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THE RELATION BETWEEN SOFT TOPOLOGICAL SPACE AND SOFT DITOPOLOGICAL SPACE

GÜZIDE ŞENEL

Abstract. Conditions related to bounds on the relations between soft spaces appear to be rare in the literature. In this paper, I study the notion of soft ditopology relates to the soft topology. Firstly, the soft ditopology via soft set theory is developed by defining soft ditopological subspace. Secondly, properties concerning to soft interior and soft closure are presented in soft ditopological subspace. In conclusion, soft subspaces of soft topology and soft ditopology being coincident have been proved, whence it is readily inferred that soft ditopological subspace can be obtained from soft topological subspace.

1. Introduction

In 1999, Molodtsov [14] proposed a new approach, viz. soft set theory for modeling vagueness and uncertainties inherent in many related concepts with the theory and the application of soft sets. After this invention, in 2002 and 2003, very readable account of this theory has been given by Maji *et al.* [10, 11] on some mathematical aspects of soft sets and fuzzy soft sets. However, over the last fifteen years, there have been many examples are defined by [1], [4], [12] and [19] in the literature related to soft topology. By using this operations, the theory of soft topological space defined by Shabir and Naz [18] over an initial universe. There are numerous results in the literature relating soft topology. In the course of analyzing the theory of soft topology I learned that the authors [9] and [13] have simultaneously obtained results similar to each other in certain respects. One of the finest work in them, Aygünoglu and Aygün [2] introduced the soft product topology and defined the soft compactness to investigate the behavior of topological structures in soft set theoretic form. In view of this and also considering the importance of topological structure in developing soft set theory, I have introduced in this paper a notion of soft ditopological subspaces. In this connection, it is worth mentioning that some

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significant works have been done on soft ditopological spaces (SDT - Spaces) by Dizman [6] and Senel [16].

They defined basic notions and concepts of soft ditopological spaces such as soft open and closed sets, soft interior, soft closure, soft basis, soft complement and established several properties of these soft notions. Rather than discuss these works in full generality, let us look at a particular situation of this kind: In the work of Dizman [6], the concept of soft ditopological space is introduced with two structures which one related to the property of openness in the space and the other one relayed on the property of closeness in the space. This is a clear contradiction of the fact that if we know the soft topology on a soft set, we can easily get soft open and soft closed sets by complement operation. In that way, in a soft ditopological space that includes same characteristic properties composed with openness and closeness of each other.

In order to make a more comprehensive research, I have decided to develop the theory; considering that rather than studying with the triple structure that is built by the spaces made through one another, it is more beneficial to study a triple structure that contains a different and a new space.

The notion of soft ditopology by Senel [16] is more general than that by Dizman [6]. The concept of soft ditopological (SDT) space on a soft set in [16] with two structures on it is being introduced - a soft topology and a soft subspace topology. The first one is used to describe soft openness properties of a soft topological space while the second one deals with its sub - soft openness properties. This structure enables to study with all soft open sets that can be obtained on a soft set. Therefore, I continue investigating the work of Senel [16] and follow this theory's notations and mathematical formalism.

In this paper, the detailed analysis of SDT - space is carried out in Section 2. In this section, I introduce a new concept called soft ditopological subspace in soft ditopological space by giving two different definitions that are not a consequence of each other. Although, these definitions run along different lines, I prove that only one and the same soft ditopological subspace can be established on the same soft set by giving examples.

I also observe relations of soft ditopological space and soft ditopological subspace in different cases with soft open and soft closed sets. It shows how soft sets in soft ditopological space can preserve their properties in soft ditopological subspaces. In this context, I serve a bridge among soft ditopological space theory and soft ditopological subspace theory.

In the last section, I analyze the relationship between soft topology and soft ditopology. The two characteristics, soft subspace and the soft subspace of a soft subspace, are connected, but the relationship is quite a complex one. Although individually these systems can still be quite complicated, a possibly more tractable task is to describe their possible joint distributions. The aim of this article is to study the relationship between the soft topology and the extent to which soft topology to be soft ditopology. In this paper I wish to renew an interest in the systematic study of the relationships between topological spaces with respect to soft set theory.

2. Preliminaries

Following the works of Molodtsov [14], Maji et al. [10, 11] and Aktas and Cagman [3] some definitions and preliminary results are presented in this section. The following basic properties of soft sets have been given in [2], [5], [14], [15], [16], [17]. Unless otherwise stated, throughout this paper, U refers to an initial universe, E is a set of parameters and $P(U)$ is the power set of U.

