the ranking values of IT2FSs with operators of arithmetic average (AA), geometric average (GA) and harmonic average (HA) and introduce MCDM method based on the ranking values of IT2FSs. In Section 3, we obtained the results of MCDM method based upon the ranking values of IT2FSs and also compared with fuzzy MCDM method rest on T1FSs for a numerical example and conclusions are given in Section 4.

2. Material and Method

In this section, firstly some basic definitions of T1FSs and T2FSs and arithmetic operations and ranking values for Trapezoidal IT2FSs are briefly given. After, MCDM method based on the ranking values of IT2FSs is introduced.

Definition 2.1. [10] A fuzzy set Z^0 in the universe of discourse X is represented by a membership function $\mu_{Z^0}(x) \in [0,1]$. The value of $\mu_{Z^0}(x)$ is called degree of membership of $x \in Z^0$.

Definition 2.2. [17]. A Type-2 fuzzy set Z^0 in the universe of discourse Z which can be characterized by a Type-2 membership function μ_{Z^0} is shown as,

$$Z^0 = \{((z, u), \mu_{Z^0}(z, u)) \mid \forall z \in Z, \forall u \in J_z \subseteq [0,1], 0 \leq \mu_{Z^0}(z, u) \leq 1\}$$

where $J_z \subseteq [0,1]$. Furthermore, the T2FS Z^0 can be shown as

$$Z^0 = \int_{z \in Z} \int_{u \in J_z} \mu_{Z^0}(z, u) / (z, u)$$

where $J_z \subseteq [0,1]$ and $\int \int$ symbolizes union over all acceptable z and u .

Definition 2.3. [17]. Let Z^0 be a T2FS in the universe of discourse Z characterized by the Type-2 membership function μ_{Z^0} . If all $\mu_{Z^0}(z, u) = 1$, then Z^0 is termed an Interval Type-2 fuzzy set. An IT2FS Z^0 can be accepted as a specific case of T2FS and represented as follows:

$$Z^0 = \int_{z \in Z} \int_{u \in J_z} 1 / (z, u)$$

where $J_z \subseteq [0,1]$.

Definition 2.4. [17]. The upper and lower membership functions of IT2FSs are Type-1 membership functions. A trapezoidal IT2FS is illustrated as

$$Z_i^0 = (Z_i^U, Z_i^L) = ((z_{i1}^U, z_{i2}^U, z_{i3}^U, z_{i4}^U; H_1(Z_i^U), H_2(Z_i^U)), (z_{i1}^L, z_{i2}^L, z_{i3}^L, z_{i4}^L; H_1(Z_i^L), H_2(Z_i^L)))$$

where Z_i^U and Z_i^L are T1FSs, $z_{i1}^U, z_{i2}^U, z_{i3}^U, z_{i4}^U, z_{i1}^L, z_{i2}^L, z_{i3}^L, z_{i4}^L$ and z_{i1}^L are the points of references of the IT2FS Z_i^0 , $H_j(Z_i^U)$ symbolised the membership degree of the element $z_{j(j+1)}^U$ in the upper trapezoidal membership function (Z_i^U) , $1 \leq j \leq 2$. $H_j(Z_i^L)$ symbolised the membership degree of the element $z_{j(j+1)}^L$ in the lower trapezoidal membership function (Z_i^L) , $1 \leq j \leq 2$, $H_1(Z_i^U) \in [0,1]$, $H_2(Z_i^U) \in [0,1]$, $H_1(Z_i^L) \in [0,1]$, $H_2(Z_i^L) \in [0,1]$ and $1 \leq i \leq n$.

2.1. Arithmetic operations on trapezoidal interval type-2 fuzzy sets

In this section, arithmetic operations on Trapezoidal IT2FSs are given [8].

Let be trapezoidal IT2FSs

$$Z_1^0 = (Z_1^U, Z_1^L) = ((z_{11}^U, z_{12}^U, z_{13}^U, z_{14}^U; H_1(Z_1^U), H_2(Z_1^U)), (z_{11}^L, z_{12}^L, z_{13}^L, z_{14}^L; H_1(Z_1^L), H_2(Z_1^L)))$$

and

$$\begin{aligned} \mathbb{Z}_2^{\otimes} = (\mathbb{Z}_2^U, \mathbb{Z}_2^L) = & \left((z_{21}^U, z_{22}^U, z_{23}^U, z_{24}^U; H_1(\mathbb{Z}_2^U), H_2(\mathbb{Z}_2^U)), \right. \\ & \left. (z_{21}^L, z_{22}^L, z_{23}^L, z_{24}^L; H_1(\mathbb{Z}_2^L), H_2(\mathbb{Z}_2^L)) \right) \end{aligned}$$

