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A NEW LIFETIME DISTRIBUTION: TRANSMUTED EXPONENTIAL POWER DISTRIBUTION

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Abstract. In this paper, we have introduced a new statistical distribution called as transmuted Exponential Power (TEP) distribution using the quadratic rank transmutation map proposed by Shaw and Buckley [25,26] in order to generate new distributions. We have also studied some statistical properties such as descriptive statistics (moments, variance, coefficient of skewness (CS) and kurtosis (CK)), point estimation (maximum likelihood estimation) and real data applications to illustrate usefulness of TEP distribution.

1. Introduction

In statistical literature, several lifetime distributions are introduced. Most of these distributions are generally obtained by compounding or mixing methodologies. In the other case, distributions are given through including an extra parameter to well-known distribution. By the way, family of distributions obtained using quadratic rank transmutation map (QRTM) proposed by Shaw and Buckley [25, 26] is defined with cumulative distribution function (cdf) and probability density function (pdf)

$$F(x) = (1 + \lambda)G(x) - \lambda \left[G(x)\right]^2 \tag{1}$$

and

$$f(x) = (1 + \lambda)g(x) - 2\lambda G(x)g(x), \tag{2}$$

respectively, where $G(x)$ denotes cdf of baseline distribution and $\lambda \in [-1, 1]$ is transmuting parameter. If $\lambda = 0$, cdf of the base distribution is obtained. In last decade, there are many studies about transmuted distributions in literature. For

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example; Aryal and Tsokos [3,4] introduced transmuted Weibull and transmuted extreme value distributions. Ashour and Eltehiwy [5,6] proposed transmuted Lomax and transmuted exponentiated Lomax distributions, Merovci [18–21] introduced transmuted Rayleigh, transmuted Lindley, transmuted generalized Rayleigh and transmuted exponentiated Exponential distributions, Mahmoud and Mandouh [17] suggested transmuted Frechet distribution, Hussian [13] has introduced transmuted exponentiated Gamma distribution, Elbatal and Aryal [10] studied transmuted additive Weibull distribution, Khan et al. [14–16] introduced transmuted Weibull distribution, transmuted Kumaraswamy distribution and transmuted generalized Gompertz distribution, Shahzad and Asghar [24] proposed transmuted Dagum distribution, Al-Babtain et.al. [2] introduced the Kumaraswamy-transmuted exponentiated modified Weibull distribution.

Over two decades before, Smith and Bain [23] introduced the Exponential Power (EP) distribution by compounding exponential and Weibull distribution functions. The cdf and pdf of EP distribution are given

$$G(x;\alpha,\beta)=1-\exp\left[1-\exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right)\right] \tag{3}$$

and

$$g(x;\alpha,\beta)=\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}\exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right)\exp\left[1-\exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right)\right], \tag{4}$$

respectively, where $\alpha>0,\beta>0$ and $x>0$. Many authors have focused on EP distribution recently. Some of these studies can be listed as follows; Chen [8], Barriga et al. [7], Akdam et al. [1].

The main purpose of this study is to suggest a new lifetime distribution as an alternative EP distribution by using QRTM. In Section 2, TEP distribution and its some statistical properties (moments, variance, CS, CK) are introduced. The maximum likelihood estimators (MLEs) for unknown parameters of introduced distribution are derived in Section 3. In Section 4, a Monte Carlo simulation study is performed to evaluate the performances of these estimators in terms of mean square errors (MSEs) and bias. In Section 5, two real data illustrations are given to show the applicability of this distribution in real life. In Section 6, the conclusion remarks are given.

2. Transmuted Exponential Power (TEP) Distribution

Let X be a random variable having TEP distribution with α,β and λ parameters denoted by TEP(α,β,λ). The cdf and pdf of this random variable are

$$\begin{aligned}F(x;\alpha,\beta,\lambda) \quad = \quad &(1+\lambda)\left[1-\exp\left(1-\exp\left(\left(\tfrac{x}{\alpha}\right)^{\beta}\right)\right)\right]\\ &-\lambda\left[1-\exp\left(1-\exp\left(\left(\tfrac{x}{\alpha}\right)^{\beta}\right)\right)\right]^{2}\end{aligned} \tag{5}$$

and

$$f(x;\alpha,\beta,\lambda) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}\exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right)\exp\left[1-\exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right)\right] \times\left[1+\lambda-2\lambda\left(1-\exp\left(1-\exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right)\right)\right)\right], \tag{6}$$

respectively. Where $-1 \leq \lambda \leq 1, \alpha, \beta > 0$ and $x > 0$. The cdf of EP distribution for $\lambda = 0$ in Eq. (5) is derived. Figure 1 shows the possible shapes of the pdf of TEP distribution for various parameter values.