Definition 2.1. [5, 14] A soft set f on the universe U is a set defined by

$$f : E \to \mathcal{P}(U) \text{ such that } f(e) = \emptyset \text{ if } e \in E \setminus A \text{ then, } f = f_A$$

Here f is also called an approximate function. A soft set f on the universe U is a set defined by

$$f = \Big\{ \big(e, f(e)\big) : e \in E \Big\}$$

We will identify any soft set f with the function $f(e)$ and we use that concept as interchangeable. Soft sets are denoted by the letters f, g, h, ... and the corresponding functions by $f(e)$, $g(e)$, $h(e)$, ...

Throughout this paper, the set of all soft sets over U will be denoted by $\mathbb{S}$. From now on, for all the undefined concepts about soft sets, we refer to: [5].

Definition 2.2. [5] Let $f \in \mathbb{S}$. Then,
If $f(e) = \emptyset$ for all $e \in E$, then f is called an empty set, denoted by Φ.
If $f(e) = U$ for all $e \in E$, then f is called universal soft set, denoted by $\tilde{E}$.

Definition 2.3. [5] Let $f, g \in \mathbb{S}$. Then,
f is a soft subset of g, denoted by $f \widetilde{\subseteq} g$, if $f(e) \subseteq g(e)$ for all $e \in E$.
f and g are soft equal, denoted by $f = g$, if and only if $f(e) = g(e)$ for all $e \in E$.

Definition 2.4. [5] Let $f, g \in \mathbb{S}$. Then, the intersection of f and g, denoted $f \widetilde{\cap} g$, is defined by

$$(f \widetilde{\cap} g)(e) = f(e) \cap g(e) \text{ for all } e \in E$$

and the union of f and g, denoted $f \widetilde{\cup} g$, is defined by

$$(f \widetilde{\cup} g)(e) = f(e) \cup g(e) \text{ for all } e \in E$$

Definition 2.5. [5] Let $f \in \mathbb{S}$. Then, the soft complement of f, denoted $f^{\widetilde{c}}$, is defined by

$$f^{\widetilde{c}}(e) = U \setminus f(e), \text{ for all } e \in E$$

Definition 2.6. [5] Let $f \in \mathbb{S}$. The power soft set of f is defined by

$$\widetilde{\mathcal{P}}(f) = \{f_i \tilde{\subseteq} f : f_i \in \mathbb{S}, i \in I\}$$

and its cardinality is defined by

$$|\widetilde{\mathcal{P}}(f)| = 2^{\sum_{e \in E} |f(e)|}$$

where $|f(e)|$ is the cardinality of $f(e)$.

Example 2.7. Let $U = \{u_1, u_2, u_3\}$ and $E = \{e_1, e_2\}$. $f \in \mathbb{S}$ and

$$f = \big\{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\big\}$$

Then,

$$\begin{aligned}
f_1 &= \{(e_1, \{u_1\})\},\\
f_2 &= \{(e_1, \{u_2\})\},\\
f_3 &= \{(e_1, \{u_1, u_2\})\},\\
f_4 &= \{(e_2, \{u_2\})\},\\
f_5 &= \{(e_2, \{u_3\})\},\\
f_6 &= \{(e_2, \{u_2, u_3\})\},\\
f_7 &= \{(e_1, \{u_1\}), (e_2, \{u_2\})\},\\
f_8 &= \{(e_1, \{u_1\}), (e_2, \{u_3\})\},\\
f_9 &= \{(e_1, \{u_1\}), (e_2, \{u_2, u_3\})\},\\
f_{10} &= \{(e_1, \{u_2\}), (e_2, \{u_2\})\},\\
f_{11} &= \{(e_1, \{u_2\}), (e_2, \{u_3\})\},\\
f_{12} &= \{(e_1, \{u_2\}), (e_2, \{u_2, u_3\})\},\\
f_{13} &= \{(e_1, \{u_1, u_2\}), (e_2, \{u_2\})\},\\
f_{14} &= \{(e_1, \{u_1, u_2\}), (e_2, \{u_3\})\},\\
f_{15} &= f,\\
f_{16} &= \Phi
\end{aligned}$$

are all soft subsets of f. So $|\widetilde{\mathcal{P}}(f)| = 2^4 = 16$.

