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A MONTE CARLO SIMULATION STUDY ROBUSTNESS OF MANOVA TEST STATISTICS IN BERNOULLI AND UNIFORM DISTRIBUTION

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Abstract

The aim of this study is to compare the robustness of Manova test statistics against the Type I error rate using the Monte Carlo simulation in Bernoulli and Uniform distribution. In the method, numbers have been generated according to constant and increasing variance for g= 3, 4, 5 group p= 3, 5, 7 dependent variables n= 10, 30, 60 sample size using the RStudio. The specified combinations have been repeated 10,000 times. As the result Pillai Trace test statistic has been the least deviating from the nominal α =0.05 value. Wilk Lambda and Hotelling-Lawley Trace test statistics results have been close to each other. Researchers can get help from these suggested results during their own study.

Keywords: Monte-Carlo, Simulation, Bernoulli, Uniform

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1. Introduction

The one-way multivariate analysis of variance (one-way MANOVA) is used to determine whether there are any differences between independent groups on more than one dependent variable. The most important assumptions are multivariate normality and homogeneity of variance-covariance matrices. The most well known and widely used MANOVA test statistics are Wilk's Λ, Pillai, Lawley-Hotelling, and Roy's test.

1.1. Wilk's Λ

Wilks' lambda (Wilks, 1932) is a test statistic used in multivariate analysis of variance (MANOVA) to test whether there are differences between the means of identified groups of subjects on a combination of dependent variables. Wilks' lambda is the oldest multivariate test statistic, and is the most widely used (Johnson and Wichern, 1982)

Let,
T: Total sums of squares and cross-products matrix
B: Between-group sums of squares and cross-products matrix
W: Within-group sums of squares and cross-products matrix
p: Number of dependent variables in each group
g: The number of groups $g \geq 2$.
$\bar{x}$: Overall sample mean vectors
n_i : sample size for the i th group
S_i : sample covariance matrix for the i th sample
Thus B and W matrix can be expressed by;

$$B = \sum_{i=1}^{g} n_i(\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})' \quad W = \sum_{i=1}^{g}(n_i - 1)S_i \quad (1)$$

The Wilks' Lambda statistic is the ratio of the within generalized dispersion to the total generalized dispersion;

$$\Lambda = \frac{|W|}{|B+W|} = \frac{|W|}{|T|} \quad (2)$$

takes values between zero and one. The Wilks' Lambda can be obtained as a product of eigenvalues which can be obtained by the eigenvalues of the matrix of BW^{-1} by following method;

$$\Lambda = \prod_{i=1}^{s} \frac{1}{1+\lambda_i} \quad (3)$$

where s=min(p, g−1) and the rank of the B matrix and the expression λ_i are eigenvalues of the BW^{-1} matrix. According to Johnson and Wichern the Wilks' Lambda performs, in a multivariate setting, with a combination of dependent variables (the same role as the F-test) performs in a one-way analysis of variance. Bartlett (1954) using a Chi-square test instead of an F-distribution test. Bartlett's test is a modification of the corresponding likelihood ratio test designed to make the approximation of the chi-square distribution better at all stages as formuled;

$$V = -[N - 1 - (p + g)/2] ln\Lambda \quad (4)$$

denotes the χ^2 distribution of $p(g$-1) degrees of freedom if $V > \chi^2_{[p(g-1)];\alpha}$ there is a difference between the mean vectors. The Wilks Lambda statistic can salso be calculated with the help of the F distribution. In different groups, variables and observation numbers, approach to F distribution and degrees of freedom are available.

1.2. Hotelling-Lawley Trace (T)

The Hotelling and Lawley Trace statistic which defined as follows;

$$T = trace(BW^{-1}) = \sum_{i=1}^{s} \lambda_i \quad (5)$$

The F distribution can be used to test the T statistic (Stevens 1986). T is the trace of the BW^{-1} matrix (Hotelling 1931; Lawley 1939).

1.3. Pillai's Trace Statistics (V)

Pillai (1955) trace statistic can be interpreted as the proportion of variance in the dependent variables which is accounted for by variation in the independent variables. The V statistics where s, m, n parameters are as follows;

s=min(g-1,p),

$$m = \frac{|p-(g-1)|-1}{2}, n = \frac{N-p-g-1}{2}, \quad \frac{2n+s+1}{2m+s+1} \times \frac{V}{s-V} \quad (6)$$

closed F distribution with $s(2m+s+1)$ and $(2n+s+1)$ degrees of freedom (Morrison 1976).

1.4. Roy's Largest Root (R)

If the big eigenvalue of the matrix of BW^{-1} is denoted by $\lambda_{\max}$ Roy's R statistic is given by;

$$R = \sum_{i=1}^{s} \frac{\lambda_{max}}{1+\lambda_{max}} \quad (7)$$

This value is compared to the Heck graph value with parameter s, m, n. If the R statistic is greater than the Heck graph value, it is said to be the difference between the mean vectors (Alpar 2013). When s=1, R shows exact F distribution (Kanık 1999).

2. Material and Method

This investigation deals mainly to assess the robustness of MANOVA. To do is the Multivariate Normality assumption is violated to see if that will affect Type I error rate. In order to evaluate the robustness of MANOVA the virtual experiment was designed in the following way. For the significance test of difference between the groups, the number of groups was determined as g=3, g=4, g=5.
Dependent variable numbers were set at p=3, p=5, p=7 for each group. Sample size determined as n=10, 30 and 60. That simulation was based on 10,000 replications. The Monte Carlo study manipulated in equal variance ($\sigma_1^2 = \sigma_2^2 = ... = \sigma_g^2$) and unequal variance ($\sigma_1^2 < \sigma_2^2 ... < \sigma_g^2$). When establishing the unequal variance, the variance of a dependent variable was first set, then the other dependent variables were multiplied by 3 that mean variance ratio is (1:3). All of the statistical methods were conducted using R (MVNormTest written by Slawomir on 04/12/2012: Normality test for multivariate variables package). In order to test the hypothesis used to compare the mean of more than two groups the Wilks' Lambda (W), Pillai's Trace (V), Hotelling-Lawley Trace (T), Roy's Largest Root test (R) statistics values and their Type I error rate were calculated. If p-value was less than 0.05,

the nominal alpha level, the null hypothesis was rejected. The data are produced in the Bernoulli and Uniform distribution. Scenarios were prepared in 54 different combinations for each test statistic. These operations were repeated 10,000 times and the number of null hypothesis rejections was determined for each test statistic. Experimental Type I error rates were calculated for each test statistic with dividing the rejection number by the repeat number.

