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Measuring Dependence between Electricity Consumption and Economic Indicators via Copulas: Turkish Case

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Abstract

This paper implements copulas to identify the dependence structure between electricity consumption and its cofounding indicators. To achieve this, Turkish electricity demand, its economic and sectoral indicators are taken into account. As a first step, bivariate copulas are used to identify the best fitting copula and the degree of the dependence. Thereafter, multivariate model is established using vine copulas using highly correlated variables. The empirical results confirm the added value of the proposed approach in determining numerous tail properties. We indicate that the copulas are useful to underline, especially, the tail properties of indicators in the market for decision makers.

1. INTRODUCTION

Energy is one of the most prominent and crucial input for the sustainable economic growth which is primarily based on the efficient use of the energy sources. As a most commonly used one, electricity market requires a clear understanding of the factors which contribute to the changes in the price and the demand. Due to its significant role in sustainable economic growth, theoretical and empirical studies are done in identifying the relation between electricity consumption and economic indicators using time series and econometric models [1-5]. In addition to depict the relation between electricity demand and related variables, the degree of the relation among them is also an important concern for decision makers in the market. Widely used conventional econometric tools like Granger-Causality, Johansen Co-integration, Autoregressive Distributed Lag (ARDL) bounds tests search for the relation between variables incorporating the impact of time which may yield disparities on the findings due to the time interval chosen. However, a significant relation between electricity consumption and economic indicators, such as GDP is one of the common indicators in previous studies. On the other hand, literature on modeling electricity supply counts to few; the causal relation between electricity supply and factors such as, economic growth, exports, electricity prices, employment are studied using similar techniques to demand analysis whose results point out the dependence between energy supply and economic indicators [6,7].

Copulas or copula functions are mainly the expressions for joint distributions of random variables having no restrictions on the marginal distributions especially, when the multivariate normality assumption is not justifiable [8]. Apart from the classical techniques for the analysis of macroeconomic variables, copulas provide an efficient and flexible tool to measure the dependence among the financial time series. One of the most important characteristics of financial data is the tail dependence structure, that accounts for the

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joint behavior of variables having strong dependence on the extreme observations, which are successfully captured by different copula families [9]. Apart from the extension of classical copula families, as a novel improvement, vine copulas came into play for detecting multivariate complex dependencies between variables conditional to their dependence ranks [10]. For instance, pair copula construction is used on macroeconomic variables such as oil revenue, economic growth, total consumption and investment with Bayesian framework [11]. A general review of the growing literature on copula based economic and financial time series models can be found in [9].

In Turkey, the energy demand, especially for electricity has increased in last decades. Recent strategic planning in Turkish energy sector aims several objectives, including the compensation of long term demand and energy requirements by adding suitable new and efficient energy resources; to diversify energy resources for avoiding dependence on external sources; to reduce all losses related to the electricity sector with the help of liberalized energy markets. To achieve these purposes, it is required to understand the relation between the factors, contributing on the price and demand changes in the electricity market. The majority of previous studies analyzed the association between energy consumption, income, aggregate output, exports, capital, labor supply, industrial production and growth using time series methods investigated in short and long run time frames [12-14]. Additionally, [15] determines the association between main macroeconomic and sectorial indicators with electricity consumption and production, and conducts a partial equilibrium analysis for determining the equilibrium prices with the indication of the existence of co-integration among variables in consideration for the time interval between 1970 and 2010.

On the basis of all these, as a main goal of this study, to capture the dependence pattern between the macroeconomic variables and the energy consumption we implement copulas alternative to the conventional econometric techniques. Turkish electricity market data, economic and sectorial indicators are used to illustrate the implementation of the copula framework. The analyses are performed in two phases: (i) Implementation of different bivariate copula families to measure the association between electricity demand and selected macroeconomic/sectoral variables. The coefficient of tail dependencies is calculated to display the joint behavior of parameters and the size of tail dependencies. (ii) Introducing vine copula (R- and C- vine) framework to detect the association between all considered variables, in a multivariate setting. R-vine and C-vine copulas with different pair copulas are investigated and the most plausible vine copula model is identified using GOF tests.

The rest of paper is organized as follows: Section 2 summarizes the theoretical background of copulas and vine copulas briefly. The pre-processing part before copula modeling with the descriptions of the data set is presented in Section 3. Thereafter, section 4 summarizes the findings of the implementation of the copula approach on measuring dependence between electricity demand and selected indicators on Turkish data. Finally, in Section 5, the main conclusions of the study are discussed with the benefits and drawbacks of copulas.

2. BIVARIATE and VINE COPULAS

The motivation behind the copula framework is based on the attempts of answering the questions about construction of multivariate distributions with different margins and separation of the dependence structure from those margins. A copula is a multivariate distribution whose marginal distributions are all uniform over $[0,1]$, any p-dimensional one can be constructed by following Sklar's Theorem, given below [8].