Definition 2.8. [17] The soft set f is called a soft point in $\mathbb{S}$, if for the parameter $e_i \in E$ such that $f(e_i) \neq \emptyset$ and $f(e_j) = \emptyset$, for all $e_j \in E \setminus \{e_i\}$ is denoted by $\left(e_{i_f}\right)_j$ for all $i, j \in \mathbb{N}^+$.

Note that the set of all soft points of f will be denoted by $SP(f)$.

Example 2.9. [17] Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ and $E = \{e_1, e_2, e_3\}$. $f \in \mathbb{S}$ and

$$f = \Big\{(e_1, \{u_1, u_3\}), (e_2, \{u_2, u_3\}), (e_3, \{u_1, u_2, u_3\})\Big\}$$

Then the soft points for the parameter e_1 are;

$$\begin{aligned}
\left(e_{1_f}\right)_1 &= \big(e_1, \{u_1\}\big)\\
\left(e_{1_f}\right)_2 &= \big(e_1, \{u_3\}\big)\\
\left(e_{1_f}\right)_3 &= \big(e_1, \{u_1, u_3\}\big)
\end{aligned}$$

For the the parameter e_2 one of three occasions can be chosen as soft point likewise;

$$\begin{aligned} \big(e_{2_f}\big)_1 &= \big(e_2, \{u_2\}\big) \\ \big(e_{2_f}\big)_2 &= \big(e_2, \{u_3\}\big) \\ \big(e_{2_f}\big)_3 &= \big(e_2, \{u_2, u_3\}\big) \end{aligned}$$

The soft points for the parameter e_3 are;

$$\begin{aligned} \big(e_{3_f}\big)_1 &= \big(e_3, \{u_1\}\big) \\ \big(e_{3_f}\big)_2 &= \big(e_3, \{u_2\}\big) \\ \big(e_{3_f}\big)_3 &= \big(e_3, \{u_3\}\big) \\ \big(e_{3_f}\big)_4 &= \big(e_3, \{u_1, u_2\}\big) \\ \big(e_{3_f}\big)_5 &= \big(e_3, \{u_1, u_3\}\big) \\ \big(e_{3_f}\big)_6 &= \big(e_3, \{u_2, u_3\}\big) \\ \big(e_{3_f}\big)_7 &= \big(e_3, \{u_1, u_2, u_3\}\big) \end{aligned}$$

Definition 2.10. [2] Let $f \in \mathbb{S}$. A soft topology on f, denoted by $\tilde{\tau}$, is a collection of soft subsets of f having the following properties:

i.: $f, \Phi \in \tilde{\tau}$,
ii.: $\{g_i\}_{i\in I} \subseteq \tilde{\tau} \Rightarrow \tilde{\bigcup}_{i\in I} g_i \in \tilde{\tau}$,
iii.: $\{g_i\}_{i=1}^{n} \subseteq \tilde{\tau} \Rightarrow \tilde{\bigcap}_{i=1}^{n} g_i \in \tilde{\tau}$.

The pair $(f, \tilde{\tau})$ is called a soft topological space.

Example 2.11. Refer to Example 2.7, $\tilde{\tau}^1 = \tilde{\mathcal{P}}(f)$, $\tilde{\tau}^0 = \{\Phi, f\}$ and $\tilde{\tau} = \{\Phi, f, f_2, f_{11}, f_{13}\}$ are soft topologies on f.

Definition 2.12. [2] Let $(f, \tilde{\tau})$ be a soft topological space. Then, every element of $\tilde{\tau}$ is called soft open set. Clearly, Φ and f are soft open sets.

Theorem 2.13. [15] *If $\tilde{F}$ is a collection of soft closed sets in a soft topological space $(f, \tilde{\tau})$, then*

i.: *The universal soft set $\tilde{E}$ is soft closed.*
ii.: *Any intersection of members of $\tilde{F}$ belongs to $\tilde{F}$.*
iii.: *Any finite union of members of $\tilde{F}$ belongs to $\tilde{F}$.*

Remark 2.14. [15] Since $\tilde{E}^{\tilde{c}} = \Phi \in \tilde{\tau}$, $\tilde{E}$ is soft closed. But, Φ and f need not to be soft closed. The following example shows that:

Example 2.15. Consider the topology $\tilde{\tau} = \{\Phi, f, f_2, f_{11}, f_{13}\}$ is defined in Example 2.7. Here, f and Φ are not soft closed sets because
$f^{\tilde{c}} = \Big\{\big(e_1, \{u_3\}\big), \big(e_2, \{u_1\}\big)\Big\} \notin \tilde{\tau}$ and $\Phi^{\tilde{c}} = \tilde{E} \notin \tilde{\tau}$.

