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Some Prešić Type Results in $b-$Dislocated Metric Spaces

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ABSTRACT. In this paper, we obtain a Prešić type common fixed point theorem for four maps in b-dislocated metric spaces. We also present one example to illustrate our main theorem. Further, we obtain two more corollaries.

Keywords: b - Dislocated metric spaces, Jointly $2k$ - weakly compatible pairs, Prešić type theorem.

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1. INTRODUCTION AND PRELIMINARIES

There are several generalizations of the Banach contraction principle in literature on fixed point theory. Recently, very interesting results regarding fixed point are presented in the papers ([3, 4, 5, 7]. One of the generalization is a famous Prešić type fixed point theorem. There are a lot of generalizations of mentioned theorem (more on this topic see [1]-[2], [7]-[15]). Hitzler and Seda [6] introduced the concept of dislocated metric spaces (metric like spaces in [5], [15]) and established a fixed point theorem in complete dislocated metric spaces to generalize the celebrated Banach contraction principle. Recently Hussain et al. [7] introduced the definition of b - dislocated metric spaces to generalize the dislocated metric spaces introduced by [6] and proved two common fixed point theorems for four self mappings.

In this paper we have proved Prešić type common fixed point theorem for four mappings in b - dislocated metric spaces. One numerical example is also presented to illustrate our main theorem. We also obtained two corollaries for three and two maps in b - dislocated metric spaces.

Now we give some known definitions, lemmas and theorems which are needful for further discussion. Throughout this paper, N denotes the set of all positive integers.

Prešić [10] generalized the Banach contraction principle as follows.

Theorem 1.1. [10] *Let (X, d) be a complete metric space, k be a positive integer and $T : X^k \to X$ be a mapping satisfying*

$$d(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1})) \leq \sum_{i=1}^{k} q_i d(x_i, x_{i+1}), \tag{1.1}$$

for all $x_1, x_2, \dots, x_{k+1} \in X$, where $q_i \geq 0$ and $\sum\limits_{i=1}^{k} q_i < 1$. Then there exists a unique point $x \in X$ such that $T(x, x, ..., x) = x$. Moreover, if $x_1, x_2, \dots, x_k$ are arbitrary points in X and for

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$n \in \mathbb{N}$, $x_{n+k} = T(x_n, x_{n+1}, \ldots, x_{n+k-1})$, *then the sequence* $\{x_n\}$ *is convergent and* $\lim_{n\to\infty} x_n = T(\lim_{n\to\infty} x_n, \lim_{n\to\infty} x_n, \ldots, \lim_{n\to\infty} x_n)$.

Inspired by the Theorem 1.1, Ćirić and Prešić [8] proved the following theorem.

Theorem 1.2. [8] *Let* (X, d) *be a complete metric space,* k *a positive integer and* $T : X^k \to X$ *be a mapping satisfying*

$$d(T(x_1, x_2, \cdots, x_k), T(x_2, x_3, \cdots, x_{k+1})) \leq \lambda \max\{d(x_i, x_{i+1}) : 1 \leq i \leq k\} \tag{1.2}$$

for all $x_1, x_2, \cdots, x_k, x_{k+1}$ *in* X*, and* $\lambda \in (0, 1)$*. Then there exists a point* $x \in X$ *such that* $x = T(x, x, ..., x)$*.*

Moreover, if $x_1, x_2, \ldots, x_k$ *are arbitrary points in* X *and for* $n \in \mathbb{N}$, $x_{n+k} = T(x_n, x_{n+1}, \ldots, x_{n+k-1})$, *then the sequence* $\{x_n\}$ *is convergent and* $\lim_{n\to\infty} x_n = T(\lim_{n\to\infty} x_n, \lim_{n\to\infty} x_n, \ldots, \lim_{n\to\infty} x_n)$*. If in addition, we suppose that on diagonal* $\Delta \subset X^k$, $d(T(u, u, ..., u), T(v, v, ..., v)) < d(u, v)$ *holds for* $u, v \in X$ *with* $u \neq v$*, then* x *is the unique fixed point satisfying* $x = T(x, x, ..., x)$*.*

Later Rao et al. [11, 12] obtained some Presić fixed point theorems for two and three maps in metric spaces.