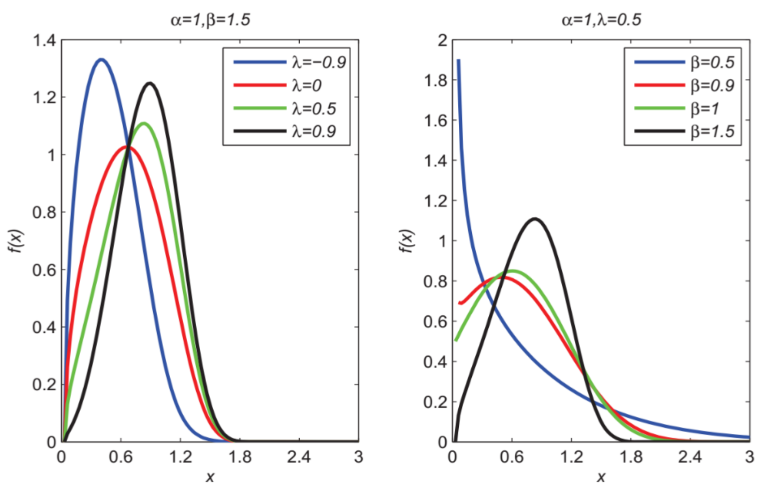

FIGURE 1. Plots of the TEP density function for various values of α, β and λ

The reliability function (rf) and hazard function (hf) of TEP distribution are defined as

$$R(t) = 1-(1+\lambda)\left[1-\exp\left(1-\exp\left(\left(\frac{t}{\alpha}\right)^{\beta}\right)\right)\right] +\lambda\left[1-\exp\left(1-\exp\left(\left(\frac{t}{\alpha}\right)^{\beta}\right)\right)\right]^{2} \tag{7}$$

and

$$h(t) = \frac{\frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}\exp\left(\left(\frac{t}{\alpha}\right)^{\beta}\right)k(t,\alpha,\beta)}{1-(1+\lambda)[1-k(t,\alpha,\beta)]+\lambda[1-k(t,\alpha,\beta)]^{2}} \times\left[1+\lambda-2\lambda\left(1-k(t,\alpha,\beta)\right)\right], \tag{8}$$

respectively. Where $k(t,\alpha,\beta) = \exp\left[1-\exp\left(\left(\frac{t}{\alpha}\right)^{\beta}\right)\right]$. Figure 2 shows that the possible shapes of (hf) for TEP distribution at different parameter values.

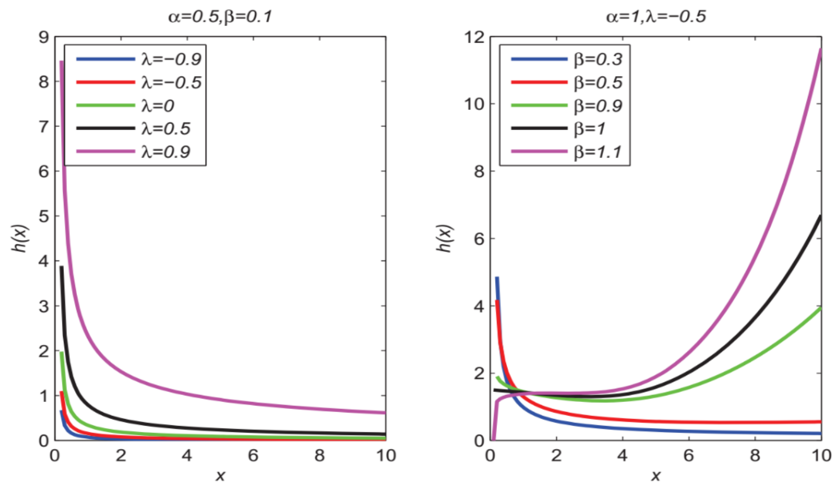

FIGURE 2. Plots of the TEP hazard function for various values of α, β and λ

2.1. **Moments of TEP distribution.** The rth moment of TEP distribution is

$$\begin{aligned} E(X^r) &= \int_0^\infty x^r \left((1+\lambda)g(x) - 2\lambda G(x)g(x)\right) dx \\ &= \left((1+\lambda)I_1 - 2\lambda I_2\right) \end{aligned} \tag{9}$$

