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Research Article

On the Forgetten Topological Index and Co Index

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Abstract

The forgotten topological index of G is specified the degrees d_i and d_j. In this paper, we find some bounds for the eigenvalues of forgotten topological matrix and we establish some inequalities about forgotten Estrada topological index.

Keywords: Forgotten topological index, Estrada index.

Unutulmuş Topolojiksel İndeks ve Eş İndeks

Öz

G' nin unutulmuş topolojiksel indeksi d_i ve d_j dereceleriyle tanımlanır. Bu makalede, unutulmuş topolojiksel indeksin özdeğerleri için bazı sınırlar bulunmuştur ve unutulmuş Estrada topolojiksel indeks ile ilgili bazı eşitsizlikler belirlenmiştir.

Anahtar Kelimeler: Unutulmuş topolojiksel indeks, Estrada indeks.

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1. Introduction

Let G be a simple, connected graph on the vertex set V(G) and the edge set E(G). For $v_i \in V(G)$, the degree of the vertex v_i denoted by d_i.

The $[F](G)$ forgotten topological matrix of graphs is defined as

$$[F]_{ij} = \begin{cases} d_i^2 + d_j^2 & if\ i\ adjacent\ to\ j \\ 0 & otherwise. \end{cases}$$

The eigenvalues of $[F](G)$ is denoted by f_i, $i = 1,2,\ldots,n$. Considering these eigenvalues, we apply some known lemmas and we obtain some bounds including the first eigenvalue. In addition, we set some conclusions using the complement of the first eigenvalue.

The Forgotten Topological Index (F index) of G is described in [4], [5] as

$$\sum_{v_i v_j \in E(G)} d_i^2 + d_j^2.$$

The Estrada index of the graph G in [3], [9] as

$$EE(G) = \sum_{i=1}^{n} e^{\lambda_i}$$

where λ is the eigenvalue of the adjacency matrix of G.

The plan of this paper is as follows: In section 2, we give some known lemmas. In section 3, we have some inequalities in terms of the degrees, the edges and the vertices. Also, we define forgotten Estrada topological index and we find different relations concepting these indices and its eigenvalues.

(See [1], [6], [8] for more details.)

2. Preliminaries

In this section, we will give some lemmas and theorems to be utilized in main results.

Lemma 2.1. [7]

Let $M = (m_{ij})$ be an nxn irreducible nonnegative matrix and $\lambda_1(M)$ be the greatest eigenvalue with $R_i(M) = \sum_{j=1}^{m} m_{ij}$. Then,

$$(minR_i(M): 1 \leq i \leq n) \leq \lambda_1(M) \leq (maxR_i(M): 1 \leq i \leq n).$$

Lemma 2.2. [2]

If G is a simple, connected graph and $\lambda_1(G)$ is the spectral radius, then

$$\lambda_1(G) \leq max\left(\sqrt{m_i m_j}: 1 \leq i, j \leq n, v_i, v_j \in E\right).$$

3. Main Results

Firstly, we set a relation for the largest eigenvalue of F matrix including the degrees, the edges and the vertices. After, we will set some inequalities concerned with F Estrada index.

Definition 3.1.

The Estrada F index of the graph G is described as

$$E^F(G) = \sum_{i=1}^{n} e^{f_i}$$

where $f_1 \geq f_2 \geq \cdots \geq f_n$ are the eigenvalues of $[F](G)$.

Definition 3.2.

The Estrada F coindex of the graph G is specified as

$$E^F(G) = \sum_{i=1}^{n} e^{\overline{f_i}}.$$

where $\overline{f_1} \geq \overline{f_2} \geq \cdots \geq \overline{f_n}$ are the eigenvalues of the complement of $[F](G)$.

Theorem 3.1.