Theorem 2.16. [15] *Let $(f, \tilde{\tau})$ be a soft topological space and $g \tilde{\subseteq} f$. Then, the collection*

$$\tilde{\tau}_g = \{h \tilde{\cap} g : h \in \tilde{\tau}\}$$

is a soft topology on g and the pair $(g, \tilde{\tau}_g)$ is a soft topological space.

Definition 2.17. [15] Let $(f, \tilde{\tau})$ be a soft topological space and $g \tilde{\subseteq} f$. Then, the collection

$$\tilde{\tau}_g = \{h \tilde{\cap} g : h \in \tilde{\tau}\}$$

is called a soft subspace topology on g and $(g, \tilde{\tau}_g)$ is called a soft topological subspace of $(f, \tilde{\tau})$.

Definition 2.18. [16] Let f be a nonempty soft set over the universe U, $g \tilde{\subseteq} f$, $\tilde{\tau}$ be a soft topology on f and $\tilde{\tau}_g$ be a soft subspace topology on g. Then, $(f, \tilde{\tau}, \tilde{\tau}_g)$ is called a soft ditopological space which is abbreviated as SDT-space.

A pair $\tilde{\delta} = (\tilde{\tau}, \tilde{\tau}_g)$ is called a soft ditopology over f and the members of $\tilde{\delta}$ are said to be $\tilde{\delta}$-soft open in f.

The complement of $\tilde{\delta}$-soft open set is called $\tilde{\delta}$-soft closed set.

Example 2.19. [16] Let us consider all soft subsets on f in the Example 2.7. Let $\tilde{\tau} = \{\Phi, f, f_2, f_{11}, f_{13}\}$ be a soft topology on f. If $g = f_9$, then $\tilde{\tau}_g = \{\Phi, f_5, f_7, f_9\}$, and $(g, \tilde{\tau}_g)$ is a soft topological subspace of $(f, \tilde{\tau})$.
Hence, we get soft ditopology over f as $\tilde{\delta} = \{\Phi, f, f_2, f_5, f_7, f_9, f_{11}, f_{13}\}$.

Definition 2.20. [16] Let $h \tilde{\subseteq} f$. Then, $\tilde{\delta}$ - interior of h, denoted by $(g)^{\circ}_{\tilde{\delta}}$, is defined by

$$(h)^{\circ}_{\tilde{\delta}} = \tilde{\bigcup}\{h : k \tilde{\subset} h, k \text{ is } \tilde{\delta}\text{-soft open}\}$$

The $\tilde{\delta}$ - closure of h, denoted by $(\overline{h})_{\tilde{\delta}}$, is defined by

$$(\overline{h})_{\tilde{\delta}} = \tilde{\bigcap}\{k : h \tilde{\subset} k, k \text{ is } \tilde{\delta}\text{-soft closed}\}$$

Note that $(h)^{\circ}_{\tilde{\delta}}$ is the biggest $\tilde{\delta}$-soft open set that contained in h and $(\overline{h})_{\tilde{\delta}}$ is the smallest $\tilde{\delta}$-soft closed set that containing h.

3. Soft Ditopological Subspace

In this section, the detailed analysis of SDT - space is carried out by introducing a new concept called soft ditopological subspace (SDT-subspace) in SDT-space. I give two different definitions of SDT-subspace that are not a consequence of each other. Although, these definitions run along different lines, I prove that only one and the same soft ditopological subspace can be established on the same soft set using two different definitions.

I also observe relations of soft ditopological space and soft ditopological subspace in different cases with soft open and soft closed sets. It shows how soft sets in soft ditopological space can preserve their properties in soft ditopological subspaces. In this context, I serve a bridge among soft ditopological space theory and soft ditopological subspace theory.

Theorem 3.1. *Let $(f, \tilde{\delta})$ be a SDT-space and $t \tilde{\subseteq} f$. Then, the collection*

$$\tilde{\delta}_t = \{k \tilde{\cap} t : k \in \tilde{\delta}\}$$

is a soft topology on t and the pair $(t, \tilde{\delta}_t)$ is a soft topological space.