Monte Carlo test result for *R*,*V*,*T* and *W* test statistics were given in Table 1, Table 2, Table 3, respectively and the comments were given below.

When group number is *g*=3, for all values of *p*, observations are interpreted in Bernoulli and Uniform distribution according to sample size for Roy Largest Root test statistics with Figure 1.

3. Results and Discussion

Table1. For *g*=3, *p*=3, 5, 7; sample size n=10, 30, 60 experimental Type I error rate with 10000 replicate

g	p	variance	n	Roy (R)		Pillai Tracks (V)		Hotelling-Lawley (T)		Wilks Lambda (W)	
				B	U	B	U	B	U	B	U
3	3	constant	10	0,0592	0.0507	0,0544	0.0504	0,0522	0.0551	0,0572	0.0525
			30	0,0521	0.0496	0,0542	0.0497	0,0519	0.0483	0,0502	0.0479
			60	0,0537	0.0504	0,0538	0.0477	0,0542	0.0488	0,0512	0.0511
		Increase	10	0,0563	0.0501	0,0549	0.0563	0,0551	0.0537	0,0608	0.0504
			30	0,0541	0.0512	0,0549	0.0444	0,0548	0.0498	0,0533	0.0514
			60	0,055	0.0497	0,0494	0.0476	0,0568	0.0497	0,0503	0.0466
	5	constant	10	0,0549	0.0537	0,0521	0.0474	0,058	0.0533	0,0517	0.0497
			30	0,0526	0.0478	0,0575	0.0495	0,0584	0.0501	0,0558	0.0521
			60	0,0517	0.0489	0,0543	0.0506	0,0484	0.0519	0,0511	0.0474
		Increase	10	0,0563	0.0509	0,0522	0.0529	0,0533	0.0506	0,0548	0.0507
			30	0,0527	0.0475	0,0549	0.0512	0,0498	0.0503	0,0524	0.0527
			60	0,0545	0.0477	0,0503	0.0468	0,0525	0.0478	0,0549	0.0491
	7	constant	10	0,0515	0.0486	0,0509	0.0488	0,052	0.0546	0,0529	0.0498
			30	0,0556	0.0529	0,055	0.048	0,0545	0.0497	0,0518	0.0478
			60	0,054	0.0449	0,0534	0.0489	0,053	0.0466	0,0491	0.0462
		Increase	10	0,058	0.0494	0,0568	0.0546	0,0561	0.0521	0,0547	0.0513
			30	0,053	0.0468	0,0573	0.0527	0,0535	0.0477	0,0542	0.0494
			60	0,0517	0.0502	0,0511	0.0471	0,0528	0.0478	0,0494	0.0461

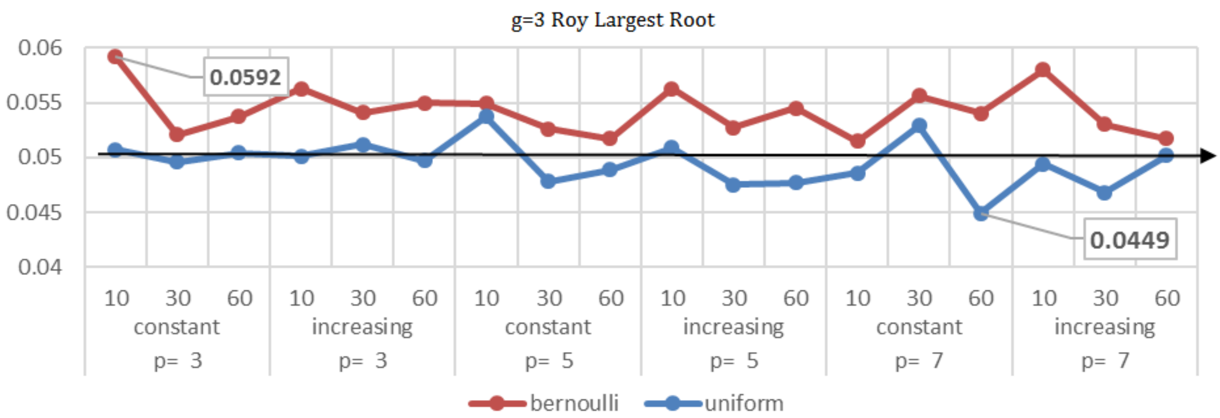

Figure 1. Type I error rates for Roy Largest Root test statistic.

For the Roy in Bernoulli test statistic, constant and increasing variance; when the sample size and the number of variables increased, it was seen that deviations from Type I error decreased. For Roy test statistic in *g*=3, the highest deviation was seen in all scenarios when *p*=3 *n*=10, constant variance with 0.0592 value. In uniform distribution it was observed that deviations from 0.05 are low when p and g are small and large when *p* and *g* are big. The highest deviation in uniform distribution was seen in all scenarios when p=7 n=60, constant variance with 0.0449 value.

When group number is g=3, for all values of p, observations are interpreted in Bernoulli and Uniform distribution according to sample size of Pillai's Trace test statistics with Figure 2.