Theorem 1 (Sklar's Theorem, 1959). Let F be a p-dimensional distribution function with univariate margins $F_1, F_2, \ldots, F_p$ defined on their corresponding domains. Then, there exists a copula function C such that

$$F(u_1, u_2, \ldots, u_p) = C(F_1(u_1), F_2(u_2), \ldots, F_p(u_p)) \quad (1)$$

Here, C is unique whenever $F_1, F_2, \ldots, F_p$ are continuous marginal distributions [16].
To illustrate, elliptical (implicit) copulas are defined shortly as follows:

Definition 2. Let F be the multivariate Cumulative Distribution Function (CDF) of an elliptical distribution. Let F_i and F_i^{-1} for $i = 1,2,\ldots,p$ denote the CDF of the i'th marginal and its inverse, respectively. The elliptical, F, is expressed as,

$$C(u_1,\ldots,u_p) = F(F_1^{-1}(u_1) + \ldots + F_p^{-1}(u_p)) \tag{2}$$

The widely used and known elliptical families are normal (Gaussian) and Student-t copulas, representing different tail dependence properties. In addition to elliptical families, many of other well-known copula families can be classified as archimedean copulas based on their construction such as Frank, Joe, Clayton and Gumbel family. Different copula families stand for ways of deriving distinct dependence structures. For example, elliptical copulas and Frank copula are preferable to examine the symmetric dependence structures. On the other hand, Clayton and Gumbel are useful to identify the tail dependencies at lower and upper quantiles, respectively. Joe copula family behaves similar to Clayton one in terms of tail dependence. Naturally, if there is no significant dependence between variables, independent copula appears for the modeling part. Along with one parameter copula families, other copula families are constructed based on the rotations or extensions of the existed ones, called as associated copulas. Among those special families, rotated copulas, especially survival ones, deserves more attention since they allow to measure negative dependence between variables over various tail structures. Above mentioned copula families with their bivariate density functions and the corresponding parameter space presented in Table 1.

Table 1. *The forms of the most widely used copula families and their parameter spaces*

Copula Family	$C_\theta(u,v)$	Parameter (θ)
Normal	$\emptyset_\rho(\emptyset^{-1}(u),\emptyset^{-1}(v)) = \int_{-\infty}^{\emptyset^{-1}(u)}\int_{-\infty}^{\emptyset^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{s^2-2\rho st+t^2}{2(1-\rho^2)}\right) dsdt$	$\rho\epsilon(-1,1)$
Student-t	$t_{d,\rho}\left(t_d^{-1}(u),t_d^{-1}(v)\right) = \int_{-\infty}^{t_d^{-1}(u)}\int_{-\infty}^{t_d^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \left(1+\frac{s^2-2\rho st+t^2}{d(1-\rho^2)}\right)^{-\frac{d+2}{2}} dsdt$	$\rho\epsilon(-1,1)$; $d\epsilon(0,\infty)$
Frank	$\frac{-1}{\theta}\ln\left[1+\frac{(exp^{-\theta u}-1)(exp^{-\theta v}-1)}{(exp^{-\theta}-1)}\right]$	$\theta\epsilon(-\infty,\infty)$ /{0}
Joe	$1-\left((1-u)^\theta+(1-v)^\theta-(1-u)(1-v)^\theta\right)^{\frac{1}{\theta}}$	$\theta\epsilon[1,\infty)$
Clayton	$\left(u^{-\theta}+v^{-\theta}-1\right)^{\frac{-1}{\theta}}$	$\theta\epsilon(0,\infty)$
Gumbel	$\exp\left[-\left(u^{-\theta}+v^{-\theta}\right)^{\frac{-1}{\theta}}\right]$	$\theta\epsilon[1,\infty)$
Independent	$u.v$	$u,v\epsilon(0,1)$

Even if they are very practical to model the dependence structure for bivariate case, the use of copulas is still challenging in higher dimensions, where standard multivariate copulas suffer from rather inflexible structures. One of the recent approaches for copula construction in higher dimensions is vine copula methodology which measures the dependencies in multivariate case by allowing a tractable graphical illustration. In this respect, vine copulas overcome limitations of direct extension of copula families for higher dimensions and are able to model complex dependence patterns by benefiting from the rich variety of bivariate copulas simultaneously.

A vine copula structure is defined as a nested set of trees describing the pairwise copula functions unconditional at the first tree and conditional at the rest of connected trees described as follows:

Definition 4. Let $V = T_1,\ldots,T_{p-1}$ denote the regular vine for p variables, where T_i is a connected tree with nodes N_i and edges E_i for $i = 1,2,\ldots,p$. In this tree structure, T_i represents tree with nodes such that $N_i = E_{i-1}$

A regular vine with p variables is a vine where two edges in tree i are connected by an edge in tree $i+1$, only if these edges share a common node. In general, there are totally $p(p-1)/2$ possible edges in a regular vine for p variables [17]. Formally, vine copulas are multivariate distribution functions built on bivariate copulas and this modeling setup is called as Pair Copula Construction (PCC) generally [18]. The procedure of PCC for a multivariate distribution function in p-dimension is defined as follows [10]:

Definition 5. Given

$$f(u_1,\ldots,u_p) = \left(\sum_{t=2}^{p} f(u_t \mid u_1,\ldots,u_{t-1}) \right) f_1(u_1) \tag{3}$$

and for distinct values of i , j , $i_1,\ldots,i_k$ with $i < j$ and $i_1 < \ldots < i_k$ define

$$c_{i,j|i_1,\ldots,i_k} = c_{i,j|i_1,\ldots,i_k}(F(u_i \mid u_{i_1},\ldots,u_{i_k}), F(u_j \mid u_{i_1},\ldots,u_{i_k})) \tag{4}$$

where f and c denote probability density function (pdf.) of original margins and copula density function, respectively. Then, one can rewrite the conditional pdf., $f(u_t \mid u_1,\ldots,u_{t-1})$, in terms of conditional copulas as,

$$f(u_t \mid u_1,\ldots,u_{t-1}) = c_{1,t|2,\ldots,t-1} f(u_t \mid u_2,\ldots,u_{t-1}) = \left(\sum_{s=1}^{t-2} c_{s,t|s+1,\ldots,t-1} \right) c_{t-1,t} f_t(u_t) \tag{5}$$

Then, rewriting equation (3) with the help of equation (5) for $s = i$, $t = i + j$ results in,

$$f(u_1,\ldots,u_p) = \left(\sum_{j=1}^{p-1} \sum_{i=1}^{p-j} c_{i,i+j|i+1,\ldots,i+j-1} \right) \left(\sum_{k=1}^{p} f_k(u_k) \right) \tag{6}$$

The decomposition in Equation (5) suggests that, there is no unique way of deriving multivariate copula density. For this reason, the full specification of a vine copula requires the choice of vine tree structure, selection of copula families for each pair and estimation of the corresponding parameters [19]. In multivariate case, there are two special tree structures among the others, called Canonical (C)- and Drawable (D)-vine copulas. C-vine is a type of regular vine distribution in which each tree has a unique node that is connected to edges. It uses only star like trees and it is useful for ordering of variables by importance. On the other hand, D-vine requires no node in any tree is connected to more than two edges. It uses only path like trees and beneficial for temporal ordering of variables.

