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Prediction of Critical Parameters of Cholesterol and its Binary Interaction Coefficient in the Supercritical Carbon Dioxide

Hatem Ksibi *, Ali Ben Moussa
Sfax University, IPEIS, P. O. Box 805 Sfax 3018, Tunisia
CFDTP, ENIS, P. O. Box <W> Sfax 3038, Tunisia
hatem.ksibi@ipeis.rnu.tn

Abstract

In this work, the cholesterol concentration in the supercritical (SC) CO_2, under working conditions, is calculated by implementing the modified Peng Robinson equation of state (EoS) combined with the Van Der Waals mixing rules. It was shown that this model achieves good correlation of the cholesterol solubility in SC carbon dioxide with an adjustable parameter called binary interaction coefficient (k_{ij}) and gives new estimation of the cholesterol critical parameter. A comparison of the experimental data given in literature with the commonly used Peng–Robinson equation reveals a good agreement of the empirical correlation of k_{ij} written as a function of the operating temperature.

Keywords: Cholesterol, critical parameters, binary interaction, solubility, supercritical.

1. Introduction

A supercritical (SC) fluid is defined as any substance that is above its critical temperature T_c and its critical pressure P_c where gas and liquid states can no longer co-exist. Supercritical fluids can display liquid-like densities and the ability to dissolve substances which gaseous substances cannot do. Therefore, relatively high supercritical fluid density gives it a good solvent power. As a result, a sudden reduction in temperature or pressure often causes the loss of these special characteristics of the supercritical solvent. Therefore a solute can be extracted at supercritical conditions of the solvent, and separated from it by reducing pressure or temperature below the critical parameters of the solvent, yielding a solvent-free extract. By using supercritical solvents having low critical temperatures such as carbon dioxide, it is possible to extract thermally labile compounds, especially pharmaceutical and food products. The solubility of high molecular weight compounds in supercritical carbon dioxide is of great importance for a number of applications and processes.

The solubility of most of lipids in CO_2 is rather very low. In supercritical fluids, solubilities of organic solutes can be varied carefully and systematically by pressure and temperature (i.e. density) to extract undesirable compounds which have a high molecular weight or a long chain. Therefore selective extraction and fractionation are possible from a multi-component mixture.

Cholesterol ($C_{27}H_{46}O$) is a sterol that can be found in many animal tissues. It is the most abundant sterol in human tissues. It is transported in the plasma and is delivered to the tissues in the form of lipoprotein particles. The deposition of cholesterol in tissues can cause the narrowing of blood vessels, known as atherosclerosis.

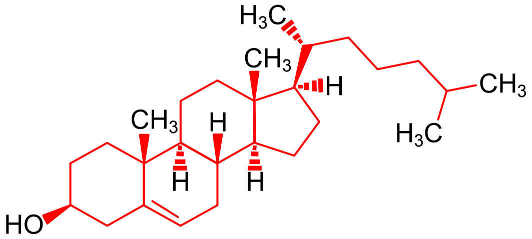

Figure 1. Cholesterol molecule

Recently, many supercritical applications concerning cholesterol have been published by several authors. Among them, we notice the work of Mohamed et al. (2001) who has studied the reduction in the cholesterol content of butter oil using supercritical ethane extraction and adsorption on alumina. Zeidler et al. (1996) presented a US patent where cholesterol can be removed from food products using supercritical solvents resulting in minimal protein degradation. Concerning the

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*Author to whom correspondence should be extraction of the cholesterol contained in shellfish, Kang et al. (2005) has refined squid oil using supercritical carbon dioxide with ethanol as a cosolvent in a semi-continuous flow extractor. In addition, Higuera-Ciapara et al. (2005) has carried out how a low-cholesterol shrimp can be produced using supercritical extraction. Also, Vedaraman et al. (2005) has extracted cholesterol from cow brain using a small supercritical carbon dioxide extractor. Indeed, the extracted cholesterol was analysed for its structure to confirm it by different characterisation techniques.

Moreover, very often the solubility of cholesterol in carbon dioxide is extremely low. Therefore, it is worthwhile investigating the possibility of applying numerical optimisation of the dissolved mole fraction in SC carbon dioxide when the extraction of these compounds is very difficult and extract analysis requires instrumentations of high performance.

The calculation of the solubility of an organic compound in the supercritical fluid has included both correlative and predictive approaches. Three models are well used in the literature to establish correlations for binary systems solubility data. These models are summarised in empirical correlations as the Chrastil relation (Chrastil, 1982) and cubic EoS such as Redlich-Kwong-Soave (RKS) (Soave, 1972) and the modified Peng-Robinson (PR) introduced by Schmitt and Reid (1986).

