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Accretive Canonical Type Quasi-Differential Operators for First Order

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Abstract. In this work, using the method Calkin-Gorbachuk all maximal accretive extensions of the minimal operator generated by linear canonical type quasi-differential operator expression in the weighted Hilbert space of the vector functions defined at right semi-axis are described. Lastly, geometry of spectrum set of these type extensions will be investigated.

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1. Introduction

A linear closed densely defined operator in any Hilbert space is called accretive if its real part is non-negative and maximal accretive if it is accretive and it does not have any proper accretive extension [10]. Note that the study of abstract extension problems for operators on Hilbert spaces goes at least back to J.von Neumann [12] such that in [12] a full characterization of all selfadjoint extensions of a given closed symmetric operator with equal deficiency indices was investigated. Class of accretive operators is an important class of non-selfadjoint operators in the operator theory. Note that spectrum set of the accretive operators lies in right half-plane. The maximal accretive extensions of the minimal operator generated by regular differential-operator expression in Hilbert space of vector-functions defined in one finite interval case and their spectral analysis have been investigated by V. V. Levchuk [11].

In [6], the complete description of all maximal accretive extensions of the minimal operator generated by any linear ordinary differential expression for any order in weighted Hilbert space of functions at finite interval has been given. In paper [1] for accretive or α-sectorial extensions of a positive linear relation some criteria are established.

Description of all maximal sectorial extensions of a given closed densely defined sectorial operator in terms of abstract boundary conditions is investigated in [2].

And also the complete representation of all maximal accretive extensions of sectorial operators via boundary triplets was given in [3].

The description of the domains of all maximal accretive and sectorial quasi-selfadjoint extensions of a closed densely defined nonnegative operator in some Hilbert space was investigated in [4].

A description of all the maximal accretive extensions for a densely defined closed sectorial operator in terms of

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abstract boundary conditions was researched in [5].

Note that the detail informations in this topics can be obtained in famous work [7].

2. Statement of The Problem

Let H be a separable Hilbert space and $a \in \mathbb{R}$. Moreover assumed that $\alpha : (a, \infty) \to (0, \infty)$, $\alpha \in C(a, \infty)$ and $\frac{1}{\alpha} \in L^1(a, \infty)$. In the weighted Hilbert space $L^2_\alpha(H, (a, \infty))$ of $H-$ valued vector-functions defined on the right semi-axis consider the following linear quasi-differential expression with operator coefficient for first order in form

$$l(x) = J(\alpha x)'(\xi) + Ax(\xi),$$

where $A : H \to H$ is a bounded selfadjoint operator with condition $A \geq 0$, $J \in L(H)$, $J^* = J$, $J^2 = E$, $JA = AJ$.

By a known method the minimal L_0 and maximal L operators corresponding to quasi-differential expression $l(\cdot)$ in $L^2_\alpha(H, (a, \infty))$ can be described (see [9]). In this case the minimal operator L_0 is accretive, but it is not maximal in $L^2_\alpha(H, (a, \infty))$.

The main aim of this work is to describe of all maximal accretive extensions of the minimal operator L_0 in terms of boundary condition in $L^2_\alpha(H, (a, \infty))$. Secondly, geometry of the spectrum set of these extensions will be investigated.

3. Description of Maximal Accretive Extensions

Note that in similar way the minimal operator L_0^+ generated by quasi-operator expression

$$l^+(y) = -J(\alpha y)'(\xi) + Ay(\xi)$$

can be defined in $L^2_\alpha(H, (a, \infty))$ (see [9]). In this instance the operator $L^+ = (L_0)^*$ in $L^2_\alpha(H, (a, \infty))$ is called the maximal operator generated by $l^+(\cdot)$. It is obviously that $L_0 \subset L$ and $L_0^+ \subset L^+$.

If $\widetilde{L}$ is any maximal accretive extension of the minimal operator L_0 in $L^2_\alpha(H, (a, \infty))$ and $\widetilde{M}$ is corresponding extension of the minimal operator M_0 generated by quasi-differential expression

$$m(x) = iJ(\alpha x)'(\xi)$$

in $L^2_\alpha(H, (a, \infty))$, then it is clear that

$$\begin{aligned}
\widetilde{L}x &= J(\alpha x)'(\xi) + Ax(\xi) \\
&= i(-iJ(\alpha x)')(\xi) + Ax(\xi) \\
&= i(-\widetilde{M})x(\xi) + Ax(\xi) \\
&= i\left(-\left(Re\widetilde{M} + iIm\widetilde{M}\right)\right)x(\xi) + Ax(\xi) \\
&= \left(Im\widetilde{M}\right)x(\xi) - i\left(Re\widetilde{M}\right)x(\xi) + Ax(\xi) \\
&= \left[\left(Im\widetilde{M}\right) + A\right]x(\xi) - i\left(Re\widetilde{M}\right)x(\xi).
\end{aligned}$$

