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Forced Vibration Analysis of Warping Considered Curved Composite Beams Resting on Viscoelastic Foundation

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Abstract

The forced vibration analysis of warping considered curved composite Timoshenko beams resting on viscoelastic foundation is investigated via the mixed finite element method. Rocking is considered both for Winkler and Pasternak viscoelastic foundations. Two nodded curved element has 12 degrees of freedom. Problems are solved in frequency domain via Laplace transform and modified Durbin's algorithm is used for back transformation to time domain. Warping considered average torsional rigidities of the composite cross-sections are calculated numerically by ANSYS and verified by the literature. After the verification of the algorithms, as benchmark examples, curved composite beams on rocking considered viscoelastic Pasternak foundation are solved.

1. INTRODUCTION

The composite materials have a wide range application area in recent technology such as aerospace, medical, mechanical, civil engineering, etc. The increasing use of composite materials due to their advantages in stiffness, strength and lightness has resulted a growing demand for the investigation of their structural behaviors.

In the case of non-circular composite cross-sections, torsional rigidity has a great influence on the static and dynamic analysis of curved beams. In such a case, torsional rigidity may be determined numerically in order to overcome the difficulties of analytical approaches. For example; finite element formulation of Saint-Venant torsion problem based on Prandtl stress functions or warping function let simple solutions. Although there is a great number of studies in the case of Saint-Venant torsion problem of isotropic composite sections, e.g. displacement type elements [1-6], stress functions [7-13] or some other numerical approaches [14-23], the studies of orthotropic composite sections are limited [24].

In spite of the vast number of studies examining static and free vibration analyses of curved beams resting on elastic foundation [25-40], the number of researches concerning the forced vibration analysis of curved beam on elastic or viscoelastic foundation is very limited. Thin circular ring resting on a tensionless Winkler foundation is considered under time dependent in-plane loads in [41]. A planar isotropic curved beam resting on Pasternak foundation is solved under time dependent load in Laplace domain by using mixed finite element method in [42]. [43,44] investigated curved Timoshenko beam on the rocking effect considered Winkler type elastic and viscoelastic foundation under triangle type impulsive loading in Laplace domain by using the complementary functions method.

In this study, the forced vibration analysis of planar curved composite beams is investigated by the mixed finite element formulation based on Timoshenko beam theory. The warping considered torsional rigidity

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of composite sections is calculated by the commercial finite element package ANSYS and verified by the finite element analysis given by [5]. In order to solve the problems in frequency domain, the element matrices of the two nodded curvilinear elements are transformed into Laplace domain. After the analysis, the results are transformed back to the time domain by Modified Durbin's transformation algorithm [45,46]. Regarding the presented numerical results; first, the mixed finite element formulation is verified through the forced vibration analysis of a planar curved isotropic beam resting on viscoelastic Winkler foundation. Here, the rocking effect is considered and the results are compared with those available in the literature. Then, the forced vibration analysis of a planar curved composite beam resting on viscoelastic Pasternak foundation is conducted. As a benchmark example, curved beams resting on rocking considered viscoelastic Pasternak foundation is handled, which is completely original for the literature where the effects of some parameters (e.g. the opening angle of curved beam, foundation type, time-dependent load type) are discussed.

2. FORMULATION

2.1. Constitutive Relations for Laminates of a General Curved Beam

The constitutive equations of linear elasticity theory yields $\boldsymbol{\sigma} = \mathbf{E} : \boldsymbol{\varepsilon}$, where $\boldsymbol{\sigma}$ is the stress tensor, $\boldsymbol{\varepsilon}$ is the strain tensor and $\mathbf{E}$ is the function of elastic constants. The derivation of the constitutive equations of a composite beam exists in [47,48]. Thus, in Frenet coordinate system (see Fig. 1), paying attention to $\sigma_n = \sigma_b = \tau_{nb} = 0$, the constitutive relations yield $\{\sigma_t \quad \tau_{bt} \quad \tau_{tn}\}^T = [\boldsymbol{\beta}]\{\varepsilon_t \quad \gamma_{bt} \quad \gamma_{tn}\}^T$ where the matrix $[\boldsymbol{\beta}]_{3\times3}$ is a function of orthotropic material constants. Timoshenko beam theory requires shear correction factors and it is assumed to be $5/6$ for a general rectangular cross-section. Letting u_t, u_n, u_b the displacements on the beam axis and Ω_t, Ω_n, Ω_b the rotations of the beam cross-section around the *t*, *n*, *b* Frenet coordinates, respectively, by means of the kinematic equations strain terms are expressed as

$$\begin{Bmatrix} \varepsilon_t \\ \gamma_{bt} \\ \gamma_{tn} \end{Bmatrix} = \begin{Bmatrix} u_{t,t} \\ u_{t,b} + u_{b,t} \\ u_{t,n} + u_{n,t} \end{Bmatrix} + b \begin{Bmatrix} \Omega_{n,t} \\ 0 \\ -\Omega_{t,t} \end{Bmatrix} + n \begin{Bmatrix} -\Omega_{b,t} \\ \Omega_{t,t} \\ 0 \end{Bmatrix} \tag{1}$$