3. DEPENDENCE STRUCTURES IN THE MARKET

In this paper, bivariate and multivariate models mentioned earlier with numerous pair copula families are considered to detect the most significant model. Besides, tail dependence properties of the relevant pairs are investigated based on both bivariate and vine copulas. Various factors which have impacts on electricity prices are chosen such as industrial value added, GDP, production, investments, labor supply, gross profit, consumption and sectoral influence such as urbanization ratio, transmission and distribution losses. Annually collected data set retrieved from Turkish Electricity Transmission (TEIAS) and World Bank between years 1970-2012 [20,21] for dependence analysis. The summary of selected variables, their abbreviations and units are listed in Table 2. Some of the economic variables are adjusted to per capita and all computations are reproducible in Cran-R.

Table 2. *Variables used in the electricity demand modeling*

Variable	Abbreviation	Unit
Average Annual Electricity Prices	AEPR	USD
Industrial Value Added per capita	IVAL	USD
Transmission and Distribution Losses per capita	TDLOS	%
Urbanization Ratio per capita	UR	%
Gross Domestic Product per capita	GDP	USD
Total Annual Electricity Production per capita	AELPR	kWh
Investments in Electricity Sector	INV	USD
Labor Supply per capita	LSUP	%
Gross Profit in Electricity Market	GRPEM	cent/kWh
Electricity Consumption per capita	ECON	kWh

Spearman's (ρ) and Kendall's (τ) coefficients are calculated depending on the bivariate copula family to measure the association between the selected variables. Such measurement allows us to describe the pairwise dependencies of standardized time series data presented in Table 3. Based on both correlation measures (ρ and τ), almost each pair shows significant dependence except the pair (AEPR; TDLOS). Besides, the most significant positive correlation occurs for the pairs (UR; AELPR), (UR; ECON) and (AELPR; ECON) indicating that price have influence on urbanization and consumption. Such result is coherent with the fact that the urbanization ratio with the electricity production and consumption measures are directly related to each other.

Table 3. *Pairwise correlation coefficients: Upper triangle values (.) for Kendall's τ , lower triangle values [.] for Spearman's ρ*

	AEPR	IVAL	TDLOS	UR	GDP	AELPR	LSUP	ECON
AEPR	1	(-0.2)	(-0.08)	(-0.23)	(-0.25)	(-0.23)	(0.67)	(-0.24)
IVAL	[-0.37]	1	(0.55)	(0.88)	(0.93)	(0.88)	(-0.4)	(0.88)
TDLOS	[-0.19]	[0.77]	1	(0.61)	(0.53)	(0.61)	(-0.19)	(0.61)
UR	[-0.42]	[0.97]	[0.81]	1	(0.88)	(0.99)	(-0.44)	(0.99)
GDP	[-0.42]	[0.99]	[0.76]	[0.97]	1	(0.88)	(-0.46)	(0.88)
AELPR	[-0.42]	[0.97]	[0.81]	[1]	[0.97]	1	(-0.43)	(1)
LSUP	[0.86]	[-0.59]	[-0.43]	[-0.61]	[-0.63]	[-0.61]	1	(-0.43)
ECON	[-0.42]	[0.97]	[0.81]	[1]	[0.97]	[1]	[-0.61]	1

To derive uncorrelated residuals for each variable before implementing copula analysis, seasonality and trends should be detected and eliminated by a suitable ARIMA(p, d, q) model to the stationary series. Having no seasonality, the increasing trend in standardized series is analyzed through stationarity and unit root tests which agree on the non-stationarity at 5% significance level (Figure 1).