$$R_{(3)}(\mathbb{Z}) = \left(\frac{2z_{11}^U z_{14}^U}{z_{11}^U + z_{14}^U} + \frac{4}{\sum_{k=L}^U \frac{H_1(\mathbb{Z}_1^k) + H_2(\mathbb{Z}_1^k)}{H_1(\mathbb{Z}_1^k) H_2(\mathbb{Z}_1^k)}} \right) \times \frac{8}{\sum_{i=1}^4 \left(\frac{1}{z_{1i}^U} + \frac{1}{z_{1i}^L} \right)} \quad (3)$$

Addition:

$$\begin{aligned} \mathbb{Z}_1^{\otimes} \oplus \mathbb{Z}_2^{\otimes} = & (\mathbb{Z}_1^U, \mathbb{Z}_1^L) \oplus (\mathbb{Z}_2^U, \mathbb{Z}_2^L) \\ = & \left((z_{11}^U + z_{21}^U, z_{12}^U + z_{22}^U, z_{13}^U + z_{23}^U, z_{14}^U + z_{24}^U; \right. \\ & \min(H_1(\mathbb{Z}_1^U); H_1(\mathbb{Z}_2^U)), \min(H_2(\mathbb{Z}_1^U); H_2(\mathbb{Z}_2^U))), \\ & (z_{11}^L + z_{21}^L, z_{12}^L + z_{22}^L, z_{13}^L + z_{23}^L, z_{14}^L + z_{24}^L; \\ & \left. \min(H_1(\mathbb{Z}_1^L); H_1(\mathbb{Z}_2^L)), \min(H_2(\mathbb{Z}_1^L); H_2(\mathbb{Z}_2^L))) \right) \end{aligned}$$

Subtraction:

$$\begin{aligned} \mathbb{Z}_1^{\otimes} - \mathbb{Z}_2^{\otimes} = & (\mathbb{Z}_1^U, \mathbb{Z}_1^L) - (\mathbb{Z}_2^U, \mathbb{Z}_2^L) \\ = & \left((z_{11}^U - z_{21}^U, z_{12}^U - z_{22}^U, z_{13}^U - z_{23}^U, z_{14}^U - z_{24}^U; \right. \\ & \min(H_1(\mathbb{Z}_1^U); H_1(\mathbb{Z}_2^U)), \min(H_2(\mathbb{Z}_1^U); H_2(\mathbb{Z}_2^U))), \\ & (z_{11}^L - z_{21}^L, z_{12}^L - z_{22}^L, z_{13}^L - z_{23}^L, z_{14}^L - z_{24}^L; \\ & \left. \min(H_1(\mathbb{Z}_1^L); H_1(\mathbb{Z}_2^L)), \min(H_2(\mathbb{Z}_1^L); H_2(\mathbb{Z}_2^L))) \right) \end{aligned}$$

Multiplication:

$$\begin{aligned} \mathbb{Z}_1^{\otimes} \otimes \mathbb{Z}_2^{\otimes} = & (\mathbb{Z}_1^U, \mathbb{Z}_1^L) \otimes (\mathbb{Z}_2^U, \mathbb{Z}_2^L) \\ = & \left((z_{11}^U \times z_{21}^U, z_{12}^U \times z_{22}^U, z_{13}^U \times z_{23}^U, z_{14}^U \times z_{24}^U; \right. \\ & \min(H_1(\mathbb{Z}_1^U); H_1(\mathbb{Z}_2^U)), \min(H_2(\mathbb{Z}_1^U); H_2(\mathbb{Z}_2^U))), \\ & (z_{11}^L \times z_{21}^L, z_{12}^L \times z_{22}^L, z_{13}^L \times z_{23}^L, z_{14}^L \times z_{24}^L; \\ & \left. \min(H_1(\mathbb{Z}_1^L); H_1(\mathbb{Z}_2^L)), \min(H_2(\mathbb{Z}_1^L); H_2(\mathbb{Z}_2^L))) \right) \end{aligned}$$

Multiplication with Crisp Scalar k:

$$\begin{aligned} k \times \mathbb{Z}_1^{\otimes} = & \left((k \times z_{11}^U, k \times z_{12}^U, k \times z_{13}^U, k \times z_{14}^U; H_1(\mathbb{Z}_1^U), H_2(\mathbb{Z}_1^U)), \right. \\ & \left. (k \times z_{11}^L, k \times z_{12}^L, k \times z_{13}^L, k \times z_{14}^L; H_1(\mathbb{Z}_1^L), H_2(\mathbb{Z}_1^L)) \right) \end{aligned}$$