where I_1 and I_2 are

$$\begin{aligned} I_1 &= \int_0^\infty x^r g(x)dx \\ &= \int_0^\infty x^r \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right) s(x,\alpha,\beta)dx \\ &= \sum_{n=0}^{\infty} \frac{1}{n!}\alpha^r \int_1^\infty (\ln(u))^{n+(r/\beta)} \frac{e^{1-u}}{u} du \end{aligned} \tag{10}$$

and

$$\begin{aligned} I_2 &= \int_0^\infty x^r G(x)g(x)dx \\ &= \int_0^\infty x^r \left[1 - s(x,\alpha,\beta)\right] \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right) s(x,\alpha,\beta)dx \\ &= I_1 - \sum_{n=0}^{\infty} \frac{1}{n!}\alpha^r \int_1^\infty (\ln(u))^{n+(r/\beta)} \frac{e^{2-2u}}{u} du, \end{aligned} \tag{11}$$

respectively. Where $s(x, \alpha, \beta) = \exp\left(1 - \exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right)\right)$. Figure 3 illustrates the plots of some descriptive statistics such as expected value, variance, CS and CK at selected parameter values for TEP distribution.

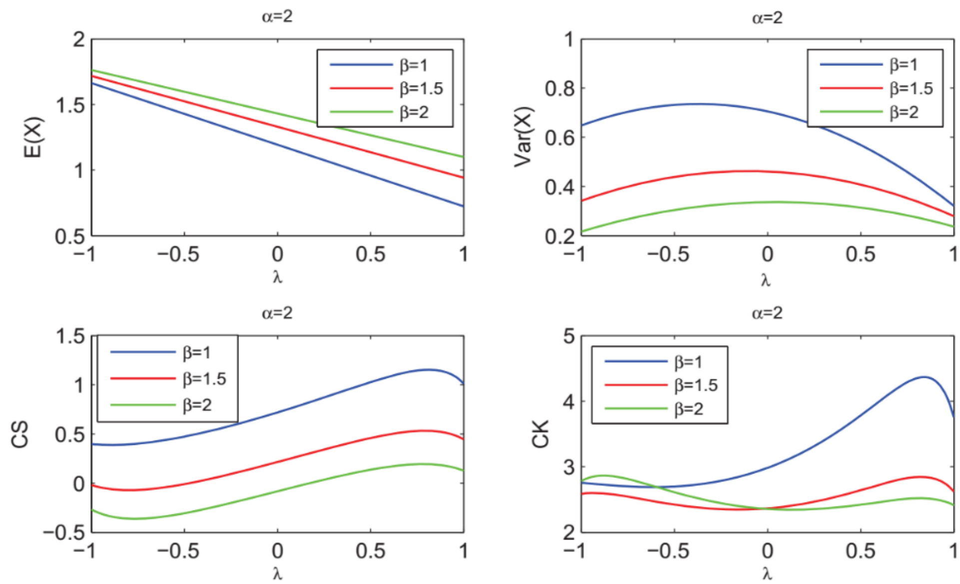

FIGURE 3. Descriptive statistics for $\alpha = 2, \beta \in \{1, 1.5, 2\}, \lambda \in [-1, 1]$.

2.2. **Random Number Generator.** The method of inversion transformation to generate random numbers from TEP distribution with parameters α, β and λ is used as follows;

$$F(x; \alpha, \beta, \lambda) = (1 + \lambda)[1 - s(x, \alpha, \beta)] - \lambda [1 - s(x, \alpha, \beta)]^2 = u, \qquad (12)$$

where $s(x, \alpha, \beta) = \exp\left(1 - \exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right)\right)$ and u is a number generated from Uniform distribution shown as $U(0, 1)$. Solution of Eq. (12) is given by

$$x = \alpha \left[\ln \left(1 - \ln \left(1 - \left(\frac{1 + \lambda - \sqrt{(\lambda + 1)^2 - 4\lambda u}}{2\lambda} \right) \right) \right) \right]^{1/\beta}. \qquad (13)$$

Maximum Likelihood Estimation

Let $X_1, X_2, ..., X_n$ be a random sample having TEP distribution with parameters α, β and λ. Then the log-likelihood function is given by

$$\begin{aligned}\ell\left(\alpha,\beta,\lambda\middle|\underline{x}\right) &= n\ln(\beta) - n\ln(\alpha) + \textstyle\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta} + n \\ &\quad - \left(\textstyle\sum_{i=1}^{n}\exp\left(\left(\frac{x_i}{\alpha}\right)^{\beta}\right)\right) + (\beta-1)\textstyle\sum_{i=1}^{n}\ln\left(\frac{x_i}{\alpha}\right) \\ &\quad + \textstyle\sum_{i=1}^{n}\ln\left(\frac{x_i}{\alpha}\right)\ln\left(1+\lambda-2\lambda\left(1-s(x_i,\alpha,\beta)\right)\right)\end{aligned} \tag{14}$$