If G is a simple, connected graph then

$$f_1(G) \leq \sqrt{(nd_i^2 + 4m^2)(nd_j^2 + 4m^2)}.$$

Proof. Let $D(G)^{-1}F(G)D(G) = F^+(G)$ and $X = (x_1, x_2, \dots, x_n)^T$ be an eigenvector of $F^+(G)$. Also, $x_i = 1$ and $0 < x_i \leq 1$ for every k. Let $x_j = max_k\ (x_k: v_i v_k \in E)$ where i is adjacent to k. Let $F^+(G)X = f_1(G)X$. If we get i-th equation from above equation, then

$$f_1(G)x_i = \sum_k (d_i^2 + d_k^2)x_k \leq (nd_i^2 + (2m)^2)x_k.$$

By Lemma 2.1, we have

$$f_1(G)x_i \leq (nd_i^2 + 4m^2)x_k.$$

The j-th equation of the same equation,

$$f_1(G)x_j \leq (nd_j^2 + 4m^2)x_k.$$

From Lemma 2.2,

$$f_1(G) \leq \sqrt{(nd_i^2 + 4m^2)(nd_j^2 + 4m^2)}.$$

Theorem 3.2.

Let G be a graph on n vertices and m edges. Then,

$$\left(\overline{f}_1(G)\right) \leq \sqrt{\mathcal{L} - \left(nd_i^2 + 4m^2\right)\left(nd_j^2 + 4m^2\right)}$$

where $\mathcal{L} = \left(d_i^2 + d_j^2\right)[(n-1)^2 + n^3(n-1)^2 - 4mn^2(n-1)] + 2n^2 d_i d_j\left[d_i d_j + 2(n-1)^2\right] - \left(2\left(d_i + d_j\right)(n-1)\right)\left[((n-1)n)^2 + d_i d_j + n^2 m(n-1)^2 - 4mn(n-1) + 4m^2\right] + (n-1)^4 n^2[2m + n^2] + (n-1)^3 mn\left[-8 - \frac{n^2}{2}\right] + (n-1)^2 m^2\left[8m + 2n^2 - \frac{n^2}{2}\right] - 2m^3 n(n-1) + 2m^4$.

Proof. Cauchy-Schwarz inequality gives that

$$\begin{aligned}\left(f_1(G) + \overline{f}_1(G)\right)^2 &\leq (f_1(G))^2 + \left(\overline{f}_1(G)\right)^2 \\ &\leq \left(nd_i^2 + 4m^2\right)\left(nd_j^2 + 4m^2\right) + \left((n-1-d_i)^2 + 4\overline{m}^2\right)\left(\left(n-1-d_j\right)^2 + 4\overline{m}^2\right).\end{aligned}$$

It is implies that

$$\begin{aligned}(f_1(G))^2 + \left(\overline{f}_1(G)\right)^2 &\leq \left(n^2 d_i^2 d_j^2 + 4m^2 n\left(d_i^2 + d_j^2\right) + 16m^4\right) + \left(n^2(n-1-d_i)^2\left(n-1-d_j\right)^2\right. \\ &\quad + 4\overline{m}^2 n\left[(n-1-d_i)^2 + \left(n-1-d_j\right)^2\right] + 16\overline{m}^4 \\ &= (n^2\left((n-1-d_i)^2\left(n-1-d_j\right)^2 + d_i^2 d_j^2\right) + 16[m^4 + \overline{m}^4] \\ &\quad + 4n(d_i^2 + d_j^2)(m^2 + \overline{m}^2) + 8\overline{m}^2 n[(n-1) - d_i - d_j)].\end{aligned}$$

Since $m + \overline{m} = \frac{n^2 - n}{2}$ then,

$$\begin{aligned}m^4 + \overline{m}^4 &= (m^2 + \overline{m}^2)^2 - 2m^2\overline{m}^2 \\ &= ((m + \overline{m})^2 - 2m\overline{m})^2 - 2(m\overline{m})^2 \\ &= \left(\left(\frac{n^2 - n}{2}\right)^2 - 2m(\frac{n^2 - n}{2} - m)\right)^2 - 2\left(m\left(\frac{n^2 - n}{2} - m\right)\right)^2.\end{aligned}$$

So, we get

$$\begin{aligned}(f_1(G))^2 + \left(\overline{f}_1(G)\right)^2 &\leq \left(d_i^2 + d_j^2\right)[(n-1)^2 + n^3(n-1)^2 - 4mn^2(n-1)] \\ &\quad + 2n^2 d_i d_j[d_i d_j + 2(n-1)^2] - 2(d_i + d_j)(n-1) \\ &\quad + [((n-1)n)^2 + d_i d_j + n^2 m(n-1)^2 - 4mn(n-1) + 4m^2] \\ &\quad + (n-1)^4 n^2[2m + n^2] + (n-1)^3 mn\left[-8 - \frac{n^2}{2}\right] \\ &\quad + (n-1)^2 m^2\left[8m + 2n^2 - \frac{n^2}{2}\right] - 2m^3 n(n-1) + 2m^4.\end{aligned}$$

