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## APPLICATION OF AHP METHOD FOR SOLVING THE UNIT COMMITMENT PROBLEM IN A DAY AHEAD MARKET

**ABSTRACT**

The aim of unit commitment problem in power systems has been converted from cost minimization to profit maximization with the liberalization of power markets. The generation companies (GENCOs) schedule the units to maximize their profit for the forecasted prices in day ahead market (DAM). The generation scheduling of generators in deregulated environment is called as Profit Based Unit Commitment (PBUC). In this paper, an application of Analytic Hierarchy Process (AHP) is proposed to solve PBUC problem. The method is applied to 3-units power system. The results are compared with the methods in the literature. As shown in the study, the proposed AHP method introduces its applicability and efficiency for solving the unit commitment problem in a day ahead market.

**Keywords:** Price Based Unit Commitment, Analytic Hierarchy Process, Decision Making, Power Systems, Day ahead Market

### 1. INTRODUCTION

In the traditional regulated energy industry, unit commitment aims to optimizing generation units to fulfill load demand with minimum cost. However, the countries worldwide have liberalized their electricity markets for increasing economic efficiencies and reliability of the system. There is a competition among generation companies (GENCOs) in the energy industry so the structure of the power system is altering. In the traditional unit commitment, the objective is to minimize the operation cost and it is commonly defined as cost-based unit commitment [1]. Now, the generators are scheduled to maximize profit of GENCOs contrary to regulated market. It has different objective and referred as price or profit-based unit commitment (PBUC) [1]. In PBUC, it is not necessary to satisfy power demand while committing the units. Independent System Operator (ISO) monitor the power system operation. The PBUC evaluates power and reserve that are offered in the day-ahead market to get the maximum profit [2]. There have been many solution techniques presented in the literature for solving PBUC problems. Some of them are Lagrange Relaxation-Differential Evolution [2], Binary fireworks algorithm [3], Particle Swarm Optimization (PSO) [4], Genetic Algorithm [5], hybrid Lagrangian Relaxation-Particle Swarm Optimization [6], Memetic Algorithm [7], Shuffled Frog Leaping Algorithm [8], Artificial Immune System [9], hybrid Binary Successive Approximation (BSA) and Civilized

Swarm Optimization (CSO) [10], hybrid Lagrangian relaxation (LR)-secant method-invasive weed optimization (IWO) [11], evolutionary particle swarm optimization (EPSO) [12]. In this study, AHP is developed to solve PBUC problem that aims to maximize GENCOs profit. 3-units, 12-hours power system is used for the application. The results are compared with LR-GA, LR-EP, LR-HF and LR-Secant-DE methods in the literature with respect to total profit. The results are encouraging and beneficial in deregulated power markets.

## 2. RESEARCH SIGNIFICANCE

The planning and economic operation of electric power generation have an important position in the electric power industry. After the restructuring and deregulation of electric power systems, an open-market environment is created with marked-based competition among GENCOs. PBUC problem optimizes generation units of power systems to maximize the total profit of GENCOs. The significance of present study is the finding the planning of power generation that maximizes GENCOs profit with AHP method. PBUC is a crucial problem for power companies which want to maximize the profit of the company. Therefore, PBUC has gained considerable interest and researchers have examined the problem [13]. AHP method is not used to solve this problem in the literature so it makes the study original.

