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Original Article

Comment (2) on Soft Set Theory and uni-int Decision Making [European Journal of Operational Research, (2010) 207, 848-855]

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Abstract − The uni-int decision-making method, which selects a set of optimum elements from the alternatives, was defined by Çağman and Enginoğlu [Soft set theory and uni-int decision making, European Journal of Operational Research 207 (2010) 848-855] via soft sets and their soft products. Lately, this method constructed by and-product/or-product has been configured by Enginoğlu and Memiş [A configuration of some soft decision-making algorithms via $fpfs$-matrices, Cumhuriyet Science Journal 39 (4) (2018) In Press] via fuzzy parameterized fuzzy soft matrices ($fpfs$-matrices), faithfully to the original, because a more general form is needed for the method in the event that the parameters or objects have uncertainties. In this study, we configure the method via $fpfs$-matrices and andnot-product/ornot-product, faithfully to the original. However, in the case that a large amount of data is processed, the method still has a disadvantage regarding time and complexity. To deal with this problem and to be able to use this configured method effectively denoted by CE10n, we suggest two new algorithms in this paper, i.e. EMA18an and EMA18on, and prove that CE10n constructed by andnot-product (CE10an) and constructed by ornot-product (CE10on) are special cases of EMA18an and EMA18on, respectively, if first rows of the $fpfs$-matrices are binary. We then compare the running times of these algorithms. The results show that EMA18an and EMA18on outperform CE10an and CE10on, respectively. Particularly in problems containing a large amount of parameters, EMA18an and EMA18on offer up to 99.9966% and 99.9964% of time advantage, respectively. Latterly, we apply EMA18on to a performance-based value assignment to the methods used in the noise removal, so that we can order them in terms of performance. Finally, we discuss the need for further research.

Keywords − *Fuzzy sets, Soft sets, Soft decision-making, Soft matrices, $fpfs$-matrices*

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1 Introduction

The concept of soft sets was produced by Molodtsov [1] to deal with uncertainties, and so far many theoretical and applied studies from algebra to decision-making problems [2–24] have been conducted on this concept.

Recently, some decision-making algorithms constructed by soft sets [3, 5, 25, 26], fuzzy soft sets [2, 8, 27–29], fuzzy parameterized soft sets [9, 30], fuzzy parameterized fuzzy soft sets ($fpfs$-sets) [7, 31], soft matrices [5, 32] and fuzzy soft matrices [10, 33] have been configured [34] via fuzzy parameterized fuzzy soft matrices ($fpfs$-matrices) [11], faithfully to the original, because a more general form is needed for the method in the event that the parameters or objects have uncertainties.

One of the configured methods above-mentioned is CE10 [5, 34] constructed by and-product (CE10a) or constructed by or-product (CE10o). Since the authors point to a configuration of these methods by using a different product such as andnot-product and ornot-product, in this study, we configure the uni-int decision-making method constructed by andnot-product/ornot-product via $fpfs$-matrices, faithfully to the original. However, in the case that a large amount of data is processed, this configured method denoted by CE10n has a disadvantage regarding time and complexity. It can be overcome this problem via simplification of the algorithms but in the event that first rows of the $fpfs$-matrices are binary, though there exist simplified versions of CE10n constructed by andnot-product (CE10an) and constructed by ornot-product (CE10on), no exist in the other cases. Therefore, in this study, we aim to develop two algorithms which have the ability of CE10an and CE10on and are also faster than them.

In Section 2 of the present study, we introduce the concept of $fpfs$-matrices. In Section 3, we configure the uni-int decision-making method constructed by andnot-product/ornot-product via $fpfs$-matrices. In Section 4, we suggest two new algorithms in this paper, i.e. EMA18an and EMA18on, and prove that CE10an and CE10on are special cases of EMA18an and EMA18on, respectively, if first rows of the $fpfs$-matrices are binary. A part of this section has been presented in [35]. In Section 5, we compare the running times of these algorithms. In Section 6, we apply EMA18on to the decision-making problem in image denoising. Finally, we discuss the need for further research.

2 Preliminary

In this section, we present the definition of $fpfs$-sets and $fpfs$-matrices. Throughout this paper, let E be a parameter set, $F(E)$ be the set of all fuzzy sets over E, and $\mu \in F(E)$. Here, $\mu := \{^{\mu(x)}x : x \in E\}$.

Definition 2.1. [7, 11] Let U be a universal set, $\mu \in F(E)$, and α be a function from μ to $F(U)$. Then the graphic of α, denoted by α, defined by

$$\alpha := \{(^{\mu(x)}x, \alpha(^{\mu(x)}x)) : x \in E\}$$

that is called fuzzy parameterized fuzzy soft set ($fpfs$-set) parameterized via E over U (or briefly over U).

In the present paper, the set of all $fpfs$-sets over U is denoted by $FPFS_E(U)$.

Example 2.2. Let $E = \{x_1, x_2, x_3, x_4\}$ and $U = \{u_1, u_2, u_3, u_4, u_5\}$. Then

$$\alpha = \{({}^1x_1, \{{}^{0.3}u_1, {}^{0.7}u_3\}), ({}^{0.8}x_2, \{{}^{0.2}u_1, {}^{0.2}u_3, {}^{0.9}u_5\}), ({}^{0.3}x_3, \{{}^{0.5}u_2, {}^{0.7}u_4, {}^{0.2}u_5\}), ({}^{0}x_4, \{{}^{1}u_2, {}^{0.9}u_4\})\}$$

is a $fpfs$-set over U.

Definition 2.3. [11] Let $\alpha \in FPFS_E(U)$. Then $[a_{ij}]$ is called the matrix representation of α (or briefly $fpfs$-matrix of α) and defined by

$$[a_{ij}] = \begin{bmatrix} a_{01} & a_{02} & a_{03} & \cdots & a_{0n} & \cdots \\ a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \end{bmatrix} \text{ for } i = \{0, 1, 2, \cdots\} \text{ and } j = \{1, 2, \cdots\}$$

such that

$$a_{ij} := \begin{cases} \mu(x_j), & i = 0 \\ \alpha({}^{\mu(x_j)}x_j)(u_i), & i \neq 0 \end{cases}$$

Here, if $|U| = m - 1$ and $|E| = n$ then $[a_{ij}]$ has order $m \times n$.

From now on, the set of all $fpfs$-matrices parameterized via E over U is denoted by $FPFS_E[U]$.

Example 2.4. Let's consider the $fpfs$-set α provided in Example 2.2. Then the $fpfs$-matrix of α is as follows:

$$[a_{ij}] = \begin{bmatrix} 1 & 0.8 & 0.3 & 0 \\ 0.3 & 0.2 & 0 & 0 \\ 0 & 0 & 0.5 & 1 \\ 0.7 & 0.2 & 0 & 0 \\ 0 & 0 & 0.7 & 0.9 \\ 0 & 0.9 & 0.2 & 0 \end{bmatrix}$$

Definition 2.5. [11] Let $[a_{ij}], [b_{ik}] \in FPFS_E[U]$ and $[c_{ip}] \in FPFS_{E^2}[U]$ such that $p = n(j-1) + k$. For all i and p,

If $c_{ip} = \min\{a_{ij}, b_{ik}\}$, then $[c_{ip}]$ is called and-product of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}] \wedge [b_{ik}]$.