2.2. The ranking values of trapezoidal IT2FSs

Let \mathbb{Z}^{\otimes} be an IT2FSs, then three kinds of ranking values of \mathbb{Z}^{\otimes} proposed by [1] are identified as follows:

$$R_{(1)}(\mathbb{Z}) = \left(\frac{z_{11}^U + z_{14}^U}{2} + \frac{\sum_{k=L}^U (H_1(\mathbb{Z}_1^k) + H_2(\mathbb{Z}_1^k))}{4} \right) \times \frac{\sum_{i=1}^4 z_{1i}^U + z_{1i}^L}{8} \quad (1)$$

$$R_{(2)}(\mathbb{Z}) = \left(\sqrt{z_{11}^U + z_{14}^U} + \left(\prod_{k=L}^U H_1(\mathbb{Z}_1^k) H_2(\mathbb{Z}_1^k) \right)^{\frac{1}{4}} \right) \times \sqrt[8]{\prod_{i=1}^4 z_{1i}^U z_{1i}^L} \quad (2)$$

where; $R_{(1)}(\mathbb{Z}^{\otimes})$, $R_{(2)}(\mathbb{Z}^{\otimes})$ and $R_{(3)}(\mathbb{Z}^{\otimes})$ are the arithmetic average (AA) ranking value, the geometric average (GA) ranking value and the harmonic average (HA) ranking value of IT2FSs \mathbb{Z}^{\otimes} , respectively.

2.3. Multi-criteria decision making method based on the trapezoidal interval type-2 fuzzy sets

Let $D = \{D_1, D_2, \dots, D_l\}$ be the set of Decision Makers (DMs), $A = \{A_1, A_2, \dots, A_m\}$ be set of alternatives and $C = \{C_1, C_2, \dots, C_n\}$ be set of criteria. Assume that $\mathbb{Z}_{ijk}^{\otimes} = ((z_{ijk1}^U, z_{ijk2}^U, z_{ijk3}^U, z_{ijk4}^U; H_{ijk1}(\mathbb{Z}_{ijk}^U), H_{ijk2}(\mathbb{Z}_{ijk}^U)), (z_{ijk1}^L, z_{ijk2}^L, z_{ijk3}^L, z_{ijk4}^L; H_{ijk1}(\mathbb{Z}_{ijk}^L), H_{ijk2}(\mathbb{Z}_{ijk}^L)))$ is the criteria value given by the DM D_k (where $\mathbb{Z}_{ijk}^{\otimes}$ is a trapezoidal IT2 fuzzy number for the alternative A_i with regard to the criteria C_j) and $w_{jk} = ((w_{jk1}^U, w_{jk2}^U, w_{jk3}^U, w_{jk4}^U; H_{jk1}(\mathbb{Z}_{jk}^U), H_{jk2}(\mathbb{Z}_{jk}^U)), (w_{jk1}^L, w_{jk2}^L, w_{jk3}^L, w_{jk4}^L; H_{jk1}(\mathbb{Z}_{jk}^L), H_{jk2}(\mathbb{Z}_{jk}^L)))$ be the criteria weight given by the k th decision maker, (where $\mathbb{Z}_{jk}^{\otimes}$ is also a trapezoidal IT2 fuzzy number). We utilize assessment of the alternatives and criteria, and Eqs.(1)-(3) to obtain ranking values of alternatives.

3. Results

We use the data from [18] and the linguistic values from [15] to illustrate the MCDM method by using the ranking values of IT2FSs. Assume that a Telecommunication Company wants to select a manager from four candidates called as A_1, A_2, A_3, A_4 for R&D department. The decision makers DM_1, DM_2, DM_3 evaluate the four relevant candidates according to five criteria that are Proficiency in identifying research areas (C_1), Proficiency in administration (C_2), Personality (C_3), Past experience (C_4) and Self-confidence (C_5).

The hierarchical structure of this decision problem is shown as in Fig. 1. The MCDM method based on the ranking values of IT2FSs is utilized to handle this problem and the method is given as follows.

Step 1: The decision makers evaluate the criteria by using the linguistic terms and IT2FSs given in [15] and presented in Table 1.

Step 2: The decision makers evaluate alternatives according to each criterion by using the linguistic terms and IT2FSs in [15] and given in Table 2.