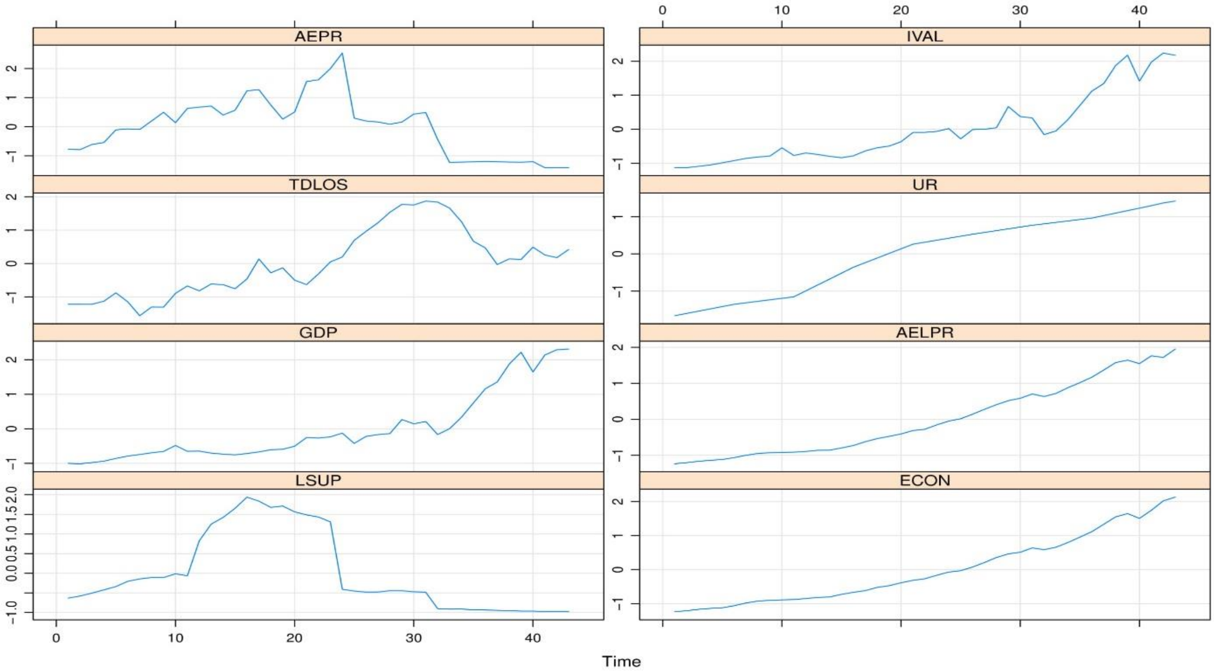

Figure 1. *Standardized time series plots of the variables*

Table 4. *The best fitting ARIMA models and diagnostic test results on residuals of the models*

Variable	Model	BP (p-value)	LB (p-value)
AEPR	ARIMA(0,1,0)	(0.8288)	(0.8227)
IVAL	ARIMA(0,1,0)	(0.4337)	(0.4174)
TDLOS	ARIMA(0,1,0)	(0.2041)	(0.1883)
UR	ARIMA(1,1,0)	(0.9764)	(0.9755)
GDP	ARIMA(0,1,0)	(0.6382)	(0.6262)
AELPR	ARIMA(0,1,0)	(0.9330)	(0.9306)
LSUP	ARIMA(0,1,0)	(0.2145)	(0.1985)
ECON	ARIMA(0,1,0)	(0.0704)	(0.0609)

To eliminate the stochastic trend term from the series, the first ordered differences of all series are taken and the resulting series are employed in further analysis. Among all plausible models, the best fitting ARIMA with associated orders are chosen based on BP and LB statistics. ARIMA models with fitting orders based on the corresponding p-values based on differenced standardized time series data are found to be stationary yielding the residuals being independent (p-value>0.05) as can be seen Table 4. Except urbanization ratio (UR), all other variables follow $ARIMA(0,1,0)$ based on ACF and PACF plots and relevant test statistics. After making each univariate series stationary and fitting plausible time series models, the copula framework comes into play to describe the bivariate dependence structure. Based on residuals, the details of pairwise are visualized in Figures 2 and 3.

In Figure 2, pair plots of uniform margins on $[0,1]$ with respect to the τ has three layers. Bivariate contour plots are represented on the lower panel in the matrix, whereas the histograms for each data are given on the diagonal, and the scatter plots and τ correlations are visualized on the upper panel. Based on τ values, the most correlated pairs are (IVAL; GDP), (AELPR; ECON) (IVAL; AELPR), (IVAL; ECON), (GDP; AELPR) and (GDP; ECON) have positive correlations, the last four pairs being more significant. However, (IVAL; TDLOS), (TDLOS; GDP), (TDLOS; ECON) and (AELPR; LSUP) depict less significant negative correlations. For the variables AEPR, UR and LSUP, no significant positive or negative relationship is captured with other indicators, so it is the main reason why copula modeling is considered only for the variables that have noteworthy associations. These are listed as (IVAL, TDLOS, GDP, AELPR and ECON) which yield 10 different pairs for the construction of bivariate copulas.