If $c_{ip} = \max\{a_{ij}, b_{ik}\}$, then $[c_{ip}]$ is called or-product of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}] \vee [b_{ik}]$.

If $c_{ip} = \min\{a_{ij}, 1 - b_{ik}\}$, then $[c_{ip}]$ is called andnot-product of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}] \overline{\wedge} [b_{ik}]$.

If $c_{ip} = \max\{a_{ij}, 1 - b_{ik}\}$, then $[c_{ip}]$ is called ornot-product of $[a_{ij}]$ and $[b_{ik}]$ and is denoted by $[a_{ij}] \underline{\vee} [b_{ik}]$.

3 A Configuration of the uni-int Decision-Making Method

In this section, we configure the uni-int decision-making method [5] constructed by andnot-product/ornot-product via $fpfs$-matrices.

Algorithm Steps

Step 1. Construct two $fpfs$-matrices $[a_{ij}]$ and $[b_{ik}]$

Step 2. Find andnot-product/ornot-product $fpfs$-matrix $[c_{ip}]$ of $[a_{ij}]$ and $[b_{ik}]$

Step 3. Find andnot-product/ornot-product $fpfs$-matrix $[d_{it}]$ of $[b_{ik}]$ and $[a_{ij}]$

Step 4. Obtain $[s_{i1}]$ denoted by $\max\text{-}\min(c_{ip}, d_{it})$ defined by

$$s_{i1} := \max\{\max_j \min_k (c_{ip}), \max_k \min_j (d_{it})\}$$

such that $i \in \{1, 2, \dots, m-1\}$, $I_a := \{j \mid a_{0j} \neq 0\}$, $I_b := \{k \mid b_{0k} \neq 0\}$, $I_a^* := \{j \mid 1 - a_{0j} \neq 0\}$, $I_b^* := \{k \mid 1 - b_{0k} \neq 0\}$, $p = n(j-1)+k$, $t = n(k-1)+j$, and

$$\max_j \min_k (c_{ip}) := \begin{cases} \max\limits_{j\in I_a} \left\{ \min\limits_{k\in I_b^*} c_{0p} c_{ip} \right\}, & I_a \neq \emptyset \text{ and } I_b^* \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\max_k \min_j (d_{it}) := \begin{cases} \max\limits_{k\in I_b} \left\{ \min\limits_{j\in I_a^*} d_{0t} d_{it} \right\}, & I_a^* \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

Step 5. Obtain the set $\{u_k \mid s_{k1} = \max\limits_i s_{i1}\}$

Preferably, the set $\{^{s_{i1}}u_i \mid u_i \in U\}$ or $\{^{\frac{s_{k1}}{\max s_{i1}}}u_k | u_k \in U\}$ can be attained.

4 The Soft Decision-Making Methods: EMA18an and EMA18on

In this section, firstly, we present a fast and simple algorithm denoted by EMA18an [35].

EMA18an's Algorithm Steps

Step 1. Construct two $fpfs$-matrices $[a_{ij}]$ and $[b_{ik}]$

Step 2. Obtain $[s_{i1}]$ denoted by $\max\text{-}\min(a_{ij}, b_{ik})$ defined by

$$s_{i1} := \max\{\max_j \min_k (a_{ij}, b_{ik}), \max_k \min_j (b_{ik}, a_{ij})\}$$

such that $i \in \{1, 2, \dots, m-1\}$, $I_a := \{j \mid a_{0j} \neq 0\}$, $I_b := \{k \mid b_{0k} \neq 0\}$, $I_a^* := \{j \mid 1 - a_{0j} \neq 0\}$, $I_b^* := \{k \mid 1 - b_{0k} \neq 0\}$, and

$$\max_j \min_k (a_{ij}, b_{ik}) := \begin{cases} \min \left\{ \max\limits_{j\in I_a}\{a_{0j} a_{ij}\}, \min\limits_{k\in I_b^*}\{(1-b_{0k})(1-b_{ik})\} \right\}, & I_a \neq \emptyset \text{ and } I_b^* \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\max_k \min_j (b_{ik}, a_{ij}) := \begin{cases} \min \left\{ \max\limits_{k\in I_b}\{b_{0k} b_{ik}\}, \min\limits_{j\in I_a^*}\{(1-a_{0j})(1-a_{ij})\} \right\}, & I_a^* \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

Step 3. Obtain the set $\{u_k \,|\, s_{k1} = \max_i s_{i1}\}$

Preferably, the set $\{{}^{s_{i1}}u_i \,|\, u_i \in U\}$ or $\{{}^{\frac{s_{k1}}{\max s_{i1}}}u_k | u_k \in U\}$ can be attained.

Secondly, we propose a fast and simple algorithm denoted by EMA18on.

EMA18on's Algorithm Steps

Step 1. Construct two $fpfs$-matrices $[a_{ij}]$ and $[b_{ik}]$

Step 2. Obtain $[s_{i1}]$ denoted by $\max\text{-}\min(a_{ij}, b_{ik})$ defined by

$$s_{i1} := \max\{\max_j \min_k(a_{ij}, b_{ik}), \max_k \min_j(b_{ik}, a_{ij})\}$$

such that $i \in \{1, 2, \ldots, m-1\}$, $I_a := \{j \,|\, a_{0j} \neq 0\}$, $I_b := \{k \,|\, b_{0k} \neq 0\}$, $I_a^* := \{j \,|\, 1 - a_{0j} \neq 0\}$, $I_b^* := \{k \,|\, 1 - b_{0k} \neq 0\}$, and

$$\max_j \min_k(a_{ij}, b_{ik}) := \begin{cases} \max\left\{\max\limits_{j\in I_a}\{a_{0j}a_{ij}\}, \min\limits_{k\in I_b^*}\{(1-b_{0k})(1-b_{ik})\}\right\}, & I_a \neq \emptyset \text{ and } I_b^* \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\max_k \min_j(b_{ik}, a_{ij}) := \begin{cases} \max\left\{\max\limits_{k\in I_b}\{b_{0k}b_{ik}\}, \min\limits_{j\in I_a^*}\{(1-a_{0j})(1-a_{ij})\}\right\}, & I_a^* \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

Step 3. Obtain the set $\{u_k \,|\, s_{k1} = \max_i s_{i1}\}$

Preferably, the set $\{{}^{s_{i1}}u_i \,|\, u_i \in U\}$ or $\{{}^{\frac{s_{k1}}{\max s_{i1}}}u_k | u_k \in U\}$ can be attained.

Theorem 4.1. [35] CE10an is a special case of EMA18an provided that first rows of the $fpfs$-matrices are binary.