Step 3: Construct the weighted assessment value $\tilde{\mathcal{R}}_{ij}^k$ of the alternative i with respect to the criteria j evaluated by the k th DM, which is calculated by multiplying the IT2FSs corresponding to the linguistic assessments of the alternatives and criteria made by the DMs. The weighted assessment values are given in Table 3. For example, weighted assessment values ($\tilde{\mathcal{R}}_{11}^k, k = 1, 2, 3$) of the Alternative 1 (A_1) with respect to the Criterion 1 (C_1) for each DM are calculated by

$$\begin{aligned} \tilde{\mathcal{R}}_{11}^1 &= VG \otimes VO \\ &= ((0.9, 1, 1, 1; 1, 1), (0.9, 1, 1, 1; 1, 1)) \otimes \\ &\quad ((9, 10, 10, 10; 1, 1), (9, 10, 10, 10; 1, 1)) \\ &= ((8.1, 10, 10, 10; 1, 1), (8.10, 10, 10, 10; 1, 1)) \end{aligned}$$

$$\begin{aligned} \tilde{\mathcal{R}}_{11}^2 &= VG \otimes O \\ &= ((0.9, 1, 1, 1; 1, 1), (0.9, 1, 1, 1; 1, 1)) \otimes \\ &\quad ((7, 9, 9, 10; 1, 1), (7, 9, 9, 10; 1, 1)) \\ &= ((6.30, 9, 9, 10; 1, 1), (6.30, 9, 9, 10; 1, 1)) \end{aligned}$$

$$\begin{aligned} \tilde{\mathcal{R}}_{11}^3 &= VG \otimes MO \\ &= ((0.9, 1, 1, 1; 1, 1), (0.9, 1, 1, 1; 1, 1)) \otimes \\ &\quad ((5, 7, 7, 9; 1, 1), (5, 7, 7, 9; 1, 1)) \\ &= ((4.50, 7, 7, 9; 1, 1), (4.50, 7, 7, 9; 1, 1)) \end{aligned}$$

For example, average assessment value of the Alternative 1 (A_1) in terms of the Criterion 1 (C_1) is calculated by

$$\begin{aligned} \tilde{\mathcal{R}}_{11} &= \left(\frac{\tilde{\mathcal{R}}_{11}^1 \oplus \tilde{\mathcal{R}}_{11}^2 \oplus \tilde{\mathcal{R}}_{11}^3}{3} \right) \\ &= \frac{1}{3} \left(((8.1, 10, 10, 10; 1, 1), (8.1, 10, 10, 10; 1, 1)) \oplus \right. \\ &\quad \left. ((6.3, 9, 9, 10; 1, 1), (6.3, 9, 9, 10; 1, 1)) \oplus \right. \\ &\quad \left. ((4.5, 7, 7, 9; 1, 1), (4.5, 7, 7, 9; 1, 1)) \right) \\ &= ((6.3, 8.67, 8.67, 9.67; 1, 1), (6.3, 8.67, 8.67, 9.67; 1, 1)). \end{aligned}$$

The results are given in Table 4.

Table 1. Assessments of the criteria by the decision makers

Criteria	Decision Makers		
	DM_1	DM_2	DM_3
C_1	VG	VG	VG
C_2	G	G	MG
C_3	G	MG	MG
C_4	VG	G	VG
C_5	F	MG	F

Table 2. Assessments of alternatives according to each criterion by the decision makers

Criteria	Alternatives	Decision Makers		
		DM_1	DM_2	DM_3
C_1	A_1	VO	O	MO
	A_2	O	O	MO
	A_3	VO	O	VO
	A_4	O	VO	MO
C_2	A_1	VO	MO	N
	A_2	VO	VO	MO
	A_3	MO	O	VO
	A_4	N	N	VO
C_3	A_1	VO	O	O
	A_2	VO	VO	O
	A_3	O	MO	VO
	A_4	N	MO	MO
C_4	A_1	VO	O	VO
	A_2	VO	VO	MO
	A_3	O	VO	VO
	A_4	O	N	VO
C_5	A_1	VO	VO	VO
	A_2	MO	MO	O
	A_3	O	O	MO
	A_4	MO	O	N

Step 4: Construct the ranking matrix R_1, R_2 and R_3 based on Eqs.(1)-(3), respectively.