## 3. ANALYTICAL STUDY AND SUBJECT
### 3.1. Profit Based Unit Commitment

PBUC problem is an optimization problem that maximizes the profit of GENCOs and its objective function can be formulated as shown in Equation 1 [2].

$$Maximize.\ PR = RE - TOC \tag{1}$$

where PR is profit, RE is revenue, TOC is total operation cost. The revenue and total operation cost can be found from Equation 2 and Equation 3 [2].

$$RE = \sum_{i=1}^{N}\sum_{t=1}^{T}(P_{it}\ SP_t)\ X_{it} + \sum_{i=1}^{N}\sum_{t=1}^{T} r\ RP_t\ R_{it}\ X_{it} \tag{2}$$

where

$P_{it}$ : Power output of i-th unit at t-th hour (MW)
$SP_t$: Forecasted spot price at t-th hour ($/MWh)
$X_{it}$ : Status of unit (ON/OFF)
$r$ : Probability of calling and generating the reserve
$RP_t$ : Forecasted reserve price at t-th hour ($/MWh)
$R_{it}$ : Reserve power of i- unit at t-th hour (MW)
N: Total number of units
T: Total number of hours

$$TOC = (1-r)\sum_{i=1}^{N}\sum_{t=1}^{T}F(P_{it})\ X_{it} + r\ \sum_{i=1}^{N}\sum_{t=1}^{T}F(P_{it}+R_{it})\ X_{it} + STU.X_{it} \tag{3}$$

where STU is startup cost ($) and $F(P_{it})$ indicates the fuel cost of i-th unit during t-th interval and it is described as shown in Equation 4 [2].

$$F(P_{it}) = a_i + b_i\ P_{it} + c_i\ P_{it}^2 \tag{4}$$

where $a_i$, $b_i$, and $c_i$ are the fuel cost coefficients of i-th unit. The various system and unit constraints are considered while solving the PBUC problem. Some constraints are explained below:

### 3.1.1. Power Demand Constraint

This constraint suggests that the units can generate less than or equal to the forecasted load as shown in Equation 5 [2].

$$\sum_{i=1}^{N} P_{it}\ X_{it} \leq PD_t \qquad t = 1, \dots, T \tag{5}$$

where $PD_t$ is the forecasted power at t-th hour (MW).

### 3.1.2. Reserve Constraint

The generators can be scheduled to generate less than or equal to forecasted system reserve as shown in Equation 6 [2].

$$\sum_{i=1}^{N} R_{it} X_{it} \leq PR_t \quad t = 1, \dots, T \tag{6}$$

where $PR_t$ is the total reserve power demand.

### 3.1.3. Generator and Reserve Limits

The generation units have to generate power between their lower and upper limits [2].

$$P_{imin} \leq P_i \leq P_{imax} \quad i = 1, \dots, N \tag{7}$$
$$0 \leq R_i \leq P_{imax} - P_{i,min} \quad i = 1, \dots, N \tag{8}$$
$$R_i + P_i < P_{imax} \quad i = 1, \dots, N \tag{9}$$

where $P_{imin}$ is lower limit and $P_{imax}$ is upper limit of i-th generator (MW).

### 3.1.4. Minimum Up and Down Time Constraints

The generating unit that is in OFF status can be committed provided that minimum down time of it has expired and the generating unit that is in ON status can be shut down provided that minimum up time of it has expired.

$$T_{i,on} \geq T_{i,up} \tag{10}$$
$$T_{i,off} \geq T_{i,down} \tag{11}$$

where $T_{i,up}$ is the minimum up time of the i-th generator and $T_{i,down}$ is the minimum down time of the i-the generator.

### 3.2. AHP

The AHP is a decision-making approach [13]. It offers alternatives and criteria, evaluates trade-off and applies a synthesis to reach a final decision [14]. The PBUC problem can be solved with this approach by making decision effectively of ranking units in term of their values. The steps of the AHP methodology are explained as follows:

- **Step 1:** The problem is formed by determining the aim, criteria and alternatives.
- **Step 2:** The judgment matrix (A) is formed by making comparisons among alternatives for each criterion and converting into quantitative numbers. It is also done for among criteria. While doing it, gradation scale that is adapted from [15] is used for quantitative comparison of alternatives. The pairwise comparisons of criteria are arranged into a square matrix. The diagonal elements of the matrix are 1. If the value of element (i,j) is more than 1, it means that the criterion in the i-th row is better than criterion in the j-th column; otherwise the criterion in the j-th column is better than in the i-th row. The (j,i) element of the matrix is the reciprocal of the (i,j) element [15].
- **Step 3:** The maximal eigenvalue and the corresponding eigenvector of the judgment matrix are calculated. They can be found by using sum method [14]:
  Firstly, every column in the judgment matrix is normalized as shown in Equation 12 [14].

