Available online: March 25, 2019

Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.
Volume 68, Number 2, Pages 1576–1585 (2019)
DOI: 10.31801/cfsuasmas.543297
ISSN 1303–5991 E-ISSN 2618-6470
http://communications.science.ankara.edu.tr/index.php?series=A1

SOME CHARACTERIZATIONS FOR SPACELIKE INCLINED CURVES

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Abstract. In this paper by establishing the Frenet frame $\{T, N, B_1, B_2\}$ for a spacelike curve we give some characterizations for the spacelike inclined curves and B_2-slant helices in R_2^4.

1. Introduction

In the classical differential geometry inclined curves and slant helices are well known. A general helix or an iclined curve in E_1^3 defined as a curve whose tangent lines make a constant angle with a fixed direction called the axis of the helix. A helix curve is characterized by the fact that the ratio $\frac{k_1}{k_2}$ is constant along the curve, where k_1 and k_2 denote the first curvature and the second curvature(torsion), respectively. Analogue to that A. Magden has given a characterization for a curve $x(s)$ to be a helix in Euclidean 4-space E^4. He characterizes a helix iff the function

$$\left(\frac{k_1(s)}{k_2(s)}\right)^2 + \frac{1}{k_3^2(s)}\left\{\frac{d}{ds}\left(\frac{k_1(s)}{k_2(s)}\right)\right\}^2$$

is constant where k_1, k_2 and k_3 are first, second and third curvatures of Euclidean curve $x(s)$, respectively and they are not zero anywhere [2]. Similar characterizations of timelike helices in Minkowski 4-space E_1^4 were given by H. Kocayigit and M. Onder [6].

S. Yilmaz and M. Turgut presented necessary and sufficient conditions to be inclined for spacelike and timelike curves in terms of Frenet equations in Minkowski spacetime E_1^4 [12]. A. T. Ali and R. Lopez studied the generalized timelike helices in Minkowski 4-space and gave some characterizations for these curves[3].

M. Onder, H. Kocayigit and M. Kazaz gave the differential equations characterizing the spacelike helices and also gave the integral characterizations for these curves in E_1^4 [7].

Received by the editors: April 18, 2018; Accepted: October 08, 2018.
2010 *Mathematics Subject Classification.* Primary 53C99, Secondary 53A35.
Key words and phrases. B_2-slant helix, inclined curve, spacelike curve.

Izumiya and Takeuchi have introduced the concept of slant helix by considering that the normal lines make a constant angle with a fixed direction. They characterized a slant helix if and only if the function

$$\frac{\kappa^2}{(\kappa^2+\tau^2)^{\frac{3}{2}}}(\frac{\tau}{\kappa})'$$

is constant [10].

A. T. Ali and R. Lopez gave different characterizations of slant helices in terms of their curvature functions [4]. Kula and Yayli investigated spherical images, the tangent indicatrix and the binormal indicatrix of a slant helix and they obtained that the spherical images are spherical helices [9].

M. Onder, H. Kocayigit and M. Kazaz gave the characterizations of spacelike B_2-slant helix by means of curvatures of the spacelike curve in Minkowski 4-space. Moreover they gave the integral characterizations of the spacelike B_2-slant helix [8].

In this study we investigate the conditions for spacelike curves to be inclined or B_2-slant helix in R_2^4 and we give some characterizations and theorems for these curves.

2. Preliminaries

The Semi-Euclidean space R_2^4 is the standart vector space equipped with an indefinite flat metric $\langle , \rangle$ given by

$$\langle , \rangle = dx_1^2 + dx_2^2 - dx_3^2 - dx_4^2 \tag{1}$$

where (x_1, x_2, x_3, x_4) is a rectangular coordinate system of R_2^4. A vector v in R_2^4 is called a spacelike, timelike or null(lightlike) if respectively hold $\langle v, v\rangle > 0$, $\langle v, v\rangle < 0$ or $\langle v, v\rangle = 0$ and $v \neq 0 = (0, 0, 0, 0)$. The norm of a vector v is given by $\|v\| = \sqrt{|\langle v, v\rangle|}$. Two vectors v and w are said to be orthogonal if $\langle v, w\rangle = 0$.

An arbitrary curve $\alpha : I \to R_2^4$ can locally be spacelike, timelike or null if respectively all of its velocity vectors $\alpha'(s)$ are spacelike, timelike or null.

Let a and b be two spacelike vectors in R_2^4. Then there is unique real number $0 < \delta < \Pi$, called angel between a and b, such that $\langle a, b\rangle =\| a \| . \| b \| .cos\delta$.

