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Normal Stress Analysis In An Infinite Elastic Body With A Locally Curved Carbon Nanotube

Kadriye Şimşek Alan

ABSTRACT

In this paper, the results of normal stress values in unidirectional fibrous composite with locally curved carbon nanotube (CNT) were obtained as much as the second approximation and the obtained results were analyzed.The boundary form perturbation method is used to solve the problem. This investigation is made within the framework of a piecewise homogeneous body model by using the three-dimensional geometrically nonlinear exact equations of elasticity theory. The concentration of carbon nanotubes in the composite is assumed to be low and the interaction between them is neglected. Numerous results are obtained for the normal stress distribution and the effect of the problem parameters on this stress distribution is analyzed.

Keywords: Carbon Nanotube, stress distribution, geometric nonlinearity, locally curved

1. INTRODUCTION

One of the most important factors determining the stress state in fiber composite materials is the curvature of the fibers , [1,5,6,9,10]. For this reason, Akbarov and Guz [2] focused on the stress state in unidirectional composite. The investigation in [2] is made within the framework of a piecewise homogeneous body model, by using the exact three-dimensional equations of elasticity theory. But the unidirectional fibrous composites are traditional materials. Nowadays, nanotechnology is used in the world and the importance of nano materials is increasing day by day. For this reason researchers have concentrated their work on this field. One of these studies Alan and Akbarov [3] studied the normal stresses state in the nanocomposite with a locally curved covered nanofibers. In [4], a method was given for the investigation of the stress distribution in the nanocomposites with unidirectional locally curved and hollow nanofiber. Coban and Kösker [8] was considered the stress distribution in the infinite elastic body containing a single locally curved carbon nanotube (CNT). However, results for the first approximation were obtained only. Increasing the number of approaches is crucial in order to increase the sensitivity of the results.

In this study, a mathematical formulation was developed to determine the normal stress distribution of the infinite elastic body containing a single local curved carbon nanotube (CNT) as much as up to the second approximation on the Carbon nanotube and matrix interface . The problem is solved using the boundary form perturbation method. In addition, an Algorithm is designed to solve the related problem. We obtained the numerous numerical results on the normal stress distribution on the surface between the CNT and matrix . The influence of the problem parameters on this distribution were analyzed.

The investigation is made in the framework of the three-dimensional geometrically nonlinear exact equations of elasticity theory. The model " an infinite body containing a single CNT " regards the case where the concentration of carbon nanotubes in the composite is assumed to be low, and the interaction between the carbon nanotubes is neglected.

We acknowledge that the statements of some results of normal stress distribution in an infinite elastic body with a locally curved carbon nanotube in this paper were presented at 2nd International Conference of Mathematical Sciences (ICMS 2018) [11].

2. FORMULATION OF THE PROBLEM

Infinite elastic body with local curved carbon nanotube is given as in figure 1 .

In this work, the cross section of the carbon nanotube normal to its axial line is described two circles of constant radius R_1 and R_2 along the entire length and the body is compressed or stretched of the uniformly distributed normal forces with intensity p acting along Ox_3 axis direction. With the middle line of the carbon nanotube , we associate Lagrangian rectilinear $Ox_1x_2x_3$ and cylindrical $Or\theta z$ system of coordinates (Figure 1). The carbon nanotube and matrix materials are homogeneous, isotropic and linear elastic.

The equation of the carbon nanotube middle line is given as follows:

$$x_1 = F(x_3) = \varepsilon\delta(x_3),\ x_2=0\ ;$$

$$x_1 = \mathrm{A}\exp\left(-\left(\frac{x_3}{\mathrm{L}}\right)^2\right)\cos\left(m\frac{x_3}{\mathrm{L}}\right)$$

$$= \varepsilon\mathrm{L}\exp\left(-\left(\frac{x_3}{\mathrm{L}}\right)^2\right)\cos\left(m\frac{x_3}{\mathrm{L}}\right) = \varepsilon\delta(x_3)$$

$$\varepsilon = \frac{A}{L} \qquad (1)$$

Where A and L geometrical parameter were shown in Figure 1. We assume that A is smaller than L, we describe a small parameter $\varepsilon=\frac{\mathrm{A}}{\mathrm{L}}$ $(0 \le \varepsilon < 1)$. The function δ_n^j is the local curving form of the carbon nanotube . Assume that on the contact surface between the carbon nanotube and matrix material is denoted by S. Then S satisfies the following equations.