Diferentiating the $\ell\left(\alpha,\beta,\lambda\middle|\underline{x}\right)$ with respect to α, β and λ parameters, then equating to zero, non-linear equations is obtained as follows;

$$\begin{aligned}\frac{\partial\ell\left(\alpha,\beta,\lambda/\underline{x}\right)}{\partial\alpha} = 0 \;\Rightarrow\; & -\frac{n}{\alpha} - \textstyle\sum_{i=1}^{n}\left(\frac{\beta}{\alpha}\right)\left(\frac{x_i}{\alpha}\right)^{\beta} \\ & - \textstyle\sum_{i=1}^{n}\left(\frac{\beta}{\alpha}\right)\left(\frac{x_i}{\alpha}\right)^{\beta}\exp\left(\left(\frac{x_i}{\alpha}\right)^{\beta}\right) - \frac{n(\beta-1)}{\alpha} \\ & + \textstyle\sum_{i=1}^{n} 2\lambda\frac{\beta}{\alpha}\left(\frac{x_i}{\alpha}\right)^{\beta}\exp\left(\left(\frac{x_i}{\alpha}\right)^{\beta}\right) \\ & \times s(x_i,\alpha,\beta)\left(1+\lambda-2\lambda\left(1-s(x_i,\alpha,\beta)\right)\right)^{-1} = 0\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{\partial\ell\left(\alpha,\beta,\lambda/\underline{x}\right)}{\partial\beta} = 0 \;\Rightarrow\; & \frac{n}{\beta} + \textstyle\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta}\ln\left(\frac{x_i}{\alpha}\right) \\ & - \textstyle\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta}\ln\left(\frac{x_i}{\alpha}\right)\exp\left(\left(\frac{x_i}{\alpha}\right)^{\beta}\right) \\ & + \textstyle\sum_{i=1}^{n}\ln\left(\frac{x_i}{\alpha}\right) \\ & - \textstyle\sum_{i=1}^{n} 2\lambda\left(\frac{x_i}{\alpha}\right)^{\beta}\ln\left(\frac{x_i}{\alpha}\right)\exp\left(\left(\frac{x_i}{\alpha}\right)^{\beta}\right) \\ & \times s(x_i,\alpha,\beta)\left(1+\lambda-2\lambda\left(1-s(x_i,\alpha,\beta)\right)\right)^{-1} = 0\end{aligned} \tag{16}$$

$$\frac{\partial\ell\left(\alpha,\beta,\lambda/\underline{x}\right)}{\partial\lambda} = 0 \Rightarrow -1 + 2\left(1-\exp\left(\left(\frac{x_i}{\alpha}\right)^{\beta}\right)\right)\left(1+\lambda-2\lambda\left(1-s(x_i,\alpha,\beta)\right)\right)^{-1} = 0. \tag{17}$$

The MLEs of α, β and λ are obtained by solving of Eqs. [15-17] via some numerical methods.

Table 1. The MSEs and biases of α,β and λ.

		bias			MSE		
n	parameter values	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$
5		-0.4738	0.7255	-0.8020	0.5023	1.1866	0.7368
10		-0.3492	0.2620	-0.6064	0.2502	0.1958	0.4855
20		-0.3460	0.1159	-0.4095	0.1858	0.0498	0.2452
50	(2,0.8,0.1)	-0.3439	0.0578	-0.2558	0.1510	0.0126	0.0796
100		-0.3314	0.0433	-0.2167	0.1320	0.0061	0.0532
300		-0.2863	0.0275	-0.1834	0.1052	0.0022	0.0428
5		0.0533	0.5609	0.6402	0.0274	0.7886	0.4485
10		0.0698	0.3201	0.5680	0.0195	0.2037	0.3830
20		0.0689	0.2132	0.4810	0.0138	0.0872	0.3277
50	(0.4,0.6,-0.8)	0.0582	0.1328	0.3541	0.0099	0.0398	0.2559
100		0.0424	0.0886	0.2503	0.0071	0.0241	0.1916
300		0.0198	0.0382	0.1148	0.0035	0.0101	0.0961
5		-0.3532	0.7178	-0.3549	0.3446	2.5125	0.2176
10		-0.2487	0.2176	-0.3094	0.1959	0.5131	0.2248
20		-0.1759	0.0386	-0.2589	0.1183	0.1932	0.2212
50	(3,2,0.5)	-0.1123	-0.0391	-0.1976	0.0746	0.0882	0.1997
100		-0.0832	-0.0620	-0.1645	0.0608	0.0610	0.1821
300		-0.0273	-0.0457	-0.0754	0.0403	0.0308	0.1183
5		-0.0513	0.1908	-0.6024	0.0257	0.1503	0.4549
10		-0.0563	0.0490	-0.4850	0.0173	0.0248	0.3634
20		-0.0671	0.0091	-0.4005	0.0137	0.0080	0.3027
50	(0.3,0.4,0.8)	-0.0630	-0.0041	-0.2968	0.0104	0.0031	0.2163
100		-0.0516	-0.0052	-0.2235	0.0082	0.0017	0.1495
300		-0.0310	-0.0047	-0.1314	0.0058	0.0006	0.0783

