Proceedings of International Conference on Mathematics
and Mathematics Education (ICMME 2018)
Turk. J. Math. Comput. Sci.
10(2018) 263–268

http://dergipark.gov.tr/tjmcs

MATDER

Investigation of solutions of the Pade-II equation by MEFM

Tolga Akturk [a,*], Eda Gunaydin [b]

[a] *Department of Mathematics and Science Education, Faculty of Education, Ordu University, Turkey.*
[b] *Department of Mathematics, Faculty of Science, Ordu University, Turkey.*

Received: 15-08-2018 • Accepted: 11-12-2018

Abstract. In this study, singular soliton, topological, nontopological and rational solutions of the Pade-II equation, which is a type of nonlinear partial differential equations, are obtained. By selecting the appropriate parameters, solutions were obtained which provided the equations. Two and three dimensional graphics of the solutions were drawn. It appears that the resulting solution functions and graphics have similar features. Hyperbolic and trigonometric functions were also seen to have the same properties in their graphs since they are periodic functions at the same time. The obtained solutions were found by using the modified expansion method. All solutions using this method are controlled by the Mathematica software program which provides the Pade-II equation. Using the modified expansion function method, the nonlinear partial differential equation with travelling wave transformation takes the form of nonlinear ordinary differential equation. According to the balancing principle, there are also the degree of the solution function containing the exponential function.

2010 AMS Classification: 35D99, 35L05, 65K99, 65L80.

Keywords: The modified expansion function method (MEFM), the Pade-II equation, topological solution.

1. Introduction

There are various methods in the literature to obtain solutions of nonlinear partial differential equations. Some of these, respectively the sine-Gordon expansion method [3], [6], [7], the trial equation method [10], [12], the extended trial equation method [13], [14], the new function methods [1], [2], [5], [9] and so on. In this study, we apply the modified expansion function method (MEFM) [4], [8], [15], [16] to solve a nonlinear Pade-II equation.
The Pade-II equation can be defined as follow [11],

$$u_t + u_x + uu_x - \frac{9}{10}u_{xxx} - \frac{19}{10}u_{xxt} = 0.$$

*Corresponding Author
Email addresses: tolgaakturkk@gmail.com (T. Akturk), gunaydin@windowslive.com (E. Gunaydin)

2. Modified Expansion Function Method

We consider the following nonlinear partial differential equation;

$$P(u_x, uu_x, u_t, u_{xxx}, u_{xxt}) = 0. \tag{2.1}$$

where $u(x,t)$ is unknown function.
The following travelling wave transformation:

$$u(x,t) = U(\xi), \xi = x - ct. \tag{2.2}$$

where c is constant.
Substituting Eq. (2.2) into Eq. (2.1), gives the following nonlinear ordinary differential equation;

$$N(U, U^2, U'') = 0. \tag{2.3}$$

According to the MEFM, the desired solution is as follows:

$$U(\xi) = \frac{\sum_{i=0}^{m} A_i [e^{-\vartheta(\xi)}]^i}{\sum_{j=0}^{n} B_j [e^{-\vartheta(\xi)}]^j} = \frac{A_0 + A_1 e^{-\vartheta} + ... + A_m e^{-m\vartheta}}{B_0 + B_1 e^{-\vartheta} + ... + B_n e^{-n\vartheta}}. \tag{2.4}$$

where A_i,B_j, $(0 \le i \le m, 0 \le j \le n)$.
Using the homogeneous balance principle, the m and n positive integer values are obtained.

$$\upsilon'(\xi) = e^{-\vartheta(\xi)} + \mu e^{\vartheta(\xi)} + \lambda. \tag{2.5}$$

Eq.(2.5) has the following families of solutions [11]:
Family 1: When $\mu \neq 0, \lambda^2 - 4\mu > 0$,

$$\vartheta(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right).$$

Family 2: When, $\mu \neq 0, \lambda^2 - 4\mu < 0$,

$$\vartheta(\xi) = \ln\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu} \tan\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right).$$

Family 3: When, $\mu = 0, \lambda \neq 0, \lambda^2 - 4\mu > 0$,

$$\vartheta(\xi) = -\ln\left(\frac{\lambda}{e^{\lambda(\xi+E)} - 1}\right).$$

Family 4: When, $\mu \neq 0, \lambda \neq 0, \lambda^2 - 4\mu = 0$,

$$\vartheta(\xi) = \ln\left(-\frac{2\lambda(\xi + E) + 4}{\lambda^2(\xi + E)}\right).$$

Family 5: When, $\mu = 0, \lambda = 0, \lambda^2 - 4\mu = 0$,

$$\ln(\xi + E).$$

Substituting Eq. (2.4) and its derivatives into Eq. (2.3), we get algebraic equation system. This system was solved by using the Mathematica software program and then the solutions of the Pade-II equation were obtained.