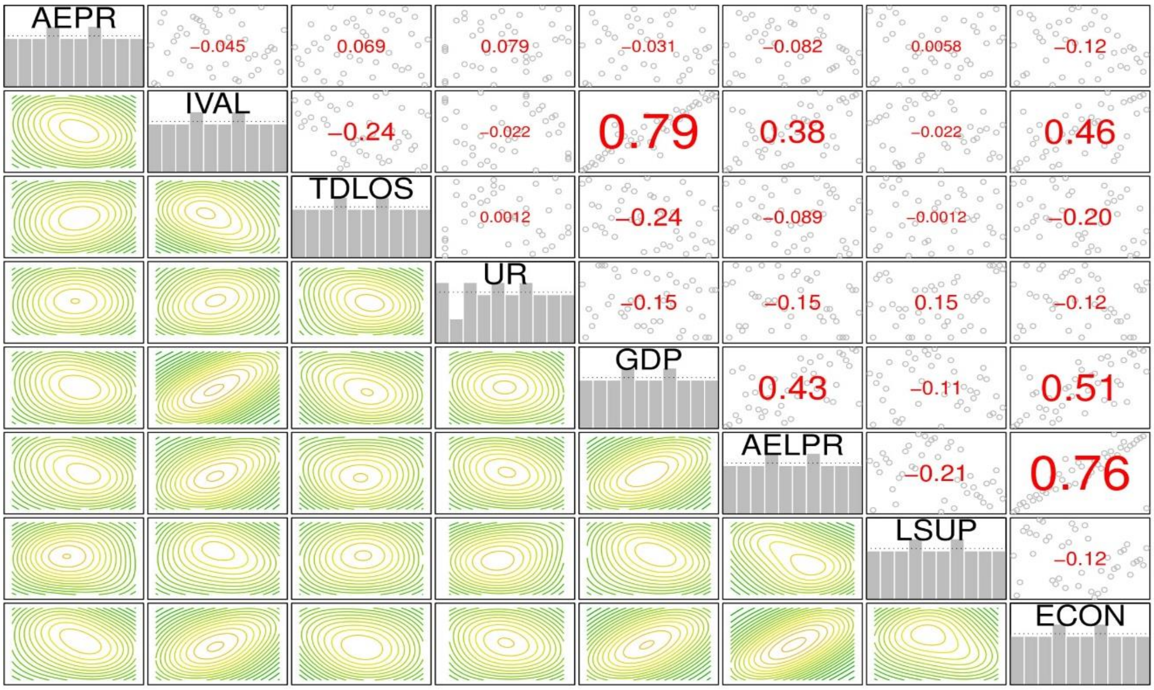

Figure 2. *Pair plots for transformed data with respect to Kendall's* τ

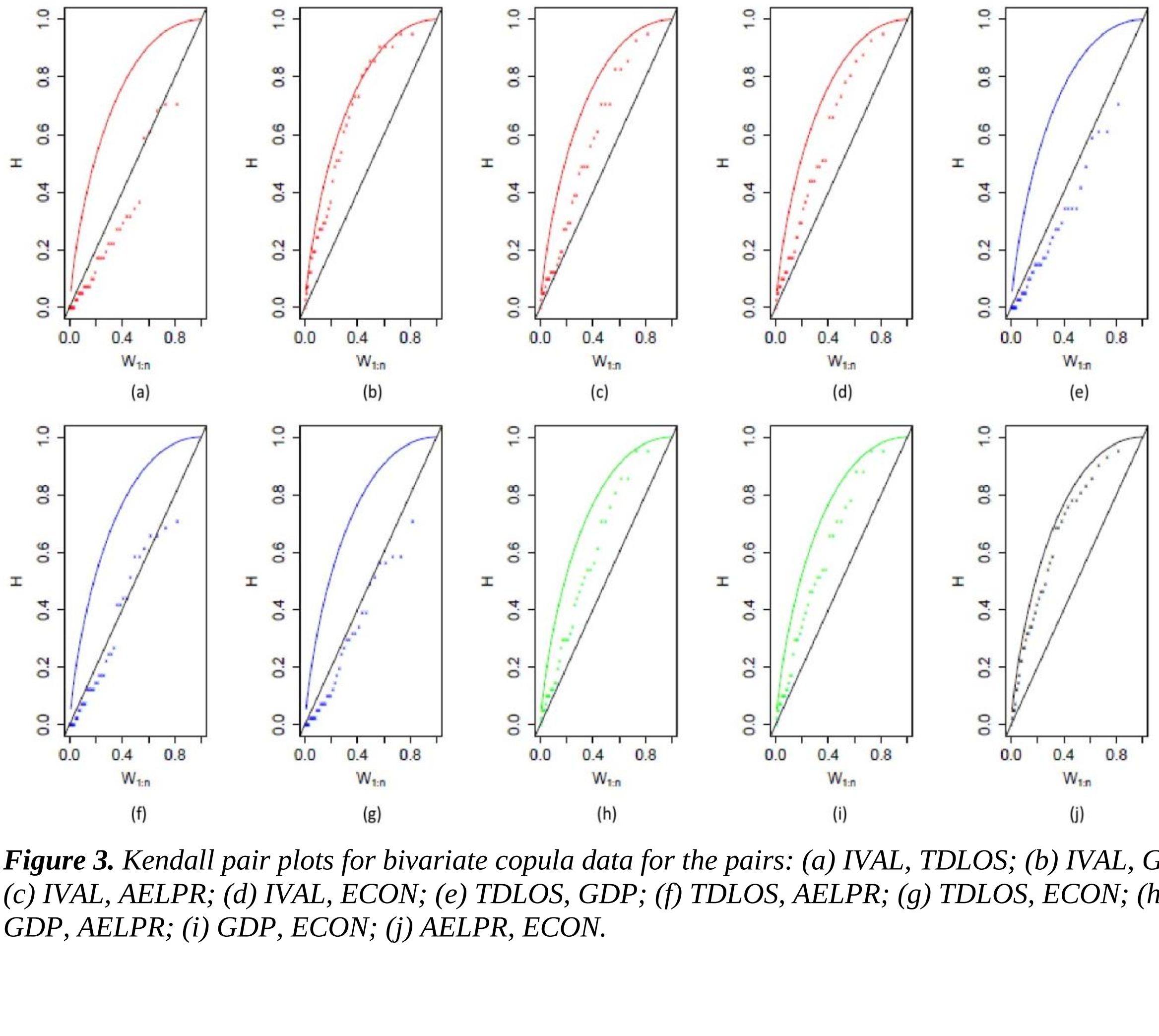

Figure 3. *Kendall pair plots for bivariate copula data for the pairs: (a) IVAL, TDLOS; (b) IVAL, GDP; (c) IVAL, AELPR; (d) IVAL, ECON; (e) TDLOS, GDP; (f) TDLOS, AELPR; (g) TDLOS, ECON; (h) GDP, AELPR; (i) GDP, ECON; (j) AELPR, ECON.*

In Figure 3, visualizations of Kendall Pair plots (K-plots), the possible positive and negative dependencies and their magnitudes are described based on whether the curve is located above or below the main diagonal line. For instance, the perfect negative dependence corresponds to the data points lying on the x-axis or closer. Conversely, for the perfect positive dependence, the points must be located around the curve above the line where $y = x$. Accordingly, there exists significant positive dependence between (IVAL, GDP), (IVAL, AELPR), (IVAL, ECON), (GDP, AELPR), (GDP, ECON) and (AELPR, ECON). On the other hand, for the pairs (IVAL, TDLOS), (TDLOS, GDP), (TDLOS, AELPR) and (TDLOS, ECON) no significant dependence is captured. Besides, (TDLOS, AELPR) and (TDLOS, ECON) are almost independent because of grouped data points around the main diagonal line.