Therefore

$$\left(Re\widetilde{L}\right) = \left(Im\widetilde{M}\right) + A.$$

Furthermore

$$\left(Re\widetilde{L}\right) = \left(Im\widetilde{M}\right) + A = Im\left(\widetilde{M} + A\right).$$

Hence for the describe all accretive extension of the minimal operator L_0 in $L^2_\alpha(H, (a, \infty))$ the necessary and sufficient is representative of all dissipative extensions of the minimal operator K_0 generated by quasi-differential expression

$$k(x) = iJ(\alpha x)'(\xi) + Ax(\xi)$$

in $L^2_\alpha(H, (a, \infty))$.

Moreover, by K will be denoted the maximal operator generated by quasi-differential expression $k(\cdot)$ in $L^2_\alpha(H, (a, \infty))$.

In this chapter firstly, using Calkin-Gorbachuk method will be researched the general representation of all maximal dissipative extensions of the minimal operator in K_0 in $L^2_\alpha(H, (a, \infty))$.

Lemma 3.1. *The deficiency indices of the minimal operator K_0 in $L^2_\alpha(H, (a, \infty))$ are in form*

$$(n_+(K_0), n_-(K_0)) = (dimH, dimH).$$

Proof. For the simplicity we will take $A = 0$. It is obvious that the general solutions of differential equations

$$iJ(\alpha x_{\pm})'(\xi) \pm ix_{\pm}(\xi) = 0,\ \xi > a$$

in the $L^2_\alpha(H,(a,\infty))$ are in form

$$x_{\pm}(\xi) = \frac{1}{\alpha(\xi)} exp\left(\mp J\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)f,\ f \in H,\ \xi > a.$$

From these representations we have

$$\begin{aligned}
\|x_+\|^2_{L^2_\alpha(H,(a,\infty))} &= \int_a^\infty \alpha(\xi)\|x_+(\xi)\|^2_H d\xi \\
&= \int_a^\infty \|\frac{1}{\alpha(\xi)} exp\left(-J\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)f\|^2_H \alpha(\xi)d\xi \\
&\leq \int_a^\infty \frac{1}{\alpha(\xi)}\|exp\left(-J\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)\|^2_H d\xi\|f\|^2_H \\
&\leq \int_a^\infty \frac{1}{\alpha(\xi)}\left(exp\left(\|J\|\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)\right)^2 d\xi\|f\|^2_H \\
&\leq \int_a^\infty \frac{1}{\alpha(\xi)}d\xi\left(exp\left(\int_a^\infty \frac{1}{\alpha(\varsigma)}d\varsigma\right)\right)^2 \|f\|^2_H < \infty.
\end{aligned}$$

Consequently $n_+(K_0) = dim\ ker(K + iE) = dimH$.
Similarly $n_-(K_0) = dim\ ker(K - iE) = dimH$.
Accordingly, the minimal operator K_0 has a maximal dissipative extension (see [8]).
In order to describe these extensions we need to obtain the space of boundary values.

Definition 3.2 ([8]). Let H be any Hilbert space and $K : D(K) \subset \mathsf{H} \to \mathsf{H}$ be a closed densely defined symmetric operator in the Hilbert space H having equal finite or infinite deficiency indices. A triplet $(\mathbf{H}, \gamma_1, \gamma_2)$, where $\mathbf{H}$ is a Hilbert space, γ_1 and γ_2 are linear mappings from $D(K^*)$ into $\mathbf{H}$, is called a space of boundary values for the operator K if for any $f_1, f_2 \in D(K^*)$

$$< K^* f_1, f_2 >_{\mathsf{H}} - < f_1, K^* f_2 >_{\mathsf{H}} = < \gamma_1(f_1), \gamma_2(f_2) >_{\mathbf{H}} - < \gamma_2(f_1), \gamma_1(f_2) >_{\mathbf{H}}$$

while for any $F_1, F_2 \in \mathbf{H}$, there exists an element $f_1 \in D(K^*)$ such that $\gamma_1(f_1) = F_1$ and $\gamma_2(f_1) = F_2$.