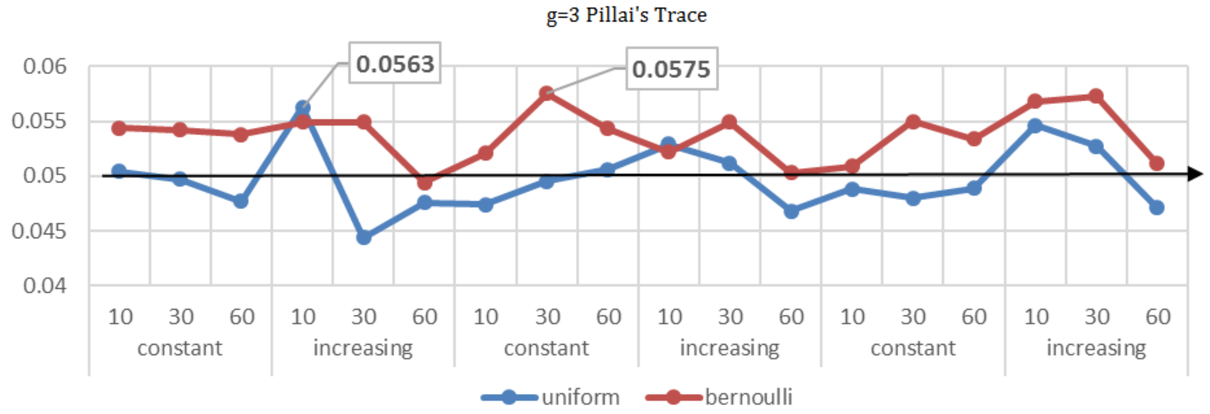

Figure 2. Type I error rates for Pillai's Trace test statistic.

For Pillai in Bernoulli distribution when p=3, both constant and increasing variance, deviations from nominal significance level, α =0.05, decrease as the sample size (n value) increase. For Pillai test statistic in g=3, the highest deviation was seen in all scenarios when p=5, n=30, in constant variance with 0.0575 value. In uniform distribution for Pillai test statistic in g=3, the highest deviation was seen in all scenarios when p=3, n=10, in increasing variance with 0.0563 value.

When group number is g=3, for all values of p, observations are interpreted in Bernoulli and Uniform distribution according to sample size of Hotelling-Lawley test statistics with Figure 3.

For Hotelling-Lawley in Bernoulli distrubition when p=3, most deviations was seen when the sample size n=60 both in case of constant and increasing variance. When p = 5, the greatest deviation was seen when n = 30, both in case of constant and increasing variance again. As the number of variables p = 7 the highest deviation was seen; when n = 30 for the constant variance and when n = 10 for the increasing variance. For Hotelling-Lawley test statistic in g=3 in bernoulli distribution, the highest deviation was seen in all scenarios when p=5, n=30, in constant variance with 0.0584 value.

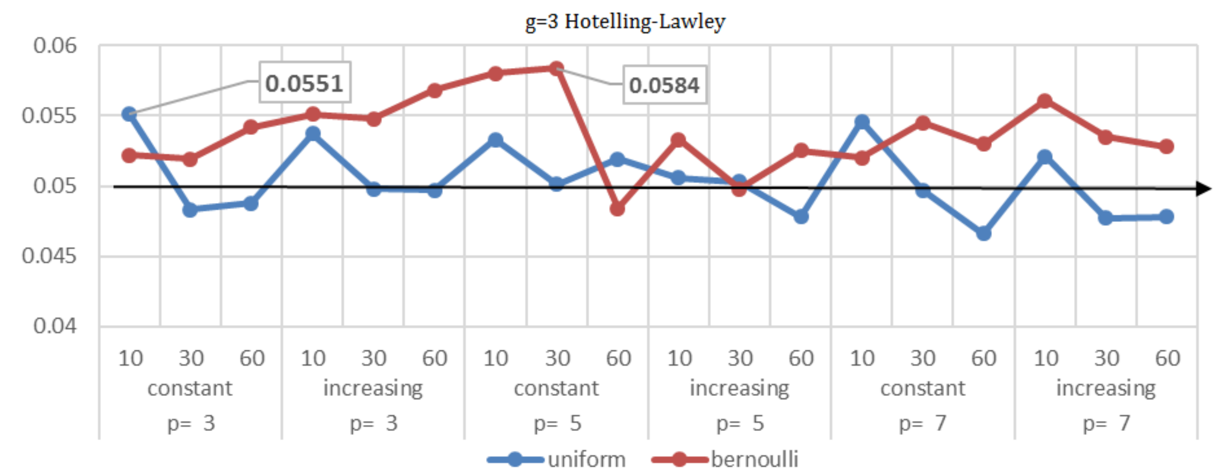

Figure 3. Type I error rates for Hotelling-Lawley test statistic.

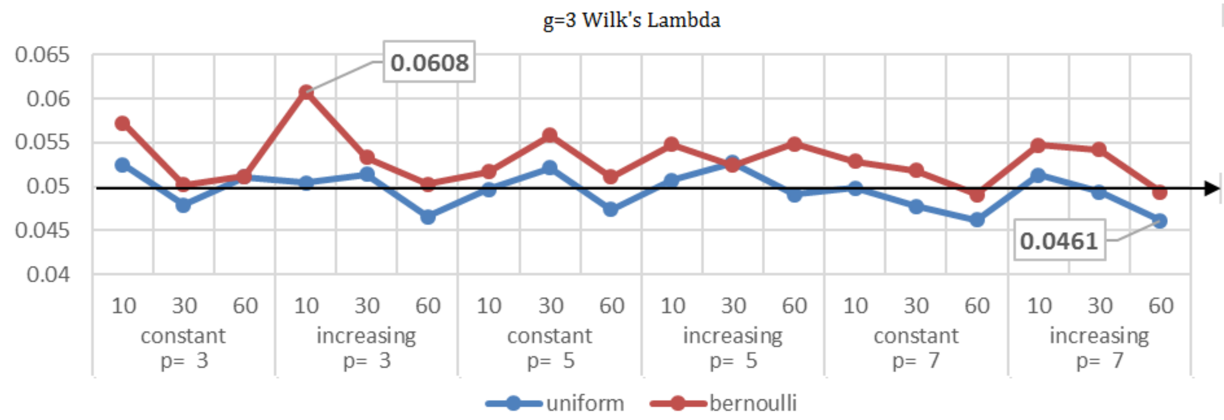

Figure 4. Type I error rates for Wilks'Lambda test statistic.