The resultant forces and couple moments at a cross-section can be derived by analytical integration of the stresses and their moments in each layer through the thickness of the cross-section, respectively.

$$\begin{aligned} T_t &= \textstyle\sum_{L=1}^{N}\left(\int_{-0.5n_L}^{0.5n_L}\left(\int_{b_{L-1}}^{b_L} \sigma_t \mathrm{d}b\right)\mathrm{d}n\right) \\ T_b &= \textstyle\sum_{L=1}^{N}\left(\int_{-0.5n_L}^{0.5n_L}\left(\int_{b_{L-1}}^{b_L} \tau_{bt} \mathrm{d}b\right)\mathrm{d}n\right) \\ T_n &= \textstyle\sum_{L=1}^{N}\left(\int_{-0.5n_L}^{0.5n_L}\left(\int_{b_{L-1}}^{b_L} \tau_{tn} \mathrm{d}b\right)\mathrm{d}n\right) \end{aligned} \tag{2}$$

$$\begin{aligned} M_t &= \textstyle\sum_{L=1}^{N}\left(-\int_{-0.5n_L}^{0.5n_L}\left(\int_{b_{L-1}}^{b_L} b\,\tau_{tn} \mathrm{d}b\right)\mathrm{d}n\right) + \sum_{L=1}^{N}\left(\int_{b_{L-1}}^{b_L}\left(\int_{-0.5n_L}^{0.5n_L} n\,\tau_{tb} \mathrm{d}n\right)\mathrm{d}b\right) \\ M_n &= \textstyle\sum_{L=1}^{N}\left(\int_{-0.5n_L}^{0.5n_L}\left(\int_{b_{L-1}}^{b_L} b\sigma_t \mathrm{d}b\right)\mathrm{d}n\right) \\ M_b &= -\textstyle\sum_{L=1}^{N}\left(\int_{b_{L-1}}^{b_L}\left(\int_{-0.5n_L}^{0.5n_L} n\sigma_t \mathrm{d}n\right)\mathrm{d}b\right) \end{aligned} \tag{3}$$

N is the number of the layers, n_L is the width of the layer, b_L and b_{L-1} are the directed distances to the bottom and the top of the L^{th} layer where *b* is positive in upwards direction. Finally, in accordance with the above equations, the constitutive equation in a matrix form yields to the form

$$\{\varepsilon_t \quad \gamma_{tn} \quad \gamma_{bt} \mid \kappa_t \quad \kappa_n \quad \kappa_b\}^T = \begin{bmatrix} \mathbf{C}_m & \mathbf{C}_{mf} \\ \mathbf{C}_{fm} & \mathbf{C}_f \end{bmatrix} \{T_t \quad T_n \quad T_b \mid M_t \quad M_n \quad M_b\}^T \tag{4}$$

where $\kappa_t, \kappa_n, \kappa_b$ are curvatures and $\mathbf{C}_m$, $\mathbf{C}_f$, $\mathbf{C}_{mf}$, $\mathbf{C}_{fm}$ are compliance matrices of the elastic material where $\mathbf{C}_{mf}$, $\mathbf{C}_{fm}$ are coupling matrices [49,50].

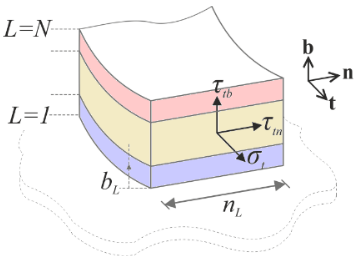


Figure 1. *The stresses in the Frenet Coordinate System (N: Total number of layers)*

2.2. Functional In Laplace Domain and Mixed Finite Element Method

In Frenet coordinate system, the field equations and functional based on the Gâteaux differential and potential operator concept [51] for the isotropic homogenous spatial Timoshenko beam exist in [52,53,6]. The field equations and the functional are extended to laminated composite beams in [54,55]. The formulation of the elastic Pasternak and rocking foundation exists in [56,57]. In this study, as an original inclusion, rocking considered viscoelastic Pasternak foundation is formulated. Forced vibration problem of curved composite beams is examined in frequency domain and the field equations are transformed into Laplace space and the functional in frequency domain yields:

$$\begin{aligned} \mathbf{I}(\bar{\mathbf{y}}) = & -[\bar{\mathbf{u}}, \bar{\mathbf{T}}_{,s}] + [\mathbf{t}\times\bar{\mathbf{\Omega}}, \bar{\mathbf{T}}] - [\bar{\mathbf{M}}_{,s}, \bar{\mathbf{\Omega}}] - \tfrac{1}{2}\Big\{\big[\bar{\mathbf{C}}_m\bar{\mathbf{T}}, \bar{\mathbf{T}}\big] + \big[\bar{\mathbf{C}}_{mf}\bar{\mathbf{M}}, \bar{\mathbf{T}}\big] \\ & + \big[\bar{\mathbf{C}}_{fm}\bar{\mathbf{T}}, \bar{\mathbf{M}}\big] + \big[\bar{\mathbf{C}}_f\bar{\mathbf{M}}, \bar{\mathbf{M}}\big]\Big\} + \tfrac{1}{2}\Sigma\big(\rho A z^2 [\bar{\mathbf{u}}, \bar{\mathbf{u}}]\big) + \tfrac{1}{2}\Sigma\big(\rho I z^2 [\bar{\mathbf{\Omega}}, \bar{\mathbf{\Omega}}]\big) \\ & - [\bar{\mathbf{q}}, \bar{\mathbf{u}}] - [\bar{\mathbf{m}}, \bar{\mathbf{\Omega}}] + \tfrac{1}{2}\big[(\bar{\mathbf{k}}_W)^T\bar{\mathbf{u}}, \bar{\mathbf{u}}\big] + \tfrac{1}{2}\big[(\bar{\mathbf{k}}_G)^T\bar{\mathbf{u}}_{,s}, \bar{\mathbf{u}}_{,s}\big] + \tfrac{1}{2}\big[(\bar{\mathbf{k}}_R)^T\bar{\mathbf{\Omega}}, \bar{\mathbf{\Omega}}\big] \\ & + \Big(\big[(\bar{\mathbf{T}} - \hat{\mathbf{T}}), \bar{\mathbf{u}}\big] + \big[(\bar{\mathbf{M}} - \hat{\mathbf{M}}), \bar{\mathbf{\Omega}}\big] - \big[\bar{\mathbf{k}}_G\hat{\mathbf{u}}_{,s}, \bar{\mathbf{u}}\big]\Big)_\sigma + \Big(\big[\hat{\mathbf{u}}, \bar{\mathbf{T}}\big] + \big[\hat{\mathbf{\Omega}}, \bar{\mathbf{M}}\big] - \big[\bar{\mathbf{k}}_G\bar{\mathbf{u}}_{,s}, (\bar{\mathbf{u}} - \hat{\mathbf{u}})\big]\Big)_\varepsilon \end{aligned} \tag{5}$$

where s is the arc axis of the spatial beam, z is Laplace transformation parameter. Laplace transformed variables are denoted by the over bars [45,46]. $\bar{\mathbf{k}}_W(\bar{k}_{Wt}, \bar{k}_{Wn}, \bar{k}_{Wb})$ and $\bar{\mathbf{k}}_G(\bar{k}_{Gt}, \bar{k}_{Gn}, \bar{k}_{Gb})$ are the viscoelastic foundation vectors of Winkler and Pasternak, respectively. $\bar{\mathbf{k}}_R(\bar{k}_{Rt}, \bar{k}_{Rn}, \bar{k}_{Rb})$ is the viscoelastic foundation rocking stiffness vector. If $\bar{\mathbf{k}}_R = 0$, (5) yields to viscoelastic Pasternak foundation. If we also let $\bar{\mathbf{k}}_G = 0$, (5) yields to viscoelastic Winkler foundation. In (5), if we let only $\bar{\mathbf{k}}_G = 0$, while $\bar{\mathbf{k}}_R$ is non-zero, then we obtain rocking considered viscoelastic Winkler foundation. In (5), the square brackets indicate the inner product, the terms with hats are known values on the boundary and the subscripts ε and σ represent the geometric and dynamic boundary conditions, respectively. Two-nodded 2×12 degrees of freedom curved element is employed [52] for the discretization.

2.3. Numerical Inverse Laplace Transformation Algorithm

The finite element analysis of curved beam resting on viscoelastic Pasternak foundation is carried out in the Laplace domain and modified Durbin's algorithm is used for the inversion of the results back to time space. This algorithm is developed from Durbin's numerical inverse Laplace transformation method [58-60]. The parameters used in the analysis for inverse Laplace transformation algorithm were discussed in [45].

3. NUMERICAL EXAMPLES

As a verification example, forced vibration results of a planar curved isotropic beam resting on rocking effect considered viscoelastic Winkler foundation is compared with [44]. Next, forced vibration analysis of a planar curved composite beam resting on rocking effect considered viscoelastic Pasternak foundation is performed.