3. Application

Consider the following travelling wave transformation:

$$u(x,t) = U(\xi), \xi = x - ct.$$

The following nonlinear ordinary differential equation is get,

$$5U^2 + 10(-c+1) + (-9+19c)U'' = 0. \tag{3.1}$$

If the balancing procedure is applied to equation (3.1),

$$m = n + 2.$$

Choosing $n = 1$ we get $m = 3$,the Eq. (2.4) is obtained for m and n values as follows;

$$U(\xi) = \frac{A_0 + A_1 e^{-\vartheta} + A_2 e^{-2\vartheta} + A_3 e^{-2\vartheta}}{B_0 + B_1 e^{-\vartheta}} \tag{3.2}$$

When the derivative expressions in equation (3.1) are obtained from (3.2) equation and written instead, the system of algebraic equations get. The equation system found is solved with Mathematica software program and the necessary coefficients in equation (3.1) are obtained. Substituting these coefficients into Eq. (3.2), the following solutions:

$$A_0 = \frac{\sqrt{c}\lambda B_0}{\sqrt{\lambda^2 - 4\mu}}, A_1 = \frac{\sqrt{c}\,(2B_0 + \lambda B_1)}{\sqrt{\lambda^2 - 4\mu}}, A_2 = \frac{2\sqrt{c}B_1}{\sqrt{\lambda^2 - 4\mu}}, k = \sqrt{\frac{2}{-d\,(\lambda^2 - 4\mu)}}$$

Substituting these coefficients into Eq. (3.5), we have the following solutions:
Family 1: When, $k \neq 0, \lambda^2 - 4k > 0$,

$$u_1(x,t) = \frac{\Big(-20(\lambda^2 - 4\mu)\sec h[\psi]^2\big(-4\mu + (\lambda^2 - 2\mu)\cosh[2\psi] + \lambda\sqrt{\lambda^2 - 4\mu}\sinh[2\psi]\big)\Big)}{\Big((10 + 19\lambda^2 - 76\mu)(\lambda + \sqrt{\lambda^2 - 4}\tanh[\psi])^2\Big)} \tag{3.3}$$

where $\psi = \frac{1}{2}\sqrt{\lambda^2 - 4\mu}(EE + \xi)$.

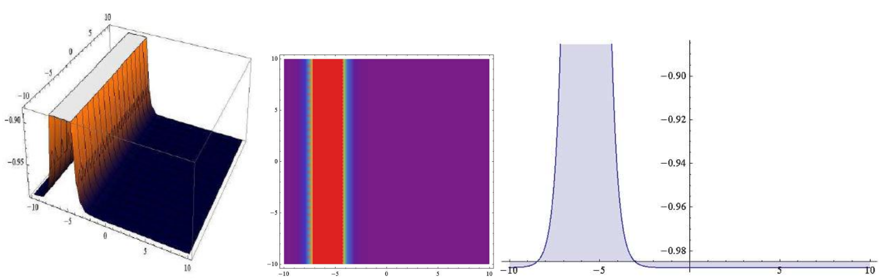

Figure 1. The 3D, density graphic and the 2D surface of Eq. (3.3) in $\lambda = 3, \mu = 0.25, c = -4, B_0 = 0.55, E = 0.5$ and $t = 1$

Family 2: When, $k \neq 0, \lambda^2 - 4k < 0$,

$$u_2(x,t) = \frac{\Big(20(\lambda^2 - 4\mu)\sec[\upsilon]^2(4\mu(\lambda^2 - 2\mu)\cos[2\upsilon] + \lambda\sqrt{-\lambda^2 + 4\mu}\sin[2\upsilon])\Big)}{\Big((10 + 19\lambda^2 - 76\mu)(\lambda + \sqrt{-\lambda^2 + 4}\tan[\upsilon])^2\Big)} \tag{3.4}$$

where $\upsilon = \frac{1}{2}\sqrt{-\lambda^2 + 4\mu}(EE + \xi)$.

