

## A computational approach for testing equality of coefficients of variation in $k$ normal populations

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### Abstract

In this article, a testing procedure based on computational approach testis proposed for the equality of coefficients of variation in  $k$  normal populations. We compare this procedure to some of the existing tests; the likelihood ratio, modified Bennett's, score, generalized  $p$ -value tests in terms of the estimated type I error rates and powers by using Monte Carlo simulation. Furthermore, applications of these tests are given on a real dataset.

**Keywords:** Computational approach test, Generalized  $p$ -value test, Likelihood ratio test, Modified Bennett's test, Score test

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### 1. Introduction

The ratio of population standard deviation to the population mean is called coefficient of variation (CV) of a population which is free from the unit of measurement. CV has a wide range of applications in many physical, biological, medical sciences, etc. For example, in haematology and serology, CV values of the measurement of the blood sample taken from the different laboratories are compared to determine performances of these laboratories [19]. Also the usage of CV includes other applications such as the determination of instrumental precision and the homogeneity of test samples.

A very common problem in applied statistics is to test equality of coefficients of variation. Many methods are developed for this problem. Miller and Karson [14] proposed a test for the equality of coefficients of variation in two normal populations. Doornbos and Dijkstra [6] presented two tests which are called likelihood ratio test and non-central  $t$  test for testing equality of coefficients of variation in  $k$  normal populations. Likelihood ratio test involves estimators of parameters which can be only obtained from iteration method. Gupta and Ma [11] used a better iteration method called bisection method to

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obtain these estimators of parameters. Bennet [1] proposed a test for  $k$  normal populations using transformed sample CV's. Afterwards, this test was modified by Shafer and Sullivan [17]. Gupta and Ma [11] developed score test for the case of  $k$  normal populations and compared their test to some of the existing tests. Fung and Tsang [8] compared parametric and nonparametric tests by using simulation studies for the equality of coefficients of variation in  $k$  normal populations. All of the test statistics mentioned above have asymptotic chi-squared distributions with  $k-1$  degrees of freedom. Thus, there is no exact statistical test for the equality of coefficients of variation in  $k$  normal populations. Approximate methods have been applied to solve a number of problems when conventional methods are difficult to apply or fail to provide exact solutions. Exact distributions of approximate methods are not known and the p-values can be only found by simulation. One of these methods is introduced by Tsui and Weerahandi [21], which is called the generalized  $p$ -value approach. Many researchers developed test procedures based on the generalized  $p$ -value for hypothesis testing [22, 12, 18, 24, 13]. Liu et al. [13] applied the generalized  $p$ -value approach for the equality of coefficients of variation in  $k$  normal populations and compared this approach to some of the existing tests; the likelihood ratio, modified Bennett's, score tests.

In this paper, a computational approach test (CAT) which is one of the most popular approximate methods is proposed for the equality of coefficients of variation in  $k$  normal populations. The CAT method based on simulation and numerical computations uses the maximum likelihood estimates (MLEs), and does not require the knowledge of any sampling distribution. This approach provides an algorithmic framework based on the Monte-Carlo simulation and numerical computations when a suitable parametric model is assumed for a given dataset [16]. Some papers related to CAT can be given as follows. Chang et al. [3, 5] showed how the CAT can be applied to Poisson and Gamma models for hypothesis testing. Chang and Pal [2] applied CAT to test the equality of two population means when the variances are unknown and arbitrary. Also Chang et al. [4] demonstrated that the CAT is as powerful as the classical F test for one-way ANOVA under homoscedasticity. Gokpinar and Gokpinar [9] modified CAT to test the equality of  $k$  population means when the variances are unequal. Also Gokpinar et al. [10] proposed a test based on CAT for the equality of several inverse Gaussian means under heterogeneity.

This article is organized as follows. In Section 2, the likelihood ratio, modified Bennett's, score tests used for the equality of coefficients of variation in  $k$  normal populations, are presented. In Section 3, the general CAT procedure and its algorithm are given in detail. Also in the third section the application of the CAT procedure for equality of coefficients of variation in  $k$  normal populations is presented. In Section 4, simulation results of estimated type I error rates and powers are obtained by using Monte Carlo studies. In section 5, CAT procedure is applied to a real life dataset. Concluding remarks are summarized in Section 6.

## 2. Tests for equality of coefficients of variation

In this section, the likelihood ratio, modified Bennett's, score, generalized  $p$ -value tests for testing equality of coefficients of variation in  $k$  normal populations, are presented. Initially, we need to give some notations and assumptions.

Let  $X_{i1}, X_{i2}, \dots, X_{in_i}$  be a random sample with size  $n_i$  from  $N(\mu_i, \sigma_i^2)$ ,  $i = 1, \dots, k$  where  $\mu_i$  and  $\sigma_i^2$  are the mean and variance of  $i$ th population, respectively and the coefficients of variation for  $i$ th population are defined as  $R_i = \sigma_i/\mu_i$ ,  $i = 1, \dots, k$ . The problem of interest involves testing:

$$(2.1) \quad H_0 : R_1 = R_2 = \dots = R_k = R \quad \text{against} \quad H_A : \exists R_i \neq R_j \quad i = 1, \dots, k.$$

Let  $\bar{X}_i = \sum_{j=1}^{n_i} X_{ij}/n_i$ ,  $S_i^2 = \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2 / n_i$  and  $r_i = S_i/\bar{X}_i$  denote the  $i$ th sample mean, variance and coefficients of variation for  $i = 1, \dots, k$ , respectively. Let  $\bar{x}_i$  and  $s_i^2$  denote the  $i$ th observed sample mean and variance for  $i = 1, \dots, k$ , respectively. Similar to Liu et. al. [13], we assume,

- (1)  $\mu_i > 0$ ;
- (2)  $P(\bar{X}_i < 0)$ , for each of  $i = 1, \dots, k$  is very small.