Lemma 3.3. *The triplet* (H, γ_1, γ_2),

$$\begin{aligned}
\gamma_1 : D(K) \to H,\ \gamma_1(x) &= \frac{1}{\sqrt{2}}\left(J(\alpha x)(\infty) - J(\alpha x)(a)\right) \quad and \\
\gamma_2 : D(K) \to H,\ \gamma_2(x) &= \frac{1}{i\sqrt{2}}\left((\alpha x)(\infty) + (\alpha x)(a)\right),\ x \in D(K)
\end{aligned}$$

is a space of boundary values of the minimal operator K_0 *in* $L^2_\alpha(H,(a,\infty))$.

Proof. For any $x,\ y \in D(K)$

$$\begin{aligned}
& (Kx, y)_{L^2_\alpha(H,(a,\infty))} - (x, Ky)_{L^2_\alpha(H,(a,\infty))} \\
= \ & (iJ(\alpha x)' + Ax, y)_{L^2_\alpha(H,(a,\infty))} - (x, iJ(\alpha y)' + Ay)_{L^2_\alpha(H,(a,\infty))} \\
= \ & (iJ(\alpha x)', y)_{L^2_\alpha(H,(a,\infty))} - (x, iJ(\alpha y)')_{L^2_\alpha(H,(a,\infty))} \\
= \ & \int_a^\infty (iJ(\alpha x)'(\xi), y(\xi))_H \, \alpha(\xi) d\xi - \int_a^\infty (x(\xi), iJ(\alpha y)'(\xi))_H \, \alpha(\xi) d\xi \\
= \ & i \left[\int_a^\infty (J(\alpha x)'(\xi), (\alpha y)(\xi))_H d\xi + \int_a^\infty (J(\alpha x)(\xi), (\alpha y)'(\xi))_H d\xi \right] \\
= \ & i \int_a^\infty (J(\alpha x)(\xi), (\alpha y)(\xi))'_H d\xi \\
= \ & i \left[(J(\alpha x)(\infty), (\alpha y)(\infty))_H - (J(\alpha x)(a), (\alpha y)(a))_H \right] \\
= \ & (\gamma_1(x), \gamma_2(y))_H - (\gamma_2(x), \gamma_1(y))_H.
\end{aligned}$$

Now for any given elements $f_1,\ f_2 \in H$ find the function $x \in D(K)$ such that

$$\gamma_1(x) = \frac{1}{\sqrt{2}}(J(\alpha x)(\infty) - J(\alpha x)(a)) = f_1 \ \text{ and } \ \gamma_2(x) = \frac{1}{i\sqrt{2}}((\alpha x)(\infty) + (\alpha x)(a)) = f_2$$

From this it is obtained that

$$(\alpha x)(\infty) = (Jf_1 + if_2)/\sqrt{2} \ \text{ and } \ (\alpha x)(a) = (-Jf_1 + if_2)/\sqrt{2}.$$

If choose the functions $x(\cdot)$ in following forms

$$x(\xi) = \frac{1}{\alpha(\xi)}(1 - e^{a-\xi})(Jf_1 + if_2)/\sqrt{2} + \frac{1}{\alpha(\xi)} e^{a-\xi}(-Jf_1 + if_2)/\sqrt{2},$$

then it is obvious that $x \in D(K)$ and $\gamma_1(x) = f_1,\ \gamma_2(x) = f_2$.

The following result can be set up by using the method given in [8].

Theorem 3.4. *If $\widetilde{K}$ is a maximal dissipative extension of the minimal operator K_0 in $L^2_\alpha(H,(a,\infty))$, then it is generated by the differential-operator expression $k(\cdot)$ and boundary condition*

$$(V - E)(J(\alpha x)(\infty) - J(\alpha x)(a)) + (V + E)((\alpha x)(\infty) + (\alpha x)(a)) = 0,$$

where $V : H \to H$ is contraction operator. Additionally, the contraction operator V in H is determined uniquely by the extension $\widetilde{K}$, i.e. $\widetilde{K} = K_V$ and vice versa.