$$r(\theta,t_3)=\frac{\varepsilon\delta(t_3)\left(1+\varepsilon^2(\delta'(t_3))^2\right)\cos\theta}{1+\varepsilon^2(\delta'(t_3))^2\cos^2\theta}+$$

$$\left\{\begin{array}{l}\dfrac{\varepsilon^2(\delta(t_3))^2\left(1+\varepsilon^2(\delta'(t_3))^2\right)^2\cos\theta}{(1+\varepsilon^2(\delta'(t_3))^2\cos^2\theta)^2}\\ +R^2-(\varepsilon^2(\delta(t_3))^2\left(1+\varepsilon^2(\delta'(t_3))^2\right)\end{array}\right\}^{\frac{1}{2}}$$

$$z(\theta,t_3)=t_3-\varepsilon\delta'(t_3)\left(r(\theta,t_3)-\varepsilon\delta(t_3)\right),\ \delta'(t_3)=\frac{d\delta(t_3)}{dt_3}$$

$$t_3 \in (-\infty,+\infty) \qquad (2)$$

We determine the components of the normal vector on the contact surface as follows.

$$n_r = r(\theta,t_3)\frac{\partial z(\theta,t_3)}{\partial t_3}\left[A(\theta,z)\right]^{-1},$$

$$n_\theta = \left[\frac{\partial z(\theta,t_3)}{\partial\theta}\frac{\partial r(\theta,t_3)}{\partial t_3} - \frac{\partial r(\theta,t_3)}{\partial\theta}\frac{\partial z(\theta,t_3)}{\partial t_3}\right]\left[A(\theta,z)\right]^{-1}$$

$$n_z = -r(\theta,t_3)\frac{\partial r(\theta,t_3)}{\partial t_3}\left[A(\theta,t_3)\right]^{-1}$$

where

$$\left[A(\theta,t_3)\right]=\left[\left(r(\theta,t_3)\frac{\partial z(\theta,t_3)}{\partial t_3}\right)^2+\left(\frac{\partial z(\theta,t_3)}{\partial\theta}\frac{\partial r(\theta,t_3)}{\partial t_3}-\frac{\partial z(\theta,t_3)}{\partial t_3}\frac{\partial r(\theta,t_3)}{\partial\theta}\right)^2 +\left(r(\theta,t_3)\frac{\partial z(\theta,t_3)}{\partial t_3}\right)^2\right]^{1/2} \qquad (3)$$

The Carbon nanotube and matrix material values are defined by superscripts (2) and (1), respectively.

Under this situation which has no motion, the following field equations must be satisfied for the carbon nanotube and matrix,

The equilibrum equations :

$$\nabla_i\left[\sigma^{(k)in}\left(g_n^j+\nabla_n u^{(k)j}\right)\right]=0, \qquad k=1,2 \qquad (4)$$

The strain-displacement relations:

$$2\varepsilon_{jm}^{(k)} = \nabla_j u_m^{(k)} + \nabla_m u_j^{(k)} + \nabla_j u^{(k)n}\nabla_m u_n^{(k)}, \qquad (5)$$

The constitutive equations (Hooke's Law):

$$\sigma_{(in)}^{(k)} = \lambda^{(k)} e^{(k)}\delta_i^n + 2\mu^{(k)}\varepsilon_{(in)}^{(k)},$$

$$e^{(k)} = \varepsilon_{(11)}^{(k)} + \varepsilon_{(22)}^{(k)} + \varepsilon_{(33)}^{(k)} \qquad (6)$$

Where λ and μ the material constants.

We use the conventional notation is used In Eqs.(4) , (5) and (6) , and $\sigma^{(k)}$ and $\varepsilon_{jm}^{(k)}$ denote the physical components of the stress tensors and the strain tensors, respectively.

For detailed explanations and formulations on these notations, we refer to Akbarov and Guz [1]

Also, perfect contact conditions are defined at the interfaces S:

$$\sigma^{(1)in}\left(g_n^j+\nabla_n u^{(1)j}\right)\Big|_{S_1} n_j=0$$

$$\sigma^{(1)in}\left(g_n^j+\nabla_n u^{(1)j}\right)\Big|_{S_2} n_j=\sigma^{(2)in}\left(g_n^j+\nabla_n u^{(2)j}\right)\Big|_{S_2} n_j$$

$$,\quad u^{(1)j}\Big|_{S_2}=u^{(2)j}\Big|_{S_2}$$

(7)

The conditions are in given eq. (8)

$$\sigma_{zz}^{(1)}\xrightarrow[r\to\infty]{}p,\ \ \sigma_{ij}^{(1)}\xrightarrow[r\to\infty]{}0\ \ (ij)\neq zz, \qquad (8)$$

where n_j are the covariant components of the unit normal vector to the surfaces S.

In this way, the mathematical formulation of the problem is completed with the solution of the equations systems (4), (5) and (6) within the contact condition (7).