### 2.1. Likelihood ratio test

Likelihood ratio test is proposed by Doornbos and Dijkstra [6]. The likelihood function under  $H_0$  is given as follows:

$$(2.2) \quad L_0 = \prod_{i=1}^k \left( \frac{1}{\sqrt{2\pi\mu_i R}} \right)^{n_i} \exp \left( - \sum_{i=1}^k \sum_{j=1}^{n_i} \frac{(X_{ij} - \mu_i)^2}{2\mu_i^2 R^2} \right)$$

Differentiating the Equation (2.2) with respect to  $\mu_i$  and  $R$  yields the following results:

$$(2.3) \quad \frac{\partial \ln L_0}{\partial R} = - \sum_{i=1}^k \frac{n_i}{R} + \sum_{i=1}^k \sum_{j=1}^{n_i} \frac{(X_{ij} - \mu_i)^2}{\mu_i^2 R^3} = 0,$$

$$(2.4) \quad \frac{\partial \ln L_0}{\partial \mu_i} = - \frac{n_i}{\mu_i} + \sum_{j=1}^{n_i} \frac{X_{ij}(X_{ij} - \mu_i)}{\mu_i^3 R^2} = 0, \quad i = 1, \dots, k.$$

The Equation (2.3) and Equation (2.4) are simplified as follows:

$$(2.5) \quad \sum_{i=1}^k \frac{n_i \left( 1 + \sqrt{1 + 4(1 + r_i^2)R^2} \right)}{2(1 + r_i^2)} - \sum_{i=1}^k n_i = 0$$

$$(2.6) \quad \mu_i = \frac{2(1 + r_i^2)\bar{X}_i}{1 + \sqrt{1 + 4(1 + r_i^2)R^2}}, \quad i = 1, \dots, k.$$

As seen from Equation (2.5) and Equation (2.6), the restricted MLEs (RMLEs) of the  $R$  and  $\mu_i$  have no closed forms. Therefore, the numerical method called bisection method, which is proposed by Gupta and Ma [11], could be used for the RMLEs of these parameters. Hence, the likelihood ratio test statistic is given as follows:

$$(2.7) \quad -2 \ln \lambda = \sum_{i=1}^k n_i \ln \left( \frac{\hat{\mu}_{i(RML)}^2 \hat{R}_{RML}^2}{S_i^2} \right) \sim \chi_{k-1}^2,$$

where  $\hat{\mu}_{i(RML)}$  and  $\hat{R}_{RML}$  are the RMLEs of  $\mu_i$  and  $R$ . For a given level  $\alpha$ , this test rejects the  $H_0$  in Equation (2.1) if  $-2 \ln \lambda > \chi_{k-1, \alpha}^2$ .

### 2.2. Modified Bennett's test

Shafer and Sullivan [17] modified Bennett's test given as

$$(2.8) \quad -2 \ln \lambda = (N - k) \ln \sum_{i=1}^k \left( \frac{d_i}{N - k} \right) - \sum_{i=1}^k (n_i - 1) \ln \left( \frac{d_i}{n_i - k} \right) \sim \chi_{k-1}^2.$$

Here  $N = \sum_{i=1}^k n_i$  and  $d_i = n_i r_i^2 / (r_i^2 + 1)$ . For a given level  $\alpha$ , this test rejects the null hypothesis in Equation (2.1) if  $-2 \ln \lambda > \chi_{k-1, \alpha}^2$ .

### 2.3. Score test

To test the null hypothesis in Equation (2.1), the explicit value of test statistic is given by

$$(2.9) \quad S = \left[ \frac{\hat{R}_{RML}^2 (2\hat{R}_{RML}^2 + 1)}{2} \right] \sum_{i=1}^k \frac{a_i^2}{n_i} \sim \chi_{k-1}^2,$$

where  $a_i = \sum_{j=1}^{n_i} (X_{ij} - \hat{\mu}_{i(RML)})^2 / \hat{\mu}_{i(RML)}^2 \hat{R}_{RML}^3 - n_i / \hat{R}_{RML}$  (see [11] for details). For a given level  $\alpha$ , this test rejects the null hypothesis in Equation (2.1) if  $-2 \ln \lambda > \chi_{k-1, \alpha}^2$ .

**2.4. Generalized p-value test**

Tsui and Weerahandi [21] presented the concept of generalized  $p$ -value for some statistical testing problems. A parallel test for the equality of coefficients of variation in  $k$  normal populations is developed as follows [13]. The null hypothesis in Equation (2.1) can be rewritten as follows:

$$(2.10) \quad H_{10}: \mu_1/\sigma_1 = \mu_2/\sigma_2 = \dots = \mu_k/\sigma_k.$$

Put  $C = (\mu_1/\sigma_1, \mu_2/\sigma_2, \dots, \mu_k/\sigma_k)'$  and

$$A = \begin{pmatrix} 1 & 0 & \dots & 0 & -1 \\ 0 & 1 & \dots & 0 & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -1 \end{pmatrix}_{(k-1) \times k}.$$

Thus, the null hypothesis in Equation (2.7) is equivalent to  $H_{20} : AC = 0$ . The generalized pivotal quantities for  $\mu_i$  and  $\sigma_i$  are given as follows:

$$R_{\mu_i} = \bar{x}_i - \frac{s_i}{S_i} (X_i - \mu_i) \text{ and } R_{\sigma_i} = \frac{S_i}{s_i} \sigma_i$$

[23]. The generalized pivotal quantity for  $AC$  could be written as follows.

$$R_{AC} = AR_c = A \left( \frac{R_{\mu_1}}{R_{\sigma_1}}, \dots, \frac{R_{\mu_k}}{R_{\sigma_k}} \right)'.$$

Here  $\frac{R_{\mu_i}}{R_{\sigma_i}} = \frac{\bar{x}_i - s_i/S_i(X_i - \mu_i)}{s_i/S_i\sigma_i} = \frac{\bar{x}_i}{\sqrt{n_i} s_i} U_i - \frac{Z_i}{\sqrt{n_i}}$  and  $U_i^2 \sim \chi_{(n_i-1)}^2$ ,  $Z_i \sim N(0, 1)$ ,  $i = 1, \dots, k$ .