Proof. It is known that each maximal dissipative extension $\widetilde{K}$ of the minimal operator K_0 is described by differential-operator expression $k(\cdot)$ with boundary condition

$$(V - E)\gamma_1(x) + i(V + E)\gamma_2(x) = 0,$$

where $V : H \to H$ is a contraction operator. Therefore from Lemma 3.3 it is obtained that

$$(V - E)(J(\alpha x)(\infty) - J(\alpha x)(a)) + (V + E)((\alpha x)(\infty) + (\alpha x)(a)) = 0,\ x \in D(\widetilde{K}).$$

Theorem 3.5. *Each maximal accretive extension $\widetilde{L}$ of the minimal operator L_0 generates by linear singular quasi-differential expression $l(\cdot)$ and boundary condition*

$$(V - E)(J(\alpha x)(\infty) - J(\alpha x)(a)) + (V + E)((\alpha x)(\infty) + (\alpha x)(a)) = 0,$$

where $V : H \to H$ is a contraction operator such that this operator is determined uniquely by the extension $\widetilde{L}$, i.e. $\widetilde{L} = L_V$ and vice versa.

4. Spectrum of the Maximal Accretive Extensions

In this chapter the geometry of spectrum set of the maximal accretive extensions of the minimal operator L_0 in $L^2_\alpha(H,(a,\infty))$ will be researched.

Theorem 4.1. *In order to $\lambda \in \sigma(L_V)$ if and only if*

$$0 \in \sigma(\Delta(\lambda)),$$

where,

$$\Delta(\lambda) = \left[(V-E)J\left(exp\left(J(\lambda E - A)\int_a^\infty \frac{1}{\alpha(\varsigma)}d\varsigma\right) - E\right) + (V+E)\left(exp\left(J(\lambda E - A)\int_a^\infty \frac{1}{\alpha(\varsigma)}d\varsigma\right) + E\right)\right].$$

Proof. Take into account the following problem to get the spectrum of the extension L_V, i.e.

$$L_V(x) = \lambda x + f,\ \lambda \in \mathbb{C},\ \lambda_r = Re\lambda \geq 0.$$

Then we have

$$\begin{aligned} &J(\alpha x)'(\xi) + Ax(\xi) = \lambda x(\xi) + f(\xi),\ \xi > a,\\ &(V-E)\left(J(\alpha x)(\infty) - J(\alpha x)(a)\right) + (V+E)\left((\alpha x)(\infty) + (\alpha x)(a)\right) = 0. \end{aligned}$$

The general solution of the last differential equation is in form

$$\begin{aligned} x(\xi;\lambda) &= \frac{1}{\alpha(\xi)}exp\left(J(\lambda E - A)\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)f_\lambda \\ &- \frac{J}{\alpha(\xi)}\int_\xi^\infty \exp\left(J(\lambda E - A)\int_\varsigma^\xi \frac{1}{\alpha(\tau)}d\tau\right)f(\varsigma)d\varsigma,\ f_\lambda \in H,\ \xi > a. \end{aligned}$$

In this case

$$\begin{aligned} &\|\frac{1}{\alpha(\xi)}exp\left(J(\lambda E - A)\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)f_\lambda\|^2_{L^2_\alpha(H,(a,\infty))} \\ &= \int_a^\infty \|\frac{1}{\alpha(\xi)}exp\left(J(\lambda E - A)\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)f_\lambda\|^2_H \alpha(\xi)d\xi \\ &= \int_a^\infty \left(\frac{1}{\alpha(\xi)}exp\left(J(\lambda E - A)\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)f_\lambda, \frac{1}{\alpha(\xi)}exp\left(J(\lambda E - A)\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)f_\lambda\right)_H \alpha(\xi)d\xi \\ &= \int_a^\infty \frac{1}{\alpha(\xi)}exp\left(2J\lambda_r\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)\left(exp\left(-AJ\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)f_\lambda, exp\left(-AJ\int_a^\xi \frac{1}{\alpha(\varsigma)}ds\right)f_\lambda\right)_H d\xi \\ &= \int_a^\infty \frac{1}{\alpha(\xi)}exp\left(2\lambda_r J\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)\|exp\left(-AJ\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)f_\lambda\|^2_H d\varsigma \\ &\leq \int_a^\infty \frac{1}{\alpha(\xi)}exp\left(2\lambda_r J\int_a^\xi \frac{1}{\alpha(\varsigma)}d\varsigma\right)d\xi\|f_\lambda\|^2_H \\ &= exp\left(2\lambda_r J\int_a^\infty \frac{1}{\alpha(\varsigma)}d\varsigma\right)\int_a^\infty \frac{d\xi}{\alpha(\xi)}\|f_\lambda\|^2_H < \infty \end{aligned}$$