3. Simulation Study

In this section, a Monte Carlo simulation study is performed to evaluate the performances of MLEs according to MSEs and biases. Algorithm steps regarding to simulation study are as follows;

Step 1. Random numbers are generated from TEP distribution with parameters α, β and λ by using Eq. (13).

Step 2. MLEs of α, β and λ are calculated by using Eqs. (15)-(17) as based on 10000 replicates.

Step 3. The biases and MSEs of these estimators are simulated for different sample sizes as 5,10, 20, 50, 100 and 300 at selected parameter values ($(\alpha, \beta, \lambda) =$ $(2, 0.8, 0.1)$, $(0.4, 0.6, -0.8)$,

$(3, 2.0.5)$, and $(0.3, 0.4, 0.8)$).The results of simulation study are presented in Table 1.

According to Table 1, it is clearly seen that the MSEs and biases of MLEs for all parameter cases decrease as sample sizes increases. This case indicates that estimate values approach to true values as sample size n increases.

4. Real Data Analysis

In this section, we aim to compare TEP distribution with other distributions in terms of goodness of fit measures to demonstrate the applicability of TEP distribution. Two real data sets have been used for these purposes. We have considered some goodness of fit measures such as the Akaike's Information Criterion (AIC), corrected Akaike's Information Criterion (AICc), -2×log-likelihood value, Kolmogorov-Smirnov (KS) statistics and its p-value to compare the fits of the distributions for two data sets. These statistics are given as follows;

$$AIC = -2\ell + 2k, \tag{18}$$

$$AICc = AIC + \left(\frac{2k(k+1)}{n-k-1}\right), \tag{19}$$

$$KS = \sup\left(|F(x) - F_n(x)|\right). \tag{20}$$

where k is number of parameters, n is sample size, ℓ is the value of log-likelihood function.

4.1. **Operation and empirical data.** The first real data set consist of 50 observations has been obtained by Dasgupta [9]. This data set relates to holes operation on jobs made of iron sheet is given by in Table 2.

Table 2. Operation and empirical data

0.04	0.02	0.06	0.12	0.14	0.08	0.22	0.12	0.08	0.26
0.24	0.04	0.14	0.16	0.08	0.26	0.32	0.28	0.14	0.16
0.24	0.22	0.12	0.18	0.24	0.32	0.16	0.14	0.08	0.16
0.24	0.16	0.32	0.18	0.24	0.22	0.16	0.12	0.24	0.06
0.02	0.18	0.22	0.14	0.06	0.04	0.14	0.26	0.18	0.16

These data have been fitted to TEP, generalized Gompertz (GG) [11], transmuted generalized Gompertz (TGG) [16], transmuted Kumaraswamy (TKw) [14], transmuted Rayleigh (TR) [18], transmuted exponentiated exponential (TEE) [21] and transmuted Weibull (TW) [3] distributions. The density functions of the fitted

Table 3. Parameter estimates(standard errors) for operation and empirical data

Distribution	MLEs
TEP	$\hat{\alpha} = 0.2400\,(0.0327)\,, \hat{\beta} = 1.5996\,(0.4081)\,,$ $\hat{\lambda} = -0.0234\,(0.7000)$
TGG	$\hat{a} = 3.0808\,(3.2536)\,, \hat{b} = 7.5224\,(4.2694)\,,$ $\hat{\alpha} = 1.3521\,(0.5060)\,, \hat{\lambda} = -0.1075\,(0.9151)$
GG	$\hat{a} = 2.5012\,(1.6916)\,, \hat{b} = 8.3737\,(3.0939)\,,$ $\hat{\alpha} = 1.2784\,(0.4692)$
TKw	$\hat{a} = 1.9335\,(0.3524)\,, \hat{b} = 30.1864\,(13.7921)\,,$ $\hat{\lambda} = -0.2911\,(0.4408)$
TR	$\hat{\sigma} = 0.1211\,(0.0110)\,, \hat{\lambda} = -0.2645\,(0.3086)$
TW	$\hat{\mu} = 1.9917\,(0.3310)\,, \hat{\sigma} = 0.1708\,(0.0236)\,,$ $\hat{\lambda} = -0.2722\,(0.4308)$
TEE	$\hat{\theta} = 2.6946\,(0.8398)\,, \hat{\alpha} = 12.4959\,(1.6663)\,,$ $\hat{\lambda} = -0.5468\,(0.3189)$