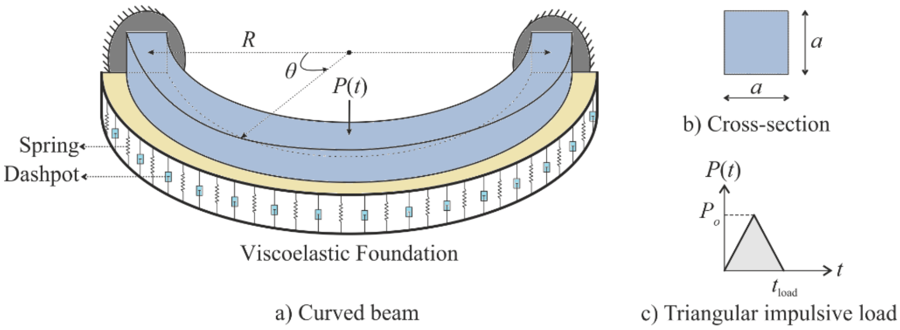


Figure 2. *A planar curved beam on viscoelastic Winkler foundation*

3.1. Curved Isotropic Beam Resting On Viscoelastic Rocking Winkler Foundation

The forced vibration problem of a planar curved isotropic square cross-sectioned beam under a triangular impulsive type load $P_z(t)$ acting at the midpoint of beam (Figure 2) is solved. The fixed-fixed end condition is employed. The radius of curved beam is $R = 7.63\text{m}$. The opening angle is $\theta = 180°$. The dimension of square cross-section is $a = 0.762\text{m}$. The modulus of elasticity of the beam is $E = 47.24\text{GPa}$, Poisson's ratio is $\upsilon = 0.2$, the density of material is $\rho = 5000\ \text{kg/m}^3$. The component of Winkler foundation parameter in the direction of b is $k_{Wb} = 23.623\text{MPa}$, the foundation rocking stiffness constant in the direction of t is $k_{Rt} = 1143\text{kNm/m}$. The intensity and the duration of the loading are $P_o = 100\,\text{kN}$ and $t_{load} = 0.064\text{s}$, respectively. The dynamic response of the beam is determined within $0 \le t \le 0.25\text{s}$.

Convergence analysis: The dynamic analysis of the semi-circular curved beam resting on viscoelastic Rocking Winkler foundation is carried out for $\eta_{Wb} = 2362.3\,\text{Ns/m}^2$ and $\eta_{Rt} = 2362.3\,\text{Ns}$ using 40, 60, 80 and 100 finite elements. The time history curve of vertical displacement $u_b(t)$ at the midpoint of the beam (Figure 2) are presented for $0 \le t \le 0.25\text{s}$ in Figure 3. When Figure 3 is examined for $t = 0.032\text{s}$, it is observed that the percentage differences for 80 elements with respect to 100 elements are zero for u_b. Thus, in the following examples, 80 elements are employed.

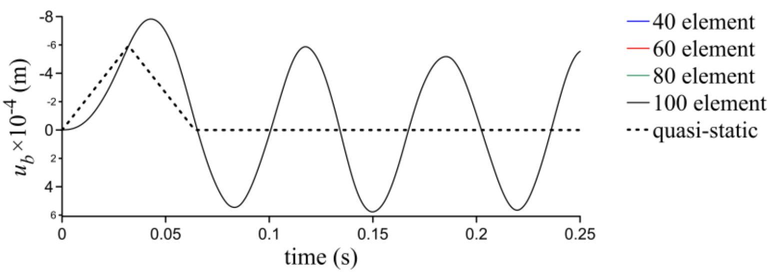


Figure 3. *Convergence analysis of the vertical displacement $u_b(t)$ of the curved beam on viscoelastic Rocking Winkler foundation*

Verification: The curved isotropic beam resting on viscoelastic Winkler foundation with rocking effect (Figure 2) is solved and the influence of the viscosity coefficients ($\eta_{Wb} = 0;\ 2362.3\,\text{Ns/m}^2$;

$23623\,\text{Ns/m}^2$; $236230\,\text{Ns/m}^2$, $\eta_{Rt}=0$; $2362.3\,\text{Ns}$; $23623\,\text{Ns}$; $236230\,\text{Ns}$) on the dynamic behavior of the curved beam is investigated. The mixed finite element results (displacement $u_b(t)$ and rotation $\Omega_t(t)$ at the midpoint of the beam and the moment $M_t(t)$ at the fixed end of the beam (Figure 2a)) are presented in Figure 4 and compared with literature [44]. It is observed that, all the results are compatible with each other. As viscosity increases, the amplitude of the displacement $u_b(t)$, the rotation $\Omega_t(t)$ and the moment $M_t(t)$ (see Figure 4) decrease. If the first extrema of $u_b(t)$, $\Omega_t(t)$ and $M_t(t)$ in each $\eta_{Wb}=\eta_{Rt}=2362.3\,\text{Ns/m}^2$, $23623\,\text{Ns/m}^2$, $236230\,\text{Ns/m}^2$ are normalized with respect to the results of $\eta_{Wb}=\eta_{Rt}=0$, the percent reductions for u_b are -0.44%, -4.28% and -30.6%; for Ω_t are -0.60%, -5.69% and -38.2%; for M_t are -0.36%, -3.68% and -34.6%, respectively.