It is conclude that,

$$\left(\overline{f}_1(G)\right)^2 \leq \mathcal{L} - (f_1(G))^2.$$

From Theorem 3.1,

$$\left(\overline{f}_1(G)\right) \leq \sqrt{\mathcal{L} - (nd_i^2 + 4m^2)\left(nd_j^2 + 4m^2\right)}.$$

Theorem 3.3.

Let *G* be graph on *n* vertices, *m* edges and E_F be Estrada *F* index. Then,

$$E_F \leq ne^{\sqrt{(nd_i^2+4m^2)\left(nd_j^2+4m^2\right)}}.$$

Proof. By Theorem 3.1, we have that

$$E_F = \sum_{j=1}^{n} e^{f_j} \leq ne^{f_1} = ne^{\sqrt{(nd_i^2+4m^2)\left(nd_j^2+4m^2\right)}}.$$

Theorem 3.4.

Let *G* be graph on *n* vertices, *m* edges and E_F be Estrada *F* index. Then,

$$E_F \leq \sqrt{n^2 - (2n-1)e^{2\sqrt{(nd_i^2+4m^2)\left(nd_j^2+4m^2\right)}}} - e^{\sqrt{(nd_i^2+4m^2)\left(nd_j^2+4m^2\right)}}.$$

Proof. We know that

$$(E_F - e^{f_n})^2 = {E_F}^2 - 2E_F e^{f_n} + (e^{f_n})^2$$

$$= \left(\sum_{j=1}^{n} e^{f_j}\right)^2 - 2\left(\sum_{j=1}^{n} e^{f_j}\right)e^{f_n} + e^{2f_n}.$$

By the Arithmetic-Geometric Mean inequality, we result that

$$\left(\sum_{j=1}^{n} e^{f_j}\right)^2 \leq \left(n\left(\prod_{j=1}^{n} e^{f_j}\right)^{\frac{1}{n}}\right)^2 = n^2\left(e^{\sum_{j=1}^{n} e^{f_j}}\right)^{\frac{2}{n}}.$$

Since $f_1 \geq f_2 \geq \cdots \geq f_n$ then,

$$(E_F - e^{f_n})^2 \leq n^2 - 2ne^{f_n}e^{f_n} + e^{2f_n}$$

$$= n^2 - (2n-1)e^{2f_n}$$

$$= n^2 - (2n-1)e^{2\sqrt{(nd_i^2+4m^2)\left(nd_j^2+4m^2\right)}}.$$

It gives that

$$E_F \leq \sqrt{n^2-(2n-1)e^{2\sqrt{(nd_i^2+4m^2)(nd_j^2+4m^2)}}+e^{\sqrt{(nd_i^2+4m^2)(nd_j^2+4m^2)}}}.$$

Theorem 3.5.

Let G be graph on n vertices, m edges and E_F be Estrada F index. Then,

$$E_F + E_{\bar{F}} \leq \sum_{k=0}^{\infty} \frac{n\sqrt{\mathcal{L}}}{k!}$$

where $E_{\bar{F}}$ is Estrada F coindex of G.

Proof. By the sum of the Estrada F index and Estrada F coindex, we find that

$$E_F + E_{\bar{F}} = \left(\sum_{j=1}^{n} e^{f_j}\right) + \left(\sum_{j=1}^{n} e^{\bar{f}_j}\right)$$
$$= \sum_{k=0}^{\infty} \frac{1}{k!}\left(\sum_{j=1}^{n} (f_j + \bar{f}_j)\right)$$
$$\leq \sum_{k=0}^{\infty} \frac{n}{k!}(f_j + \bar{f}_j).$$

Using Theorem 3.2, we obtain

$$E_F + E_{\bar{F}} \leq \sum_{k=0}^{\infty} \frac{n\sqrt{\mathcal{L}}}{k!}.$$

4. Conclusions

In this study, we expand forgotten topological index, we define F Estrada index and we find some bounds deal with this index. In the sequel, we describe F Estrada coindex and we obtain an inequalities for this index.

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