Proof: Since $\Phi \tilde{\cap} t = \Phi$ and $f \tilde{\cap} t = t$, then $t, \Phi \in \tilde{\delta}_t$.
Moreover,

$$\tilde{\bigcap}_{i=1}^{n} (k_i \tilde{\cap} t) = \left(\tilde{\bigcap}_{i=1}^{n} k_i\right) \tilde{\cap} t$$

and

$$\tilde{\bigcup}_{i \in I} (k_i \tilde{\cap} t) = \left(\tilde{\bigcup}_{i \in I} k_i\right) \tilde{\cap} t$$

for $\tilde{\delta} = \{k_i \tilde{\subseteq} f : i \in I\}$. Thus, the union of any number of soft sets in $\tilde{\delta}_t$ belongs to $\tilde{\delta}_t$ and the finite intersection of soft sets in $\tilde{\delta}_t$ belongs to $\tilde{\delta}_t$. So, $\tilde{\delta}_t$ is a soft topology on t.

Definition 3.2. Let $(f, \tilde{\delta})$ be a SDT-space and $t \tilde{\subseteq} f$. Then, the collection

$$\tilde{\delta}_t = \{k \tilde{\cap} t : k \in \tilde{\delta}\}$$

is called a soft subspace ditopology on t and $(t, \tilde{\delta}_t)$ is called a soft ditopological subspace of $(f, \tilde{\delta})$.

In order to carry out the construction, I have to make a judicious choice of soft ditopological subspace that given in the below example:

Example 3.3. Let us consider the SDT-space $(f, \tilde{\delta})$ defined in the Example 2.19. If $t = f_8$, then,

$$\begin{aligned}
\Phi \tilde{\cap} f_8 &= \Phi \\
f_2 \tilde{\cap} f_8 &= \Phi \\
f_5 \tilde{\cap} f_8 &= f_5 \\
f_7 \tilde{\cap} f_8 &= f_1 \\
f_9 \tilde{\cap} f_8 &= f_8 \\
f_{11} \tilde{\cap} f_8 &= f_5 \\
f_{13} \tilde{\cap} f_8 &= f_1 \\
f \tilde{\cap} f_8 &= f_8
\end{aligned}$$

Hence, we get soft subspace ditopology on t as $\tilde{\delta}_t = \{\Phi, f_1, f_5, f_8\}$.

In the next definition, I state a new characterization of SDT-subspace topology which seems not to be a consequence of previous SDT-subspace definition made in Definition 3.2. The reason for my attention to obtain a new different definition, is that, in certain circumstances, it provides a way to prove results for SDT-subspaces will be seen in the next section:

Definition 3.4. Let $(f, \tilde{\delta})$ be a SDT-space and $t \tilde{\subseteq} f$. If the collections

$$\tilde{\delta}_t = \{k \tilde{\cap} t : k \in \tilde{\tau}\}$$

and

$$(\tilde{\delta}_g)_t = \{z \tilde{\cap} t : z \in \tilde{\tau}_g\}$$

are two soft topologies on t, then a SDT-space $(t, \tilde{\delta}_t, (\tilde{\delta}_g)_t)$ is called a SDT-subspace of $(f, \tilde{\delta})$.

This definition is convenient for the induction on soft subspace of a soft subspace that will be used in the next section.

The usefulness and interest of this correspondence of two different definitions about SDT-subspaces made above will of course be enhanced if they are coincident. I now study to get the same soft subspace ditopology on t obtained in Example 3.3 using Definition 3.4:

Example 3.5. Let us consider the SDT-space $(f, \tilde{\delta})$ defined in the Example 2.19 where $g = f_9$ and $\tilde{\delta}_g = \{\Phi, f_5, f_7, f_9\}$. Here we get the collections

$$\tilde{\delta}_t = \{\Phi, f_1, f_5, f_8\}$$

and

$$(\tilde{\delta}_g)_t = \{\Phi, f_1, f_5, f_8\}$$

Hence, we obtain SDT-subspace $(t, \tilde{\delta}_t, (\tilde{\delta}_g)_t) = \{\Phi, f_1, f_5, f_8\}$.

The result above seems appropriate to mention that we obtain the same SDT-subspace topology with different definitions. Although, these definitions run along different lines, it is easy to deduce that I obtain exactly only one and the same soft ditopological subspace on the same soft set.
The reader is cautioned that the notation in Definition 3.2 is coincident with Definition 3.4 and will be used to serve a bridge among soft topological space and soft ditopological spaces in the next section.
On the way, I continue to investigate SDT-subspace properties:

Definition 3.6. A soft ditopological property is said to be hereditary if whenever a soft ditopological space $(f, \tilde{\delta})$ has that property, then so does every soft ditopological subspace of it.