Definition 1.1. *Let* X *be a nonempty set,* k *a positive integer and* $T : X^{2k} \to X$ *and* $f : X \to X$*. The pair* (f, T) *is said to be* $2k$*-weakly compatible if* $f(T(x, x, ..., x)) = T(fx, fx, ..., fx)$ *whenever there exists* $x \in X$ *such that* $fx = T(x, x, ..., x)$

Actully Rao et al. [11] obtained the following.

Theorem 1.3. *Let* (X, d) *be a metric space and* k *be any positive integer. Let* $S, T : X^{2k} \longrightarrow X$ *and* $f : X \longrightarrow X$ *be mappings satisfying*

(1) $d(S(x_1, x_2, ..., x_{2k}), T(x_2, x_3, ..., x_{2k+1})) \leq \lambda \max\{d(fx_i, fx_{i+1}) : 1 \leq i \leq 2k\}$
for all $x_1, x_2, ..., x_{2k}, x_{2k+1} \in X$*, where* $\lambda \in (0, 1)$*.*

(2) $d(S(u, u, ..., u), T(v, v, ..., v)) < d(fu, fv)$ *for all* $u, v \in X$ *with* $u \neq v$

(3) *Suppose that* $f(X)$ *is complete and either* (f, S) *or* (f, T) *is* $2k$*-weakly compatible pair.*

Then there exists a unique point $p \in X$ *such that* $p = fp = S(p, p, .., p, p) = T(p, p, .., p, p)$*.*

Hussain et al. [7] introduced b-dislocated metric spaces as follows.

Definition 1.2. *Let* X *be a non empty set. A mapping* $b_d : X \times X \to [0, \infty)$ *is called a* b *- dislocated metric (or simply* b_d*-metric) if the following conditions hold for any* $x, y, z \in X$ *and* $s \geq 1$*:*

(b_{d1}) : *If* $b_d(x, y) = 0$ *then* $x = y$*,*
(b_{d2}) : $b_d(x, y) = b_d(y, x)$*,*
(b_{d3}) : $b_d(x, y) \leq s[b_d(x, z) + b_d(z, y)]$*.*

The pair (X, b_d) *is called a* b*-dislocated metric space or* b_d*-metric space.*

Definition 1.3. [7]

(i) *A sequence* $\{x_n\}$ *in* b*-dislocated metric space* (X, b_d) *converges with respect to* b_d *if there exists* $x \in X$ *such that* $b_d(x_n, x)$ *converges to 0 as* $n \to \infty$*. In this case,* x *is called the limit of* $\{x_n\}$ *and we write* $x_n \to x$*.*

(ii) *A sequence $\{x_n\}$ in a b-dislocated metric space (X, b_d) is called a b_d - Cauchy sequence if given $\varepsilon > 0$, there exists $n_0 \in N$ such that $b_d(x_m, x_n) < \varepsilon$ for all $n, m \geq n_0$ or $\lim_{n,m\to\infty} b_d(x_m, x_n) = 0$.*

(iii) *A b-dislocated metric (X, b_d) is called b_d-complete if every b_d-Cauchy sequence in X is b_d - convergent.*

Lemma 1.1. [7] *Let (X, b_d) be a b-dislocated metric space with $s \geq 1$. Suppose that $\{x_n\}$ and $\{y_n\}$ are b_d-convergent to x, y respectively. Then we have*

$$\frac{1}{s^2}\, b_d(x,y) \leq \lim_{n\to\infty} inf\; b_d(x_n, y_n) \leq \lim_{n\to\infty} sup\; b_d(x_n, y_n) \leq s^2\, b_d(x,y),$$

and

$$\frac{1}{s}\, b_d(x,z) \leq \lim_{n\to\infty} inf\; b_d(x_n, z) \leq \lim_{n\to\infty} sup\; b_d(x_n, z) \leq s\, b_d(x,z)$$

for all $z \in X$.