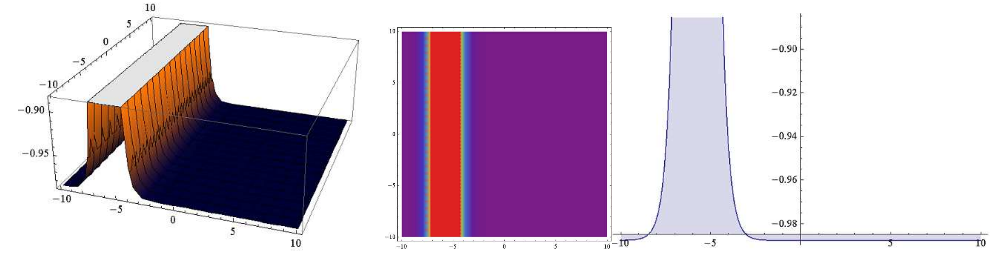

FIGURE 2. The 3D, density graphic and the 2D surface of Eq. (3.4) in $\lambda = 3, \mu = 0.25, c = -4, B_0 = 0.55, E = 0.5$ and $t = 1$

Family 3: When, $k = 0, \lambda \neq 0, \lambda^2 - 4k > 0$

$$u_3(x,t) = -\frac{10\left(2\lambda^2 + 4\mu + 3\lambda^2 \operatorname{csc} h\left[\frac{1}{2}\lambda(EE+\xi)\right]^2\right)}{10 + 19\lambda^2 - 76\mu} \tag{3.5}$$

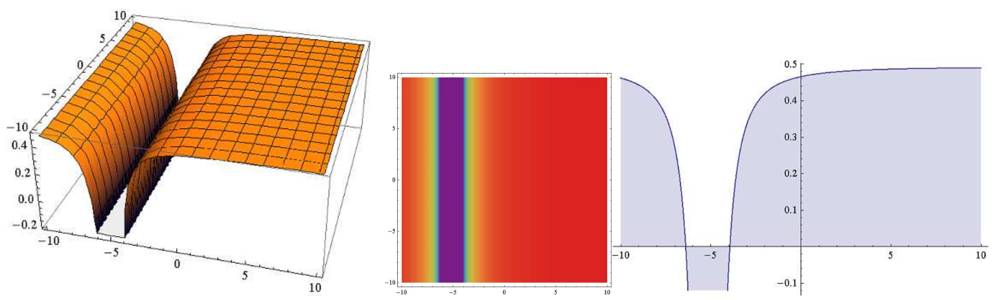

FIGURE 3. The 3D, density graphic and the 2D surface of Eq. (3.5) in $\lambda = 3, \mu = 0.25, c = -4, B_0 = 0.55, E = 0.5$ and $t = 1$

Family 4: When, $k \neq 0, \lambda \neq 0, \lambda^2 - 4k = 0$

$$u_4(x,t) = \frac{10\left(-4\mu + \lambda^2\left[1 - \frac{12}{\left(2+\lambda(EE+\xi)\right)^2}\right]\right)}{10 + 19\lambda^2 - 76\mu}$$

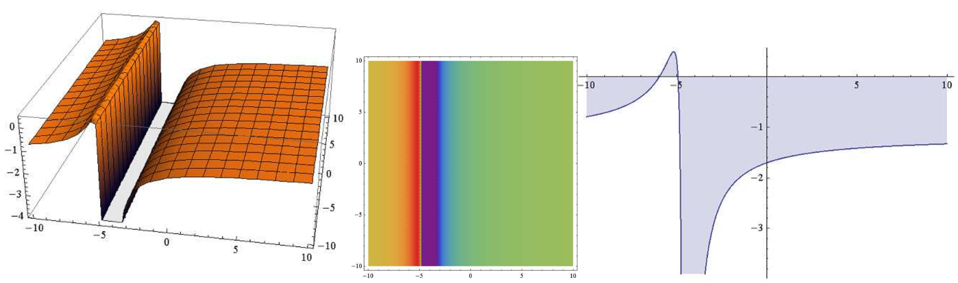

FIGURE 4. The 3D, density graphic and the 2D surface of Eq. (3.5) in $\lambda = 3, \mu = 0.25, c = -4, B_0 = 0.55, E = 0.5$ and $t = 1$

Family 5: When, $k =, \lambda = 0, \lambda^2 - 4k = 0$

$$u_5(x,t) = -\frac{20\left(6 + 6\lambda(EE + \xi) + (\lambda^2 + 2\mu)(EE + \xi\right)}{(10 + 19\lambda^2 - 76\mu)(EE + \xi)^2}$$