Definition 3.7. Let $(t, \tilde{\delta}_t)$ be a SDT-subspace of a SDT-space $(f, \tilde{\delta})$ and $z \tilde{\subseteq} t$.
z is called a soft open set in SDT-subspace t if $z = m \tilde{\cap} t$ for $m \in \tilde{\delta}$.
So, the members of $\tilde{\delta}_t$ are said to be a soft open sets in SDT-subspace $(t, \tilde{\delta}_t)$.

Theorem 3.8. *Let $(t, \tilde{\delta}_t)$ be a SDT-subspace of a SDT-space $(f, \tilde{\delta})$ and $z \tilde{\subseteq} t$. If $z \in \tilde{\delta}$ then, $z \in \tilde{\delta}_t$.*

Proof: Suppose that $z \in \tilde{\delta}$. Since $z \tilde{\subseteq} t$, $z = z \tilde{\cap} t$. Then, $z \in \tilde{\delta}_t$ by assumption $z \in \tilde{\delta}$.

Theorem 3.9. *Let $(t, \tilde{\delta}_t)$ be a SDT-subspace of a SDT-space $(f, \tilde{\delta})$. Then, the following are equivalent:*

i. $t \in \tilde{\delta}$
ii. $\tilde{\delta}_t \tilde{\subseteq} \tilde{\delta}$.

Proof: $(i) \Rightarrow (ii)$: Let $t \in \tilde{\delta}$. Take as given $\forall z \in \tilde{\delta}_t$. From the Definition 3.2, $z = m \tilde{\cap} t$, where $\exists m \in \tilde{\delta}$. Since $t \in \tilde{\delta}$ and $m \in \tilde{\delta}$ then, $z \in \tilde{\delta}$. Hence $\tilde{\delta}_t \tilde{\subseteq} \tilde{\delta}$.
$(ii) \Rightarrow (i)$: Assume that $\tilde{\delta}_t \tilde{\subseteq} \tilde{\delta}$. Since $t \in \tilde{\delta}_t$ then, $t \in \tilde{\delta}$.

Remark 3.10. A $\tilde{\delta}$-soft open set in a SDT-subspace is not need to be $\tilde{\delta}$-soft open in the SDT-space which is given in the following example:

Example 3.11. Consider the SDT-subspace $(t, \tilde{\delta}_t)$ defined in the Example 3.3. Here, $f_1 \in \tilde{\delta}_t$ but $f_1 \notin \tilde{\delta}$.

4. The Relation Between Soft Topological Space and Soft Ditopological Space

Conditions related to bounds on the relations between soft spaces appear to be rare in the literature, I study how the notion of soft ditopology relates to the soft topology.

The two characteristics, soft ditopological subspace and the soft subspace of a soft topological subspace, are coincided, but the relationship is quite a complex one. Although individually these systems can still be quite complicated, a possibly more tractable task is to describe their possible joint distributions. The relationship between the soft topology and the extent in which soft subspace topology is soft ditopological subspace is studied, obtained joint distributions.

I now exploit the relation to see what else I can say about the relation between soft topology and soft ditopology in the view of subspace topology.

Theorem 4.1. *Let $h \tilde{\subseteq} t \tilde{\subseteq} f$, $(t, \tilde{\delta}_t)$ and $(h, \tilde{\delta}_h)$ be SDT-subspaces of SDT-space $(f, \tilde{\delta})$ and $(h, (\tilde{\delta}_t)_h)$ be a soft subspace of $(t, \tilde{\delta}_t)$. Then,*