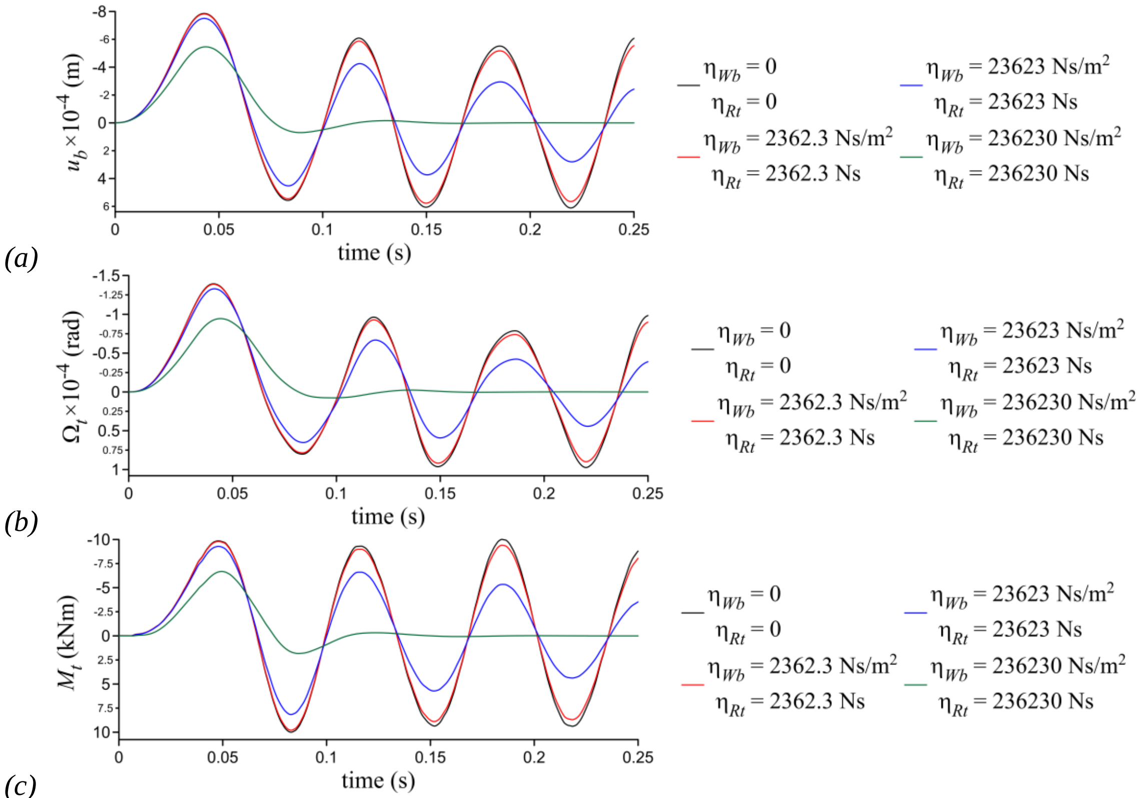


***Figure 4.** Time histories of curved beam on viscoelastic Rocking Winkler foundation for different viscosity coefficients (a) vertical displacement u_b, (b) rotation Ω_t, (c) moment M_t*

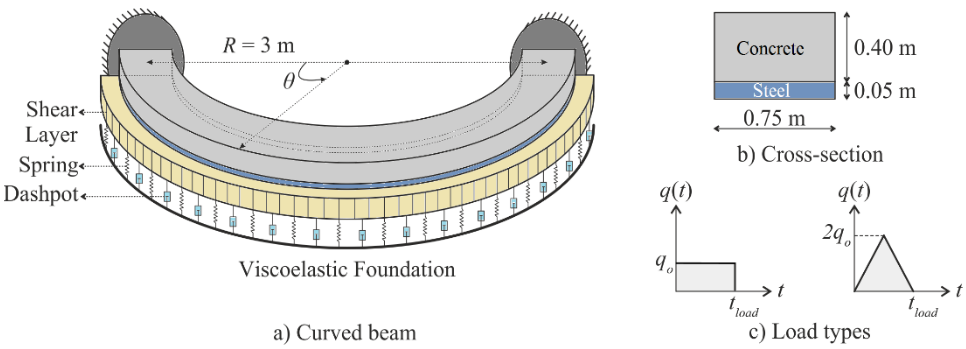
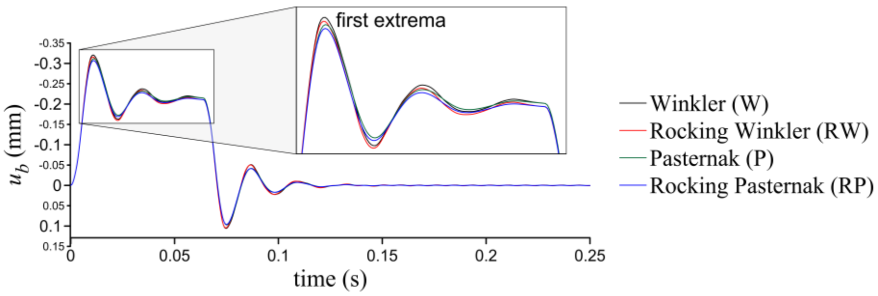


***Figure 5.** The planar curved composite beam resting on viscoelastic Pasternak foundation*

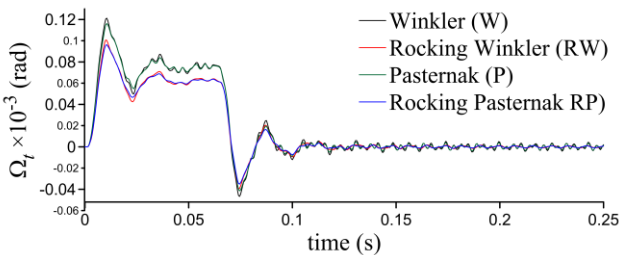
3.2. Curved Composite Beam Resting On Viscoelastic Rocking Pasternak Foundation