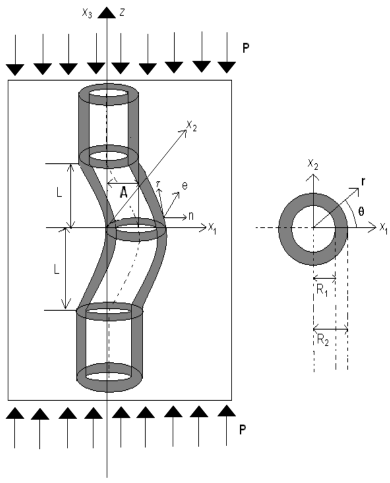


Figure 1: The geometry of a Locally Curved Carbon Nanotube and its cross section

3. SOLUTION OF THE PROBLEM

We solve our problem by using the boundary form perturbation method given in Akbarov and Guz [1]. According to this method, we can write the sought values in the series form in the small parameter ε :

$$\sigma_{rr}^{(k)}=\sum_{i=0}^{\infty}\varepsilon^i\sigma_{rr}^{(k),i},\ldots,\varepsilon_{rr}^{(k)}=\sum_{i=0}^{\infty}\varepsilon^i\varepsilon_{rr}^{(k),i},\ldots,$$

$$u_r^{(k)}=\sum_{i=0}^{\infty}\varepsilon^i u_r^{(k),i} \qquad (9)$$

The quantities r, z, n_r, n_θ and n_z are also given in series forms:

$$r=R+\sum_{i=1}^{\infty}\varepsilon^i a_{ri}(\theta,t_3),$$

$$z=t_3+\sum_{i=1}^{\infty}\varepsilon^i a_{zi}(\theta,t_3),$$

$$n_r=1+\sum_{i=1}^{\infty}\varepsilon^i b_{ri}(\theta,t_3),\quad n_\theta=\sum_{i=1}^{\infty}\varepsilon^i b_{\theta i}(\theta,t_3),$$

$$n_z=\sum_{i=1}^{\infty}\varepsilon^i b_{zi}(\theta,t_3)\quad ,\qquad t_3\in(-\infty,\infty) \quad (10)$$

by using some routine operations, we can calculate the coefficients of the ε^k in (10) , for whose details see Akbarov Guz [1].

Substituting the power series (9) into Eq.(5), it can be obtain sets of equations for each approximation . Using relations (10) we expand the values of each approximation (9) in series 3*form in the vicinity (R,θ,t_3) . If we substitute these last expressions in boundary conditions and use expressions in equation (10), we obtain boundary conditions for each approximation after some mathematical transformations. For the zeroth and first approximations see Alan [4].

Now we will discuss the second approximation. We can write the mechanical and the geometrical relations for this approximation.

$$\sigma_{(in)}^{(k),2}=\lambda^{(k)}e^{(k),2}\delta_i^n+2\mu^{(k)}\varepsilon_{(in)}^{(k),2},$$

$$e^{(k),2}=\varepsilon_{(11)}^{(k),2}+\varepsilon_{(22)}^{(k),2}+\varepsilon_{(33)}^{(k),2}$$

$$\varepsilon_{rr}^{(k),2}=\frac{\partial u_r^{(k),2}}{\partial r},$$

$$\varepsilon_{\theta\theta}^{(k),2}=\frac{\partial u_\theta^{(k),2}}{r\partial\theta}+\frac{u_r^{(k),2}}{r},\ \varepsilon_{zz}^{(k),2}=\frac{\partial u_z^{(k),2}}{\partial z} \qquad (11)$$

$$\varepsilon_{r\theta}^{(k),2}=\frac{1}{2}\left(\frac{\partial u_r^{(k),2}}{r\partial\theta}+\frac{\partial u_\theta^{(k),2}}{\partial r}-\frac{u_\theta^{(k),2}}{r}\right),$$

$$\varepsilon_{\theta z}^{(k),2}=\frac{1}{2}\left(\frac{\partial u_\theta^{(k),2}}{\partial z}+\frac{\partial u_z^{(k),2}}{r\partial\theta}\right)$$

$$\varepsilon_{zr}^{(k),2}=\frac{1}{2}\left(\frac{\partial u_z^{(k),2}}{\partial r}+\frac{\partial u_r^{(k),2}}{\partial z}\right)$$

By using Akbarov and Guz [1], the contact conditions can be writen for the second approximation.