In the present report, we study the two-parameter equations of state to predict the solubility of cholesterol in supercritical carbon dioxide. These equations of state need physical properties of the solute as critical coordinates and the acentric factor. These data are computed simultaneously with the binary interaction coefficient (k_{ij}) by fitting numerically the experimental solubility of cholesterol in carbon dioxide over a wide range of pressure and for given temperatures, see references (Wong et al., 1986), (Yun et al., 1991), (Hartono et al., 2001) and (Huang et al., 2004).

2. Numerical approach

2.1 Cubic EoS models

To predict the solubility of the cholesterol in supercritical CO_2, the compressed gas model due to Prausnitz et al. (1986) was used. The model requires critical parameters and physical properties of the solute which cannot be measured, because the solute undergoes degradation when treated. In predicting the phase equilibrium between a supercritical phase and a condensed phase, the equality of fugacity of each component in each phase is required. The solubility of a condensed (y_2) in a vapour phase at supercritical condition is defined as follow:

$$y_2 = \frac{P_2^{sat}\varphi_2^{sat}}{\varphi_2^{SF} P}\,\mathbf{exp}\left(\frac{V_2^s\left(P - P_2^{sat}\right)}{RT}\right) \qquad (1)$$

where 2 denotes the solute and ($\varphi_2^{\mathrm{sat}}, \varphi_2^{\mathrm{SF}}$) are fugacity coefficients of the saturated pure vapour and the supercritical state of the solute, respectively. The coefficient φ_2^{sat} is equal to 1, and the second one is calculated using the chosen EoS.

Usually established for pure component, relations of RKS EoS and PR EoS, could be extended to a solution at equilibrium of a multi-component system by using suitable mixing rules that define the homogenous supercritical mixture characteristics.

These equations can be written respectively as:

$$P = \frac{RT}{(V-b)} - \frac{a}{V(V+b)} f\,(T_r) \qquad (2)$$

$$P = \frac{RT}{(V-b)} - \frac{a}{V(V+b)+b(V-b)} f\,(T_r) \qquad (3)$$

R is the perfect gas constant, **a** denotes the attractive term and **b** is the covolume. The corrective terms **a** and **b** are expressed as functions of the critical coordinates of both the solvent and the solute.

The corrective function $f(T_r)$, depending on the temperature, is defined as:

$$f(T_r) = \left(1 + \kappa\left(1 - \sqrt{T_r}\right)\right)^2 \qquad (4)$$

where T_r is the reduced temperature $\left(T_r = \frac{T}{T_c}\right)$ and κ is a polynomial function of the acentric factor ω.

Using the Peng Robinson EoS to integrate the fugacity of the solute in a dilute supercritical solution, we write:

$$Ln\left(\varphi_2^{SF}\right) = \frac{b_2}{b_1}(Z-1) - Ln\left(Z - \frac{b_1 P}{RT}\right) - \frac{a_1}{2\sqrt{2}b_1 RT}\left(\frac{2a_{12}}{a_1} - \frac{b_2}{b_1}\right) Ln\left(\frac{V_1 + \left(1+\sqrt{2}\right)b_1}{V_1 + \left(1-\sqrt{2}\right)b_1}\right)$$

The conventional mixing rules of Van Der Waals for the supercritical solution are defined as:

$$\begin{cases} b = \sum_{i=1}^{2} y_i b_i \\ a = \sum_{i=1}^{2} \sum_{j=1}^{2} y_i y_j a_{ij} \end{cases} \qquad (5)$$

and $a_{i \neq j} = \sqrt{a_i a_j}(1 - k_{ij})$ (6)

The fitting aspect is based on the molecular interaction coefficient as an adjustable parameter. The solute properties necessary to effectively correlate solubilities based on the modified PR approach or the RKS EoS include the molar volume and the saturated vapour pressure. The molar volume of cholesterol V^S is given in the literature by various values, which their average is: $V^S = 373.55$ cm^3/mol.

The knowledge of the vapour pressure of the solute P_2^{sat} is also an essential step in the solubility calculation. Its equation is based on the Clausuis-Clapeyron model and given as a logarithmic function of the temperature:

$$Log_{10}\left(P^{sat}\right) = A - B/T \qquad (7)$$

For cholesterol, Singh et al. (1993) gave constants of the above relation as:

$A = 14.418$ and $B = 5633.4$

As can be seen, the EoS corrective coefficients (**a**, **b**) require the interaction parameters for phase behaviour description (eq. 6). For supercritical solution at equilibrium, this parameter can be fitted numerically from experimental solubility data.