$$\tilde{\delta}_h = (\tilde{\delta}_t)_h$$

Proof: Take as given $\forall\, w \in \tilde{\delta}_h$. From the definition of SDT-subspace, $w = w \tilde{\cap} h$, where $w \in \tilde{\delta}$. We obtain that $w \tilde{\cap} t \in \tilde{\delta}_t$. Then, by choosing $w \tilde{\cap} t = y$, $y \tilde{\cap} h \in (\tilde{\delta}_t)_h$ because of $(h, (\tilde{\delta}_t)_h) \subseteq (t, \tilde{\delta}_t)$ and $y \in \tilde{\delta}_t$.
Since $y = w \tilde{\cap} t$ then, $y \tilde{\cap} h = w \tilde{\cap} t \tilde{\cap} h \in (\tilde{\delta}_t)_h$.
$h \tilde{\subseteq} t \Leftrightarrow h = h \tilde{\cap} t$ then, $y \tilde{\cap} h = w \tilde{\cap} h \in (\tilde{\delta}_t)_h$.
Since $w = w \tilde{\cap} h \in (\tilde{\delta}_t)_h$ then, $w \in (\tilde{\delta}_t)_h$.
Hence, we get $\tilde{\delta}_h \subseteq (\tilde{\delta}_t)_h$.
Conversely, assume that $\forall z \in (\tilde{\delta}_t)_h$. From the definition of SDT-subspace, $z = k \tilde{\cap} h$, where $k \in \tilde{\delta}_t$. We obtain that $k = w \tilde{\cap} t$, where $w \in \tilde{\delta}$.
$z = k \tilde{\cap} h = w \tilde{\cap} t \tilde{\cap} h = w \tilde{\cap} h \in \tilde{\delta}_h$, so this completes the proof.

The method of this proof carries over to soft ditopological space satisfying soft topological space's properties via soft subspace topology.

Theorem 4.2. *Let $(t, \tilde{\delta}_t)$ and $(h, \tilde{\delta}_h)$ be SDT-subspaces of SDT-space $(f, \tilde{\delta})$ and $w \subseteq h \tilde{\cap} t$. Then,*

$$\tilde{\delta}_w = (\tilde{\delta}_t)_w = (\tilde{\delta}_h)_w$$

Proof: Since the entire argument is based solely upon the Theorem 4.1, the conclusion of the theorem must hold.

I underline that all the aforementioned results in this theorem rely on the conformality of the underlying construction of a relation between soft topological soft ditopological spaces.

Using Theorem 4.1, Theorem 4.2 and Definition 3.2, Definition 3.4 about SDT-subspace and soft topological subspace, I am able to deduce that their soft subtopological spaces are coincident that is proved below:

Remark 4.3. Let $(t, \tilde{\delta}_t)$ be a SDT-subspace of a SDT-space $(f, \tilde{\delta})$ and $t, g \tilde{\subseteq} f$. If we consider the notation:

$$(f, \tilde{\delta}) = (f, \tilde{\tau}, \tilde{\tau}_g)$$

The SDT-subspace of $(f, \tilde{\delta})$ is $(t, \tilde{\delta}_t)$ and the SDT-subspace of $(f, \tilde{\tau}, \tilde{\tau}_g)$ is $(t, \tilde{\tau}_t, (\tilde{\tau}_g)_t)$. If we write the equality of SDT-subspaces;

$$(t, \tilde{\delta}_t) = (t, \tilde{\tau}_t, (\tilde{\tau}_g)_t)$$

From Theorem 4.1, $\tilde{\tau}_t = (\tilde{\tau}_g)_t$, then,

$$(t, \tilde{\delta}_t) = (t, \tilde{\tau}_t)$$

Hence, SDT-subspace and soft topological subspace of t are coincident.

I have shown that soft subspaces of soft topology and soft ditopology are overlapped, whence it is readily inferred that I can obtain soft ditopological subspace via soft topological subspace.

5. Conclusion

The aim of this article is to study the relationship between the soft topology and the extent in which soft topology to be soft ditopology. Also, in this work, soft ditopological subspace on a soft set is defined and its related properties are studied. Then, the relation between soft topology and soft ditopology is presented. The concept of soft topological subspace of soft ditopological subspace have been introduced. Also, several relations of soft ditopological and soft ditopological subspace have been established and their properties with given examples have been compared. All these results present a bridge among soft topological and soft ditopological theory. In the last section, I describe how the notion of soft ditopology relates to the soft topology. A complete explication of the soft ditopological subspace is warranted, as it will likely reveal further clues to the differences between the soft topology and soft ditopology theories.

It considers some of the new results and consequences, which could be useful from the point of view of soft set theory, that were not studied at all. All these findings will provide a base to researchers who want to work in the field of soft ditopology and will help to strengthen the foundations of the theory of soft ditopological spaces.

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Current address: Department of Mathematics, Amasya University, 05100 Amasya, TURKEY
E-mail address: g.senel@amasya.edu.tr
ORCID Address: http://orcid.org/0000-0003-4052-2631