$$\left(\sigma_{rr}^{(1),2}-\sigma_{rr}^{(2),2}\right)\Big|_{(R_1,\theta,t_3)}=-f_1\left(\frac{\partial\sigma_{rr}^{(1),1}}{\partial r}-\frac{\partial\sigma_{rr}^{(2),1}}{\partial r}\right)-\gamma_z\left(\sigma_{rz}^{(1),1}-\sigma_{rz}^{(2),1}\right)$$

$$\left(\sigma_{r\theta}^{(1),2}-\sigma_{r\theta}^{(2),2}\right)\Big|_{(R_1,\theta,t_3)}=-f_1\left(\frac{\partial\sigma_{\theta r}^{(1),1}}{\partial r}-\frac{\partial\sigma_{\theta r}^{(2),1}}{\partial r}\right)$$

$$-\varphi_1\left(\frac{\partial\sigma_{\theta r}^{(1),1}}{\partial z}-\frac{\partial\sigma_{\theta r}^{(2),1}}{\partial z}\right)-\gamma_\theta\left(\sigma_{\theta\theta}^{(1),1}-\sigma_{\theta\theta}^{(2),1}\right)$$

$$\left(\sigma_{zr}^{(1),2}-\sigma_{zr}^{(2),2}\right)\Big|_{(R_1,\theta,t_3)}=-f_1\left(\frac{\partial\sigma_{zr}^{(1),1}}{\partial r}-\frac{\partial\sigma_{zr}^{(2),1}}{\partial r}\right)$$
$$-\varphi_1\left(\frac{\partial\sigma_{zr}^{(1),1}}{\partial z}-\frac{\partial\sigma_{zr}^{(2),1}}{\partial z}\right)-\gamma_r\left(\sigma_{zr}^{(1),1}-\sigma_{zr}^{(2),1}\right)$$
$$-\gamma_\theta\left(\sigma_{z\theta}^{(1),1}-\sigma_{z\theta}^{(2),1}\right)-\gamma_z\left(\sigma_{zz}^{(1),1}-\sigma_{zz}^{(2),1}\right)-\gamma_\theta\left(\sigma_{zz}^{(1),1}-\sigma_{zz}^{(2),1}\right)$$
$$\left(u_r^{(1),2}-u_r^{(2),2}\right)\Big|_{(R_1,\theta,t_3)}=-f_1\left(\frac{\partial u_r^{(1),1}}{\partial r}-\frac{\partial u_r^{(2),1}}{\partial r}\right)-\varphi_1\left(\frac{\partial u_r^{(1),1}}{\partial z}-\frac{\partial u_r^{(2),1}}{\partial z}\right)$$
$$\left(u_r^{(1),2}-u_r^{(2),2}\right)\Big|_{(R_1,\theta,t_3)}=-f_1\left(\frac{\partial u_\theta^{(1),1}}{\partial r}-\frac{\partial u_\theta^{(2),1}}{\partial r}\right)-\varphi_1\left(\frac{\partial u_\theta^{(1),1}}{\partial z}-\frac{\partial u_\theta^{(2),1}}{\partial z}\right)$$
$$\left(u_z^{(1),2}-u_z^{(2),2}\right)\Big|_{(R_1,\theta,t_3)}=-f_1\left(\frac{\partial u_z^{(1),1}}{\partial r}-\frac{\partial u_z^{(2),1}}{\partial r}\right)-\varphi_1\left(\frac{\partial u_z^{(1),1}}{\partial z}-\frac{\partial u_z^{(2),1}}{\partial z}\right) \quad (12)$$

$$\left(\sigma_{rr}^{(2),2}\right)\Big|_{(R_2,\theta,t_3)}=f_1\left(\frac{\partial\sigma_{rr}^{(2),1}}{\partial r}\right)+\gamma_z\sigma_{rz}^{(2),1}$$
$$\left(\sigma_{r\theta}^{(2),2}\right)\Big|_{(R_2,\theta,t_3)}=-f_1\left(\frac{\partial\sigma_{\theta r}^{(2),1}}{\partial r}\right)+\varphi_1\left(\frac{\partial\sigma_{\theta r}^{(2),1}}{\partial z}\right)+\gamma_\theta\sigma_{\theta\theta}^{(2),1}$$
$$\left(\sigma_{zr}^{(2),2}\right)\Big|_{(R_2,\theta,t_3)}=f_1\left(\frac{\partial\sigma_{zr}^{(2),1}}{\partial r}\right)+\varphi_1\left(\frac{\partial\sigma_{zr}^{(2),1}}{\partial z}\right)$$
$$+\gamma_r\left(\sigma_{zr}^{(2),1}\right)+\gamma_\theta\left(\sigma_{z\theta}^{(2),1}\right)+\gamma_z\sigma_{zz}^{(2),1}+\gamma_\theta\sigma_{zz}^{(2),1}$$

We used in Eq.(12) the following notation.

$$f_1=\delta(t_3)\cos\theta,$$
$$\varphi_1=-R\frac{d\delta(t_3)}{dt_3}\cos\theta,$$
$$\gamma_r=\left(\frac{\delta(t_3)}{R}-\frac{d^2\delta(t_3)}{dt_3^2}R\right)\cos\theta, \quad (13)$$
$$\gamma_\theta=\frac{\delta(t_3)}{R}\sin\theta,\quad \gamma_z=-\frac{d\delta(t_3)}{dt_3}\cos\theta,$$
$$\delta(t_3)=\exp\left(\frac{-x_3}{L}\right)^2\cos\left(m\frac{-x_3}{L}\right)$$

Similar contact conditions can be given for subsequent approximation.