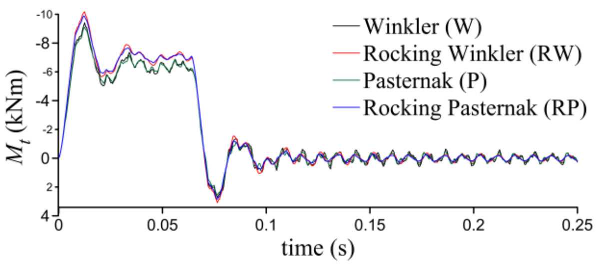
A circular-curved, composite beam made of steel and concrete rectangular cross-section is shown in Figure 5. The moduli of elasticity are $E_s = 210\,\text{GPa}$ and $E_c = 30\,\text{GPa}$, Poisson′s ratios are $\upsilon_s = 0.3$ and $\upsilon_c = 0.2$, the density of materials are $\rho_s = 7850\,\text{kg/m}^3$ and $\rho_c = 2400\,\text{kg/m}^3$. The radius of curved composite beam is $R = 3\,\text{m}$, the opening angles of the circular beam axis are chosen as $\theta = 45°, 90°, 135°, 180°$. The width of rectangular cross-section $b = 0.75\,\text{m}$, the height of steel and concrete are $h_s = 0.05\,\text{m}$ and $h_c = 0.40\,\text{m}$, respectively (see Figure 5b). The parameters of the viscoelastic Pasternak foundation $k_{Wb} = 69\,\text{MPa}$, $k_{Rt} = 13.8\,\text{MN}$, $\eta_{Wb} = 138\,\text{kNsm}^{-2}$, $\eta_{Rt} = 5520\,\text{Ns}$, and $\eta_{Gb} = 100\,\text{kNs}$. The two form of the impulsive uniform dynamic load are rectangular and triangular with a maximum intensity $q_o = 14.1\,\text{kN/m}$ applied in a time interval $t_{load} = 0.064\,\text{s}$. The time histories of $u_b(t)$ and $\Omega_t(t)$ at the midpoint, $T_b(t)$, $M_t(t)$ and $M_n(t)$ at the fixed end of the beam are evaluated within the time interval $0 \le t \le 0.25\,\text{s}$.

Influence of the viscoelastic foundation models on the dynamic behavior of structure: The planar semi-circular composite beam under rectangular impulsive type load is analyzed. The viscoelastic foundation models are Winkler (W), Rocking Winkler (RW), Pasternak (P), and Rocking Pasternak (RP). The time histories of $u_b(t)$, $\Omega_t(t)$ and $M_t(t)$ are presented in Figure 6. The results of RW, P and RP models for first extrema value of $u_b(t)$ are normalized with respect to W model as shown in Figure 6(a), where the percent reductions for RW, P and RP models are -1.5%, -2.8% and -4.2%, respectively. To investigate the effect of rocking on the viscoelastic foundation, the rocking parameters are increased by 10%, 20%, and 30% where Winkler and Pasternak parameters are kept constant. The first extrema values that occur within the forced vibration zone for RW and RP models are normalized with W model and results are shown in Figure 7. It is observed that the rocking effect is more influential in the rotation $\Omega_t(t)$.



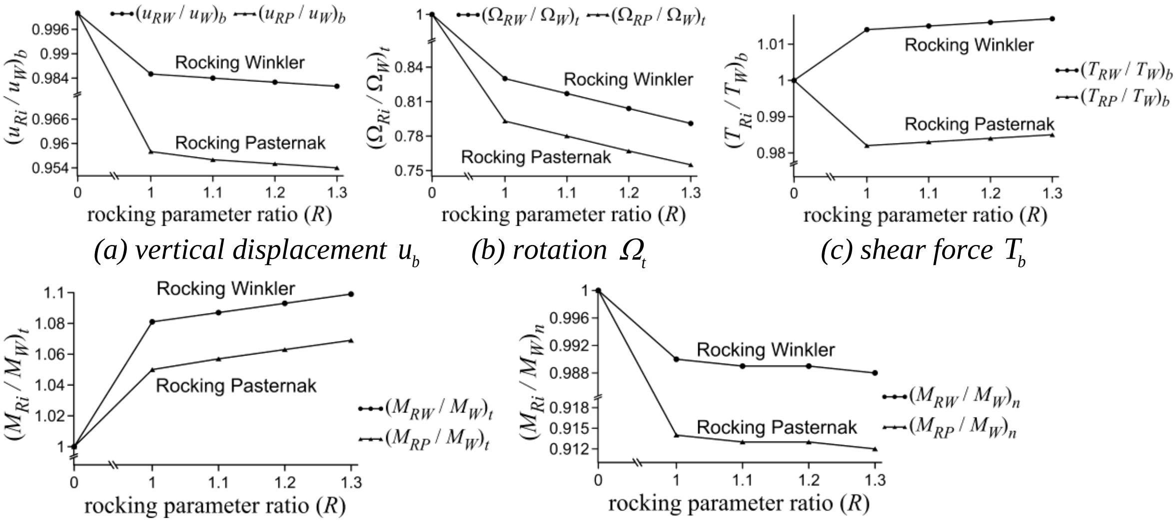
(a) $u_b(t)$ at the midpoint of the beam