4. RESULTS and DISCUSSIONS

After observing dependence patterns of variables, bivariate and multivariate copulas (vine models) are performed to investigate the tail dependence among the variables and vine copula modeling is examined with its tree structure and its interpretations.

4.1. Bivariate Copula Modeling

Appropriate copula family for the given pairwise series is estimated using the maximum likelihood (ML) estimation at which the best bivariate copula family is selected according to BIC values. For instance, the best fitted copula function is Survival Gumbel with parameter θ = 1.9231 for the pair (IVAL, ECON) given in Table 5. Here, the value of θ denotes a meaningful positive correlation, as it is positive and exceeds 1. Based on the same selection criteria, the most suitable copula model for each pair are studied and summarized in Table 5. Only the pairs (IVAL, TDLOS) and (IVAL, AELPR) fit to two-parameter copula families whose second parameters are given in parenthesis.

In Table 5, the higher parameter values of the selected copula families result in the higher dependence between two variables. As it is expected from Figure 3, the pairs (IVAL, TDLOS) and (TDLOS, GDP) has negative dependence structure, the former one is more significant based on its parameter. Furthermore, there exist a certain lower tail dependence patterns for the pairs (GDP, AELPR) and (GDP, ECON), having a reasonable interpretation from the economical point of view. At the lower tail, the impact of GDP is more significant on electricity demand because of higher parameter value presented above. In this respect, economic growth sustains its importance on the demand side of the Turkish electricity market.

Table 5. *Bivariate copula fit for each pair with their parameter estimates*

Pair	Copula Family	Parameter Estimates
(IVAL, TDLOS)	Tawn type 1 copula (270 degrees)	-6.4456 (0.1662)
(IVAL, GDP)	Joe copula	165.4306
(IVAL, AELPR)	Student-t copula	0.6640 (d=2)
(IVAL, ECON)	Survival Gumbel	1.9231
(TDLOS, GDP)	Gaussian	-0.4194
(TDLOS, AELPR)	independence	0
(TDLOS, ECON)	independence	0
(GDP, AELPR)	Survival Gumbel	1.9318
(GDP, ECON)	Survival Gumbel	2.1840
(AELPR, ECON)	Joe copula	150.2248

In examining tail behaviors to capture the behavior of extremes, the Coefficients of Tail Dependence (CoeffTD) which provide a measure of external dependence, equivalently, the level of the strength of dependence in the tails of a bivariate distribution are calculated (Table 6). The CoeffTD is defined in terms of limiting conditional probabilities of quantile exceedances for the given copula family.

Table 6. *Coefficients of tail dependence for each bivariate copula model*

Pair	Copula Family	Tail Type	CoeffTD
(IVAL, TDLOS)	Tawn type 1 copula (270 degrees)	None	-
(IVAL, GDP)	Joe copula	Upper	0.9958
(IVAL, AELPR)	Student-t copula	Lower, Upper	0.4931, 0.4931
(IVAL, ECON)	Survival Gumbel	Lower	0.5661
(TDLOS, GDP)	Gaussian	None	-
(TDLOS, AELPR)	independence	None	-
(TDLOS, ECON)	independence	None	-
(GDP, AELPR)	Survival Gumbel	Lower	0.5684
(GDP, ECON)	Survival Gumbel	Lower	0.6265
(AELPR, ECON)	Joe copula	Upper	0.9954

Based on the values given in the last column of Table 6, almost for all pairs, theoretical tail dependencies are remarkable (> 0.5 or ≈ 0.5). Naturally, no tail dependence exists for the pairs (IVAL, TDLOS), (TDLOS, GDP), (TDLOS, AELPR) and (TDLOS, ECON). For the Joe Copula selection, there exist perfect upper tail dependencies for the pairs (IVAL, GDP) and (AELPR, ECON). Besides, there exist positive dependence structure at lower tails for the pairs (IVAL, ECON), (GDP, AELPR) and (GDP, ECON). The only two tailed dependence structure is found for the pair (IVAL, AELPR), yielding much lower association compared to the other ones. The significant positive dependence in the upper tail for pairs (IVAL, GDP) and (AELPR, ECON) is meaningful as they behave similarly whenever they take higher values. Equivalently, when the value of GDP is increased, one can expect that IVAL is increased or vice versa, especially, on the higher observations. Survival Gumbel copula fits for the pairs (GDP, AELPR) and (GDP, ECON) represents that lower tail dependence structure overlaps with one of the main result of [21], GDP stimulates electricity sector by affecting both ECON and AELPR positively. Besides, another Survival Gumbel fit for the pair (IVAL, ECON) re-examines the discovered negative association between IVAL and ECON with respect to lower tail dependence, a substantial indicator for the required developments in Turkish electricity market. These findings are also supported by [15]. Besides, contour plots are illustrated to diagnose the dependencies in terms of probability measure via their corresponding probability density functions (pdf) and visualized the joint behavior of pairwise data (Figure 4). Two different tail behaviors for the pair (GDP, ECON) and (IVAL, AELPR) are depicted. The left panel in Figure 4 visualizes the Survival Gumbel Copula for the pair (GDP, ECON) which yields the significance of the lower tail dependence meaning to such joint tail behaviors re-examine the relationship between these variables with more information on extremes instead of focusing on either the association or causality among them. In a similar manner, at the right panel of Figure 4, given contour plot shows that IVAL and AELPR move together with respect to Student-t copula yielding joint behavior on both lower and upper tails. This means, both variables are likely to increase together as a result of the proper economic regulation in the long run compatible with the similar findings obtained using co-integration analysis.