The conditional expectation and covariance matrix of  $R_{AC}$  for given  $(\bar{x}, s)$  can be written as:

$$\begin{aligned} \mu_R &= A \left( E \left( \frac{R_{\mu_1}}{R_{\sigma_1}} (\bar{x}, s) \right), \dots, E \left( \frac{R_{\mu_k}}{R_{\sigma_k}} (\bar{x}, s) \right) \right)', \\ \Sigma_R &= A \text{diag} \left( \text{Var} \left( \frac{R_{\mu_1}}{R_{\sigma_1}} (\bar{x}, s) \right), \dots, \text{Var} \left( \frac{R_{\mu_k}}{R_{\sigma_k}} (\bar{x}, s) \right) \right) A', \end{aligned}$$

where

$$\begin{aligned} E \left( \frac{R_{\mu_1}}{R_{\sigma_1}} (\bar{x}, s) \right) &= \frac{\bar{x}_i}{\sqrt{n_i} s_i} E(U_i), \\ \text{Var} \left( \frac{R_{\mu_1}}{R_{\sigma_1}} (\bar{x}, s) \right) &= \frac{\bar{x}_i^2}{n_i s_i^2} \text{Var}(U_i) + \frac{1}{n_i}, \end{aligned}$$

and

$$E(U_i) = \begin{cases} \sqrt{\frac{2}{\pi}} \frac{(n_i-2)!!}{(n_i-3)!!}, & n_i \geq 3 \text{ and } n_i \text{ is odd} \\ \sqrt{\frac{\pi}{2}} \frac{(n_i-2)!!}{(n_i-3)!!}, & n_i \geq 4 \text{ and } n_i \text{ is even} \end{cases}$$

$$\text{Var}(U_i) = n_i - 1 - (E(U_i))^2.$$

The standardized expression of  $R_{AC}$  is  $D = (\Sigma_R)^{-1/2} (R_{AC} - \mu_R)$  and  $d$  is the observed value of  $D$  for  $(\bar{X}, S) = (\bar{x}, s)$ . Then  $\|D\|^2 = (R_{AC} - \mu_R)' \Sigma_R^{-1} (R_{AC} - \mu_R)$  and the observed value  $\|d\|^2$  under  $H_{20} : AC = 0$  is equal to  $\mu_R' \Sigma_R^{-1} \mu_R$ . The generalized  $p$ -value based on  $\|D\|^2$  could be given as follows:

$$\begin{aligned} p &= P(\|D\|^2 \geq \|d\|^2 / H_{20}) \\ &= P\left((R_{AC} - \mu_R)' \Sigma_R^{-1} (R_{AC} - \mu_R) \geq \mu_R' \Sigma_R^{-1} \mu_R\right) \\ &= P\left(\sum_{i=1}^k \frac{[\bar{x}_i(U_i - E(U_i)) - s_i Z_i]^2}{\bar{x}_i^2 \text{Var}(U_i) + s_i^2} - \frac{1}{\sum_{j=1}^k n_j s_j^2 / (\bar{x}_j^2 \text{Var}(U_j) + s_j^2)}\right. \\ &\quad \left. \times \left(\sum_{i=1}^k \frac{[\bar{x}_i(U_i - E(U_i)) - s_i Z_i] \sqrt{n_i} s_i}{\bar{x}_i^2 \text{Var}(U_i) + s_i^2}\right)^2\right) \\ (2.11) \quad &\geq \sum_{i=1}^k \frac{(\bar{x}_i E(U_i))^2}{\bar{x}_i^2 \text{Var}(U_i) + s_i^2} - \frac{1}{\sum_{j=1}^k n_j s_j^2 / (\bar{x}_j^2 \text{Var}(U_j) + s_j^2)} \left(\sum_{i=1}^k \frac{\bar{x}_i E(U_i) \sqrt{n_i} s_i}{\bar{x}_i^2 \text{Var}(U_i) + s_i^2}\right)^2. \end{aligned}$$

$H_{10}$  in Equation (2.10) is rejected if  $p < \alpha$ .

### 3. The computational approach test

In this section, initially we introduce the general framework of CAT procedure. By using this procedure, we give an algorithm for testing equality of coefficients of variation in  $k$  normal populations. The algorithm of CAT based on the paper of Pal et al. [16] can be given as follows:

Let  $X_1, X_2, \dots, X_n$  be random sample having a probability density function  $f(x/\theta)$ , where the functional form of  $f$  is assumed to be known and  $\theta = (\theta^{(1)}, \theta^{(2)})$  is an unknown vector in parameter space  $\Theta$ .  $\theta^{(1)}$  is the parameter of interest and  $\theta^{(2)}$  is the nuisance parameter. The problem of interest is to test  $H_0' : \theta^{(1)} = \theta_0^{(1)}$  against a suitable alternative  $H_1'^*$ . To do this, initially we express  $H_0'$  as  $H_0'^* : \eta(\theta^{(1)}, \theta_0^{(1)}) = 0$  where  $\eta$  is a scalar valued function.

The general methodology of the proposed CAT for testing  $H_0'^* : \eta(\theta^{(1)}, \theta_0^{(1)}) = 0$  against  $H_1'^*$  at a desired level  $\alpha$  is given through the following steps.

1. Calculate  $\hat{\theta}_{ML} = (\hat{\theta}_{ML}^{(1)}, \hat{\theta}_{ML}^{(2)})$ , where  $\hat{\theta}_{ML}$  is maximum likelihood estimation (MLE) of  $\theta$ . Thus,  $\hat{\eta}_{ML} = \eta(\hat{\theta}_{ML}^{(1)}, \theta_0^{(1)})$  is estimated.

2. Find the MLE of  $\theta^{(2)}$  under  $H_0'$  or  $H_0'^*$  from the data which is called the restricted MLE (RMLE) of  $\theta^{(2)}$  and denoted by  $\hat{\theta}_{RML}^{(2)}$ .

3. Generate artificial sample  $X_1, X_2, \dots, X_n$  from  $f(x/\theta_0^{(1)}, \hat{\theta}_{RML}^{(2)})$  a large number of times (say  $m$  times). For each of these data, recalculate the MLE of  $\theta^{(1)}$ , i.e.,  $\tilde{\theta}_1^{(1)}, \tilde{\theta}_2^{(1)}, \dots, \tilde{\theta}_m^{(1)}$  and  $\tilde{\eta}_{ML}^{(j)} = \eta(\tilde{\theta}_j^{(1)}, \theta_0^{(1)})$ ,  $j = 1, \dots, m$ .