2. Main Result

We introduce the definition of jointly $2k-$weakly compatible pairs as follows.

Definition 2.4. *Let X be a nonempty set, k a positive integer and $S, T : X^{2k} \to X$ and $f, g : X \to X$. The pairs (f, S) and (g, T) are said to be jointly $2k$-weakly compatible if*

$$f(S(x,x,...,x)) = S(fx, fx, ..., fx)$$

and

$$g(T(x,x,...,x)) = T(gx, gx, ..., gx)$$

whenever there exists $x \in X$ such that $fx = S(x,x,...,x)$ and $gx = T(x,x,...,x)$.

Now we give our main result. The contractive condition in the next theorem is similar with conditions in [2, 7, 10, 13].

Theorem 2.4. *Let (X, b_d) be a b_d-complete b-dislocated metric space with $s \geq 1$ and k be any positive integer. Let $S, T : X^{2k} \longrightarrow X$ and $f, g : X \longrightarrow X$ be mappings satisfying*

(2.3) $$S(X^{2k}) \subseteq g(X), T(X^{2k}) \subseteq f(X),$$

(2.4)

$$b_d(S(x_1, x_2, ..., x_{2k}), T(y_1, y_2, ..., y_{2k})) \leq \lambda \max \left\{ \begin{array}{c} b_d(gx_1, fy_1), b_d(fx_2, gy_2), \\ b_d(gx_3, fy_3), b_d(fx_4, gy_4), \\ \dots\dots\dots\dots\dots\dots \\ b_d(gx_{2k-1}, fy_{2k-1}), b_d(fx_{2k}, gy_{2k}) \end{array} \right\}$$

for all $x_1, x_2, ..., x_{2k}, y_1, y_2, .., y_{2k} \in X$,where $\lambda \in (0, \frac{1}{s^{2k}})$.

(2.5) (f, S) *and* (g, T) *are jointly* $2k-$ *weakly compatible pairs,*

(2.6) *Assume that there exists* $u \in X$ *such that* $fu = gu$ *whenever there is sequence* $\{y_{2k+n}\}_{n=1}^{\infty} \in X$ *with* $lim \lim_{n\to\infty} y_{2k+n} = fu = gu = z \in X.$

Then z is the unique point in X such that $z = fz = gz = S(z, z, .., z, z) = T(z, z, ..., z, z)$.

Proof. Suppose $x_1, x_2, ..., x_{2k}$ are arbitrary points in X, From (2.3), we can define

$$y_{2k+2n-1} = S(x_{2n-1}, x_{2n}, ..., x_{2k+2n-2}) = gx_{2k+2n-1},$$

and

$$y_{2k+2n} = T(x_{2n}, x_{2n+1}, ..., x_{2k+2n-1}) = fx_{2k+2n}, \ \ n = 1, 2, \dots$$

Let

$$\alpha_{2n} = b_d(fx_{2n}, gx_{2n+1}),$$

and

$$\alpha_{2n-1} = b_d(gx_{2n-1}, fx_{2n}) \ \ n = 1, 2, \dots$$

Write $\theta = \lambda^{\frac{1}{2k}}$ and $\mu = \max\{\frac{\alpha_1}{\theta}, \frac{\alpha_2}{(\theta)^2}, ..., \frac{\alpha_{2k}}{(\theta)^{2k}}\}$.