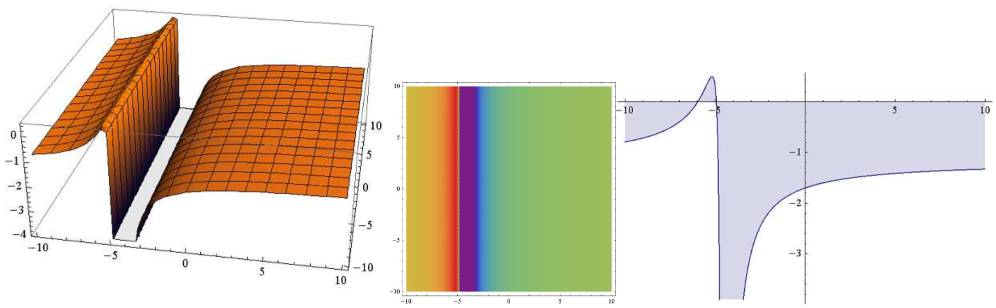

Figure 5. The 3D, density graphic and the 2D surface of Eq. (3.8) in $\lambda = 3, \mu = 0.25, c = -4, B_0 = 0.55, E = 0.5$ and $t = 1$

4. Conclusion

In this article, we get some travelling wave solutions of Pade-II equation by using modified expansion function method. The results show that the modified expansion function method is suitable mathematical method for solving nonlinear partial differential equations. The resulting solutions were checked with the Mathematica software.

References

[1] Akturk, T., Bulut, H., & Gurefe, Y., New function method to the (n+1)-dimensional nonlinear problems , An Int. Jour. of Opt. and Control: Theories & App. (2017), 7(3), 234-239. 1

[2] Akturk, T., Bulut, H., & Gurefe, Y., An application of the new function method to the Zhiber-Shabat equation , An Int. Jour. of Opt. and Control: Theories & App. (2017), 7(3), 271-274. 1

[3] Baskonus, H. M., Bulut, H., & Sulaiman, T. A. Investigation of various travelling wave solutions to the extended (2+1)-dimensional quantum ZK equation. The European Physical Journal Plus (2017), 132(11), 482. 1

[4] Baskonus, H. M., Bulut, H., & Atangana, A. On the complex and hyperbolic structures of the longitudinal wave equation in a magneto-electro-elastic circular rod. Smart Materials and Structures (2016), 25(3), 035022. 1

[5] Bulut, H., Akturk, T., & Gurefe, Y., Travelling wave solutions of the (N+1)-dimensional sine-cosine-Gordon equation, AIP Conf. Proc. (2014), 5. 1

[6] Chen, Y., & Yan, Z. New exact solutions of (2+ 1)-dimensional Gardner equation via the new sine-Gordon equation expansion method. Chaos, Solitons & Fractals (2005), 26(2), 399-406. 1

[7] Chen, Y., & Yan, Z. New exact solutions of (2+ 1)-dimensional Gardner equation via the new sine-Gordon equation expansion method. Chaos, Solitons & Fractals (2005), 26(2), 399-406. 1

[8] He, J. H., & Wu, X. H. (2006). Exp-function method for nonlinear wave equations. Chaos, Solitons & Fractals, 30(3), 700-708. 1

[9] Kudryashov, N. A., One method for finding exact solutions of nonlinear differential equations, Commun. Nonl. Sci. Numer. Simul. (2012), 17, 2248-2253. 1

[10] Liu, C. S. Trial equation method and its applications to nonlinear evolution equations, Acta. Phys. Sin., 54(6) (2005), 2505-2509. 1

[11] Naher, H., & Abdullah, F. A. New approach of (G'/G)-expansion method and new approach of generalized (G'/G)-expansion method for nonlinear evolution equation. AIP Advances (2013), 3(3), 032116. 1, 2

[12] Pandir, Y., Gurefe, Y., Kadak, U., & Misirli, E., Classification of exact solutions for some nonlinear partial differential equations with generalized evolution, Abstr. Appl. Anal. (2012), 16. 1

[13] Shen, G., Sun, Y., & Xiong, Y., New travelling-wave solutions for Dodd-Bullough equation, J. Appl. Math., (2013), 5. 1

[14] Sun, Y., New travelling wave solutions for Sine-Gordon equation, J. Appl. Math. (2014), 4. 1

[15] Shi, Y., Li, X., & Zhang, B. G. Traveling Wave Solutions of Two Nonlinear Wave Equations by-Expansion Method. Advances in Mathematical Physics (2018). 1

[16] Xu, F. Application of Exp-function method to symmetric regularized long wave (SRLW) equation. Physics Letters A (2008), 372(3), 252-257. 1