By using the method in [1] we can define governing equations for the second approximation as follows. We use the physical components of the tensors and vectors in this equation.

$$\frac{\partial\sigma_{rr}^{(k),2}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}^{(k),2}}{\partial\theta}+\frac{\partial\sigma_{rz}^{(k),2}}{\partial z}+\frac{1}{r}\left(\sigma_{rr}^{(k),2}-\sigma_{\theta\theta}^{(k),2}\right)+\sigma_{zz}^{(k),0}\frac{\partial^2u_r^{(k),2}}{\partial z^2}=0,$$
$$\frac{\partial\sigma_{r\theta}^{(k),2}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}^{(k),2}}{\partial\theta}+\frac{\partial\sigma_{\theta z}^{(k),2}}{\partial z}+\frac{2}{r}\sigma_{r\theta}^{(k),2}+\sigma_{zz}^{(k),0}\frac{\partial^2u_\theta^{(k),2}}{\partial z^2}=0 \quad (14)$$
$$\frac{\partial\sigma_{rz}^{(k),2}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta z}^{(k),2}}{\partial\theta}+\frac{\partial\sigma_{zz}^{(k),2}}{\partial z}+\frac{1}{r}\sigma_{rz}^{(k),2}+\sigma_{zz}^{(k),0}\frac{\partial^2u_z^{(k),2}}{\partial z^2}=0,$$

These equations coincide with the 3-dimensional linearized elasticity equations.

If we apply the following exponential Fourier transform with respect to z to the contact conditions in Eq. (12) and if we substitute the expressions for the first approximation in eq. (12), we obtain the Fourier transformed states of the contact conditions that include single or double integral on the right side.

$$\bar{\sigma}_{rr}=\int_{-\infty}^{+\infty}\sigma_{rr}e^{-isz}ds \quad (15)$$

To solve the contact conditions in equation (12), we apply the above Fourier transformation to the following equations (16) and (17).

$$u_r^{(m),n}=\frac{1}{r}\frac{\partial}{\partial\theta}\psi^{(m),n}-\frac{\partial^2}{\partial r\partial z}\chi^{(m),n};$$
$$u_\theta^{(m),n}=-\frac{\partial}{\partial r}\psi^{(m),n}-\frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}\chi^{(m),n}; \quad (16)$$
$$\Delta_1^{(m)}=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$
$$u_z^{(m),n}=(\lambda^{(m)}+\mu^{(m)})^{-1}\begin{pmatrix}(\lambda^{(m)}+2\mu^{(m)})\Delta_1^{(m)}+\\(\mu^{(m)}+\sigma_{zz}^{(m),0})\frac{\partial^2}{\partial z^2}\end{pmatrix}\chi^{(m),n}$$

The functions of $\psi^{(m),q}$, $\chi^{(m),q}$ satisfy the following equations:

$$\left(\Delta_1^{(m)}+\left(\xi_1^{(m)}\right)^2\frac{\partial^2}{\partial z^2}\right)\psi^{(m),q}=0; \quad (17)$$
$$\left(\Delta_1^{(m)}+\left(\xi_2^{(m)}\right)^2\frac{\partial^2}{\partial z^2}\right)\left(\Delta_1^{(m)}+\left(\xi_3^{(m)}\right)^2\frac{\partial^2}{\partial z^2}\right)\chi^{(m),q}=0$$

Where, $\xi_i^{(m)}$ (m=1,2; i=1,2,3) are given in the following equations:

$$\xi_1^{(m)}=\sqrt{\frac{\mu^{(m)}+\sigma_{zz}^{(m),0}}{\mu^{(m)}}},$$

$$\xi_2^{(m)}=\sqrt{\frac{\mu^{(m)}+\sigma_{zz}^{(m),0}}{\mu^{(m)}}},$$

$$\xi_3^{(m)}=\sqrt{\frac{\lambda^{(m)}+2\mu^{(m)}+\sigma_{zz}^{(m),0}}{\lambda^{(m)}+2\mu^{(m)}}}$$