In Uniform distribution, it was observed that the deviation increases when the number of variables increases. For Hotelling-Lawley test statistic in *g*=3 in Uniform distribution, the highest deviation was seen in all scenarios when *p*=3, n=10, in constant variance with 0.0551 value. When group number is *g*=3, for all values of *p*, observations was interpreted in Bernoulli and Uniform distribution according to sample size of Wilks' Lambda test statistics with Figure 4.

Wilks' Lambda in Bernoulli distribution when *p*=3, both constant and increasing variance, the highest deviation was seen when *n*=10. As *p*=5 the highest deviation is seen; when *n*=30 for the constant variance and when *n*=10 for the increasing variance. As *p*=7 both constant and increasing variance, the highest deviation is seen when *n*=10. For Wilks Lambda test statistic in *g*=3, the highest deviation was seen in all scenarios when *p*=3, *n*=10, in constant variance with 0.0608 value. In Uniform distribution, the highest deviation was seen in all scenarios when *p*=7, *n*=60, in constant variance with 0.0461 value.

When group number is *g*=4, for all values of *p*, observations are interpreted according to sample size of Roy Largest Root test statistics with Figure 5.

For the Roy test statistic, it was seen that when *n*=30, *p*=3 and the both constant and increasing variance, there is more deviations from nominal significance level, *α*=0.05. As *p*=5, the greatest deviation was seen when *n*=60 for the constant variance, and when *n*=10 for the increasing variance. As the number of variables *p*=7, the highest deviation is seen when *n*=10 for constant variance and when *n*=30 for the increasing variance. For Roy test statistic in *g*=4, the highest deviation was seen in all scenarios when *p*=3, *n*=30, in constant variance with 0.0599 value. The highest deviation in the uniform distribution was observed when *p*=3, *n*=10, and this deviation was the highest one in all scenarios with 0.053.

When group number is *g*=4, for all values of *p*, observations are interpreted according to sample size of Pillai's Trace test statistics with Figure 6.

For Pillai'sTrace when *p*=3 per group, most deviations is seen when the sample size *n*=30 for the constant variance and when n = 60 for the increasing variance. When *p*=5, 7 the greatest deviation is seen when *n*=30, both constant and increasing variance. For Pillai test statistic in *g*=4, the highest deviation was seen in all scenarios when *p*=7, *n*= 10 with 0.0553 value. In uniform distribution, deviations are reduced as variable values grow. The highest variance value 0.0424, while *p*=5, *n*=60 while increasing variance was observed.

When group number is *g*=4, for all values of *p*, observations are interpreted according to sample size of Hotelling-Lawley test statistics with Figure 7.

For Hotelling-Lawley in Bernoulli distribution when *p*=3 and *p*=5 per group, both constant and increasing variance, the highest deviation is seen when *n*=10. As *p*=7 both constant and increasing variance, the highest deviation is seen when *n*=30. For Hotelling-Lawley test statistic in *g*=4, the highest deviation was seen in all scenarios when *p*=7, *n*=30, in increasing variance with 0.0577 value. In Uniform distribution the closest results to the nominal *α* =0.05 value were seen when *p*=5 at constant variance. Also 0.0539 is which is the highest value in uniform distribution all scenarios.

When group number is *g*=4, for all values of *p*, observations are interpreted according to sample size of Wilks'Lambda test statistics with Figure 8.

For Wilks'Lambda in Bernoulli distribution when *p*=3, both constant and increasing variance, the highest deviation was seen when *n*=30. The number of variables *p*=5, both constant and increasing variance, the highest deviation was seen when *n*=10. The highest deviation was seen as the *p*=7, *n*=30 for the constant variance and as *n*=10 for the increasing variance. For Wilks Lambda test statistic, the highest deviation was seen in all scenarios when *p*=5, *n*=10, in increasing variance with 0.0567 value. In the uniform distribution, the Wilks' Lambda test statistic gave deviated results for all scenarios in general, except for *p*=7.

Table 2. For *g*=4, *p*=3, 5, 7; sample size *n*=10, 30, 60 experimental Type I error rate with 10000 replicate

g	p	variance	n	Roy (R)		Pillai Tracks (V)		Hotelling-Lawley (T)		Wilks Lambda (W)	
				B	U	B	U	B	U	B	U
4	3	constant	10	0.0535	0.053	0.0499	0.0505	0.0545	0.0484	0.0549	0.0527
			30	0.0599	0.0502	0.0548	0.0475	0.0505	0.0502	0.0552	0.049
			60	0.0517	0.0474	0.0497	0.0455	0.0508	0.0489	0.0537	0.046
		Increase	10	0.0526	0.0476	0.052	0.0513	0.0556	0.0494	0.0506	0.0479
			30	0.0532	0.0494	0.0519	0.0474	0.0518	0.0475	0.0528	0.0503
			60	0.0531	0.0502	0.0528	0.0481	0.0502	0.0539	0.0513	0.0441
	5	constant	10	0.0534	0.0489	0.0506	0.0501	0.0565	0.0499	0.0551	0.0518
			30	0.0543	0.0494	0.0498	0.0521	0.0533	0.0505	0.0533	0.0489
			60	0.0544	0.0491	0.0502	0.0511	0.0518	0.0501	0.0503	0.0486
		Increase	10	0.0543	0.0496	0.0541	0.0506	0.0539	0.0535	0.0567	0.0538
			30	0.0518	0.0494	0.0537	0.0462	0.0521	0.05	0.054	0.0461
			60	0.0536	0.0524	0.0508	0.0424	0.0523	0.0477	0.0542	0.0505
	7	constant	10	0.0561	0.0487	0.0553	0.0446	0.0517	0.0492	0.0508	0.047
			30	0.0559	0.0493	0.0538	0.0499	0.055	0.0504	0.0526	0.0486
			60	0.0505	0.0483	0.0537	0.0505	0.0513	0.0515	0.0508	0.0518
		Increase	10	0.0511	0.0528	0.0551	0.0503	0.0567	0.0537	0.0545	0.0514
			30	0.0581	0.0493	0.0526	0.0491	0.0577	0.0501	0.0544	0.0502
			60	0.0557	0.0493	0.052	0.0464	0.053	0.0516	0.0539	0.0494