(b) $\Omega_t(t)$ at the midpoint of the beam *(c) $M_t(t)$ at the fixed end of the beam*

Figure 6. *The influence of viscoelastic foundation type on the behavior of the semi-circular composite beam*

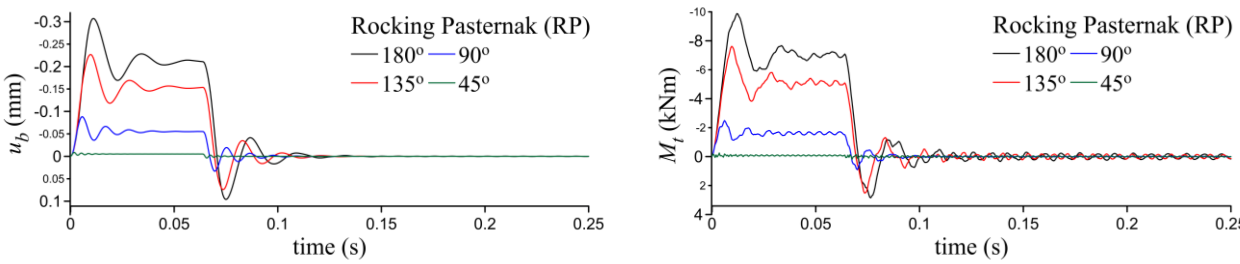


(a) vertical displacement u_b *(b) rotation Ω_t* *(c) shear force T_b*

(d) Moment M_t *(e) Moment M_n*

Figure 7. *The influence of rocking parameter on the behavior of the semi-circular composite beam ($\bar{k}_R = R\bar{k}_{Ro}$ where $\bar{k}_{Ro} = k_{Ro} + \eta_{Ro} z$ and R: rocking parameter ratio; $i: W, P$)*

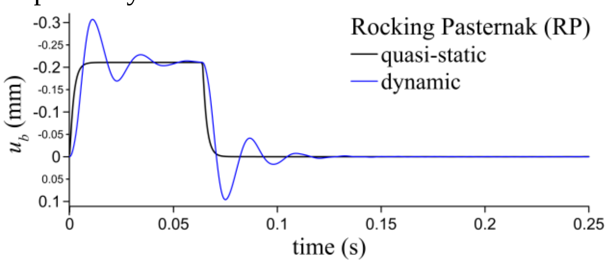
Influence of the curved beam geometry on the dynamic behavior of structure: Beams of various geometries resting on viscoelastic Rocking Pasternak foundation (RP) and subject to rectangular impulsive loading are analyzed. The time history responses of the beams are given in Figure 8. By the increasing opening angles $\theta = 45°, 90°, 135°, 180°$, u_b is observed to be in an increasing trend. If the values of first extrema u_b for the cases $\theta = 45°, 90°, 135°$ is normalized with respect to the results of $\theta = 180°$, the reductions are -97.2% , -71.2% and -26.2%, respectively. The vibration period decreases due to a decrease in opening angle (see Figure 8).



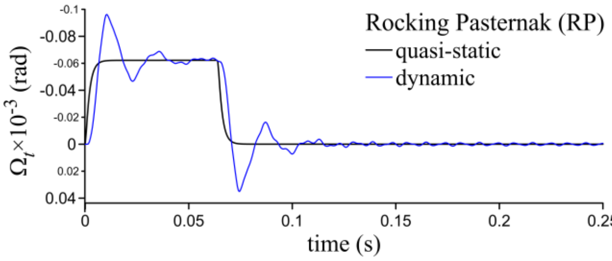
(a) $u_b(t)$ at the midpoint of the beam *(b) $M_t(t)$ at the fixed end of the beam*

Figure 8. *The time histories of curved composite beam on viscoelastic Rocking Pasternak foundation*

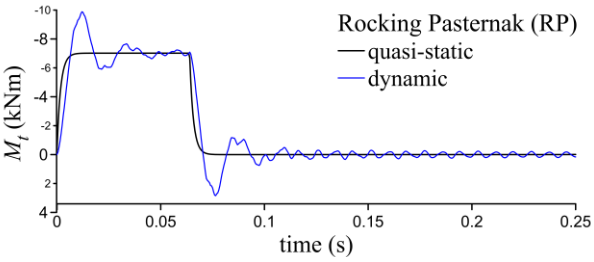
The influence of time dependent load type on the behavior of structure: The results of the semi-circular beam under triangular impulsive loading (Figure 5c) are compared with those of the beam under rectangular impulsive loading (for $\theta = 180°$ in Figure 8). The area of the forced vibration zone is kept constant for rectangular and triangular (Figure 5c) impulsive loads. The time histories of $u_b(t)$ and $\Omega_t(t)$ at the midpoint of the beam, $M_t(t)$ at the fixed end of the beam are given for both the impulsive load cases in Figures. 9-10. The values of $u_b(t)$, $\Omega_t(t)$ and $M_t(t)$ for quasi-static case that occur at $t = 0.032\text{s}$ are determined using Figure 9 and Figure 10 and the results of u_b, Ω_t, M_t for triangular type load are normalized with respect to the rectangular type load. It is observed that, the percent increases in the case of the triangular impulsive load range between $89.4\% \sim 91.8\%$. If first extrema values of $u_b(t)$, $\Omega_t(t)$ and $M_t(t)$ for dynamic case in Figure 10 are normalized with respect to the rectangular impulsive load, the percent increases in the case of the triangular impulsive load are 41.6%, 35.6%, 44.1%, respectively.