Proof. Suppose that first rows of the $fpfs$-matrices are binary. The functions s_{i1} provided in CE10an and EMA18an are equal in the event that $I_a = \emptyset$ or $I_b^* = \emptyset$. Assume that $I_a \neq \emptyset$ and $I_b^* \neq \emptyset$. Since $a_{0j} = 1$ and $b_{0k} = 0$, for all $j \in I_a := \{a_1, a_2, ..., a_s\}$ and $k \in I_b^* := \{b_1, b_2, ..., b_t\}$,

$$\begin{aligned}
\max_j \min_k(c_{ip}) &= \max_{j\in I_a}\left\{\min_{k\in I_b^*} c_{0p}c_{ip}\right\} \\
&= \max_{j\in I_a}\left\{\min_{k\in I_b^*}\{\min\{a_{0j}, 1-b_{0k}\}.\min\{a_{ij}, 1-b_{ik}\}\}\right\} \\
&= \max_{j\in I_a}\left\{\min_{k\in I_b^*}\{\min\{a_{ij}, 1-b_{ik}\}\}\right\} \\
&= \max\{\min\{\min\{a_{ia_1}, 1-b_{ib_1}\}, \min\{a_{ia_1}, 1-b_{ib_2}\}, \ldots, \min\{a_{ia_1}, 1-b_{ib_t}\}\}, \\
&\qquad \min\{\min\{a_{ia_2}, 1-b_{ib_1}\}, \min\{a_{ia_2}, 1-b_{ib_2}\}, \ldots, \min\{a_{ia_2}, 1-b_{ib_t}\}\}, \\
&\qquad \ldots, \min\{\min\{a_{ia_s}, 1-b_{ib_1}\}, \min\{a_{ia_s}, 1-b_{ib_2}\}, \ldots, \min\{a_{ia_s}, 1-b_{ib_t}\}\}\}
\end{aligned}$$

$$
\begin{aligned}
&= \max\{\min\{a_{ia_1}, \min\{1-b_{ib_1}, 1-b_{ib_2}, \dots, 1-b_{ib_t}\}\},\\
&\qquad \min\{a_{ia_2}, \min\{1-b_{ib_1}, 1-b_{ib_2}, \dots, 1-b_{ib_t}\}\}, \dots,\\
&\qquad \min\{a_{ia_s}, \min\{1-b_{ib_1}, 1-b_{ib_2}, \dots, 1-b_{ib_t}\}\}\}\\
&= \min\{\max\{a_{ia_1}, a_{ia_2}, \dots, a_{ia_s}\}, \min\{1-b_{ib_1}, 1-b_{ib_2}, \dots, 1-b_{ib_t}\}\}\\
&= \min\left\{\max_{j\in I_a}\{a_{ij}\}, \min_{k\in I_b^*}\{1-b_{ik}\}\right\}\\
&= \min\left\{\max_{j\in I_a}\{a_{0j}a_{ij}\}, \min_{k\in I_b^*}\{(1-b_{0k})(1-b_{ik})\}\right\}\\
&= \max_j \min_k (a_{ij}, b_{ik})
\end{aligned}
$$

In a similar way, $\max_k \min_j (d_{it}) = \max_k \min_j (b_{ik}, a_{ij})$. Consequently,

$$\max\text{-}\min(a_{ij}, b_{ik}) = \max\text{-}\min(c_{ip}, d_{it})$$

□

Theorem 4.2. CE10on is a special case of EMA18on provided that first rows of the $fpfs$-matrices are binary.

Proof. The proof is similar to that of Theorem 4.1. □

5 Simulation Results

In this section, we compare the running times of CE10an-EMA18an and CE10on-EMA18on by using MATLAB R2017b and a workstation with I(R) Xeon(R) CPU E5-1620 v4 @ 3.5 GHz and 64 GB RAM.

We, firstly, present the running times of CE10an and EMA18an in Table 1 and Fig. 1 for 10 objects and the parameters ranging from 10 to 100. We then give their running times in Table 2 and Fig. 2 for 10 objects and the parameters ranging from 1000 to 10000, in Table 3 and Fig. 3 for 10 parameters and the objects ranging from 10 to 100, in Table 4 and Fig. 4 for 10 parameters and the objects ranging from 1000 to 10000, in Table 5 and Fig. 5 for the parameters and the objects ranging from 10 to 100, and in Table 6 and Fig. 6 for the parameters and the objects ranging from 100 to 1000. The results show that EMA18an outperforms CE10an in any number of data under the specified condition.

Table 1. The results for 10 objects and the parameters ranging from 10 to 100

	10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
CE10an	0.02798	0.01283	0.00623	0.00531	0.01103	0.00829	0.00966	0.01325	0.01637	0.01919
EMA18an	0.01249	0.00714	0.00090	0.00052	0.00244	0.00066	0.00039	0.00035	0.00048	0.00024
Difference	0.0155	0.0057	0.0053	0.0048	0.0086	0.0076	0.0093	0.0129	0.0159	0.0189
Advantage (%)	55.3709	44.3108	85.5050	90.1242	77.8817	92.0866	95.9250	97.3876	97.0461	98.7574

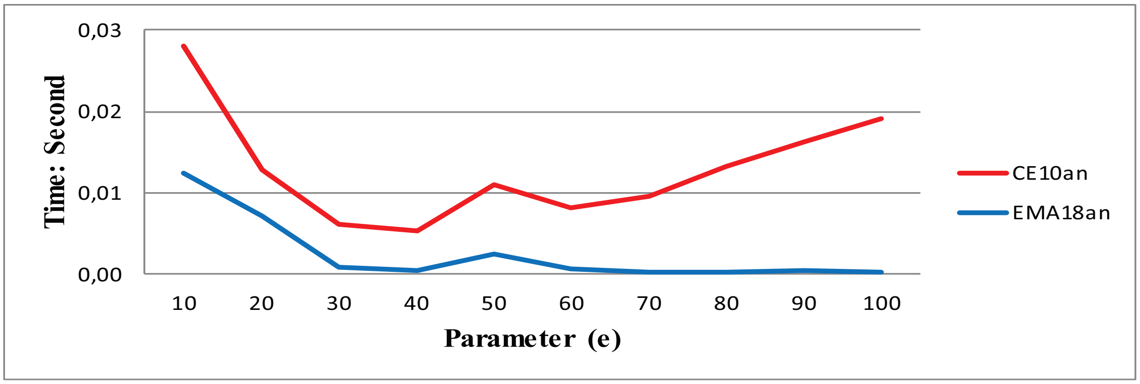


Fig. 1. The figure for Table 1

Table 2. The results for 10 objects and the parameters ranging from 1000 to 10000

	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000
CE10an	1.7420	5.9795	12.4333	21.8006	34.2186	46.9271	66.0375	88.0452	110.2487	143.4280
EMA18an	0.0140	0.0050	0.0024	0.0027	0.0051	0.0053	0.0039	0.0044	0.0048	0.0049
Difference	1.7280	5.9745	12.4310	21.7979	34.2135	46.9218	66.0336	88.0408	110.2439	143.4230
Advantage (%)	99.1965	99.9163	99.9810	99.9875	99.9850	99.9887	99.9940	99.9950	99.9957	99.9966