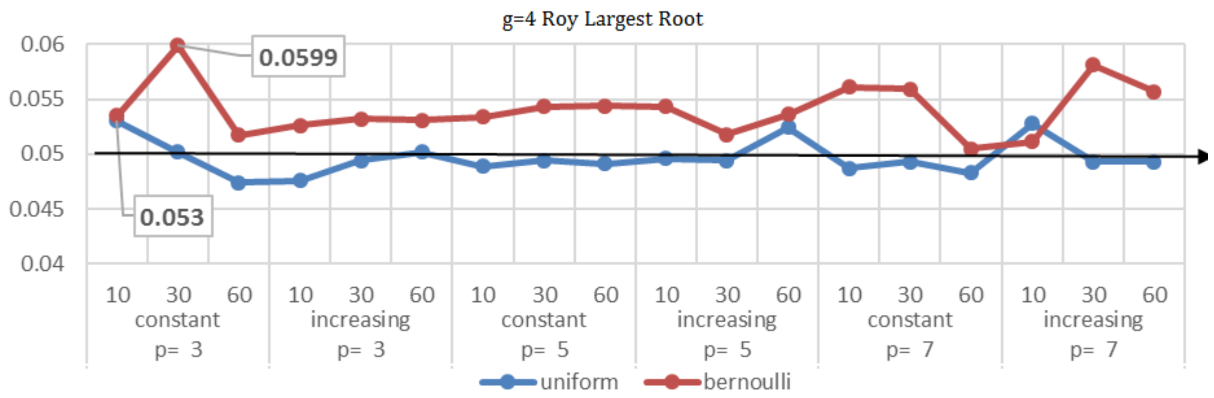

Figure 5. Type I error rates for Rooy Largest Root test statistic.

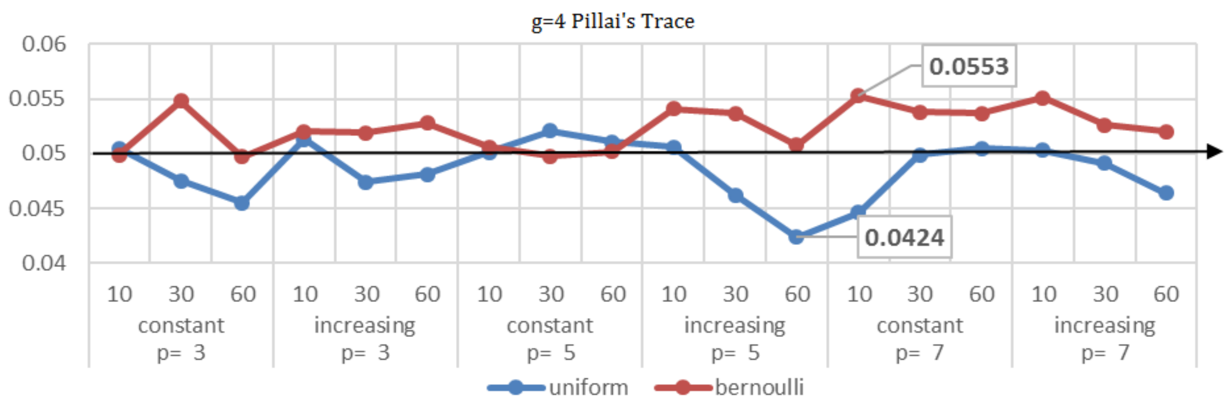

Figure 6. Type I error rates for Pillai's Trace test statistic.

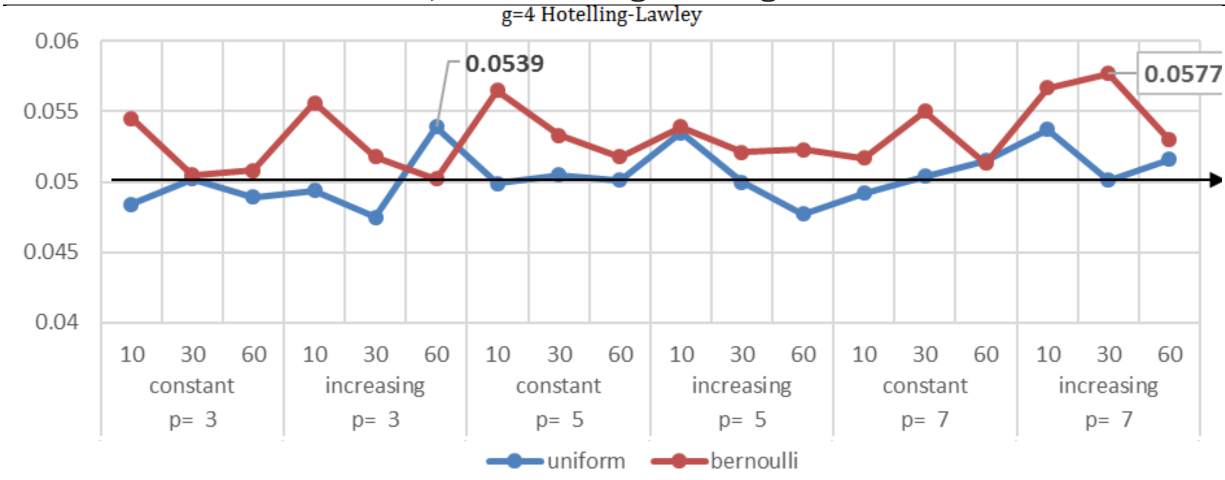

Figure 7. Type I error rates for Hotelling-Lawley test statistic.