4. For testing  $H_0'^* : \eta(\theta^{(1)}, \theta_0^{(1)}) = 0$  versus  $H_1'^* : \eta(\theta^{(1)}, \theta_0^{(1)}) > 0$ , calculate the  $p$ -value as  $p = \#(\tilde{\eta}_{ML}^{(j)} > \hat{\eta}_{ML}) / m$ . In the case of  $p < \alpha$ ,  $H_0$  is rejected.

**Remark 3.1:** The success of CAT depends heavily on the selection of  $\eta$ . For this purpose, the choice of  $\eta$  needs a little clarification. According to Chang et. al.[5], CAT works best (in terms of maintaining the desired level and attaining a high power) for two different situations as follows. When we have location parameters which can take values over the real line, we can use the standard quadratic expression for  $\eta$  as it is done in classical one-way ANOVA under normality assumption. When our parameters

are nonnegative, and so are the observations, the logarithmic transformation makes the parameters behave like location parameters as it is done in gamma model [5]. Thus, interested parameters take nonnegative values, as we have here, firstly we use the logarithmic transformation of parameters, and after that standard quadratic (or squared) expression of these transformed parameters is used for  $\eta$ , i.e.  $\eta = \sum_{i=1}^k n_i \left( \log \theta_i^{(1)} - \log \bar{\theta}^{(1)} \right)^2$ , where  $\bar{\theta}^{(1)} = \sum_{i=1}^k \theta_i^{(1)} / k$ .

**Remark 3.2:** The CAT procedure borrows ideas from the classical likelihood ratio test as well as parametric bootstrap [2]. It is well known that the classical likelihood ratio test is based on MLE under  $H_0$ , that is, RMLE. Score test is also based on RMLE. Both the classical likelihood ratio test and Score test use test statistics which are asymptotically distributed as Chi-square under  $H_0$ . However, the CAT method uses the idea of replicating data from  $f \left( x/\theta_0^{(1)}, \hat{\theta}_{RML}^{(2)} \right)$ .

In the rest of this section, a test procedure based on CAT is given for testing equality of coefficients of variation in  $k$  normal populations based on algorithm given above.

Initially, the null hypothesis given in Equation (2.1) should be expressed in terms of suitable scalar  $\eta$  based on the criteria given in Remark. Thus,  $\eta$  is defined as shown in Equation (3.1):

$$(3.1) \quad \eta = \eta(R_1, R_2, \dots, R_k) = \sum_{i=1}^k n_i (\log R_i - \log \bar{R})^2,$$

where  $\bar{R} = \sum_{i=1}^k R_i / k$ . It is clear that testing  $H_0$  against  $H_1$  is equivalent to testing  $H_0^* : \eta = 0$  against  $H_1^* : \eta > 0$ . With the general idea of CAT which is given above, its application for testing equality of coefficients of variation in  $k$  normal populations can be given as below:

1. The sample coefficient of variation for the  $i$ th group is  $r_i = S_i / \bar{X}_i$ . Therefore, the  $\hat{\eta}_{ML}$  is obtained  $\hat{\eta}_{ML} = \sum_{i=1}^k n_i (\log r_i - \log \bar{r})^2$  by using these sample coefficient of variation. Here  $\bar{r} = \sum_{i=1}^k r_i / k$ . The observed value of  $\hat{\eta}_{ML}$  is  $\hat{\eta}_{ML}^*$ .

2. Under  $H_0$  or  $H_0^*$ , the restricted MLEs  $(\hat{\mu}_{i(RML)}, \hat{R}_{RML})$  of  $(\mu_i, R)$  are obtained iteratively from Equation (2.5) and Equation (2.6) by using bisection method given in Gupta and Ma [11].

3. Generate artificial sample  $X_{i1}, X_{i2}, \dots, X_{in_i}$ ,  $1 \leq i \leq k$  i.i.d. from  $N \left( \hat{\mu}_{i(RML)}, \hat{\mu}_{i(RML)}^2 \times \hat{R}_{RML}^2 \right)$  a large of number of times (say  $m$  times). For each of these replicated samples, recalculate the values of  $\tilde{\eta}_{ML}^{(j)}$  ( $j = 1, \dots, m$ ).

4. Calculate the  $p$ -value as  $p = \# \left( \tilde{\eta}_{ML}^{(j)} > \hat{\eta}_{ML}^* \right) / m$ . In the case of  $p < \alpha$ ,  $H_0$  is rejected.

**Remark 3.3:** By generating artificial sample we are trying to mimic the null distribution of  $\hat{\eta}_{ML}$ . Thus the cut-off point  $\tilde{\eta}_C = \tilde{\eta}_{ML((1-\alpha)m)}$  is an approximation of the true critical value based on the null model.  $\hat{\eta}_{ML}$  acts as an automatic test statistic, and helps us make a decision based on the value of  $\tilde{\eta}_C$  [3].

#### 4. A simulation study

In this section for testing equality of coefficients of variation in  $k$  normal populations, the likelihood ratio test (LRT), modified Bennett's test (MBT), score test (SCT), generalized  $p$ -value test (GPT) and CAT are compared according to type I errors and powers for different combinations of parameters  $(\mu_i, \sigma_i)$  and sample sizes. For this purpose, we

consider some cases from smaller to larger sample sizes with different number of groups as  $k=3, 4, 5, 6, 7$ . For specified nominal level of  $\alpha=0.05$ , 5000 replications are used to calculate the estimated type I error rates and powers of each tests. Also 5000 replications are used to obtain the  $p$  values of GPT and CAT.

Firstly, we calculate the type I error rates of tests under null hypothesis for  $(\mu_i = 3, \sigma_i = 1, i = 1, 2, \dots, k)$ . The numerical results for estimated type I error rates are given as in Table 1 to Table 5.