$$X_{ij}^* = \frac{X_{ij}}{\sum_{k=1}^{n} X_{kj}} \quad i, j = 1, \dots, n \tag{12}$$

  where $X_{ij}$ is the i-th row, j-th column element of the judgment matrix (A), n is the number of orders of the matrix, $X_{ij}^*$ is the i-th row, j-th column element of new matrix (A*), in which each column has been normalized.

The all elements of each row in matrix A* are added as shown in Equation 13 [14].

$$W_i^* = \sum_{j=1}^{n} X_{ij}, \quad i = 1, \dots, n \tag{13}$$

The vector W* is normalized as shown in Equation 14 and the eigenvector of the judgment matrix (A) is obtained as shown in Equation 15 [14].

$$W_i = \frac{W_i^*}{\sum_{j=1}^{n} W_j^*} \quad i = 1, \dots, n \tag{14}$$

$$W = [\, W_1, W_2, \dots, W_n \,]^T \tag{15}$$

where W is the weighted vector.
The maximal eigenvalue $\lambda_{max}$ of the judgment matrix (A) is calculated as shown in Equation 16 [14].

$$\lambda_{max} = \sum_{i=1}^{n} \frac{(AW)_j}{n\,W_i} \quad j = 1, \dots, n \tag{16}$$

where (AW) is the column vector that is obtained by multiplying the elements of judgment matrix (A) with the elements of weighted vector (W).

- **Step 4:** There can be some inconsistencies in the pairwise comparisons. In such a situation, comparisons should be evaluated again. The consistency can be checked by calculating consistency ratio (CR). If CR is less than 0.1, it shows the consistency. CR is obtained by the following equations. Firstly, consistency index (CI) is calculated as shown in Equation 17.

$$CI = (\lambda_{max} - n)/(n - 1) \tag{17}$$

  where $\lambda_{max}$ is the maximum eigenvalue of the judgment matrix.

$$CR = CI/RCI \tag{18}$$

  where RCI is the Random Consistency Index and the values of RCI are adapted from [16].

- **Step 5:** The rating of each alternative is multiplied by the weights of the criteria and percentage distribution of alternatives is obtained.

## 4. RESULTS AND DISCUSSION

In this study, three generating units with 12 scheduling periods test system is used to carry out the application. The data of the system is adapted from [17]. The data contains the forecasted demand, reserve and spot prices for 3-units, 12 period system for each hour and the data of generators ($P_{max}$, $P_{min}$, fuel cost coefficients a, b and c, min up time, min down time, start-up cost, initial status) in the power system. AHP is applied for the solution of PBUC problem. Startup cost, fuel cost coefficients (a, b, c) are chosen as criteria and 3 generators are alternatives for the problem. The hierarchical structure of the problem that indicates the goal, criteria and alternatives is shown in Figure 1. The judgment matrix for each criterion among alternatives and among criteria are formed.
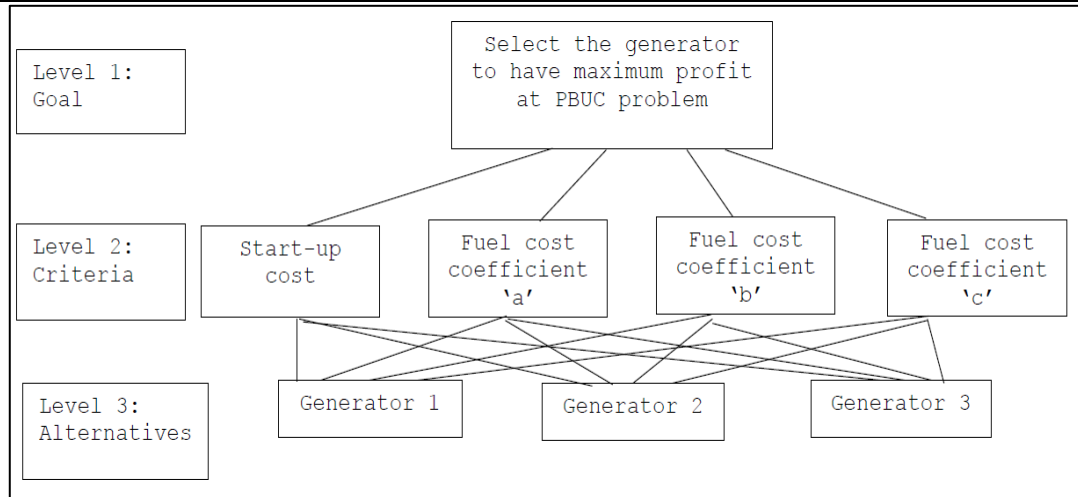
Figure 1. The AHP structural model of selecting generator for PBUC problem