distributions are given by;

$$TGG : f(x) = \alpha a e^{bx} e^{\left(-\frac{a}{b}\left(e^{(bx)-1}\right)\right)} \left[1 - e^{\left(-\frac{a}{b}\left(e^{((bx)-1)}\right)\right)}\right]^{\alpha-1} \times \left[1 + \lambda - 2\lambda \left(1 - e^{\left(-\frac{a}{b} e^{((bx)-1)}\right)}\right)^{\alpha}\right], x > 0, a, b, \alpha > 0, \lambda \in [-1, 1]$$

$$GG : f(x) = \alpha a e^{bx} e^{\left(-\frac{a}{b}\left(e^{(bx)-1}\right)\right)} \left[1 - e^{\left(-\frac{a}{b}\left(e^{((bx)-1)}\right)\right)}\right]^{\alpha-1}, x > 0, a, b, \alpha > 0$$

$$TEE : f(x) = \theta\alpha \left(1 - e^{-\alpha x}\right)^{\theta-1} e^{-\alpha x} \left[1 + \lambda - 2\lambda \left(1 - e^{-\alpha x}\right)^{\theta}\right], x > 0, a, \theta > 0, \lambda \in [-1, 1]$$

$$TKw : f\,(x) = abx^{a-1} \left(1 - x^{a}\right)^{b-1} \left[1 - \lambda + 2\lambda \left(1 - x^{a}\right)^{b}\right], \quad x \in [0, 1]\,, a, b > 0, ,\lambda \in [-1, 1]$$

$$TR : f\,(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \left[1 - \lambda + 2\lambda e^{-\frac{x^2}{2\sigma^2}}\right], \quad x > 0, \sigma > 0,\lambda \in [-1, 1]$$

$$\mathrm{TW} : f\,(x) = \frac{\mu}{\sigma} \left(\frac{x}{\sigma}\right)^{\mu-1} e^{-\left(\frac{x}{\sigma}\right)^{\mu}} \left[1 - \lambda + 2\lambda e^{-\left(\frac{x}{\sigma}\right)^{\mu}}\right], \quad x > 0, \mu, \sigma > 0, \lambda \in [-1, 1]$$

For operation and empirical data set, the MLEs (standard errors) of fitted distributions are given in Table 3 and the selection criteria statistics are given in Table 4. Furthermore, The plots which shows fits of distributions to this data set can be examined from Figure 4 and Figure 5.

4.2. **Breaking Stress data.** The second data set is with regard to breaking stress of carbon fibers of 50 mm length (GPa) obtained by Nichols and Padgett [22]. This

Table 4. Selection criteria statistics for operation and empirical data

Distribution	-2log	AIC	AICc	K-S	p-value
TEP	**-114.9401**	**-108.9401**	**-108.4184**	**0.0893**	**0.8199**
TGG	-114.5400	-106.5400	-105.6520	0.0898	0.8152
GG	-114.5924	-108.5924	-108.0707	0.0936	0.7736
TKw	-112.5020	-106.5020	-105.9800	0.1052	0.6375
TR	-112.1173	-108.1173	-107.8620	0.1070	0.6162
TW	-112.1179	-106.1179	-105.5962	0.1070	0.6163
TEE	-106.5607	-100.5607	-100.0389	0.1462	0.2357