After this employing, we obtain the following equations

$$\overline{\psi}^{(1),1}=\overline{A}_1^{(1)}(s)K_1(\xi_1^{(1)}s\frac{r}{L})\sin\theta\ ,$$

$$\overline{\chi}^{(1),1}=i\begin{bmatrix}\overline{A}_2^{(1)}(s)K_1(\xi_2^{(1)}s\frac{r}{L})+\\ \overline{A}_3^{(1)}(s)K_1(\xi_3^{(1)}s\frac{r}{L})\end{bmatrix}\cos\theta \qquad (18)$$

$$\overline{\psi}^{(2),1}=\left[\overline{A}_{11}^{(2)}(s)I_1(\xi_1^{(2)}s\frac{r}{L})+\overline{A}_{12}^{(2)}(s)K_1(\xi_1^{(2)}s\frac{r}{L})\right]\sin\theta\ ,$$

$$\overline{\chi}^{(2),1}=i\begin{bmatrix}\overline{A}_{21}^{(2)}(s)I_1(\xi_2^{(2)}s\frac{r}{L})+\overline{A}_{22}^{(2)}(s)K_1(\xi_2^{(2)}s\frac{r}{L})+\\ \overline{A}_{31}^{(2)}(s)I_1(\xi_3^{(2)}s\frac{r}{L})+\overline{A}_{32}^{(2)}(s)K_1(\xi_3^{(2)}s\frac{r}{L})\end{bmatrix}\cos\theta$$

$$\overline{\psi}^{(1),2}=\overline{A}_{12}^{(1)}(s_1)K_2(\xi_1^{(1)}s_1\frac{r}{L})\sin2\theta\ ,$$

$$\overline{\chi}^{(1),2}=i[\overline{A}_{20}^{(1)}(s_1)K_0(\xi_2^{(1)}s_1 r/L)$$
$$+\overline{A}_{30}^{(1)}(s_1)K_0(\xi_3^{(1)}s_1 r/L)+$$
$$\{\overline{A}_{22}^{(1)}(s_1)K_2(\xi_2^{(1)}s_1 r/L)+\overline{A}_{32}^{(1)}(s_1)K_2(\xi_3^{(1)}s_1 r/L)\}]\cos2\theta$$

$$\overline{\psi}^{(2),2}=\begin{bmatrix}\overline{A}_{12}^{(2)}(s_1)I_2(\xi_1^{(2)}s_1\frac{r}{L})+\\ \overline{B}_{12}^{(2)}(s_1)K_2(\xi_1^{(2)}s_1\frac{r}{L})\end{bmatrix}\sin2\theta$$

$$\overline{\chi}^{(2),2}=i\Big\{\overline{A}_{20}^{(2)}(s_1)I_0(\xi_2^{(3)}s_1\frac{r}{L})+\overline{B}_{20}^{(2)}(s_1)K_0(\xi_2^{(3)}s_1\frac{r}{L})+$$
$$\overline{A}_{30}^{(2)}(s_1)I_1(\xi_3^{(3)}s_1\frac{r}{L})+\overline{B}_{30}^{(2)}(s_1)K_1(\xi_3^{(2)}s_1\frac{r}{L})+$$
$$\begin{bmatrix}\overline{A}_{22}^{(2)}(s_1)I_2(\xi_2^{(2)}s_1\frac{r}{L})+\overline{B}_{22}^{(2)}(s_1)K_2(\xi_2^{(2)}s_1\frac{r}{L})+\\ \overline{A}_{32}^{(2)}(s_1)I_2(\xi_3^{(2)}s_1\frac{r}{L})+\overline{B}_{32}^{(2)}(s_1)K_2(\xi_3^{(2)}s_1\frac{r}{L})\end{bmatrix}\cos2\theta\Big\}$$

(19)

where $I_n(x)$ are Bessel functions of a purely imaginary argument and $K_n(x)$ are the Macdonald functions. Moreover, we used the Fourier transform parameters s and s_1 in (18) and (19) respectively. Employing the Functions (18) and (19) in boundary-value problems of the related approximations, we obtain the systems of the linear equations. If we solve this systems of the linear equations , substituting the solution of the this equation systems into the quantities of the stresses, we determine the expressions of $\overline{\sigma}_{rr}^{(1),1},..,\overline{\sigma}_{zz}^{(2),1}$. If we apply the following inverse transform for the stresses, we find the real stress values.

$$\sigma_{rr}^{(1),1}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\overline{\sigma}_{rr}^{(1),1}e^{isz}ds \qquad (20)$$

Thus, the boundary value problem related to the second approximation was solved

4. NUMERICAL RESULT AND DISCUSSION

In the numerical investigation, the improved integral (20) are calculated by using the Gauss integration algorithm. In order to calculate this improper integrals , we replaced this improper integrals by the corresponding definite integrals. Hereafter, we divide the interval of these improper integrals into certain number of short intervals .