The judgment matrix for the startup cost criterion among alternatives is formed as shown in Table 1. Unit 3 has the least startup cost, so it has the priority among the others. Unit 2 and Unit 1 follow it, respectively.

Table 1. The judgment matrix for the startup cost criterion among alternatives

| Startup cost | U-1 | U-2 | U-3 |
|---|---|---|---|
| U-1 | 1 | 1/2 | 1/4 |
| U-2 | 2 | 1 | 1/3 |
| U-3 | 4 | 3 | 1 |

The judgment matrix for the cost coefficient 'a' criterion among alternatives is formed as shown in Table 2. Unit 3 has the least cost coefficient 'a' so it has the priority among the others. Unit 2 and Unit 1 follow it, respectively. The judgment matrix for the cost coefficient 'b' criterion among alternatives is formed as shown in Table 3. Unit 3 has the least cost coefficient 'b' so it has the priority among the others. Unit 2 and Unit 1 follow it, respectively.

Table 2. The judgment matrix for the cost coefficient 'a' criterion among alternatives

| Cost coefficient 'a' | U-1 | U-2 | U-3 |
|---|---|---|---|
| U-1 | 1 | 1/2 | 1/5 |
| U-2 | 2 | 1 | 1/3 |
| U-3 | 5 | 3 | 1 |

Table 3. The judgment matrix for the cost coefficient 'b' criterion among alternatives

| Cost coefficient 'b' | U-1 | U-2 | U-3 |
|---|---|---|---|
| U-1 | 1 | 1/2 | 1/3 |
| U-2 | 2 | 1 | 1/2 |
| U-3 | 3 | 2 | 1 |

The judgment matrix for the cost coefficient 'c' criterion among alternatives is formed as shown in Table 4. Unit 1 has the least cost coefficient 'c' so it has the priority among the others. Unit 2 and Unit 3 follow it, respectively.

Table 4. The judgment matrix for the cost coefficient 'c' criterion among alternatives

| Cost coefficient 'b' | U-1 | U-2 | U-3 |
|---|---|---|---|
| U-1 | 1 | 2 | 4 |
| U-2 | 1/2 | 1 | 3 |
| U-3 | 1/4 | 1/3 | 1 |