	a_1	a_2	a_3	a_4
$R_1 = c_1$	74.79	69.54	89.78	74.79
c_2	43.90	60.82	54.52	33.35
$R_1 = c_3$	55.51	58.30	49.38	29.70
c_4	85.09	74.79	84.03	62.62
c_5	36.14	24.80	28.52	22.60
	a_1	a_2	a_3	a_4
$R_2 = c_1$	72.40	66.25	88.39	72.40
c_2	39.44	56.95	49.31	28.66
$R_2 = c_3$	50.58	54.01	44.13	23.83
c_4	83.19	72.40	81.95	59.65
c_5	32.19	19.46	23.23	17.92
	a_1	a_2	a_3	a_4
$R_3 = c_1$	15.74	14.88	17.66	15.74
c_2	10.90	13.60	12.34	8.91
$R_3 = c_3$	12.56	13.14	11.55	7.74
c_4	17.04	15.74	16.88	14.12
c_5	9.70	6.81	7.69	6.54

For example, ranking values of the Alternative 1 (A_1) according to the Criterion1 (C_1) are calculated by

$$R_1(\frac{\%}{\%}) = \left(\frac{6.3+9.67}{2} + \frac{1+1+1+1}{4} \right) \times \frac{1}{8} (6.3+8.67+8.67+9.67+6.3+8.67+8.67+9.67) = 74.79$$

$$R_2(\frac{\%}{\%}) = \left(\sqrt{6.3+9.67} + (1 \times 1 \times 1 \times 1)^{\frac{1}{4}} \right) \times \sqrt[8]{6.3 \times 8.67 \times 8.67 \times 9.67 \times 6.3 \times 8.67 \times 8.67 \times 9.67} = 72.40$$

$$R_3(\frac{\%}{\%}) = \left(\frac{2 \times 6.3 \times 9.67}{6.3+9.67} + \frac{4}{\left(\frac{1+1}{1 \times 1} + \frac{1+1}{1 \times 1} \right)} \right) \times \left(\frac{8}{\frac{1}{6.3} + \frac{1}{8.67} + \frac{1}{8.67} + \frac{1}{9.67} + \frac{1}{6.3} + \frac{1}{8.67} + \frac{1}{8.67} + \frac{1}{9.67}} \right) = 15.74$$

Step 5: Construct the rank average agreement degree AD_1, AD_2, AD_3 for each criteria by different ranking values which are arithmetic average ranking value, geometric average ranking value, harmonic average ranking value. The results are shown in Table 5.

$$AD_1 = \frac{1}{n} \sum_{j=1}^n R_1(\frac{\%}{\%}), k = 1, 2, 3$$

$$AD_2 = \left(\prod_{j=1}^n R_2(\frac{\%}{\%}) \right)^{\frac{1}{n}}, k = 1, 2, 3$$

$$AD_3 = \frac{n}{\sum_{j=1}^n \frac{1}{R_3(\frac{\%}{\%})}}, k = 1, 2, 3$$

Step 6: Compute the ranking value $R_i(x_j)$ of x_j by,

$$R_i(x_j) = \sum_{k=1}^n \frac{R_k(\frac{\%}{\%})}{AD_k}, i = 1, 2, 3$$

Results are demonstrated in Table 6.

Table 3. The weighted assessment values of the alternatives in terms of the criteria for each decision maker.