To determine the number of these intervals, we use the numerical convergence of the integral values i.e. we use the relation $\int_0^{+\infty}(.)ds\cong\int_0^{S_*}(.)ds=\sum_{i=0}^{N}\int_{S_i}^{S_{i+1}}(.)ds,\quad S_0=0.$

This process resulted in calculation of triple integral. In these calculating, we used the approximation

$$\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}(.)dsdzds_1\cong\int_0^{S_*^{(1)}}\int_0^{z_*}\int_0^{S_*^{(2)}}(.)dsdzds_1 \qquad (21)$$

$$\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\sum_{k=0}^{N_3}\int_{S_i^{(1)}}^{S_{i+1}^{(1)}}\int_{Z_J}^{Z_{j+1*}}\int_{S_k^{(2)}}^{S_{k+1}^{(2)}}(.)dsdzds_1$$

Where $S_0^{(1)}=S_0=S_0^{(2)}=0$ and the values of N_1, N_2, N_3 and $S_*^{(1)},Z_*,S_*^{(2)}$ were determined from convergence criterion. In addition, $S_{N_1}^{(1)}=S_*^{(1)},Z_{N_2}=Z_*,S_{N_3}^{(2)}=S_*^{(2)}$.

$$\int_{S_i^{(1)}}^{S_{i+1}^{(1)}}\int_{Z_J}^{Z_{j+1*}}\int_{S_k^{(2)}}^{S_{k+1}^{(2)}}(.)dsdzds_1$$

$$\int_{S_I^{(1)}}^{S_{i+1}^{(1)}}\int_0^{S_{i+1}^{(1)}}\int_{Z_J}^{Z_{j+1*}}\int_{S_k^{(2)}}^{S_{k+1}^{(2)}}(.)dsdzds_1$$

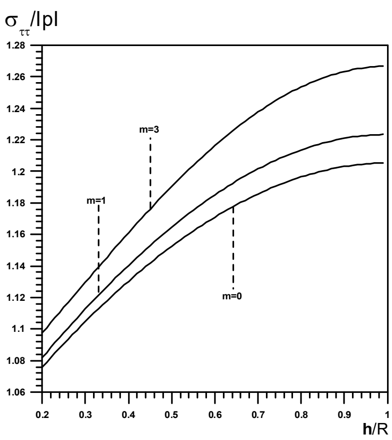
We used the Gauss integration algorithm for the calculation of triple integrals . To perform all these operations, we wrote a computer program in Fortran 77 .

The numerical results related to the normal stress are analyzed on the intersection surfaces between the CNT and matrix . Due to symmetry, we are only examine the distribution of these stresses for $x_3 \geq 0$ (and) $0 \leq \theta \leq \pi$ (Figure 1). If $\varepsilon = 0$ (i.e. if the curving is absent), the stresses $\sigma_{\tau\tau}$ coincide with σ_{zz} .

We assume that α used in all figures and table is smaller than its critical values corresponding to microbukling of the fiber in the matrix in [5]. It is assumed that $\nu^{(1)} = \nu^{(2)} = 0.3$, $\varepsilon = 0.07$, $\theta = 0$ and $\kappa = R_2/L$, bk=h/R. $\alpha = p/E^{(1)}$ shows the effect of geometrical non-linearity on the normal stresses values .

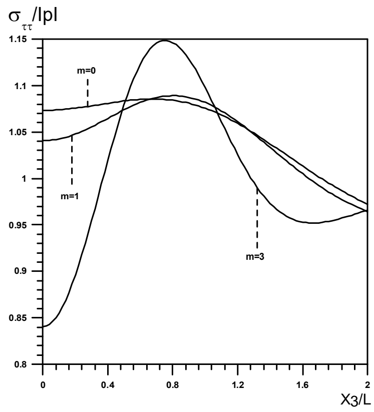
The relationship between $\sigma_{\tau\tau}/|p|$ and h/R is showed in Figure 2 . In this graph, m=0, 1, 3 at $\chi_3/L = 1.0$, $\kappa = 0.3$, $\alpha = 0.00005$ and $E^{(2)}/E^{(1)}$=500.

The same figure shows also the effect of the parameter m on the normal stresses values. From the graphs, it is seen that the maximal values of normal stresses increase monotonically with m. The results obtained when we increase the carbon nanotube thickness are the same in the case of the single local curved nanofiber in the infinite body at the same parameter values.



4*Figure 2: The relationship between the $\sigma_{\tau\tau}/p$ and h /R for $E^{(2)}/E^{(1)}$=500, $\varepsilon = 0.07$, $\kappa = 0.25$, $m = 0$, $m = 1$ and $m = 3$

Figure 3 shows the relationships between $\sigma_{\tau\tau}/|p|$ and x_3 /L for h/R=0.5 , $\kappa = 0.3$ and $E^{(2)}/E^{(1)}$=400. Likewise, the effect of the parameter m on the distribution of normal stresses can be seen. From Figure 3, we can say that absolute maximal values of normal stresses are monotonically increasing with m.