Then $0 < \theta < 1$ and by the selection of μ, we have

$$\alpha_n \leq \mu \cdot (\theta)^n, \ \ n = 1, 2, \dots, 2k \tag{2.7}$$

Consider

$$\begin{aligned} \alpha_{2k+1} = b_d(gx_{2k+1}, fx_{2k+2}) &= b_d(S(x_1, x_2, ..., x_{2k-1}, x_{2k}), T(x_2, x_3, ..., x_{2k}, x_{2k+1})) \\ &\leq \lambda \max\left\{ \begin{array}{c} b_d(gx_1, fx_2), b_d(fx_2, gx_3), \\ b_d(gx_3, fx_4), b_d(fx_4, gx_5), \\ \dots\dots\dots\dots \\ b_d(gx_{2k-1}, fx_{2k}), b_d(fx_{2k}, gx_{2k+1}) \end{array} \right\} \\ &\leq \lambda \max\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, ..., \alpha_{2k-1}, \alpha_{2k}\} \\ &\leq \lambda \max\{\mu \cdot \theta, \mu \cdot (\theta)^2, ..., \mu \cdot (\theta)^{2k}\}, \\ &= \lambda\mu \cdot \theta = \mu \cdot \theta \cdot (\theta)^{2k} = \mu \cdot (\theta)^{2k+1}. \end{aligned} \tag{2.8}$$

using (2.7),
and

$$\alpha_{2k+2} = b_d(fx_{2k+2}, gx_{2k+3}) \tag{2.9}$$

$$\begin{aligned} &= b_d(T(x_2, x_3, ..., x_{2k}, x_{2k+1}), S(x_3, x_4, ..., x_{2k+1}, x_{2k+2})) \\ &= b_d(S(x_3, x_4, ..., x_{2k+1}, x_{2k+2}), T(x_2, x_3, ..., x_{2k}, x_{2k+1})) \\ &\leq \lambda \max\left\{ \begin{array}{c} b_d(gx_3, fx_2), b_d(fx_4, gx_3), \\ b_d(gx_5, fx_4), b_d(fx_6, gx_5), \\ \dots\dots\dots\dots \\ b_d(gx_{2k+1}, fx_{2k}), b_d(fx_{2k+2}, gx_{2k+1}) \end{array} \right\} \\ &\leq \lambda \max\{\alpha_2, \alpha_3, \alpha_4, \alpha_5, ..., \alpha_{2k}, \alpha_{2k+1}\} \\ &\leq \lambda \max\{\mu \cdot (\theta)^2, \mu \cdot (\theta)^3, ..., \mu \cdot (\theta)^{2k}, \mu \cdot (\theta)^{2k+1}\}, \\ &= \lambda\mu \cdot (\theta)^2 = \mu \cdot (\theta)^2(\theta)^{2k} = \mu \cdot (\theta)^{2k+2}, \end{aligned}$$

using (2.7) and (2.8).
Continuing in this way, we get

$$\alpha_n \leq \mu \cdot (\theta)^n, \tag{2.10}$$

for $n = 1, 2, \ldots$

Consider now

$$b_d(y_{2k+2n-1}, y_{2k+2n}) = b_d(S(x_{2n-1}, x_{2n}, ..., , x_{2k+2n-2}), T(x_{2n}, x_{2n+1}, ..., x_{2k+2n-1})) \tag{2.11}$$

$$\begin{aligned}
&\leq \lambda \max \left\{ \begin{array}{c} b_d(gx_{2n-1}, fx_{2n}), b_d(fx_{2n}, gx_{2n+1}), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ b_d(gx_{2k+2n-3}, fx_{2k+2n-2}), \\ b_d(fx_{2k+2n-2}, gx_{2k+2n-1}) \end{array} \right\} \\
&\leq \lambda \max\{\alpha_{2n-1}, \alpha_{2n}, ..., \alpha_{2k+2n-3}, \alpha_{2k+2n-2}\} \\
&\leq \lambda \max\{\mu \cdot (\theta)^{2n-1}, \mu \cdot (\theta)^{2n}, ..., \mu \cdot (\theta)^{2k+2n-3}, \mu \cdot (\theta)^{2k+2n-2}\}, \\
&= \lambda\mu \cdot (\theta)^{2n-1} = \mu \cdot (\theta)^{2k}(\theta)^{2n-1} = \mu \cdot (\theta)^{2k+2n-1}
\end{aligned}$$