(a) $u_b(t)$ *at the midpoint of the beam*

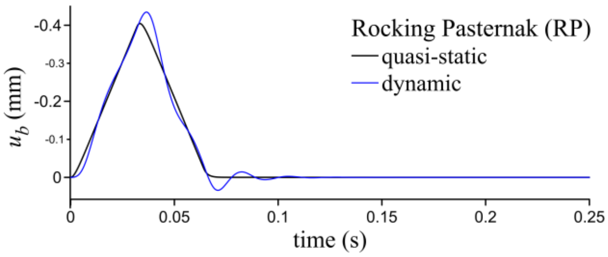


(b) $\Omega_t(t)$ *at the midpoint of the beam*

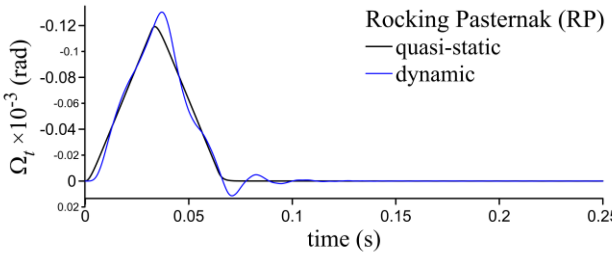


(c) $M_t(t)$ *at the fixed end of the beam*

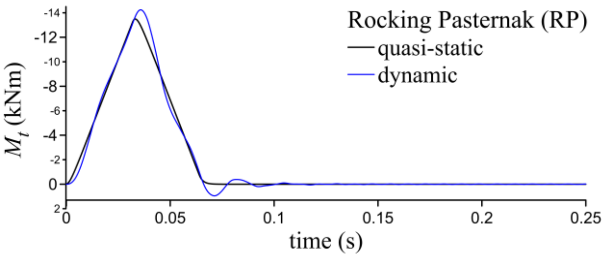
Figure 9. *Time histories of semi-circular composite beam resting on viscoelastic Rocking Pasternak foundation for rectangular impulsive type loading*



(a) $u_b(t)$ *at the midpoint of the beam*



(b) $\Omega_t(t)$ *at the midpoint of the beam*



(c) $M_t(t)$ *at the fixed end of the beam*

Figure 10. *Time histories of semi-circular composite beam resting on viscoelastic Rocking Pasternak foundation for triangular impulsive type loading*

4. CONCLUSION

Dynamic behavior of planar curved Timoshenko beams on viscoelastic Pasternak foundations having rectangular composite cross-section is investigated via the mixed finite element method. The warping effect of composite cross-section of curved beam is considered. The rocking influence is also considered in viscoelastic foundation model. The solutions are obtained in Laplace space and the results are transformed back to time space by using modified Durbin's algorithm. First, a semicircular beam having isotropic square cross-section resting on Winkler viscoelastic foundation is handled, the mixed finite element results of the planar curved beam are compared with the literature and a good agreement is observed. As benchmark examples, planar curved composite beams resting on viscoelastic Rocking Pasternak foundation are analyzed and the influence of the viscoelastic foundation type (W, RW, P and

RP), the opening angle of the curved beam and time-dependent load type is investigated. According to the authors' best knowledge, this discussion is original for the literature. The following remarks can be cited: Since shear effect is considered in viscoelastic Pasternak model (P) when compared with Winkler model (W), vertical displacements due to P model decreased with respect to W model. Consideration of the rocking effect increased the decrease trend of vertical displacement u_b, rotation Ω_t and moment M_t (see Figure 6).

The influence of the rocking parameters on the dynamic behavior of the structure are investigated in detail for viscoelastic Rocking Winkler (RW) and viscoelastic Rocking Pasternak (RP) foundation models and the results are normalized with respect to viscoelastic Winkler (W) foundation model. Pasternak and rocking parameters (viscoelastic RP model) cause reduction in the vertical displacement u_b, the rotation Ω_t, the shear force T_b and the moment M_n (except moment M_t). The influence of rocking parameters is more apparent in the rotation Ω_t (see Figure 7). It means that, the type of viscoelastic foundation should be selected in accordance with the purpose and importance of structure.

As expected an increase in the opening angles causes an increase in vertical displacement u_b, rotation Ω_t and moment M_t along the forced vibration zone and the dynamic response oscillates around zero along the free vibration zone (see Figure 8).

Type of time-dependent load is efficacious on the behavior of the structure. Simple forms of time variations of loadings are considered in the examples (see Figures. 9-10). Therefore, for a more complex form of dynamic loading, such as time history of an earthquake, the influence of the loading need to be investigated, especially for the high-tech structures.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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