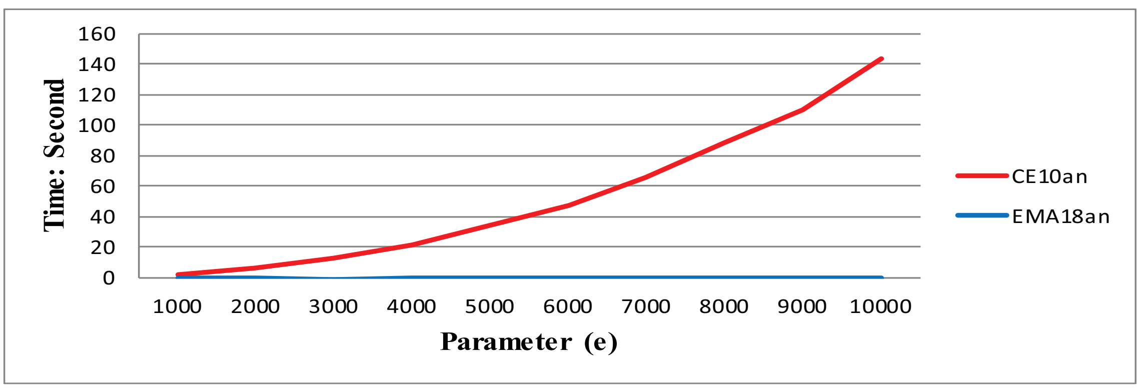


Fig. 2. The figure for Table 2

Table 3. The results for 10 parameters and the objects ranging from 10 to 100

	10	20	30	40	50	60	70	80	90	100
CE10an	0.0229	0.0087	0.0025	0.0023	0.0066	0.0095	0.0060	0.0060	0.0064	0.0072
EMA18an	0.0094	0.0040	0.0008	0.0009	0.0024	0.0023	0.0011	0.0012	0.0012	0.0018
Difference	0.0136	0.0048	0.0017	0.0015	0.0042	0.0072	0.0048	0.0048	0.0053	0.0054
Advantage (%)	59.1357	54.5995	67.8236	62.7065	63.9276	76.1559	81.2134	80.4675	81.6589	74.9437

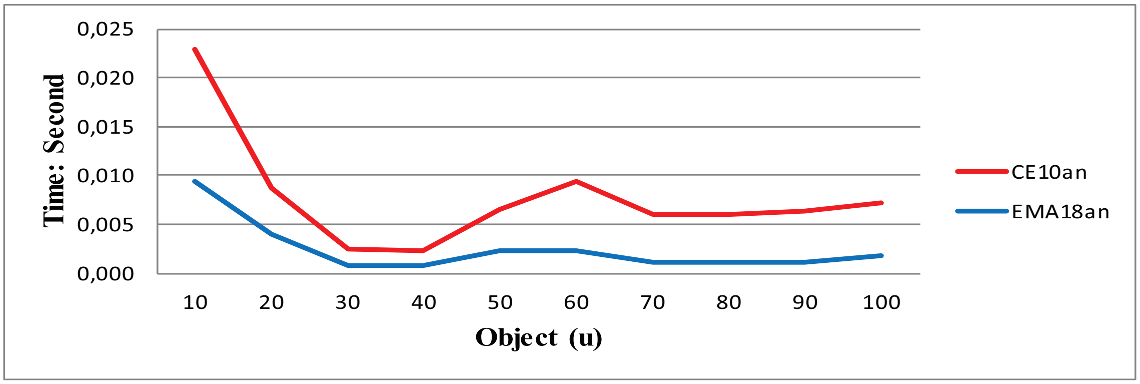


Fig. 3. The figure for Table 3

Table 4. The results for 10 parameters and the objects ranging from 1000 to 10000

	1000	**2000**	**3000**	**4000**	**5000**	**6000**	**7000**	**8000**	**9000**	**10000**
CE10an	0.1075	0.2303	0.4306	0.6850	1.0900	1.4666	1.9348	2.5576	3.1432	3.8415
EMA18an	0.0199	0.0250	0.0324	0.0447	0.0594	0.0736	0.0742	0.0993	0.1153	0.1313
Difference	0.0877	0.2053	0.3982	0.6404	1.0306	1.3930	1.8605	2.4583	3.0280	3.7102
Advantage (%)	81.5272	89.1420	92.4812	93.4776	94.5528	94.9825	96.1639	96.1160	96.3331	96.5811

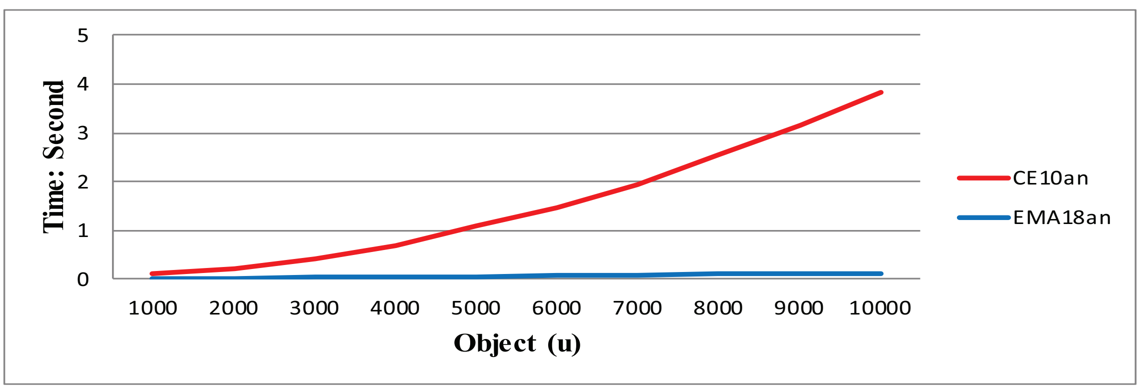


Fig. 4. The figure for Table 4

Table 5. The results for the parameters and the objects ranging from 10 to 100

	10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
CE10an	0.0213	0.0109	0.0078	0.0166	0.0378	0.0645	0.0863	0.1156	0.1665	0.2299
EMA18an	0.0093	0.0041	0.0009	0.0009	0.0048	0.0023	0.0011	0.0014	0.0014	0.0014
Difference	0.0121	0.0069	0.0069	0.0157	0.0330	0.0622	0.0851	0.1142	0.1651	0.2285
Advantage (%)	56.4639	62.7094	88.6511	94.5591	87.3928	96.4563	98.6770	98.8164	99.1380	99.3720

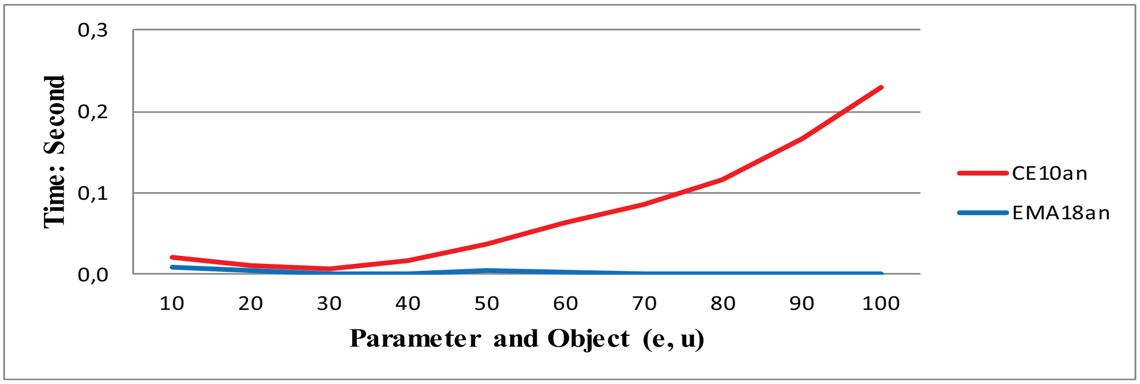


Fig. 5. The figure for Table 5

Table 6. The results for the parameters and the objects ranging from 100 to 1000

	100	200	300	400	500	600	700	800	900	1000
CE10an	0.2739	3.2532	14.0127	40.1959	93.9178	184.5333	335.5700	568.7381	914.9916	1412.0988
EMA18an	0.0113	0.0069	0.0068	0.0101	0.0162	0.0200	0.0244	0.0587	0.0396	0.0506
Difference	0.2626	3.2463	14.0060	40.1858	93.9015	184.5134	335.5456	568.6794	914.9520	1412.0482
Advantage (%)	95.8871	99.7870	99.9518	99.9748	99.9827	99.9892	99.9927	99.9897	99.9957	99.9964