Also

$$b_d(y_{2k+2n}, y_{2k+2n+1}) = b_d(T(x_{2n}, x_{2n+1}, ..., x_{2k+2n-1}), S(x_{2n+1}, x_{2n+2}, \ldots, x_{2k+2n})) \tag{2.12}$$

$$\begin{aligned}
&= b_d(S(x_{2n+1}, x_{2n+2}, \ldots, x_{2k+2n}), T(x_{2n}, x_{2n+1}, \ldots, x_{2k+2n-1})) \\
&\leq \lambda \max \left\{ \begin{array}{c} b_d(gx_{2n+1}, fx_{2n}), b_d(fx_{2n+2}, gx_{2n+1}), \\ b_d(gx_{2n+3}, fx_{2n+2}), b_d(fx_{2n+4}, gx_{2n+3}), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ b_d(gx_{2k+2n-1}, fx_{2k+2n-2}), b_d(fx_{2k+2n}, gx_{2k+2n-1}) \end{array} \right\} \\
&\leq \lambda \max\{\alpha_{2n}, \alpha_{2n+1}, \alpha_{2n+2}, \alpha_{2n+3}, ..., \alpha_{2k+2n-1}\} \\
&\leq \lambda \max\{\mu \cdot (\theta)^{2n}, \mu \cdot (\theta)^{2n+1}, ..., \mu \cdot (\theta)^{2k+2n-1}\}, \\
&= \lambda\mu \cdot (\theta)^{2n} = \mu \cdot (\theta)^{2n}(\theta)^{2k} = \mu \cdot (\theta)^{2k+2n}.
\end{aligned}$$

From (2.11) and (2.12), we have

$$b_d(y_{2k+n}, y_{2k+n+1}) \leq \mu \cdot (\theta)^{2k+n}, \quad n = 1, 2, 3, \ldots \tag{2.13}$$

Now, using (2.13), for $m > n$ and using the fact that $s > 1$ we have

$$\begin{aligned}
b_d(y_{2k+n}, y_{2k+m}) &\leq \left(\begin{array}{c} s\ b_d(y_{2k+n}, y_{2k+n+1}) + s^2\ b_d(y_{2k+n+1}, y_{2k+n+2}) \\ +s^3\ b_d(y_{2k+n+2}, y_{2k+n+3}) + ...+ \\ s^{m-n-1}\ b_d(y_{2k+m-1}, y_{2k+m}) \end{array} \right) \\
&\leq \left(\begin{array}{c} s\ \mu \cdot (\theta)^{2k+n} + s^2\ \mu \cdot (\theta)^{2k+n+1} + s^3\ \mu \cdot (\theta)^{2k+n+2} \\ +... + s^{m-n-1}\mu \cdot (\theta)^{2k+m-1}, \end{array} \right) \\
&\leq \mu \cdot \left[\begin{array}{c} (s\theta)^{2k+n} + (s\theta)^{2k+n+1} + (s\theta)^{2k+n+2} \\ +... + (s\theta)^{2k+m-1}] \end{array} \right] \\
&\leq \mu(s\theta)^{2k} \left[\frac{(s\theta)^n}{1 - s\theta} \right] \text{ since } s\theta = s\lambda^{\frac{1}{2k}} < s \cdot \frac{1}{s} = 1 \\
&\to 0 \text{ as } n \to \infty, m \to \infty.
\end{aligned}$$

Therefore, $\{y_{2k+n}\}$ is a Cauchy sequence in (X, b_d). Since X is b_d-complete, there exists $z \in X$ such that $y_{2k+n} \to z$ as $n \to \infty$.