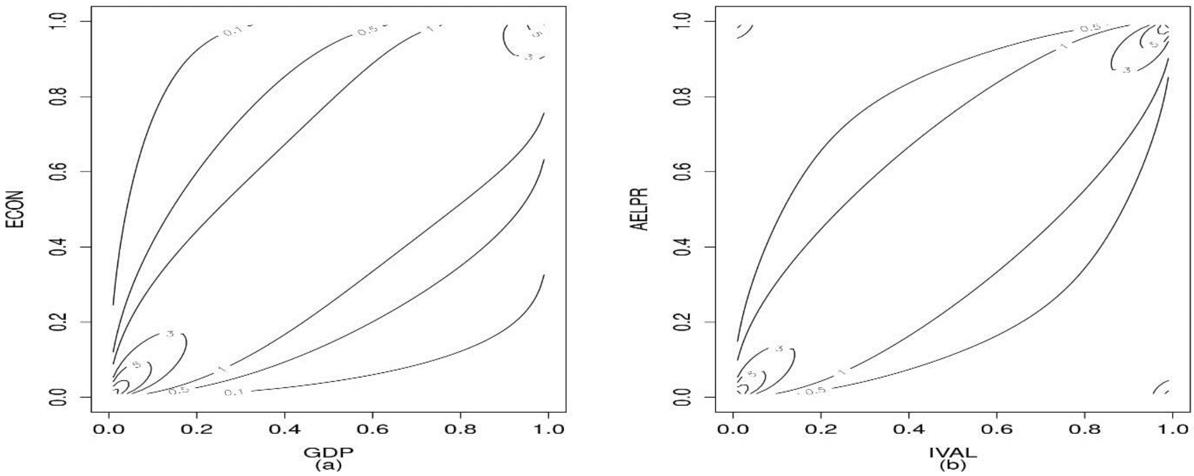

Figure 4. *Contour plots of fitted copulas: (a) Survival Gumbel for (GDP, ECON); (b) Student-t for pair (IVAL, AELPR)*

4.1. Vine Copula Modeling

To identify dependence between all selected variables in higher dimensions, the most of the correlated variables based on pairwise correlations given on the upper panel of Figure 2 are considered.

The general structure of vine copulas is based on the nested set of trees or graphs with certain properties. At each step, defined edges for the trees are generated by distinct bivariate copula families. We construct the Regular R-Vine (RV) and Canonical C-Vine (CV) copulas to capture the dependence in multivariate structure. For CV copula fit, GDP is selected as a root node in the first tree as it is one of the main indicator for economic growth. In each case, two vine structures are generated to measure the influence of assuming the independence copula on the pair copula construction. Table 7 presents Independent (I) and not independent (NI) classes for each vine models based on their selection criteria AIC or BIC which are indicated by subscripts 1 and 2, respectively.

Table 7. *The most significant scenarios selected in I and NI vine copula classes*

Model	I		NI	
	AIC	BIC	AIC	BIC
RV_1	-6899.68	-6889.25	-7469.92	-7443.85
RV_2	-6899.68	-6889.25	-7470.40	-7446.07
CV_1	-5699.87	-5685.97	-5701.67	-5675.61
CV_2	-5718.99	-5705.08	-5712.28	-5693.17

Table 8: *Comparison of RV_2 and CV_2 copula test results*

Model	p-value		AIC (p-value)		BIC (p-value)	
	Clarke	Vuong	Clarke	Vuong	Clarke	Vuong
RV_2 vs CV_2	0.0436	0.5233	0.04365	0.5261	0.0195	0.5286

Based on the values of AIC and BIC we see that RV_2(NI) performs better compared to RV_2 (I) case. Besides, the model RV_2 beats RV_1 in both I and NI scenarios. However, CV_2 is the plausible model both in I and NI cases. For both vine models, RV_i models yield lower AIC and BIC values compared to CV_i copulas for $i=1,2$, which implies that RV model is more suitable for the multivariate joint density. As a final step, to identify the most suitable vine copula model between best performing models (RV_2 and CV_2), Clarke and Vuong tests are performed ([22] and [23]) whose results are shown in Table 8. Primarily, RV_2 model is statistically different from CV_2 (p < 0,05) based on Clarke test. Besides, Vuong test implies that RV_2 is preferable to CV_2. The same result is coherent with the presented AIC and BIC values given in Table 7. Based on RV_2, for the joint density of selected parameters IVAL, TDLOS, GDP, AELPR and ECON, the vine tree structure is graphically presented in Figure 5. In this setup, the most suitable unconditional copula families for the selected pairs are given at T_1 and conditional ones for the rest of the vine copula modeling are displayed at T_2, T_3 and T_4. For instance, the most significant copula family is the Survival Gumbel for the pair (ECON; GDP) at the first tree, as an unconditional bivariate density.