Criteria	Alternatives	Decision makers		
		D_1	D_2	D_3
C_1	A_1	((8.1,10,10,10;1,1),(8.10,10,10,10;1,1))	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((4.5,7,7,9;1,1),(4.5,7,7,9;1,1))
	A_2	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((4.5,7,7,9;1,1),(4.5,7,7,9;1,1))
	A_3	((8.1,10,10,10;1,1),(8.1,10,10,10;1,1))	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((8.1,10,10,10;1,1),(8.1,10,10,10;1,1))
	A_4	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((8.1,10,10,10;1,1),(8.1,10,10,10;1,1))	((4.5,7,7,9;1,1),(4.5,7,7,9;1,1))
C_2	A_1	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((3.5,6,3,6,3,9;1,1),(3.5,6,3,6,3,9;1,1))	((1.5,3,5,3,5,6,3;1,1),(1.5,3,5,3,5,6,3;1,1))
	A_2	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((2.5,4,9,4,9,8,1;1,1),(2.5,4,9,4,9,8,1;1,1))
	A_3	((3.5,6,3,6,3,9;1,1),(3.5,6,3,6,3,9;1,1))	((4,9,8,1,8,1,10;1,1),(4,9,8,1,8,1,10;1,1))	((4,5,7,7,9;1,1),(4,5,7,7,9;1,1))
	A_4	((2,1,4,5,4,5,7;1,1),(2,1,4,5,4,5,7;1,1))	((2,1,4,5,4,5,7;1,1),(2,1,4,5,4,5,7;1,1))	((4,5,7,7,9;1,1),(4,5,7,7,9;1,1))
C_3	A_1	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((3.5,6,3,6,3,9;1,1),(3.5,6,3,6,3,9;1,1))	((3.5,6,3,6,3,9;1,1),(3.5,6,3,6,3,9;1,1))
	A_2	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((4,5,7,7,9;1,1),(4,5,7,7,9;1,1))	((3.5,6,3,6,3,9;1,1),(3.5,6,3,6,3,9;1,1))
	A_3	((4,9,8,1,8,1,10;1,1),(4,9,8,1,8,1,10;1,1))	((2,5,4,9,4,9,8,1;1,1),(2,5,4,9,4,9,8,1;1,1))	((4,5,7,7,9;1,1),(4,5,7,7,9;1,1))
	A_4	((2,1,4,5,4,5,7;1,1),(2,1,4,5,4,5,7;1,1))	((2,5,4,9,4,9,8,1;1,1),(2,5,4,9,4,9,8,1;1,1))	((2,5,4,9,4,9,8,1;1,1),(2,5,4,9,4,9,8,1;1,1))
C_4	A_1	((8.1,10,10,10;1,1),(8.1,10,10,10;1,1))	((4,9,8,1,8,1,10;1,1),(4,9,8,1,8,1,10;1,1))	((8.1,10,10,10;1,1),(8.1,10,10,10;1,1))
	A_2	((8.1,10,10,10;1,1),(8.1,10,10,10;1,1))	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((4,5,7,7,9;1,1),(4,5,7,7,9;1,1))
	A_3	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((8.1,10,10,10;1,1),(8.1,10,10,10;1,1))
	A_4	((6.3,9,9,10;1,1),(6.3,9,9,10;1,1))	((2,1,4,5,4,5,7;1,1),(2,1,4,5,4,5,7;1,1))	((8.1,10,10,10;1,1),(8.1,10,10,10;1,1))
C_5	A_1	((2,7,5,5,7;1,1),(2,7,5,5,7;1,1))	((4,5,7,7,9;1,1),(4,5,7,7,9;1,1))	((2,7,5,5,7;1,1),(2,7,5,5,7;1,1))
	A_2	((1,5,3,5,3,5,6,3;1,1),(1,5,3,5,3,5,6,3;1,1))	((2,5,4,9,4,9,8,1;1,1),(2,5,4,9,4,9,8,1;1,1))	((2,1,4,5,4,5,7;1,1),(2,1,4,5,4,5,7;1,1))
	A_3	((2,1,4,5,4,5,7;1,1),(2,1,4,5,4,5,7;1,1))	((3,5,6,3,6,3,9;1,1),(3,5,6,3,6,3,9;1,1))	((1,5,3,5,3,5,6,3;1,1),(1,5,3,5,3,5,6,3;1,1))
	A_4	((1,5,3,5,3,5,6,3;1,1),(1,5,3,5,3,5,6,3;1,1))	((3,5,6,3,6,3,9;1,1),(3,5,6,3,6,3,9;1,1))	((0,9,2,5,2,5,4,9;1,1),(0,9,2,5,2,5,4,9;1,1))

Table 4. The average assessment values of the alternatives with regard to criteria.