**Table 1.** Estimated type I error rates of tests for  $k=3$

<b>n</b>	<b>CAT</b>	<b>GPT</b>	<b>LRT</b>	<b>MBT</b>	<b>SCT</b>
6,6,6	0.058	0.026	0.109	0.070	0.053
6,8,10	0.049	0.032	0.086	0.055	0.044
10,10,10	0.048	0.039	0.073	0.055	0.047
15,15,20	0.050	0.041	0.066	0.051	0.046
20,20,20	0.050	0.042	0.061	0.053	0.050
10,15,20	0.049	0.041	0.068	0.054	0.047
10,20,30	0.046	0.038	0.063	0.051	0.047
30,30,30	0.051	0.048	0.057	0.051	0.050

**Table 2.** Estimated type I error rates of tests for  $k=4$

<b>n</b>	<b>CAT</b>	<b>GPT</b>	<b>LRT</b>	<b>MBT</b>	<b>SCT</b>
6,6,6,6	0.051	0.019	0.103	0.060	0.056
6,8,10,12	0.054	0.035	0.095	0.063	0.056
10,15,20,25	0.055	0.042	0.073	0.056	0.051
10,10,10,10	0.052	0.035	0.081	0.058	0.054
10,10,15,15	0.053	0.038	0.079	0.057	0.055
20,20,20,20	0.050	0.042	0.061	0.052	0.051
15,15,20,20	0.053	0.045	0.069	0.053	0.057
10,20,20,30	0.051	0.044	0.071	0.057	0.057
30,30,30,30	0.047	0.041	0.055	0.046	0.048

**Table 3.** Estimated type I error rates of tests for  $k=5$

<b>n</b>	<b>CAT</b>	<b>GPT</b>	<b>LRT</b>	<b>MBT</b>	<b>SCT</b>
6,6,6,6,6	0.051	0.020	0.122	0.065	0.068
6,8,10,12,14	0.051	0.033	0.091	0.058	0.059
10,15,20,25,30	0.049	0.038	0.070	0.051	0.051
10,10,10,10,10	0.050	0.031	0.092	0.058	0.065
20,20,20,20,20	0.052	0.043	0.068	0.053	0.053
15,15,15,20,20	0.053	0.043	0.067	0.054	0.056
10,10,10,15,15	0.052	0.033	0.082	0.056	0.058
10,10,20,30,30	0.053	0.042	0.075	0.055	0.057
30,30,30,30,30	0.055	0.050	0.064	0.056	0.058

**Table 4.** Estimated type I error rates of tests for  $k=6$

<b>n</b>	<b>CAT</b>	<b>GPT</b>	<b>LRT</b>	<b>MBT</b>	<b>SCT</b>
6,6,6,6,6,6	0.053	0.017	0.127	0.064	0.075
6,8,10,10,12,14	0.052	0.028	0.094	0.057	0.060
10,15,20,20,25,30	0.055	0.041	0.075	0.057	0.055
10,10,10,10,10,10	0.050	0.028	0.091	0.057	0.064
20,20,20,20,20,20	0.055	0.043	0.069	0.055	0.057
15,15,15,20,20,20	0.054	0.042	0.071	0.054	0.059
10,10,10,15,15,15	0.056	0.035	0.087	0.061	0.060
10,10,20,20,30,30	0.049	0.038	0.070	0.050	0.054
30,30,30,30,30,30	0.052	0.046	0.062	0.053	0.053

**Table 5.** Estimated type I error rates of tests for  $k=7$

<b>n</b>	<b>CAT</b>	<b>GPT</b>	<b>LRT</b>	<b>MBT</b>	<b>SCT</b>
6,6,6,6,6,6	0.052	0.016	0.136	0.066	0.079
6,8,10,10,10,12,14	0.053	0.029	0.099	0.063	0.073
10,15,20,20,20,25,30	0.051	0.041	0.071	0.054	0.064
10,10,10,10,10,10,10	0.052	0.030	0.095	0.058	0.073
20,20,20,20,20,20,20	0.055	0.044	0.074	0.057	0.061
15,15,15,20,20,20,20	0.052	0.038	0.067	0.052	0.056
10,10,10,15,15,15,15	0.053	0.033	0.082	0.058	0.066
10,10,20,20,20,30,30	0.049	0.036	0.069	0.054	0.058
30,30,30,30,30,30,30	0.048	0.044	0.058	0.048	0.055

As seen from Table 1-Table 5, the GPT seems to have lower the estimated type I error rates than nominal level, especially for small sample size. Contrary to GPT, LRT has estimated type I error rates greater than the nominal level. In the case of small sample size, the estimated type I error rates of MBT exceed the nominal level for all  $k$ . In the case of small sample size, the estimated type I error rates of SCT get larger than nominal level, especially when  $k$  is large. Also, it is observed that the MBT and SCT have the estimated type I error rates close to the nominal level for other cases. However, the CAT seems to have the estimated type I error rates close to nominal level in all cases.

After calculating the type I error rates of five methods, we calculate the estimated powers of the tests for different combinations of parameters and sample sizes.

**Table 6.** Estimated powers of tests for  $\mu_i=3$  ( $i=1,2,3$ )



<b>n</b>	<b><math>R_1, R_2, R_3</math></b>	<b>CAT</b>	<b>GPT</b>	<b>LRT</b>	<b>MBT</b>	<b>SCT</b>
6,6,6	1/9,1/6,1/6	0.109	0.057	0.207	0.138	0.102
	1/9,2/9,2/9	0.246	0.133	0.387	0.287	0.179
	1/9,1/3,1/3	0.592	0.346	0.718	0.612	0.290
	1/4,1/2,1/2	0.215	0.122	0.336	0.235	0.150
6,8,10	1/9,1/6,1/6	0.144	0.054	0.196	0.124	0.073
	1/9,2/9,2/9	0.326	0.122	0.396	0.275	0.117
	1/9,1/3,1/3	0.718	0.314	0.765	0.625	0.188
	1/4,1/2,1/2	0.267	0.118	0.366	0.252	0.116
10,10,10	1/9,1/6,1/6	0.175	0.132	0.231	0.185	0.137
	1/9,2/9,2/9	0.477	0.359	0.545	0.477	0.290
	1/9,1/3,1/3	0.885	0.797	0.913	0.880	0.617
	1/4,1/2,1/2	0.403	0.319	0.478	0.406	0.265
15,15,20	1/9,1/6,1/6	0.312	0.237	0.341	0.299	0.208
	1/9,2/9,2/9	0.748	0.646	0.764	0.728	0.536
	1/9,1/3,1/3	0.990	0.977	0.990	0.988	0.938
	1/4,1/2,1/2	0.652	0.576	0.685	0.636	0.478
20,20,20	1/9,1/6,1/6	0.382	0.334	0.410	0.382	0.309
	1/9,2/9,2/9	0.848	0.798	0.855	0.838	0.742
	1/9,1/3,1/3	0.999	0.998	0.999	0.999	0.994
	1/4,1/2,1/2	0.781	0.736	0.796	0.767	0.676
30,30,30	1/9,1/6,1/6	0.557	0.514	0.571	0.552	0.484
	1/9,2/9,2/9	0.975	0.963	0.974	0.971	0.948
	1/9,1/3,1/3	1.000	1.000	1.000	1.000	1.000
	1/4,1/2,1/2	0.940	0.925	0.945	0.936	0.900