For a complex solute molecule as cholesterol, we find in the literature a few accurate measurable data such as the burning temperature and the density. Many critical coordinates are also found in literature and given by several authors, see Table 1. Therefore, group contribution methods (GCM) (Perry et al, 1999) are used to estimate these properties. Nevertheless, estimated values of critical temperature, critical pressure and acentric factor did not provide a satisfactory fit of the solubility data and should not be interpreted as "true properties". In fact, uncertainty in these values probably contributes to fake use of the PR EoS. We notice that P_c and T_c of the solute will be adjusted, together with the interaction parameter k_{ij} to minimise the differences between the experimental and the predicted solubilities.

TABLE 1. DIFFERENT CRITICAL PROPERTIES OF CHOLESTEROL GIVEN IN THE LITERATURE.

P_c (bar)	**T_c (K)**	**ω**	**Ref.**
12.20	778.7	-	Hartono et al. (2001)
41.55	1168.2	0.950	GCM: Perry et al. (1999)
12.50	959.0	0.948	Daubert et al. (1996)

A numerical implementation based on the optimisation technique of Hook and Jeeves (Hook et al., 1961) is elaborate. The critical coordinates and k_{ij} are computed and a comparison, at each iteration, between calculated solubility with its experimental one is done. When critical pressure of the solute is accurately computed, group contribution methods are used to determine the critical temperature, the critical volume and the acentric factor.

The optimisation technique of the solubility is quantified by an absolute average relative deviation (AARD) approach which is the most widely used assessment for evaluating the success of the numerical model and calculated as:

$$AARD(\%) = \frac{1}{N}\left|\Sigma \frac{y_e - y_c}{y_e}\right| \qquad (8)$$

where y_c and y_e denote the calculated and the experimental solubilities, respectively.

2.2 The Chrastil model:

For quantitative determination of the solute mole fraction in supercritical solution at equilibrium, the Chrastil method is the most widely used technique for correlating solubilities. This model, although simple, relies much on the knowledge of the thermodynamic behaviour of the supercritical solvent rather than of the solute and it is mostly capable of correlating, rather than predicting, the solubility. The Chrastil empirical relation is written as:

$$y_2 = \rho_1^k \exp\left(\frac{a_{ch}}{T} + b_{ch}\right) \qquad (9)$$

where y_2 is the solute solubility (mol/mol), ρ_1 is the supercritical fluid density and a_{ch}, b_{ch} and k are empirical fitting parameters. For cholesterol, these constants are set as follows (Cansell et al., 1998):

$$k = 12.095\,;\ a_{ch} = -9460 \text{ and } b_{ch} = -50.488$$

3. Results & discussions

The solubility of cholesterol in supercritical carbon dioxide, as predicted by PR and RKS equations and the empirical correlation of Chrastil, are shown in Figure 2 and compared with the experimental data. According to this figure, both the PR and RKS EoS are in good agreement with the experimental data. The solubility of cholesterol calculated using the Chrastil correlation is 2 orders of magnitude higher than the experimental data.

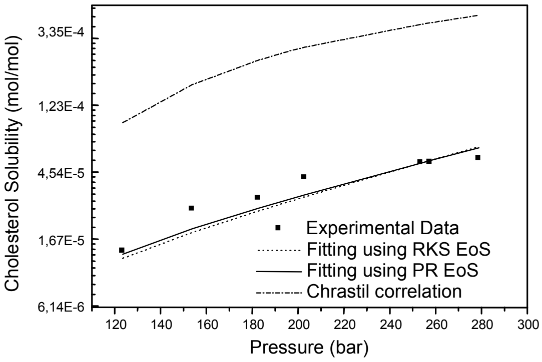

Figure 2. Comparison between experimental data [9] and these numerical optimisations of the cholesterol solubility at T=308.15 K

From this calculation, we can underline the limitation of this predictive correlation for cholesterol in carbon dioxide. Indeed specific properties and molecular characteristics of the solute are not taken into account (critical coordinates, polarity, molecular weight…).

At temperature T=318.15 K, we compared the calculated solubilities using the PR EoS for different critical properties given in the literature as presented in Table 1. We find that values of cholesterol solubility are strongly altered by these parameters (Figure 3). For example, critical data deduced from reference (Perry et al., 1999) give the lowest value at every pressure when it is compared to the experimental measurements. A little amelioration is given by implementing the solubility using critical properties given by Daubert et al. (1996), but error exceeds many orders of magnitude. The present critical values give the best fitting of the experimental data. These critical coordinates are summarised as follows:

$$P_c = 43 \text{ bars};\ T_c = 875 \text{ K};\ \omega = 0.949$$

We notice that the new value of critical temperature is close to the average value of critical temperatures given in Table 1, whereas the critical pressure is near to that estimated by GCM, Perry (1999).