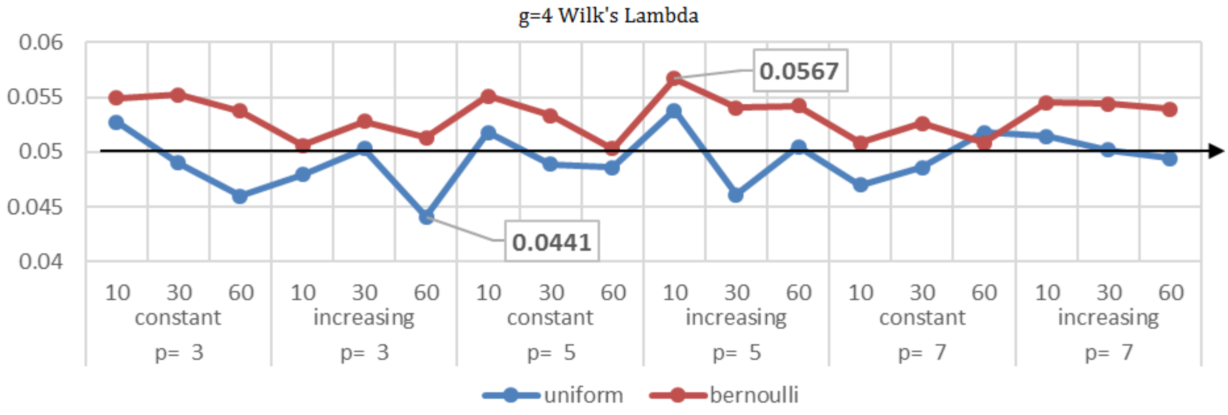

Figure 8. Type I error rates for Wilk's Lambda test statistic.

Table 3. For *g*=5, *p*=3, 5, 7; sample size *n*=10, 30, 60 experimental Type I error rate with 10000 replicate

g	p	variance	n	Roy (R)		Pillai Tracks (V)		Hotelling-Lawley (T)		Wilks Lambda (W)	
				B	U	B	U	B	U	B	U
5	3	constant	10	0.0559	0.046	0.0521	0.046	0.0517	0.0516	0.0536	0.0494
			30	0.0505	0.0489	0.0524	0.0471	0.0535	0.0466	0.051	0.0481
			60	0.0537	0.0491	0.0523	0.045	0.0539	0.0464	0.0531	0.0439
		Increase	10	0.0535	0.0507	0.053	0.049	0.0495	0.0504	0.0566	0.051
			30	0.0522	0.0507	0.0542	0.0476	0.0538	0.0533	0.054	0.0471
			60	0.0549	0.0515	0.051	0.0521	0.0544	0.0488	0.0524	0.048
	5	constant	10	0.0502	0.0492	0.0508	0.0485	0.0519	0.044	0.0537	0.0481
			30	0.0518	0.0492	0.0569	0.0502	0.0538	0.0488	0.0555	0.0468
			60	0.0551	0.0495	0.0534	0.0521	0.0531	0.0489	0.0513	0.0489
		Increase	10	0.0553	0.0493	0.0517	0.0461	0.0549	0.0475	0.0531	0.0511
			30	0.0493	0.0494	0.0462	0.0474	0.0543	0.0483	0.0585	0.0492
			60	0.0543	0.0491	0.0518	0.0455	0.0473	0.0484	0.0496	0.0489
	7	constant	10	0.0531	0.0476	0.0507	0.0517	0.0541	0.0482	0.0537	0.0502
			30	0.0508	0.0465	0.051	0.0499	0.055	0.0473	0.0542	0.0494
			60	0.0487	0.0511	0.05	0.0492	0.0508	0.0454	0.0539	0.0486
		Increase	10	0.052	0.0533	0.0521	0.0474	0.0519	0.0493	0.0565	0.048
			30	0.0524	0.0501	0.0559	0.0477	0.0583	0.0501	0.053	0.0475
			60	0.0531	0.0488	0.0518	0.0453	0.0534	0.0484	0.0556	0.0487

When group number is *g*=5, for all values of *p*, observations are interpreted according to sample size of Roy Largest Root test statistics with Figure 9.

For Roy in Bernoulli distribution as *p*=3 the greatest deviation was seen when n = 30 for the constant variance, and when *n*=60 for the increasing variance. As *p*=5, the greatest deviation was seen when n = 60 for the constant variance, and when *n*=10 for the increasing variance. As *p*=7, *n*=10, at constant variance the greatest deviation was seen and as *n*=60 for the increasing variance. For Roy test statistic in *g*=5, both Bernoulli and Uniform distribution the highest deviation was seen in all scenarios when *p*=3, *n*=10, in constant variance respectively 0.0559 and 0.046. In uniform distribution, the deviation increases as the number of variables increases.

When group number is *g*=5, for all values of *p*, observations are interpreted according to sample size of Pillai's Trace test statistics with Figure 10.

For Pillai's Trace in Bernoulli distribution when *p*=3, 5 and 7 per group, both constant and increasing variance, the highest deviation was seen when *n*=30. For Pillai test statistic in *g*=5, the highest deviation was seen in all scenarios when *p*=5, *n*=30, in constant variance with 0.0569 value. In the uniform distribution, deviations are usually below 0.05 for all variable values. Also the highest deviation was seen as *p*=7, *n*=60 in increasing variance.

When group number is *g*=5, for all values of *p*, observations are interpreted according to sample size of Hotelling-Lawley test statistics with Figure 11.