The judgment matrix among criteria is formed as shown in Table 5. The priority order among criteria is determined as startup cost, cost coefficient 'a', cost coefficient 'b' and cost coefficient 'c'. The weights of each criterion are obtained as 0.58 for startup cost, 0.25 for cost coefficient 'a', 0.12 for cost coefficient 'b' and 0.5 for cost coefficient 'c'. The values of CR are less than 0.1 for all judgment matrixes and it shows the consistency. After all the steps of AHP are applied, units are ranked with Unit 3, Unit 2 and Unit 1 respectively. Unit 3 has 60%, Unit 2 has 25% and Unit 1 has 15% percentage distribution. It is seen that Unit 3 is the best choice among all the generators with high percentage rate. Unit 2 and Unit 1 follow Unit 3, respectively. Therefore, the need of power and reserve are supplied from Unit 3 primarily. If Unit 3 does not supply enough power because of its maximum limit, Unit 2 is committed for generation. Similarly, If Unit 3 and Unit 2 does not supply enough power because of their maximum limit, Unit 1 is committed for generation. The output power, reserve and profit of 3-units 12-hours power system are shown in Table 6. While calculating the profit, the value of 'r' in Equation 2 and 3 is taken as 0.005 and reserve price is fixed at the triple times of spot price [17].

Table 5. The judgment matrix among criteria

| Criteria | Startup Cost | Cost Coefficient 'a' | Cost Coefficient 'b' | Cost Coefficient 'c' |
|---|---|---|---|---|
| Startup cost | 1 | 3 | 5 | 9 |
| Cost Coefficient 'a' | 1/3 | 1 | 2 | 7 |
| Cost Coefficient 'b' | 1/5 | 1/2 | 1 | 2 |
| Cost Coefficient 'c' | 1/9 | 1/7 | 1/2 | 1 |

Table 6. The output power, reserve and profit

| Hour | Power (MW) | | | Reserve (MW) | | | PR ($) |
|---|---|---|---|---|---|---|---|
| | U-1 | U-2 | U-3 | U-1 | U-2 | U-3 | |
| 1 | 0 | 0 | 170 | 0 | 0 | 20 | 529.8850 |
| 2 | 0 | 0 | 250 | 0 | 0 | 25 | 676.3031 |
| 3 | 0 | 0 | 400 | 0 | 0 | 40 | 301.8600 |
| 4 | 0 | 0 | 520 | 0 | 0 | 55 | 345.1406 |
| 5 | 0 | 170 | 530 | 0 | 0 | 70 | 586.1725 |
| 6 | 145 | 400 | 505 | 0 | 0 | 95 | 1520.1318 |
| 7 | 200 | 400 | 500 | 0 | 0 | 100 | 1605.4500 |
| 8 | 0 | 280 | 520 | 0 | 0 | 80 | 1216.6399 |
| 9 | 0 | 115 | 535 | 0 | 0 | 65 | 736.1094 |
| 10 | 0 | 0 | 330 | 0 | 0 | 35 | 1074.2218 |
| 11 | 0 | 0 | 400 | 0 | 0 | 40 | 1002.9100 |
| 12 | 0 | 0 | 545 | 0 | 0 | 55 | 925.8956 |
| Total | | | | | | | 1052.0720 |

The results obtained are compared with the other methods that have used the same power system in the literature and shown in Table 7.

Table 7. Comparison of PBUC results for 3-units 12-hours power system

| Method | PR ($) |
|---|---|
| LR-GA [18] | 9021.3 |
| LR-EP [17] | 9074.3 |
| LR-HF [19] | 8973.3 |
| LR-Secant-DE [2] | 9074.36 |
| AHP | 1052.0720 |

## 5. CONCLUSIONS

In this paper, AHP method is proposed to solve PBUC problem that helps GENCOs maximize their profit. AHP method is not used to solve this problem in the literature. The performance of the proposed method is applied on 3-units, 12 hours power system for a given forecasted power demand, reserve, spot prices and reserve prices that are important parameters in solving PBUC problem. The results are compared with the other methods in the literature. The proposed model has been able to achieve higher profit. As shown in the study, the proposed AHP method introduces its applicability and efficiency for solving the unit commitment problem in a day ahead market. Therefore, the proposed method can be suitable for applications in the deregulated power markets.

### NOTE

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