**Table 7.** Estimated powers of tests for  $\mu_i=3$  ( $i=1,2,3,4$ )

n	$R_1, R_2, R_3, R_4$	CAT	GPT	LRT	MBT	SCT
6,6,6,6	1/9,1/6,1/6,1/6	0.095	0.044	0.197	0.122	0.105
	1/9,2/9,2/9,2/9	0.220	0.093	0.360	0.250	0.160
	1/9,1/3,1/3,1/3	0.543	0.215	0.682	0.545	0.247
	1/4,1/2,1/2,1/2	0.195	0.092	0.330	0.213	0.159
6,8,10,12	1/9,1/6,1/6,1/6	0.147	0.054	0.204	0.125	0.082
	1/9,2/9,2/9,2/9	0.324	0.096	0.377	0.249	0.111
	1/9,1/3,1/3,1/3	0.708	0.218	0.727	0.564	0.159
	1/4,1/2,1/2,1/2	0.253	0.093	0.345	0.214	0.115
10,10,10,10	1/9,1/6,1/6,1/6	0.169	0.118	0.230	0.184	0.139
	1/9,2/9,2/9,2/9	0.450	0.299	0.521	0.444	0.255
	1/9,1/3,1/3,1/3	0.886	0.701	0.906	0.862	0.431
	1/4,1/2,1/2,1/2	0.391	0.272	0.469	0.380	0.238
15,15,20,20	1/9,1/6,1/6,1/6	0.305	0.207	0.323	0.279	0.175
	1/9,2/9,2/9,2/9	0.743	0.576	0.743	0.689	0.394
	1/9,1/3,1/3,1/3	0.994	0.967	0.992	0.990	0.769
	1/4,1/2,1/2,1/2	0.635	0.494	0.666	0.596	0.341
20,20,20,20	1/9,1/6,1/6,1/6	0.365	0.294	0.385	0.355	0.252
	1/9,2/9,2/9,2/9	0.477	0.359	0.545	0.477	0.290
	1/9,1/3,1/3,1/3	1.000	0.999	1.000	1.000	0.981
	1/4,1/2,1/2,1/2	0.769	0.694	0.783	0.747	0.562
30,30,30,30	1/9,1/6,1/6,1/6	0.551	0.476	0.552	0.528	0.412
	1/9,2/9,2/9,2/9	0.977	0.958	0.976	0.973	0.914
	1/9,1/3,1/3,1/3	1.000	1.000	1.000	1.000	1.000
	1/4,1/2,1/2,1/2	0.941	0.917	0.942	0.931	0.850

**Table 8.** Estimated powers of tests for  $\mu_i=3$  ( $i=1,2,\dots,5$ )

n	$R_1, R_2, R_3, R_4, R_5$	CAT	GPT	LRT	MBT	SCT
6,6,6,6,6	1/9,1/6,1/6,1/6,1/6	0.093	0.040	0.194	0.121	0.108
	1/9,2/9,2/9,2/9,2/9	0.190	0.076	0.338	0.221	0.161
	1/9,1/3,1/3,1/3,1/3	0.491	0.165	0.639	0.481	0.227
	1/4,1/2,1/2,1/2,1/2	0.186	0.073	0.333	0.205	0.167
6,8,10,12,14	1/9,1/6,1/6,1/6,1/6	0.143	0.047	0.184	0.114	0.075
	1/9,2/9,2/9,2/9,2/9	0.323	0.088	0.361	0.233	0.109
	1/9,1/3,1/3,1/3,1/3	0.695	0.180	0.686	0.509	0.147
	1/4,1/2,1/2,1/2,1/2	0.234	0.083	0.316	0.190	0.101
10,10,10,10,10	1/9,1/6,1/6,1/6,1/6	0.150	0.094	0.225	0.166	0.136
	1/9,2/9,2/9,2/9,2/9	0.416	0.234	0.482	0.401	0.222
	1/9,1/3,1/3,1/3,1/3	0.879	0.609	0.897	0.845	0.358
	1/4,1/2,1/2,1/2,1/2	0.345	0.220	0.425	0.328	0.216
15,15,15,20,20	1/9,1/6,1/6,1/6,1/6	0.256	0.168	0.282	0.235	0.155
	1/9,2/9,2/9,2/9,2/9	0.709	0.495	0.705	0.645	0.330
	1/9,1/3,1/3,1/3,1/3	0.993	0.948	0.991	0.984	0.606
	1/4,1/2,1/2,1/2,1/2	0.588	0.435	0.618	0.540	0.293
20,20,20,20,20	1/9,1/6,1/6,1/6,1/6	0.346	0.261	0.369	0.339	0.234
	1/9,2/9,2/9,2/9,2/9	0.844	0.729	0.840	0.814	0.541
	1/9,1/3,1/3,1/3,1/3	1.000	0.995	0.999	0.999	0.918
	1/4,1/2,1/2,1/2,1/2	0.748	0.644	0.758	0.714	0.481
30,30,30,30,30	1/9,1/6,1/6,1/6,1/6	0.531	0.449	0.535	0.506	0.377
	1/9,2/9,2/9,2/9,2/9	0.974	0.944	0.970	0.965	0.858
	1/9,1/3,1/3,1/3,1/3	1.000	1.000	1.000	1.000	0.999
	1/4,1/2,1/2,1/2,1/2	0.928	0.891	0.930	0.913	0.777