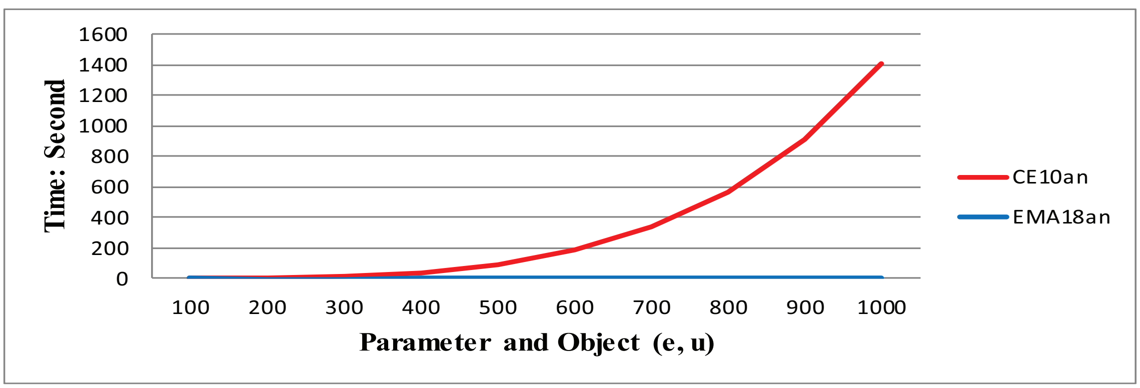


Fig. 6. The figure for Table 6

Secondly, we present the running times of CE10on and EMA18on in Table 7 and Fig. 7 for 10 objects and the parameters ranging from 10 to 100. We then give their running times in Table 8 and Fig. 8 for 10 objects and the parameters ranging from 1000 to 10000, in Table 9 and Fig. 9 for 10 parameters and the objects ranging from 10 to 100, in Table 10 and Fig. 10 for 10 parameters and the objects ranging from 1000 to 10000, in Table 11 and Fig. 11 for the parameters and the objects ranging from 10 to 100, and in Table 12 and Fig. 12 for the parameters and the objects ranging from 100 to 1000. The results show that EMA18on outperforms CE10on in any number of data under the specified condition.

Table 7. The results for 10 objects and the parameters ranging from 10 to 100

	10	20	30	40	50	60	70	80	90	100
CE10on	0, 0273	0, 0107	0, 0037	0, 0051	0, 0116	0, 0165	0, 0141	0, 0167	0, 0245	0, 0197
EMA18on	0, 0136	0, 0056	0, 0007	0, 0008	0, 0027	0, 0020	0, 0006	0, 0006	0, 0004	0, 0004
Difference	0, 0137	0, 0050	0, 0029	0, 0044	0, 0089	0, 0145	0, 0135	0, 0162	0, 0241	0, 0193
Advantage (%)	50, 1693	47, 3241	80, 2441	85, 2661	76, 7704	88, 1753	95, 9501	96, 4667	98, 3700	98, 0986

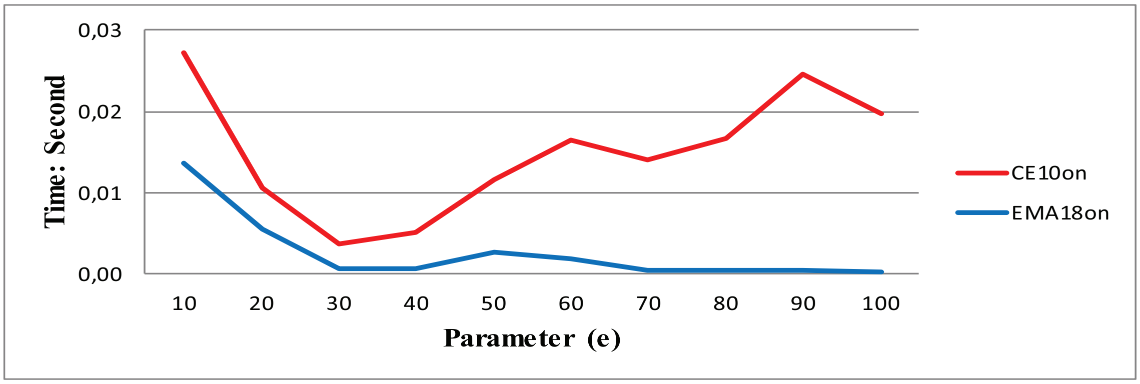


Fig. 7. The figure for Table 7

Table 8. The results for 10 objects and the parameters ranging from 1000 to 10000

	1000	2000	3000	4000	5000	6000	7000	8000	9000	10000
CE10on	1, 7399	6, 0070	12, 6272	21, 8605	34, 3464	47, 4500	68, 2726	90, 2301	112, 5468	145, 8467
EMA18on	0, 0111	0, 0061	0, 0024	0, 0028	0, 0053	0, 0052	0, 0040	0, 0042	0, 0051	0, 0053
Difference	1, 7287	6, 0009	12, 6249	21, 8577	34, 3411	47, 4448	68, 2687	90, 2259	112, 5417	145, 8414
Advantage (%)	99, 3597	99, 8982	99, 9812	99, 9872	99, 9845	99, 9890	99, 9942	99, 9954	99, 9954	99, 9964

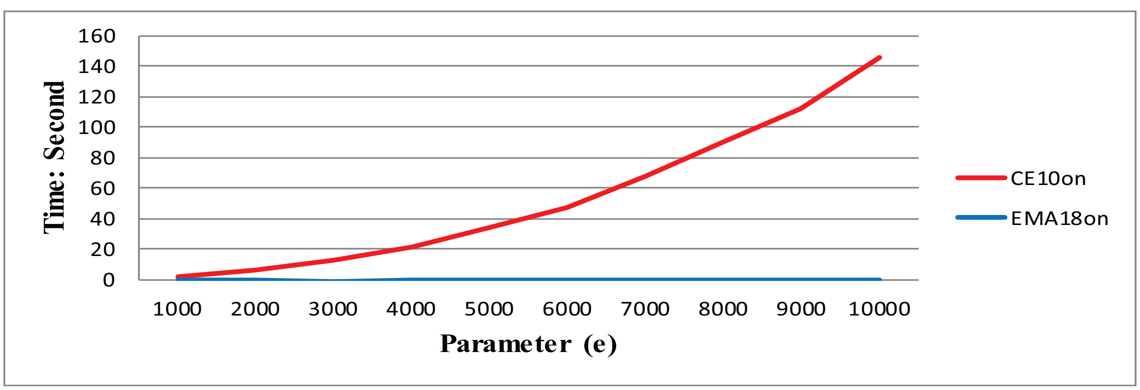


Fig. 8. The figure for Table 8

Table 9. The results for 10 parameters and the objects ranging from 10 to 100

	10	20	30	40	50	60	70	80	90	100
CE10on	0, 0225	0, 0087	0, 0022	0, 0023	0, 0066	0, 0084	0, 0044	0, 0036	0, 0043	0, 0054
EMA18on	0, 0101	0, 0048	0, 0007	0, 0008	0, 0030	0, 0023	0, 0010	0, 0009	0, 0012	0, 0013
Difference	0, 0124	0, 0039	0, 0014	0, 0015	0, 0036	0, 0062	0, 0034	0, 0027	0, 0031	0, 0041
Advantage (%)	55, 1341	44, 7449	66, 3800	66, 7695	54, 5929	73, 0843	76, 7851	74, 1328	72, 7223	75, 7520