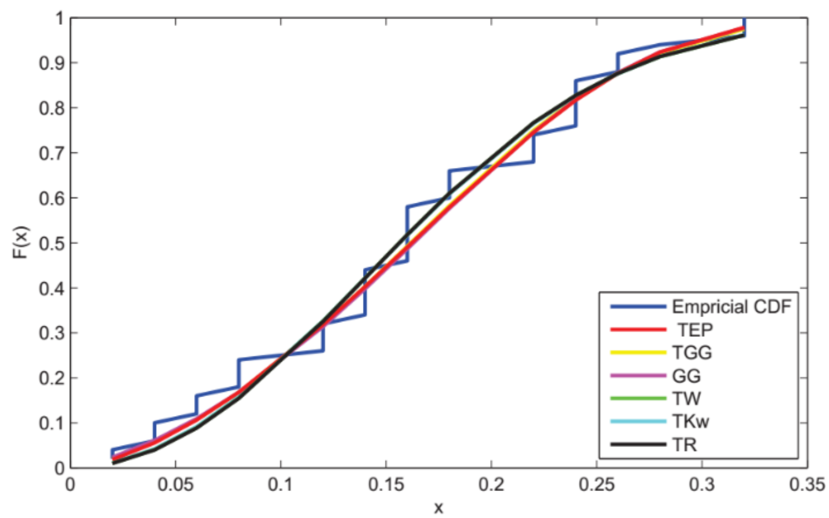

FIGURE 4. Empirical cdf and fitted cdfs for operation and empirical data set

data set consist of 66 observations has been used by Yousof et.al. [27]. The breaking stress data are presented in Table 5.

Table 5. Breaking stress data

0.39	0.85	1.08	1.25	1.47	1.57	1.61	1.61	1.69	1.80	1.84
1.87	1.89	2.03	2.03	2.05	2.12	2.35	2.41	2.43	2.48	2.50
2.53	2.55	2.55	2.56	2.59	2.67	2.73	2.74	2.79	2.81	2.82
2.85	2.87	2.88	2.93	2.95	2.96	2.97	3.09	3.11	3.11	3.15
3.15	3.19	3.22	3.22	3.27	3.28	3.31	3.31	3.33	3.39	3.39
3.56	3.60	3.65	3.68	3.70	3.75	4.20	4.38	4.42	4.70	4.90

This data set has been fitted to TEP, exponential power (EP) [23], exponentiated exponential (EE) [12], transmuted exponentiated exponential (TEE) [21] and

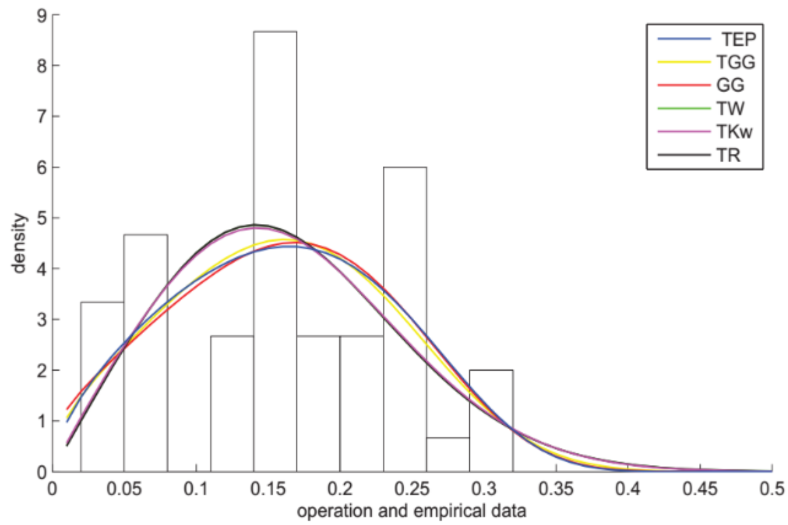

FIGURE 5. Fitted pdfs for operation and empirical data set

transmuted Rayleigh (TR) [18] distributions. The density functions of the fitted distributions are given by;

$$EP : f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right) \exp\left[1 - \exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right)\right], \quad x > 0, \alpha, \beta > 0$$

$$EE : f(x) = \theta\alpha\left(1 - e^{-\alpha x}\right)^{\theta-1} e^{-\alpha x}\ , \ x > 0, \alpha, \theta > 0$$

$$TR : f(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}\left[1 - \lambda + 2\lambda e^{-\frac{x^2}{2\sigma^2}}\right], \quad x > 0, \sigma > 0, \lambda \in [-1, 1]$$

$$\begin{aligned} TEE : f(x) &= \theta\alpha\left(1 - e^{-\alpha x}\right)^{\theta-1} \\ &\times e^{-\alpha x}\left[1 + \lambda - 2\lambda\left(1 - e^{-\alpha x}\right)^{\theta}\right], \quad x > 0, \alpha, \theta > 0, \lambda \in [-1, 1] \end{aligned}$$

The MLEs of unknown parameters for these distributions and their standard errors are shown in Table 6.

For breaking stress data, the comparison statistics of fitted distributions are given in Table 7. Also, the goodness of fit plots based on the empirical and theoretical cdfs and pdfs of fitted distributions can be seen from Figure 6 and Figure 7.