**Table 9.** Estimated powers of tests for  $\mu_i=3$  ( $i=1,2,\dots,6$ )

n	$R_1, R_2, R_3, R_4, R_5, R_6$	CAT	GPT	LRT	MBT	SCT
6,6,6,6,6,6	1/9,1/6,1/6,1/6,1/6,1/6	0.083	0.036	0.195	0.112	0.118
	1/9,2/9,2/9,2/9,2/9,2/9	0.196	0.067	0.359	0.226	0.176
	1/9,1/3,1/3,1/3,1/3,1/3	0.465	0.139	0.637	0.448	0.237
	1/4,1/2,1/2,1/2,1/2,1/2	0.165	0.058	0.320	0.176	0.156
6,8,10,10,12,14	1/9,1/6,1/6,1/6,1/6,1/6	0.132	0.051	0.196	0.114	0.092
	1/9,2/9,2/9,2/9,2/9,2/9	0.273	0.075	0.331	0.207	0.107
	1/9,1/3,1/3,1/3,1/3,1/3	0.620	0.150	0.641	0.454	0.145
	1/4,1/2,1/2,1/2,1/2,1/2	0.222	0.076	0.305	0.177	0.110
10,10,10,10,10,10	1/9,1/6,1/6,1/6,1/6,1/6	0.147	0.089	0.209	0.159	0.129
	1/9,2/9,2/9,2/9,2/9,2/9	0.402	0.214	0.476	0.384	0.230
	1/9,1/3,1/3,1/3,1/3,1/3	0.863	0.534	0.876	0.810	0.338
	1/4,1/2,1/2,1/2,1/2,1/2	0.329	0.182	0.418	0.310	0.206
15,15,15,20,20,20	1/9,1/6,1/6,1/6,1/6,1/6	0.246	0.153	0.278	0.224	0.151
	1/9,2/9,2/9,2/9,2/9,2/9	0.682	0.449	0.678	0.612	0.305
	1/9,1/3,1/3,1/3,1/3,1/3	0.991	0.923	0.987	0.979	0.513
	1/4,1/2,1/2,1/2,1/2,1/2	0.559	0.395	0.586	0.506	0.271
20,20,20,20,20,20	1/9,1/6,1/6,1/6,1/6,1/6	0.306	0.218	0.330	0.292	0.203
	1/9,2/9,2/9,2/9,2/9,2/9	0.680	0.439	0.671	0.605	0.290
	1/9,1/3,1/3,1/3,1/3,1/3	0.999	0.991	0.998	0.997	0.818
	1/4,1/2,1/2,1/2,1/2,1/2	0.718	0.590	0.727	0.673	0.418
30,30,30,30,30,30	1/9,1/6,1/6,1/6,1/6,1/6	0.507	0.407	0.499	0.471	0.331
	1/9,2/9,2/9,2/9,2/9,2/9	0.971	0.919	0.964	0.955	0.776
	1/9,1/3,1/3,1/3,1/3,1/3	1.000	1.000	1.000	1.000	0.999
	1/4,1/2,1/2,1/2,1/2,1/2	0.922	0.863	0.919	0.894	0.695

**Table 10.** Estimated powers of tests for  $\mu_i=3$  ( $i=1,2,\dots,7$ )

n	$R_1, R_2, R_3, R_4, R_5, R_6, R_7$	CAT	GPT	LRT	MBT	SCT
6,6,6,6,6,6,6	1/9, 1/6,1/6,1/6,1/6,1/6,1/6	0.084	0.035	0.203	0.113	0.124
	1/9,2/9,2/9,2/9,2/9,2/9,2/9	0.168	0.059	0.339	0.203	0.173
	1/9,1/3,1/3,1/3,1/3,1/3,1/3	0.439	0.120	0.607	0.422	0.226
	1/4,1/2,1/2,1/2,1/2,1/2,1/2	0.157	0.060	0.309	0.172	0.167
6,8,10,10,10,12,14	1/9, 1/6,1/6,1/6,1/6,1/6,1/6	0.119	0.044	0.182	0.109	0.084
	1/9,2/9,2/9,2/9,2/9,2/9,2/9	0.265	0.071	0.323	0.192	0.111
	1/9,1/3,1/3,1/3,1/3,1/3,1/3	0.584	0.134	0.597	0.407	0.141
	1/4,1/2,1/2,1/2,1/2,1/2,1/2	0.200	0.067	0.281	0.151	0.106
10,10,10,10,10,10,10	1/9, 1/6,1/6,1/6,1/6,1/6,1/6	0.134	0.077	0.212	0.151	0.137
	1/9,2/9,2/9,2/9,2/9,2/9,2/9	0.364	0.183	0.440	0.341	0.201
	1/9,1/3,1/3,1/3,1/3,1/3,1/3	0.835	0.462	0.849	0.774	0.312
	1/4,1/2,1/2,1/2,1/2,1/2,1/2	0.302	0.163	0.388	0.283	0.189
15,15,15,20,20,20,20	1/9, 1/6,1/6,1/6,1/6,1/6,1/6	0.239	0.145	0.267	0.214	0.154
	1/9,2/9,2/9,2/9,2/9,2/9,2/9	0.665	0.409	0.645	0.577	0.273
	1/9,1/3,1/3,1/3,1/3,1/3,1/3	0.987	0.877	0.981	0.969	0.455
	1/4,1/2,1/2,1/2,1/2,1/2,1/2	0.539	0.357	0.566	0.470	0.246
20,20,20,20,20,20,20	1/9, 1/6,1/6,1/6,1/6,1/6,1/6	0.303	0.217	0.323	0.284	0.202
	1/9,2/9,2/9,2/9,2/9,2/9,2/9	0.818	0.624	0.799	0.760	0.420
	1/9,1/3,1/3,1/3,1/3,1/3,1/3	0.999	0.986	0.998	0.997	0.713
	1/4,1/2,1/2,1/2,1/2,1/2,1/2	0.705	0.550	0.715	0.648	0.372
30,30,30,30,30,30,30	1/9, 1/6,1/6,1/6,1/6,1/6,1/6	0.493	0.383	0.489	0.458	0.301
	1/9,2/9,2/9,2/9,2/9,2/9,2/9	0.963	0.905	0.954	0.948	0.714
	1/9,1/3,1/3,1/3,1/3,1/3,1/3	1.000	1.000	1.000	1.000	0.993
	1/4,1/2,1/2,1/2,1/2,1/2,1/2	0.904	0.827	0.898	0.875	0.631

The numerical results for estimated powers of the tests are presented as above in Table 6 to Table 10. In most cases, the LRT can be disregarded because of its estimated type I error rates exceeding the nominal level. Although the estimated type I error rates of MBT and SCT are close to each other, the MBT performs better than the SCT in terms of their powers.