From (2.6), there exists $u \in X$ such that

$$z = fu = gu. \tag{2.14}$$

Now consider

$$\begin{aligned} b_d(S(u,u,...,u), y_{2k+2n}) &= b_d(S(u,u,...,u), T(x_{2n}, x_{2n+1}, ..., x_{2n+2k-1})) \\ &\leq \lambda \max \left\{ \begin{array}{l} b_d(gu, fx_{2n}), b_d(fu, gx_{2n+1}), \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ b_d(gu, fx_{2k+2n-2}), b_d(fu, gx_{2k+2n-1}) \end{array} \right\}. \end{aligned}$$

Letting $n \to \infty$ and using (2.14), we get

$$\frac{1}{s}\, b_d(S(u,u,...,u), fu) \leq 0 \quad \text{so that} \quad S(u,u,...,u) = fu. \tag{2.15}$$

Similarly we have

$$T(u,u,...,u) = gu. \tag{2.16}$$

Since (f, S) and (g, T) are jointly $2k$-weakly compatible pairs and from (2.15) and (2.16), we have

$$fz = f(fu) = f(S(u,u,...,u)) = S(fu, fu, ..., fu) = S(z,z,...,z), \tag{2.17}$$

and

$$gz = T(z,z,...,z,z). \tag{2.18}$$

Now using (2.16) and (2.17), we get

$$\begin{aligned} b_d(fz, z) &= b_d(fz, gu) \\ &= b_d(S(z,z,...,z,z), T(u,u,...,u,u)) \\ &\leq \lambda \max \left\{ \begin{array}{l} b_d(gz, fu), b_d(fz, gu), \\ b_d(gz, fu), b_d(fz, gu), \\ \dots\dots\dots\dots\dots \\ b_d(gz, fu), b_d(fz, gu) \end{array} \right\} \\ &\leq \lambda \max\{b_d(gz, z), b_d(fz, z)\}. \end{aligned}$$

Thus

$$b_d(fz, z) \leq \lambda \max\{b_d(gz, z), b_d(fz, z)\}. \tag{2.19}$$

Similarly, we have

$$b_d(gz, z) \leq \lambda \max\{b_d(gz, z), b_d(fz, z)\}. \tag{2.20}$$

From (2.19) and (2.20), we have

$$\max\{b_d(gz, z), b_d(fz, z)\} \leq \lambda \max\{b_d(gz, z), b_d(fz, z)\}.$$

which in turn yields that

$$fz = z = gz. \tag{2.21}$$

From (2.17), (2.18) and (2.21), we have

$$fz = z = gz = S(z,z,...,z,z) = T(z,z,...,z,z). \tag{2.22}$$

Suppose there exists $z' \in X$ such that
$z' = fz' = gz' = S(z', z', ..., z', z') = T(z', z', ..., z', z')$.
Then from (2.4), we have

$$\begin{aligned} b_d(z, z') &= b_d(S(z, z, ..., z, z), T(z', z', ..., z', z')) \\ &\leq \lambda \max \left\{ \begin{array}{l} b_d(gz, fz'), b_d(fz, gz'), \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ b_d(gz, fz'), b_d(fz, gz') \end{array} \right\} \\ &\leq \lambda b_d(z, z'). \end{aligned}$$

This implies that $z' = z$.
Thus z is the unique point in X satisfying (2.22). □

Now we give an example to illustrate our main Theorem 2.4.

Example 2.1. *Let $X = [0, 1]$ and $b_d(x, y) = |x + y|^2$ and $k = 1$.*
Define $S(x, y) = \frac{3x^2+2y}{\sqrt{4608}}$, $T(x, y) = \frac{2x+3y^2}{\sqrt{4608}}$, $fx = \frac{x}{6}$ and $gx = \frac{x^2}{4}$ for all $x, y \in X$. Then clearly $s = 2$. Then for all $x_1, x_2, y_1, y_2 \in X$, we have