A \ C	A ₁	A ₂	A ₃	A ₄
C ₁	((6.3,8.67,8.67,9.67;1,1), (6.3,8.67,8.67,9.67;1,1))	((5.7,8.33,8.33,9.67;1,1), (5.7,8.33,8.33,9.67;1,1))	((7.5,9.67,9.67,10;1,1), (7.5,9.67,9.67,10;1,1))	((6.3,8.67,8.67,9.67;1,1), (6.3,8.67,8.67,9.67;1,1))
C ₂	((3.77,6.27,6.27,8.43;1,1), (3.77,6.27,6.27,8.43;1,1))	((5.03,7.63,7.63,9.37;1,1), (5.03,7.63,7.63,9.37;1,1))	((4.3,7.13,7.13,9.33;1,1), (4.3,7.13,7.13,9.33;1,1))	((2.9,5.33,5.33,7.67;1,1), (2.9,5.33,5.33,7.67;1,1))
C ₃	((4.43,7.2,7.2,9.33;1,1), (4.43,7.2,7.2,9.33;1,1))	((4.76,7.43,7.43,9.33;1,1), (4.76,7.43,7.43,9.33;1,1))	((3.96,6.67,6.67,9.03;1,1), (3.96,6.67,6.67,9.03;1,1))	((2.37,4.77,4.77,7.33;1,1), (2.37,4.77,4.77,7.33;1,1))
C ₄	((7.03,9.37,9.37,10;1,1), (7.03,9.37,9.37,10;1,1))	((6.3,8.67,8.67,9.67;1,1), (6.3,8.67,8.67,9.67;1,1))	((6.9,9.33,9.33,10;1,1), (6.9,9.33,9.33,10;1,1))	((5.5,7.84,7.84,9;1,1), (5.5,7.84,7.84,9;1,1))
C ₅	((3.3,5.67,5.67,7.67;1,1), (3.3,5.67,5.67,7.67;1,1))	((2.03,4.3,4.3,7.13;1,1), (2.03,4.3,4.3,7.13;1,1))	((2.37,4.77,4.77,7.43;1,1), (2.37,4.77,4.77,7.43;1,1))	((1.97,4.1,4.1,6.73;1,1), (1.97,4.1,4.1,6.73;1,1))

Step 7: Since $R_1(x_3) > R_1(x_1) > R_1(x_2) > R_1(x_4)$, the preferred order of the alternatives is $A_3 > A_1 > A_2 > A_4$. Thus, the best alternative is A_3 according to arithmetic average ranking method. Since $R_2(x_1) > R_2(x_3) > R_2(x_2) > R_2(x_4)$ and $R_3(x_1) > R_3(x_3) > R_3(x_2) > R_3(x_4)$, ranking order of the alternatives is obtained as $A_1 > A_3 > A_2 > A_4$ according to the methods of geometric average ranking and harmonic average ranking.

We also solved this problem with TOPSIS method based upon T1FSs in order to compare our results. The results of the T1FS-based TOPSIS method and the different average ranking results based on IT2FS are shown in Table 6. The MCDM method based on arithmetic average operator and Type-1 fuzzy TOPSIS, give the same ranking results.

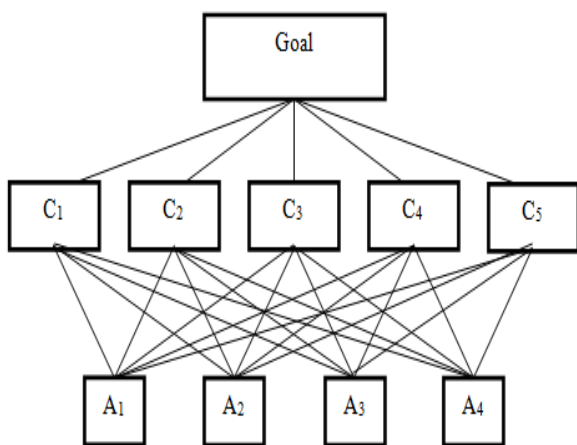


Figure 1. The hierarchical structure of numerical example

4. Discussion and Conclusion

In this paper, we presented a novel MCDM method based on the different ranking values of Interval Type 2 fuzzy sets. An application on a manager selection

for a Telecommunication Company to demonstrate the steps of this method is given. In addition, we compared the results of presented method and Type-1 fuzzy TOPSIS method. According to the Section 3, it can be said that this method ensures a beneficial tool for dealing with fuzzy MCDM problems in further flexibility and further smart way. The reason for this is that the method utilizes IT2FSs instead of conventional T1FSs to represent the assessment values and the weights of criteria.

In IT2FS based MCDM problems, how to rank IT2FSs is a very important and difficult problem. Various ranking methods have been proposed in the literature such as possibility ranking value method [19] and possibility degree method [20], but these methods have difficulty in the calculations and theoretically incomplete [1]. However, the ranking methods, which are based on arithmetic average, the geometric average and the harmonic average, used in this study are quite simple and easy to implement.

As a future research, other MCDM methods such as DEMATEL, TOPSIS, VIKOR, ARAS, etc., combining with different ranking methods for interval type 2 fuzzy sets (IT2FSs), could be suggested. These methods could be applied to various real-world decision-making problems which have multiple criteria and alternatives.

Table 5. The rank average agreement degree.

Criteria	AD ₁	AD ₂	AD ₃
C ₁	77.2	74.43	15.94
C ₂	48.15	42.21	11.16
C ₃	48.22	41.17	10.77
C ₄	76.53	73.66	15.86
C ₅	28.02	22.60	7.51

Table 6. The ranking values of the alternatives.