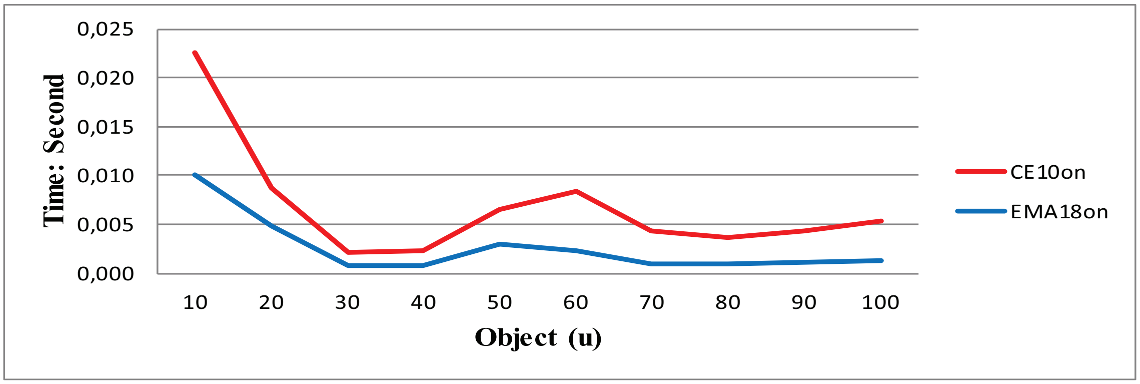


Fig. 9. The figure for Table 9

Table 10. The results for 10 parameters and the objects ranging from 1000 to 10000

	1000	**2000**	**3000**	**4000**	**5000**	**6000**	**7000**	**8000**	**9000**	**10000**
CE10on	0, 1106	0, 2317	0, 4371	0, 6954	1, 0671	1, 5367	1, 9939	2, 5698	3, 2157	4, 0413
EMA18on	0, 0220	0, 0241	0, 0299	0, 0409	0, 0550	0, 0676	0, 0779	0, 0908	0, 1062	0, 1203
Difference	0, 0886	0, 2076	0, 4072	0, 6545	1, 0121	1, 4691	1, 9160	2, 4790	3, 1094	3, 9210
Advantage (%)	80, 1058	89, 5835	93, 1572	94, 1181	94, 8425	95, 5995	96, 0947	96, 4649	96, 6968	97, 0232

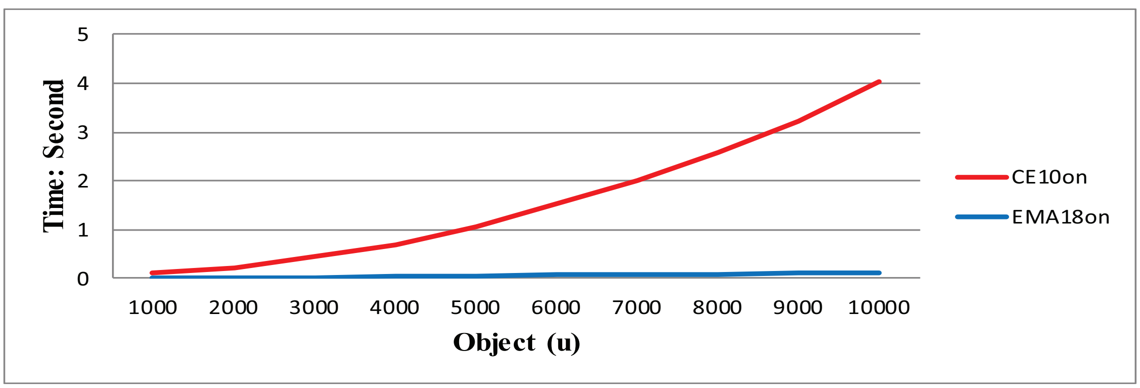


Fig. 10. The figure for Table 10

Table 11. The results for the parameters and the objects ranging from 10 to 100

	10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
CE10on	0, 0207	0, 0108	0, 0084	0, 0170	0, 0343	0, 0629	0, 0872	0, 1145	0, 1688	0, 2283
EMA18on	0, 0108	0, 0045	0, 0016	0, 0011	0, 0031	0, 0024	0, 0012	0, 0013	0, 0017	0, 0016
Difference	0, 0099	0, 0063	0, 0068	0, 0159	0, 0312	0, 0605	0, 0860	0, 1132	0, 1670	0, 2267
Advantage (%)	47, 7823	58, 1857	81, 1154	93, 7711	90, 8651	96, 1574	98, 6538	98, 8966	98, 9715	99, 3208

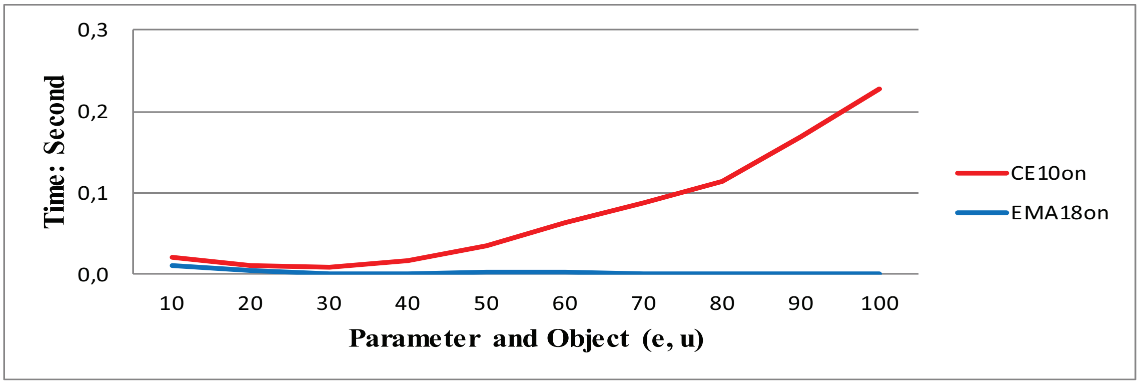


Fig. 11. The figure for Table 11

Table 12. The results for the parameters and the objects ranging from 100 to 1000

	100	200	300	400	500	600	700	800	900	1000
CE10on	0, 2714	3, 2456	14, 0665	40, 5834	93, 4571	182, 9835	325, 6018	545, 3415	870, 2537	1329, 9690
EMA18on	0, 0116	0, 0079	0, 0062	0, 0094	0, 0163	0, 0187	0, 0236	0, 0295	0, 0381	0, 0463
Difference	0, 2598	3, 2377	14, 0603	40, 5741	93, 4408	182, 9648	325, 5782	545, 3119	870, 2156	1329, 9228
Advantage (%)	95, 7199	99, 7562	99, 9556	99, 9769	99, 9826	99, 9898	99, 9927	99, 9946	99, 9956	99, 9965