For Hotelling-Lawley in Bernoulli distribution when *p*=3 per group, both constant and increasing variance, the highest deviation is seen when *n*=60. As *p*=5 the highest deviation is seen; when n = 30 for the constant variance and when *n*=10 for the increasing variance. As the number of variables *p*=7 the highest deviation is seen; when *n*=30 for both constant and increasing variance. For Hotelling-Lawley test statistic in *g*=5, the highest deviation was seen in all scenarios when *p*=7, *n*=30, in increasing variance with 0.0583 value. In uniform distribution, the deviation increases as the number of variables increases and also 0.044 is the highest deviation as p=5,n=10 in constant variance.

When group number is *g*=4, for all values of *p*, observations are interpreted according to sample size of Wilks'Lambda test statistics with Figure 12.

For Wilks'Lambda in Bernoulli distribution when p=3 the highest deviation was seen; when *n*=10 for both constant and increasing variance. When *p*=5, the greatest deviation was seen when *n*=30 for the constant variance and when *n*=10 for the increasing variance. When *p*=7 the highest deviation was seen; when *n*=30 for both constant and increasing variance. For Wilks Lambda test statistic in *g*=5, the highest deviation was seen in all scenarios when *p*=5, *n*=30, in increasing variance with 0.0585 value. In Uniform distribution, deviations from the nominal value were less than 0.05 as the variable values increased. In large variable values deviations are small. The smallest deviation observed in all scenarios was 0.0439 when *p*=3, *n*=60 in constant variance.

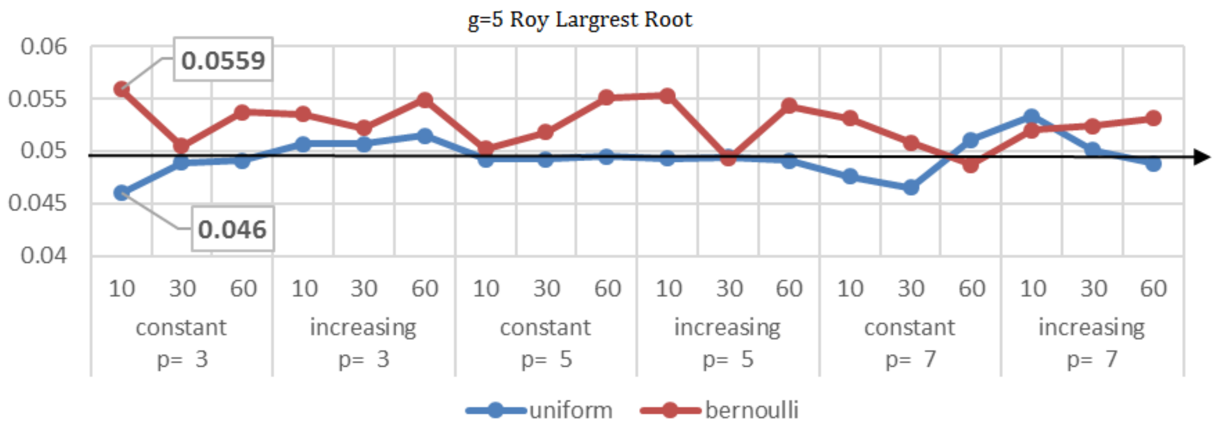

Figure 9. Type I error rates for Roy Largest Root test statistic.

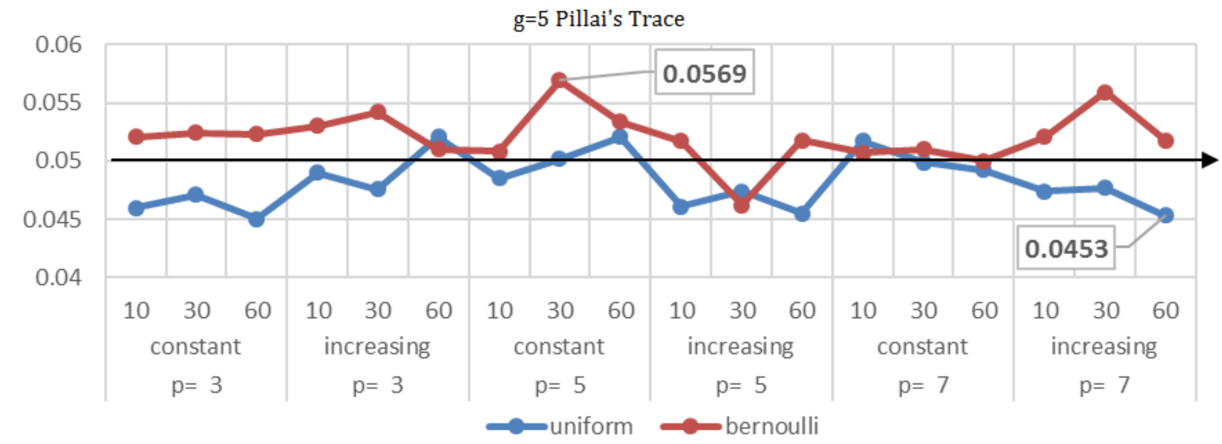

Figure 10. Type I error rates for Pillai's Trace test statistic.