and

$$
\begin{aligned}
& \| -\frac{J}{\alpha(\xi)} \int_{\xi}^{\infty} \exp\left(J(\lambda E - A) \int_{\varsigma}^{\xi} \frac{1}{\alpha(\tau)} d\tau \right) f(\varsigma) d\varsigma \|^2_{L^2_\alpha(H,(a,\infty))} \\
= & \int_a^\infty \| \frac{J}{\alpha(\xi)} \int_{\xi}^{\infty} \exp\left(J(\lambda E - A) \int_{\varsigma}^{\xi} \frac{1}{\alpha(\tau)} d\tau \right) f(\varsigma) d\varsigma \|^2_H \alpha(\xi) d\xi \\
= & \int_a^\infty \frac{\|J\|}{\alpha(\xi)} \| \int_{\xi}^{\infty} \exp\left(J(\lambda E - A) \int_{\varsigma}^{\xi} \frac{1}{\alpha(\tau)} d\tau \right) f(\varsigma) d\varsigma \|^2_H d\xi \\
= & \int_a^\infty \frac{\|J\|}{\alpha(\xi)} \| \int_{\xi}^{\infty} \exp\left(\lambda J \int_{\varsigma}^{\xi} \frac{1}{\alpha(\tau)} d\tau \right) \left[\exp\left(-AJ \int_{\varsigma}^{\xi} \frac{1}{\alpha(\tau)} d\tau \right) f(\varsigma) \right] d\varsigma \|^2_H d\xi \\
= & \int_a^\infty \frac{\|J\|}{\alpha(\xi)} \| \int_{\xi}^{\infty} \exp\left((\lambda_r + i\lambda_i) J \int_{\varsigma}^{\xi} \frac{1}{\alpha(\tau)} d\tau \right) \left[\exp\left(-AJ \int_{\varsigma}^{\xi} \frac{1}{\alpha(\tau)} d\tau \right) f(\varsigma) \right] d\varsigma \|^2_H d\xi \\
\leq & \int_a^\infty \frac{\|J\|}{\alpha(\xi)} \left(\int_{\xi}^{\infty} \exp\left(\lambda_r J \int_{\varsigma}^{\xi} \frac{1}{\alpha(\tau)} d\tau \right) \frac{\sqrt{\alpha(\varsigma)}}{\alpha(\varsigma)} (\sqrt{\alpha(\varsigma)} \| f(\varsigma) \|) d\varsigma \right)^2 d\xi \\
\leq & \int_a^\infty \frac{\|J\|}{\alpha(\xi)} \left(\int_a^\infty \frac{1}{\alpha(\varsigma)} \exp\left(2\lambda_r J \int_{\varsigma}^{\xi} \frac{1}{\alpha(\tau)} d\tau \right) d\varsigma \right) \left(\int_a^\infty \alpha(\varsigma) \| f \|^2_H d\varsigma \right) d\xi \\
= & \int_a^\infty \frac{\|J\|}{\alpha(\xi)} \left(\int_a^\infty \frac{1}{\alpha(\varsigma)} \exp\left(2\lambda_r J \int_a^\infty \frac{1}{\alpha(\tau)} d\tau \right) d\varsigma \right) d\xi \| f \|^2_{L^2_\alpha(H,(a,\infty))} \\
= & \exp\left(2\lambda_r J \int_a^\infty \frac{1}{\alpha(\tau)} d\tau \right) \left(\int_a^\infty \frac{1}{\alpha(\xi)} dt \right)^2 \|J\| \| f \|^2_{L^2_\alpha(H,(a,\infty))} < \infty.
\end{aligned}
$$

Hence $x(\,\cdot\,,\lambda) \in L^2_\alpha(H,(a,\infty))$ for $\lambda \in \mathbb{C},\ Re\lambda \geq 0$.
Furthermore from boundary condition, we get

$$
\begin{aligned}
& \left[(V - E)J \left(exp\left(J(\lambda E - A) \int_a^\infty \frac{1}{\alpha(\varsigma)} d\varsigma \right) - E \right) + (V + E) \left(exp\left(J(\lambda E - A) \int_a^\infty \frac{1}{\alpha(\varsigma)} d\varsigma \right) + E \right) \right] f_\lambda \\
= \ & [(V + E)J - (V - E)] \int_a^\infty \exp\left(J(\lambda E - A) \int_{\varsigma}^{a} \frac{1}{\alpha(\tau)} d\tau \right) f(\varsigma) d\varsigma
\end{aligned}
$$

From last equation it is obtained the validity of claim of this theorem.

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