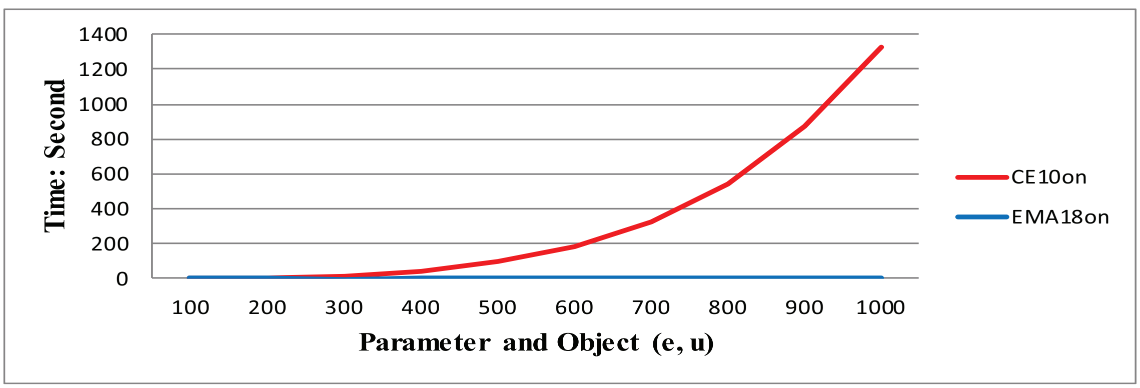


Fig. 12. The figure for Table 12

6 An Application of EMA18on

Being one of the most important topics in image processing, the noise removal directly affects the success rate of the procedures such as segmentation and classification. For this reason, the determining of the methods which perform better than the others is worthwhile to study.

In this section, in Table 13, we present the mean results of some well-known salt-and-pepper noise (SPN) removal methods Decision Based Algorithm (DBA) [36], Modified Decision Based Unsymmetrical Trimmed Median Filter (MDBUTMF) [37], Noise Adaptive Fuzzy Switching Median Filter (NAFSMF) [38]), Different Applied Median Filter (DAMF) [39], and Adaptive Weighted Mean Filter (AWMF) [40] by using 15 traditional images (Cameraman, Lena, Peppers, Baboon, Plane, Bridge, Pirate, Elaine, Boat, Lake, Flintstones, Living Room, House, Parrot, and Hill) with

512×512 pixels, ranging in noise densities from 10% to 90%, and an image quality metrics Structural Similarity (SSIM) [41], which is more preferred than the others. Secondly, in Table 14, we present the mean running times of these algorithms for the images. Finally, we then apply EMA18on to a performance-based value assignment to the methods used in the noise removal, so that we can order them in terms of performance.

Table 13. The mean-SSIM results of the algorithms for the 15 Traditional Images

Algorithm	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
DBA	0.9655	0.9211	0.8605	0.7837	0.6915	0.5895	0.4846	0.3864	0.3138
MDBUTMF	0.9428	0.7961	0.8380	0.8391	0.7830	0.6322	0.3228	0.0969	0.0213
NAFSM	0.9753	0.9506	0.9244	0.8968	0.8660	0.8312	0.7888	0.7308	0.6094
DAMF	0.9865	0.9715	0.9538	0.9330	0.9083	0.8788	0.8412	0.7883	0.6975
AWMF	0.9738	0.9639	0.9507	0.9343	0.9133	0.8857	0.8481	0.7943	0.7044

Table 14. The mean running-time results of the algorithms for the 15 Traditional Images

Algorithm	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
DBA	3.7528	3.7727	3.7827	3.7688	3.7734	3.7911	3.7954	3.7824	3.7866
MDBUTMF	2.4964	3.9101	5.5882	6.6506	7.2925	7.7194	7.9863	8.1317	8.1729
NAFSM	1.2528	2.4664	3.6903	4.8807	6.0873	7.3017	8.4870	9.6226	10.7410
DAMF	0.1567	0.3008	0.4478	0.5929	0.7399	0.8903	1.0464	1.2319	1.5205
AWMF	3.9340	3.2274	2.9008	2.7226	2.6228	2.5688	2.5946	2.7314	3.1366

Let's suppose that the success in low or high-noise density is more important than in the others. Furthermore, the long running time is a drawback. In that case, the values in Table 13 can be represented as an $fpfs$-matrices as follows:

$$[a_{ij}] := \begin{bmatrix} 0.9 & 0.7 & 0.5 & 0.3 & 0.1 & 0.3 & 0.5 & 0.7 & 0.9 \\ 0.9655 & 0.9211 & 0.8605 & 0.7837 & 0.6915 & 0.5895 & 0.4846 & 0.3864 & 0.3138 \\ 0.9428 & 0.7961 & 0.8380 & 0.8391 & 0.7830 & 0.6322 & 0.3228 & 0.0969 & 0.0213 \\ 0.9753 & 0.9506 & 0.9244 & 0.8968 & 0.8660 & 0.8312 & 0.7888 & 0.7308 & 0.6094 \\ 0.9865 & 0.9715 & 0.9538 & 0.9330 & 0.9083 & 0.8788 & 0.8412 & 0.7883 & 0.6975 \\ 0.9738 & 0.9639 & 0.9507 & 0.9343 & 0.9133 & 0.8857 & 0.8481 & 0.7943 & 0.7044 \end{bmatrix}$$

Similarly, the values given in Table 14 can be represented as an $fpfs$-matrices via the function $f : [0, 15] \to [0, 1]$ defined by $f(x) = 1 - x/15$, as follows:

$$[b_{ij}] := \begin{bmatrix} 0.1 & 0.3 & 0.5 & 0.7 & 0.9 & 0.7 & 0.5 & 0.3 & 0.1 \\ 0.7498 & 0.7485 & 0.7478 & 0.7487 & 0.7484 & 0.7473 & 0.7470 & 0.7478 & 0.7476 \\ 0.8336 & 0.7393 & 0.6275 & 0.5566 & 0.5138 & 0.4854 & 0.4676 & 0.4579 & 0.4551 \\ 0.9165 & 0.8356 & 0.7540 & 0.6746 & 0.5942 & 0.5132 & 0.4342 & 0.3585 & 0.2839 \\ 0.9896 & 0.9799 & 0.9701 & 0.9605 & 0.9507 & 0.9406 & 0.9302 & 0.9179 & 0.8986 \\ 0.7377 & 0.7848 & 0.8066 & 0.8185 & 0.8251 & 0.8287 & 0.8270 & 0.8179 & 0.7909 \end{bmatrix}$$

If we apply EMA18on to the $fpfs$-matrices $[a_{ij}]$ and $[b_{ij}]$, then the score matrix and the decision set are as follows:

$$[s_{i1}] = [0.8689 \;\; 0.8485 \;\; 0.8778 \;\; 0.8879 \;\; 0.8764]^T$$

and

$$\{^{0.9786}\text{DBA},\ ^{0.9556}\text{MDBUTMF},\ ^{0.9886}\text{NAFSM},\ ^{1}\text{DAMF},\ ^{0.9871}\text{AWMF}\}$$

The scores show that DAMF outperforms the other methods and the order DAMF, NAFSM, AWMF, DBA, and MDBUTMF is valid.