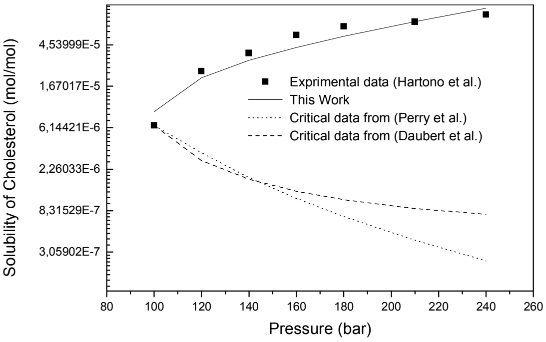

Figure 3. Cholesterol solubility calculated through several critical coordinates optimisations-comparison with experimental data at T=318.15 K

In Figure 4, the experimental solubilities given in literature are fitted using new critical parameters and the optimized binary interaction factor. A good agreement between both calculated and experimental values of the cholesterol solubility is obtained for different isotherms as a function of the carbon dioxide density.

TABLE 2. BINARY INTERACTION COEFFICIENTS ESTIMATED AT DIFFERENT TEMPERATURES

T	k_{ij}	AARD (%)
308.15	0.4380	5.9
313.15	0.4143	5.7
318.15	0.4161	9.0
323.15	0.4180	17.5
333.15	0.4219	12.3

The numerical values of the k_{ij} parameter at different isotherms are correlated using the empirical equation of Ksibi (2004) for the system (cholesterol/carbon dioxide). Indeed, it is shown as a function of characteristic parameters of the supercritical solution and the temperature.

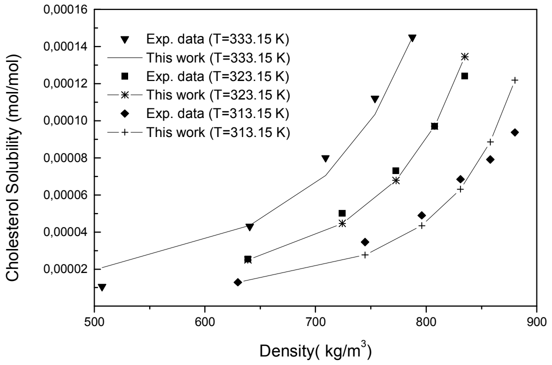

Figure 4. Isotherm network of cholesterol solubility – Comparison between experimental data and numerical computations of this work

This empirical relation is written, for the dissolved cholesterol in CO_2, as:

$$k_{ij} = -0.39414.e^{-T_r} + 0.55416 \qquad (10)$$

A good agreement of these calculations is shown via Figure 5 where the binary interaction is depicted as a function of the reduced temperature. We note that constant values of correlation (10) are strongly related to the nature of the solute. By means of the empirical equation (10), we can extrapolate estimation of the dissolved quantity of cholesterol in supercritical carbon dioxide for wide ranges of temperature and pressure.

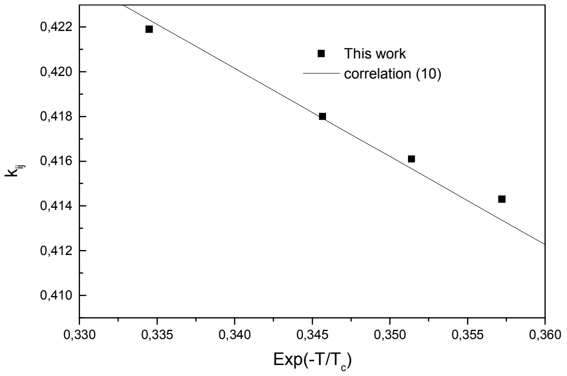

Figure 5. Variation of binary interaction k_{ij} as a function of reduced temperature - Comparison between numerical optimization and correlation (Ksibi, 2004).

4. Conclusion

We present in this work a numerical model for estimating the solubility of a high molecular weight compound in supercritical carbon dioxide. Different comparisons of the obtained results with both experimental data and other numerical predictions of the solubility at equilibrium are shown. The accuracy of the present numerical technique is noticed by giving new critical coordinate data of the cholesterol and an accurate binary interaction coefficient for the system cholesterol / carbon dioxide.

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