Table 7. Selection criteria statistics for breaking stress data

Table 6. Parameter estimates(standard errors) for breaking stress data

Distribution	MLEs
TEP	$\hat{\alpha} = 4.0683\,(0.2181)\,, \hat{\beta} = 2.8374\,(0.2716)\,,$ $\hat{\lambda} = 0.7487\,(0.2503)$
EP	$\hat{\alpha} = 3.6807\,(0.1182)\,, \hat{\beta} = 2.3799\,(0.2304)$
EE	$\hat{\theta} = 9.1992\,(2.1491)\,, \hat{\alpha} = 1.0076\,(0.1002)$
TEE	$\hat{\theta} = 7.4605\,(2.1903)\,, \hat{\alpha} = 1.1195\,(0.1089)\,,$ $\hat{\lambda} = -0.7773\,(0.1812)$
TR	$\hat{\sigma} = 1.6957\,(0.0825)\,, \hat{\lambda} = -0.9587\,(0.0930)$

Distribution	-2log	AIC	AICc	K-S	p-value
TEP	**172.4577**	**178.4577**	**178.8448**	**0.0913**	**0.6408**
EP	174.7949	178.7949	178.9854	0.1126	0.3724
EE	190.7447	194.7447	194.9352	0.1550	0.0840
TEE	185.0412	191.0412	191.4283	0.1344	0.1844
TR	177.7488	183.7488	184.1359	0.1410	0.1447

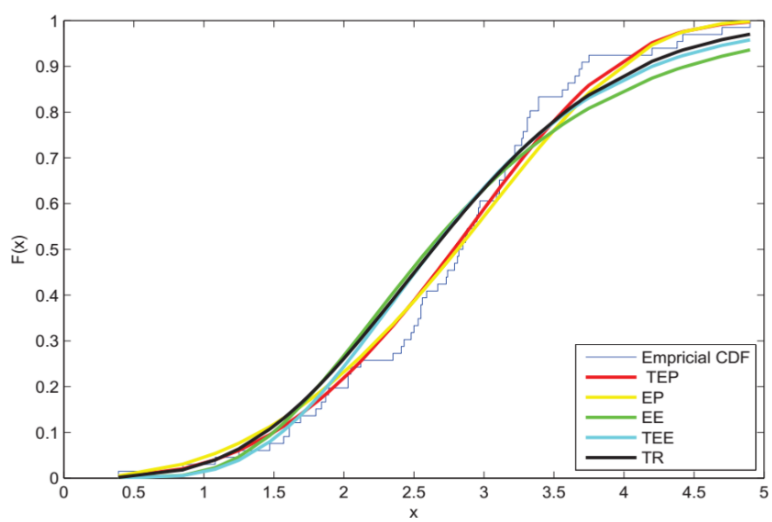

FIGURE 6. Empirical cdf and fitted cdfs for breaking stress data

5. CONCLUSION

In this study, we have proposed a new lifetime distribution which can be used as an alternative of EP distribution called as TEP. This new distribution having increasing, decrasing and bathtube hazard rate function has more flexibility than

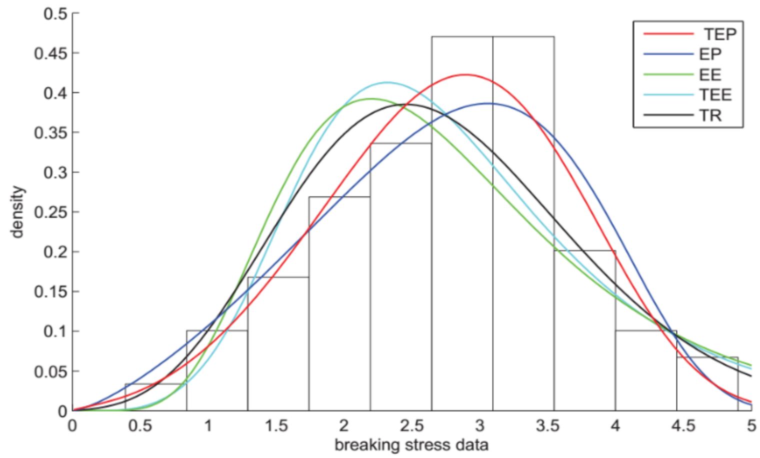

FIGURE 7. Fitted pdfs for breaking stress data

EP distribution. From two real data applications, it can have been concluded that TEP distribution has the best fitting among other fitted distributions. These real data applications show that TEP distribution is usefullness for modelling real data such as carbon fibres, operation and empirical data sets. These areas of application can be extended by using various real data sets which show fitting to TEP distribution.

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