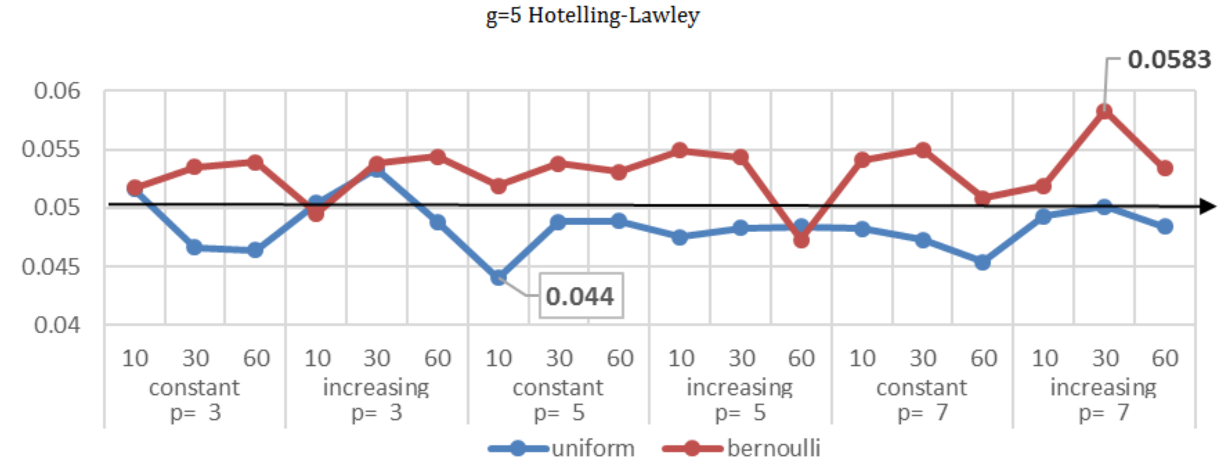

Figure 11. Type I error rates for Hotelling-Lawley test statistic.

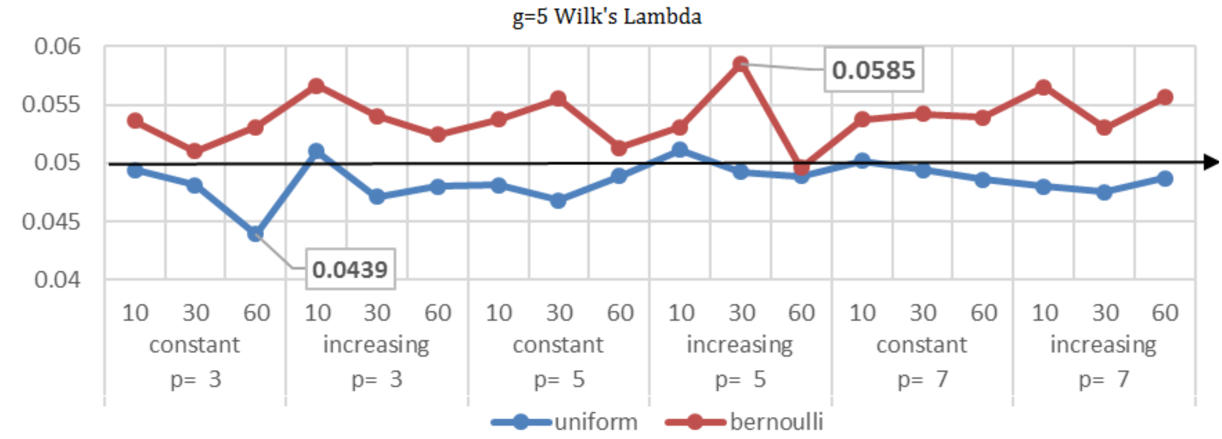

Figure 12. Type I error rates for Wilk's Lambda test statistic.

4. Conclusions

In this study, 54 design points were created for 10, 30 and 60 observations with 3, 4, 5 variable numbers 3, 5, 7, constant and increasing variance groups for each test statistic. The results of the Monte Carlo simulation run 10,000 times with each design and numbers are produced from Bernoulli and uniform distributions. Results are as follows. In Bernoulli distribution in cases where the deviation from the Type I error rate deviates from the value of 0.05, it is mostly observed in the R test statistic followed by W and T statistics. W and T statistics were given close results in terms of the maximum bias. In the V statistic, the maximum deviation scenarios are less common than the other test statistics. This study suggests that the Pillai Trace statistic works well in the Bernoulli distribution. Other studies are that found the Pillai Trace test statistic to be reliable in the form of Olson (1974), Hopkins and Clay (1963), Holloway and Dunn (1967), Ito (1969), Seber (1984), Korin (1972) and Davis (1980,1982). The details of the test statistics which give the best results in constant and increasing variance cases with different sample sizes, group numbers and variable numbers according to the derivation when comparing the scenarios for both distributions are presented below.

In case of constant variance in Bernoulli distribution;

When group number is 3, Wilks' Lambda statistic,

When the group number is 4, the Pillai's Trace,

When the group number is 5, Roy's Largest Root statistic can be suggested. However, in the case of constant variance, it can be said that Wilks' Lambda and Pillai's Trace gave better results regardless of the sample and variable numbers.

In case of increasing variance; when the group number is 3, Pillai's Trace, when the group number is 4, the statistics Pillai's Trace, when the group number is 5 Pillai's Trace, can be suggested. However, it can be said that in general, the Hotelling's Trace and the Pillai's Trace (Pillai's Trace) gave better results regardless of the sample and variant number.

In uniform distribution, in the case of constant variance; when the group number is 3, Pillai's Trace statistic, when the group number is 4 Roy's Largest Root statistic, when the group number is 5, Roy's Largest Root statistic can be suggested.

In the case of constant variance, it can be said that Roy's Largest Root Statistics and Pillai's Trace statistic gave better results regardless of the sample and variable numbers.

In case of increasing variance; when group number is 3, Wilks' Lambda statistic, when the group number is 4, Roy's largest Root statistic, when the group number is 5, the Wilks' Lambda statistic can be suggested.

However, in the case of increasing variance, it can be said that Roy's Largest Root Statistics in general and Wilks 'Lambda Statistic (Wilks' Lambda) statistic are better, regardless of the number and variety of statistics.

In general, when all the test statistics are examined, the Type 1 error ratios of the Pillai test statistic are the least deviating from the nominal $\alpha = 0.05$ value, as in many studies. However, the theoretical distribution of this statistic is not known precisely. Using the Monte Carlo method, researchers can produce critical values at some Type I error rates and degrees of freedom, and they can present a comparative chart of the literature.

Conflict of interest

The authors declare that there is no conflict of interest.

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