5*Figure 3: The relationship between the $\sigma_{\tau\tau}/p$ and x_3/L for $E^{(2)}/E^{(1)}$=400, $\varepsilon = 0.07$, $\kappa = 0.25$, $m = 0$, $m = 1$ and $m = 3$

Table 1 shows the effect of various values of $E^{(2)}/E^{(1)}$, m and α on $\sigma_{\tau\tau}/|p|$ normal stress values.

In this table, $\sigma_{\tau\tau}/|p|$ normal stress values are calculated for m = 0, 1 and 3 respectively. In this case the values of $\sigma_{\tau\tau}/|p|$ are calculated under $\kappa = 0.3$, $\chi_3/L = 1.0$ for $m = 0$, 1 and 3 respectively. From this table it is seen that the absolute maximal values of $\sigma_{\tau\tau}/|p|$ normal stresses increase monotonically with m.

Table 1: The values of $\sigma_{\tau\tau}/p$ obtained for various α and $E^{(2)}/E^{(1)}$

m	$E^{(2)}/E^{(1)}$	A.N	$\alpha = \frac{p}{E^{(1)}}$							
			Tension				compression			
			0.0005	0.005	0.05	0.03	-0.0005	-0.005	-0.01	-0.015
0	300	1	1.0551	1.0559	1.0554	1.0566	-1.0550	-1.0537	-1.0517	-1.0488
		2	1.2328	1.2337	1.2308	1.0671	-1.2328	-1.2312	-1.0654	-1.0632
	500	1	1.0312	1.0359	1.0488	1.0466	-1.0311	-1.0248	-1.0160	-1.0035
		2	1.2309	1.0236	1.2486	1.0573	-1.2308	-1.2234	-1.0310	-1.0195
	1000	1	0.9897	1.0029	1.0403	1.0327	-0.9894	-0.9706	-0.9422	-0.8966
		2	1.2198	1.2348	1.2742	1.0438	-1.2195	-1.1980	-0.9589	-0.9149
1	300	1	1.1593	1.1571	1.1380	1.1460	-1.1593	-1.1613	-1.1631	-1.1644
		2	1.2996	1.2967	1.2715	1.1555	-1.2997	-1.3025	-1.1758	-1.1778
	500	1	1.1419	1.1428	1.1340	1.1394	-1.1419	-1.1399	-1.1361	-1.1295
		2	1.3017	1.3019	1.2871	1.1493	-1.3016	-1.3002	-1.1501	-1.1445
	1000	1	1.1093	1.1172	1.1281	1.1293	-1.1091	-1.0968	-1.0767	-1.0424
		2	1.2955	1.3031	1.3084	1.1395	-1.2953	-1.2829	-1.0923	-1.0595
3	300	1	1.4986	1.4866	1.4070	1.4370	-1.4989	-1.5118	-1.5260	-1.5413
		2	1.4940	1.4822	1.4044	1.4487	-1.4942	-1.5069	-1.5376	-1.5528
	500	1	1.5052	1.4932	1.4120	1.4428	-1.5055	-1.5836	-1.5320	-1.5465
		2	1.5042	1.4924	1.4126	1.4544	-1.5044	-1.5171	-1.5433	-1.5565
	1000	1	1.5064	1.4954	1.4154	1.4468	-1.5066	-1.5177	-1.5282	-1.5374
		2	1.5104	1.4995	1.4204	1.4578	-1.5106	-1.5216	-1.5392	-1.5477

In this work, a method was developed to study the normal stress distribution in an infinite elastic body with a locally curved carbon nanotube. In order to examine the normal stress distribution, the mathematical formulation of the relevant boundary value problem is given.
In this case, we assume that the concentration of an infinite elastic body containing a single locally curved carbon nanotube is low. We neglect the interaction between the carbon 6*nanotubes. For the investigation, we used the three-dimensional geometrically nonlinear exact equations of the theory of elasticity in the piecewise homogeneous model. We have developed a method that obtains normal stress values as much as up to the second approximation on the interface the carbon nanotube and matrix material. The numerical results were presented for a single locally curved and carbon nanotube. As a result of this research, the following were obtained :

(i) When the radius of hollow approach to 0 as a limit , The normal stresses values in the carbon nanotube are the same as to the locally curved nanofiber in an nanocomposite material in same parameter values.

(ii) The absolute maximal values of the normal stresses $\sigma_{\tau\tau}/|p|$ increase monotonically with m.

The numerical results obtained agree with well-known mechanical consideration and, in some particular cases, coincide with known results.

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