$$\begin{aligned} b_d(S(x_1, x_2), T(y_1, y_2)) &= |\frac{3x_1^2 + 2x_2}{\sqrt{4608}} + \frac{2y_1 + 3y_2^2}{\sqrt{4608}}|^2 \\ &= \left(\frac{x_1^2}{16\sqrt{2}} + \frac{x_2}{24\sqrt{2}} + \frac{y_1}{24\sqrt{2}} + \frac{y_2^2}{16\sqrt{2}}\right)^2 \\ &= \frac{1}{2}\left(\left(\frac{x_1^2}{16} + \frac{y_1}{24}\right) + \left(\frac{x_2}{24} + \frac{y_2^2}{16}\right)\right)^2 \\ &= \frac{1}{32}\left(\left(\frac{x_1^2}{4} + \frac{y_1}{6}\right) + \left(\frac{x_2}{6} + \frac{y_2^2}{4}\right)\right)^2 \\ &= \frac{1}{8}\left(\frac{\left(\frac{x_1^2}{4} + \frac{y_1}{6}\right) + \left(\frac{x_2}{6} + \frac{y_2^2}{4}\right)}{2}\right)^2 \\ &\leq \frac{1}{8}\left(\max\left\{\frac{x_1^2}{4} + \frac{y_1}{6}, \frac{x_2}{6} + \frac{y_2^2}{4}\right\}\right)^2 \\ &= \frac{1}{8}\max\left\{\left(\frac{x_1^2}{4} + \frac{y_1}{6}\right)^2, \left(\frac{x_2}{6} + \frac{y_2^2}{6}\right)^2\right\} \end{aligned}$$

where used the following:

$$\frac{a+b}{2} \leq \max\{a, b\}, \ (\max(a, b))^2 = \max\{a^2, b^2\},$$

for non-negative a and b. Here $\lambda = \frac{1}{8} \in (0, \frac{1}{4}) = (0, \frac{1}{2^2}) = (0, \frac{1}{s^{2k}})$.
One can easily verify the remaining conditions of Theorem 2.4. Clearly 0 is the unique point in X such that $f0 = 0 = g0 = S(0, 0) = T(0, 0)$.

Corollary 2.1. *Let (X, b_d) be a b_d−complete b - dislocated metric space with $s \geq 1$ and k be any positive integer. Let $S, T : X^{2k} \longrightarrow X$ and $f : X \longrightarrow X$ be mappings satisfying*

(2.23) $$S(X^{2k}) \subseteq g(X), T(X^{2k}) \subseteq f(X),$$

(2.24) $$b_d(S(x_1, x_2, ..., x_{2k}), T(y_1, y_2, ..., y_{2k})) \leq \lambda \max\{b_d(fx_i, fy_i) : 1 \leq i \leq 2k\}$$
$$\textit{for all} \quad x_1, x_2, ..., x_{2k}, y_1, y_2, .., y_{2k} \in X, where\ \lambda \in (0, \frac{1}{s^{2k}})$$

(2.25) $$f(X) \quad \textit{is} \quad ab_d - \textit{complete subspace of} \quad X$$

(2.26) $$(f, S) \quad \textit{or} \quad (f, T) \quad \textit{is} \quad 2k - \textit{dweakly compatible pair}.$$

Then there exists a unique point $u \in X$ such that $u = fu = S(u, u, .., u, u) = T(u, u, .., u, u)$.

Corollary 2.2. *Let (X, b_d) be a b_d-complete b - dislocated metric space with $s \geq 1$ and k be any positive integer. Let $S, T : X^{2k} \longrightarrow X$ be mappings satisfying*

(2.27) $$b_d(S(x_1, x_2, ..., x_{2k}), T(y_1, y_2, ..., y_{2k})) \leq \lambda \ \max\{b_d(x_i, y_i) : 1 \leq i \leq 2k\}$$
$$\textit{for all} x_1, x_2, ..., x_{2k}, y_1, y_2, .., y_{2k} \in X, where\ \lambda \in (0, \frac{1}{s^{2k}})$$

Then there exists a unique point $u \in X$ such that $u = S(u, u, .., u, u) = T(u, u, .., u, u)$.

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Competing interests

Authors declare that they have no any conflict of interest regarding the publication of this paper.

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