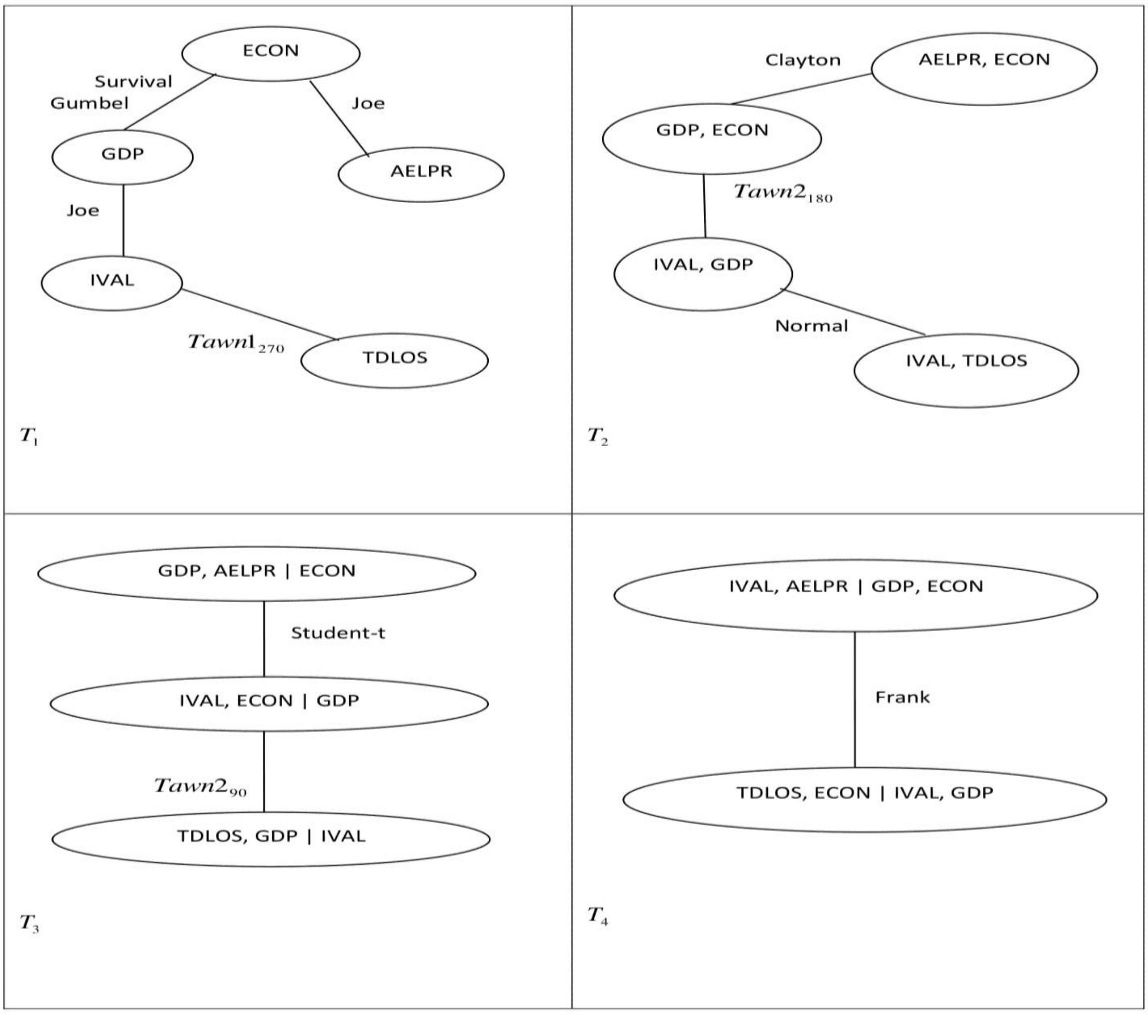

Figure 5. *Vine tree structures of selected R-vine model for* T_1, T_2, T_3 *and* T_4

Such a copula function exhibits strong right tail dependence, more precisely, both variables ECON and GDP behave similarly at the lower tail which is also justified in bivariate copula model. The observed Tawn copula families at tree T_1, T_2 and T_3 indicate different correlation structure. For instance, the rotated Tawn type copulas, $Tawn1_{270}$ at tree T_1 and $Tawn2_{90}$ at tree T_3 are the reasons for the existence of negative correlation for the unconditional pair (IVAL, TDLOS) and pair (ECON, TDLOS) conditional to (GDP, IVAL), respectively. On the other hand, the survival Tawn copula ($Tawn2_{180}$), exhibits a positive correlation structure for the pair (ECON, IVAL) conditional to GDP. The strong positive dependence patterns are captured for (GDP, IVAL) and (ECON, AELPR) by Joe Copula at tree T_1, meanwhile Clayton family is the clue of strong left tail dependence for the conditional pair (GDP, AELPR) given ECON at tree T_2. At the final tree, T_4, the conditional density of Frank copula for the conditional pair (AELPR, TDLOS) given (GDP, ECON, IVAL) identifies the symmetric positive correlation in both upper and lower tails.

5. CONCLUSIONS

Alternative to the conventional econometric techniques, copula approach is considered to explain the association between the electricity consumption and selected economic and sectorial indicators under the framework of bivariate and vine copulas. For this motivation, annual observations from Turkish electricity market between years 1970 and 2012 are utilized in this study. Especially, tail dependencies of selected variables are investigated and interpreted with the help of pair copula construction, which provides a useful guide for the joint behavior of series at extremes. Bivariate copula modeling of electricity consumption and other variables shows that Survival Gumbel comes out to be the best model for the pairs, (IVAL, ECON), (GDP, AELPR) and (GDP, ECON), which characterize lower tail dependency pattern precisely. R-vine and C-vine models are implemented with different scenarios to determine the best model; RV_2, with no independence pair copula. Taking into account the existence of tail properties, unconditional and conditional densities of the related pairs are examined and the joint behaviors of each pair can be interpreted conditionally. It is found that Clayton family describes the joint behavior of GDP with annual electricity price, AELPR, conditioned on electricity consumption, ECON. In terms of its tree structure, Clayton family depicts the existence of lower values for the pair GDP and price whenever the consumption decreases. The main advantage of the copula approach is focusing on the joint tail behaviors for the interested pair variables. Instead of only deriving the direction of the relationship and its duration (in the short or long run) with the help of classical econometric techniques, copulas allow to consider the movement of tail properties of both variables flexibly. The results of the study yield a strong evidence for the tail properties of the selected pairs, which simply imply the vital importance of a developed energy policy integrated with sustainable economic growth for an economy.

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CONFLICT OF INTEREST

No conflict of interest was declared by the author.

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