Let's suppose that the success in medium-noise density is more important than in the others. Furthermore, the long running time is a drawback. In that case, the values in Table 13 can be represented as an $fpfs$-matrices as follows:

$$[c_{ij}] := \begin{bmatrix} 0.1 & 0.3 & 0.5 & 0.7 & 0.9 & 0.7 & 0.5 & 0.3 & 0.1 \\ 0.9655 & 0.9211 & 0.8605 & 0.7837 & 0.6915 & 0.5895 & 0.4846 & 0.3864 & 0.3138 \\ 0.9428 & 0.7961 & 0.8380 & 0.8391 & 0.7830 & 0.6322 & 0.3228 & 0.0969 & 0.0213 \\ 0.9753 & 0.9506 & 0.9244 & 0.8968 & 0.8660 & 0.8312 & 0.7888 & 0.7308 & 0.6094 \\ 0.9865 & 0.9715 & 0.9538 & 0.9330 & 0.9083 & 0.8788 & 0.8412 & 0.7883 & 0.6975 \\ 0.9738 & 0.9639 & 0.9507 & 0.9343 & 0.9133 & 0.8857 & 0.8481 & 0.7943 & 0.7044 \end{bmatrix}$$

and

$$[d_{ij}] := \begin{bmatrix} 0.9 & 0.7 & 0.5 & 0.3 & 0.1 & 0.3 & 0.5 & 0.7 & 0.9 \\ 0.7498 & 0.7485 & 0.7478 & 0.7487 & 0.7484 & 0.7473 & 0.7470 & 0.7478 & 0.7476 \\ 0.8336 & 0.7393 & 0.6275 & 0.5566 & 0.5138 & 0.4854 & 0.4676 & 0.4579 & 0.4551 \\ 0.9165 & 0.8356 & 0.7540 & 0.6746 & 0.5942 & 0.5132 & 0.4342 & 0.3585 & 0.2839 \\ 0.9896 & 0.9799 & 0.9701 & 0.9605 & 0.9507 & 0.9406 & 0.9302 & 0.9179 & 0.8986 \\ 0.7377 & 0.7848 & 0.8066 & 0.8185 & 0.8251 & 0.8287 & 0.8270 & 0.8179 & 0.7909 \end{bmatrix}$$

If we apply EMA18on to the $fpfs$-matrices $[c_{ij}]$ and $[d_{ij}]$, then the score matrix and the decision set are as follows:

$$[s_{i1}] = [0.6748 \ \ 0.7502 \ \ 0.8248 \ \ 0.8906 \ \ 0.8220]^T$$

and

$$\{^{0.7577}\text{DBA},\ ^{0.8424}\text{MDBUTMF},\ ^{0.9262}\text{NAFSM},\ ^{1}\text{DAMF},\ ^{0.9229}\text{AWMF}\}$$

The scores show that DAMF performs better than the other methods and the order DAMF, NAFSM, AWMF, MDBUTMF, and DBA is valid.

7 Conclusion

The uni-int decision-making method was defined in 2010 [5]. Afterwards, this method has been configured [34] via $fpfs$-matrices [11]. However, the method suffers from a drawback, i.e. its incapability of processing a large amount of parameters on a standard computer, e.g. with 2.6 GHz i5 Dual Core CPU and 4GB RAM. For this reason, simplification of such methods is significant for a wide range of scientific and industrial processes. In this study, firstly, we have proposed two fast and simple soft decision-making methods EMA18an and EMA18on. Moreover, we have proved that these two methods accept CE10 as a special case, under the condition that the first rows of the $fpfs$-matrices are binary. It is also possible to investigate the simplifications of the other products such as andnot-product and ornot-product (see Definition 2.5).

We then have compared the running times of these algorithms. In addition to the results in Section 4, the results in Table 15 and 16 too show that EMA18an and EMA18on outperform CE10an and CE10on, respectively, in any number of data under the specified condition. Furthermore, other decision-making methods constructed by a different decision function such as minimum-maximum (min-max), max-max, and min-min can also be studied by the similar way.

Table 15. The mean/max advantage and max difference values of EMA18an over CE10an

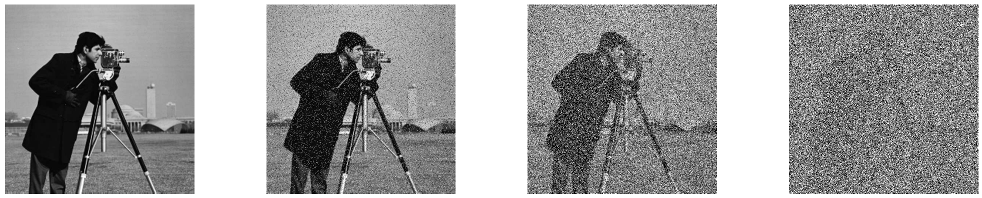
Location	Objects	Parameters	Mean Advantage%	Max Advantage%	Max Difference
Table 1	10	$10-100$	83.4395	98.7574	0.0189
Table 2	10	$1000-10000$	99.9036	99.9966	143.4230
Table 3	$10-100$	10	70.2632	81.6589	0.0136
Table 4	$1000-10000$	10	93.1357	96.5811	3.7102
Table 5	$10-100$	$10-100$	88.2236	99.3720	0.2285
Table 6	$100-1000$	$100-1000$	99.5547	99.9964	1412.0482

Table 16. The mean/max advantage and max difference values of EMA18on over CE10on

Location	Objects	Parameters	Mean Advantage%	Max Advantage%	Max Difference
Table 1	10	$10-100$	81.6835	98.3700	0.0241
Table 2	10	$1000-10000$	99.9181	99.9964	145.8414
Table 3	$10-100$	10	66.0098	76.7851	0.0124
Table 4	$1000-10000$	10	93.3686	97.0232	3.9210
Table 5	$10-100$	$10-100$	86.3720	99.3208	0.2267
Table 6	$100-1000$	$100-1000$	99.5361	99.9965	1329.9228

Finally, we have applied EMA18on to the determination of the performance of the known methods. It is clear that EMA18on, which is a fast and simple method, can be successfully applied to the decision-making problems in various areas such as machine learning and image enhancement.

Although we have no proof about the accuracy of the results of such methods, the results are in compliance with our observations. In order to help in checking the accuracy of the comparison made by a soft decision-making method, we give, in Fig. 13-16, the Cameraman image with different SPN ratios and show the denoised images via the above-mentioned filters. It must be noted that these images has no information of their running times. Whereas, the use of a filter in a software depends on its running time is short. In other words, the running time of a filter is so significant that it can not be ignored. As a result, it is understood that $fpfs$-matrices are an effective mathematical tool to deal with the situations in which more than one parameter or objects are used.



(a) **(b)** **(c)** **(d)**

Fig. 13. (a) Original image "Cameraman" (b) Noisy image with SPN ratio of 10%, (c) Noisy image with SPN ratio of 50%, and (d) Noisy image with SPN ratio of 90%

(a) DBA **(b)** MDBUTMF **(c)** NAFSM **(d)** DAMF **(e)** AWMF

Fig. 14. The images having with SPN ratio of 10% before denoising.

(a) DBA **(b)** MDBUTMF **(c)** NAFSM **(d)** DAMF **(e)** AWMF

Fig. 15. The images having with SPN ratio of 50% before denoising.

(a) DBA **(b)** MDBUTMF **(c)** NAFSM **(d)** DAMF **(e)** AWMF

Fig. 16. The images having with SPN ratio of 90% before denoising.

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