Alt.	T1FS		T2FS					
	TOPSIS		Arithmetic average		Geometric average		Harmonic average	
	R	Rank	R_1	Rank	R_2	Rank	R_3	Rank
A_1	0.69	2	5.43	2	5.69	1	5.50	1
A_2	0.68	3	5.23	3	5.40	3	5.27	3
A_3	0.70	1	5.44	1	5.57	2	5.37	2
A_4	0.60	4	3.90	4	3.83	4	4.27	4

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References

[1] Quin, J., Liu, X. 2015. Multi-attribute group decision making using combined ranking value under interval type-2 fuzzy environment. *Information Sciences* 297, 293-315.

[2] Chen, S. M. 1988. A new approach to handling fuzzy decision making problems. *IEEE Transactions on Systems, Man, and Cybernetics*, 18(6), 1012-1016.

[3] Chen, C. T. 2000. Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Systems*, 114(1), 1-9.

[4] Wang, W., Liu X., Quin, Y. 2012. Multi-attribute group decision making models under interval type-2 fuzzy environment. *Knowledge-Based System* 30 (2012),121-128.

[5] Liginlal, D., Ow, T. T. 2006. Modeling attitude to risk in human decision processes: An application of fuzzy measures. *Fuzzy Sets and Systems*, 157(23), 3040-3054.

[6] Wang T. C., Chang, T. H. 2007. Application of TOPSIS in evaluating initial training aircraft under a fuzzy environment. *Expert Systems with Applications*, 33(4), 870-880.

[7] Fu, G. 2008. A fuzzy optimization method for multicriteria decision making: An application to reservoir flood control operation. *Expert Systems with Applications*, 34(1), 145-149.

[8] Hua, Z., Gong, B., Xu, X. 2008. A DS-AHP approach for multi-attribute decision making problem with incomplete information. *Expert Systems with Applications*, 34(3), 2221-2227.

[9] Lin, Y. H., Lee, P. C., Ting, H. I. 2008. Dynamic multi-attribute decision making model with grey number evaluations. *Expert Systems with Applications*, 35(4), 1638-1644.

[10] Zadeh, L. A. 1965. Fuzzy sets. *Information and Control*, 8, 338-353.

[11] Zadeh, L. A. 1975. The concept of a linguistic variable and its application to approximate reasoning—I. *Information sciences*, 8(3), 199-249.

[12] Lee, L. W. , Chen, S. M. 2008. A new method for fuzzy multiple attributes group decision-making based on the arithmetic operations of interval type-2 fuzzy sets. *International Conference on Machine Learning and Cybernetic*, Kunming, China. 3084-3089.

[13] Chen, S. M., Lee, L. W. 2010a. Fuzzy multiple attributes group decision-making based on ranking values and the arithmetic operations of interval type-2 fuzzy sets. *Expert Systems with Applications*, 37(1), 824-833.

[14] Chen, S. M., Lee, L. W. 2010b. Fuzzy multiple attributes group decision-making based on interval type-2 TOPSIS method. *Expert Systems with Applications*, 37(4), 2790-2798.

[15] Chen, S. M., Yang, M. Y., Lee, L. W., Yang, S.W. 2012. Fuzzy multiple attributes group decision-making based on ranking interval type-2 fuzzy sets. *Expert Systems with Applications*, 39, 5295-5308.

[16] Sahin, A. , Yapıcı Pehlivan, N., 2017. Evaluation of life quality by integrated method of AHP and TOPSIS based on interval type-2 fuzzy sets. *Hacettepe Journal of Mathematics and Statistics*, 46 (3) , 511- 523.

[17] Mendel, J. M., John, R. I., & Liu, F. L. 2006. Interval type-2 fuzzy logical systems made simple. *IEEE Transactions on Fuzzy Systems*, 14(6), 808-821.

[18] Ashtiani, B., Haghghirad, F., Makui, A., Montazer, G. 2009. Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets. *Applied Soft Computing*, 9, 457-461.

[19] Chen, S.-M., Yang, M.-W., Lee, L.-W., Yang, S.-W., 2012. Fuzzy multiple attributes group decision-making based on ranking interval type-2 fuzzy sets. *Expert Systems with Applications*, 39 (5), 5295-5308.

[20] Hu, J., Zhang, Y., Chen, X., Liu, Y, 2013. Multi-criteria decision making method based on possibility degree of interval type-2 fuzzy number, *Knowledge-Based System* 43,21-29.