MBT performs slightly better than the CAT in terms of powers for small sample size. However the estimated type I error rates of MBT exceed the estimated type I error rates of CAT for in this case. If both of the MBT and CAT are compared for other all cases, the CAT appears to be more powerful than MBT does.

## 5. An application with real life dataset

In this section, the LRT, MBT, SCT, GPT and CAT are applied for two real life datasets given as follows.

**Example 5.1.** The first analysis uses data collected by Nairy and Rao [15]. The data related to survival times of patients collected from 4 hospitals, which was a part of the data by given Fleming and Harrington [7]. The data containing failure time of the patients and their summary statistics are presented in Table 11. 5000 replications are used to obtain the p values of GPT and CAT. The obtained test statistics are given in Table 12.

**Table 11.** Survival time of patients from 4 hospitals and their means, standard deviations and coefficient of variations

Hospitals	Survival time of patients	$\bar{x}_i$	$s_i^2$	$r_i = s_i/\bar{x}_i$
1	176, 105, 266, 227, 66	168	74.19	0.051
2	24, 5, 155, 54	59.5	57.84	0.128
3	58, 64, 15	45.7	21.82	0.102
4	147, 42, 305, 92, 30, 82, 265, 237, 208, 147	155.5	90.53	0.062

**Table 12.** The results of tests statistics

Tests	Values of test statistics	$p$
LRT	1.753	0.625
MBT	1.396	0.707
SCT	2.064	0.559
GPT	-	0.699
CAT	1.621	0.754

The values in Table 12 indicate that the tests do not reject the  $H_0$  given in Equation (2.1) at nominal level 0.05.

**Example 5.2.** The second analysis uses data collected by Tsou [20]. Table 13 gives the respective numbers of birth in 1978 on Monday, Thursday, and Saturday in the United Kingdom and their means, standard deviations and coefficient of variations. 5000 replications are used to obtain the p values of GPT and CAT. 5000 replications are used to obtain the p values of GPT and CAT. The obtained test statistics are given in Table 14.

**Table 13.** Numbers of birth in 1978 on Monday, Thursday, and Saturday in the United Kingdom and their means, standard deviations and coefficient of variations

Number of birth on Monday				Number of birth on Thursday				Number of birth on Saturday			
7527	9172	9458	9252	9043	9259	9226	9387	8084	8299	7954	7946
9184	9225	8966	9021	9218	9247	9103	9268	8065	8144	8167	8313
9262	9294	9022	9135	9304	9218	9327	9159	8008	8144	7965	7874
9100	9114	8870	8702	8902	8696	8724	8582	8069	7890	7527	7787
9017	8900	8987	9195	8839	8672	8903	9044	7750	7718	7762	8064
9089	7780	9127	9201	9180	9435	9075	9175	8005	7971	8040	8233
9543	9348	9284	9877	9405	9630	10184	9984	8122	8209	8773	8859
10026	9960	9890	10206	10386	10192	10128	10284	9062	8677	8738	8951
10127	9967	9998	8481	10377	10152	9489	10292	9023	9170	8735	8648
9927	9765	9531	9425	9949	9824	9502	9501	8605	8554	8411	8415
9457	9507	9606	9592	9245	9609	9568	7915	8246	8352	8432	8275
9825	8676	9686	10196	9396	9480	9524	9398	8528	8335	8507	7939
10154	10304	10414	7846	10265	10499	10175	10177	8904	8782	8580	8474
$\bar{x}_1 = 9350.3$				$\bar{x}_2 = 9471.5$				$\bar{x}_3 = 8309.3$			
$s_1^2 = 376030$				$s_2^2 = 307890$				$s_3^2 = 152300$			
$r_1 = s_1/\bar{x}_1 = 0.066$				$r_2 = s_2/\bar{x}_2 = 0.056$				$r_3 = s_3/\bar{x}_3 = 0.047$			

**Table 14.** The results of tests statistics

Tests	Values of test statistics	$p$
LRT	5.708	0.058
MBT	5.598	0.061
SCT	5.220	0.074
GPT	-	0.064
CAT	3.014	0.057

The Table 14 shows that all of tests lead to the same conclusion, that is, all of tests do not reject the  $H_0$  given in Equation (2.1) at nominal level 0.05.

## 6. Conclusion

In this article, we propose the CAT for testing the equality of coefficients of variation in  $k$  normal populations. We compare the CAT to some of the existing tests; the LRT, MBT, SCT, GPT. For a different sample sizes and number of groups, we investigate the performance of these tests using Monte Carlo simulation.

It could be observed from the simulation results that for small sample size the LRT approach seems to have the estimated type I error rates exceeding the nominal level and the GPT performs contrary to the LRT that its estimated type I error rates are lower than the nominal level. However, the estimated type I error rates of CAT are generally more conservative than other tests for all the sample size. Therefore, we could mention that the CAT is not affected from the changes in the sample size. Furthermore, according to power comparison results, the CAT appears to be more powerful than the other tests when the differences between coefficients of variation in  $k$  normal populations are increased.

Consequently, in respect to our simulation study, even when comparing different number of groups (as  $k=3, 4, 5, 6, 7$ ), CAT could be suggested as a good alternative for testing the equality of coefficients of variation in  $k$  normal populations.

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