Let $\{T(s), N(s), B_1(s), B_2(s)\}$ be the moving Frenet frame along the curve $\alpha(s)$ in R_2^4. Then T, N, B_1, B_2 are the tangent, the principal normal, the first binormal and the second binormal fields respectively and let $\nabla_T T$ is spacelike.

Let α be a spacelike curve in R_2^4, parametrized by arclength function of s. The following cases occur for the spacelike curve α. Let the vector N is spacelike, B_1 and B_2 be timelike. In this case there exists only one Frenet frame $\{T, N, B_1, B_2\}$ for which $\alpha(s)$ is a spacelike curve with Frenet equations

$$\begin{aligned} \nabla_T T &= k_1 N \\ \nabla_T N &= -k_1 T + k_2 B_1 \\ \nabla_T B_1 &= k_2 N + k_3 B_2 \end{aligned} \tag{2}$$

$$\nabla_T B_2 = -k_3 B_1$$

where T, N, B_1 and B_2 are mutually orthogonal vectors satisfying the equations

$$\langle N,N\rangle = \langle T,T\rangle = 1, \quad \langle B_1,B_1\rangle = \langle B_2,B_2\rangle = -1 \tag{3}$$

Recall that the functions $k_1 = k_1(s)$, $k_2 = k_2(s)$ and $k_3 = k_3(s)$ are called the first, the second and the third curvature of the spacelike curve $\alpha(s)$, respectively and we will assume throughout this work that all the three curvatures satisfy $k_i(s) \neq 0$, $1 \leq i \leq 3$.

3. Some Characterizations for Spacelike Inclined Curves and B_2-Slant Helices in R^4_2

Let $\alpha(s)$ be a non-geodesic spacelike curve in R^4_2 and let $\{T, N, B_1, B_2\}$ denotes the Frenet frame of the curve $\alpha(s)$. A spacelike curve in R^4_2 is said to be an inclined curve if its tangent vector forms a constant angle with a constant vector U. From the definition of the inclined curve we can write

$$T.U = cos\theta \tag{4}$$

where U is a spacelike constant vector. Differentiating both sides of this equations we have

$$k_1 N.U = 0 \tag{5}$$

Thus we arrive $N \perp U$. Considering this we can compose U as

$$U = u_1 T + u_2 B_1 + u_3 B_2 \tag{6}$$

where u_i, $1 \leq i \leq 3$ are arbitrary functions. Differentiating (6) and considering Frenet equations, we have

$$0 = u_1' T + (u_1 k_1(s) + u_2 k_2(s))N + (u_2' - u_3 k_3(s))B_1 + (u_3' + u_2 k_3(s))B_2 \tag{7}$$

From (7) we find the equations

$$\begin{cases} u_1' = 0 \\ u_1 k_1(s) + u_2 k_2(s) = 0 \\ u_2' - u_3 k_3(s) = 0 \\ u_3' + u_2 k_3(s) = 0 \end{cases} \tag{8}$$

By using the equations above we have $u_1 = c = cons$,

$$u_2 = -c\frac{k_1(s)}{k_2(s)} = -\frac{1}{k_3(s)}\frac{du_3}{ds} \tag{9}$$

and

$$u_3 = -\frac{c}{k_3(s)}\frac{d}{ds}\frac{k_1(s)}{k_2(s)} \tag{10}$$

From the equation $u_2' - u_3 k_3(s) = 0$ we have

$$\frac{du_2}{ds} = k_3(s) u_3 \tag{11}$$

Differentiating u_2 we have

$$\frac{d}{ds}\left(-\frac{1}{k_3(s)}\frac{du_3}{ds}\right) = k_3(s)u_3. \tag{12}$$

By a direct computation we have the differential equation

$$\frac{d}{ds}\left(\frac{1}{k_3(s)}\frac{du_3}{ds}\right) + k_3(s)u_3 = 0 \tag{13}$$

By using exchange variable $t = \int_0^s k_3(s)ds$ in (13) we find

$$\frac{d^2u_3}{dt^2} + u_3 = 0 \tag{14}$$

The general solution of (14) is

$$u_3 = m_1 cost + m_2 sint \tag{15}$$

where $m_1, m_2 \in R$. Replacing variable $t = \int_0^s k_3(s)ds$ in (15) we have

$$u_3 = -\frac{c}{k_3(s)}\frac{d}{ds}\left(\frac{k_1(s)}{k_2(s)}\right) = m_1 cos\left(\int_0^s k_3(s)ds\right) + m_2 sin\left(\int_0^s k_3(s)ds\right) \tag{16}$$

Considering equation (16) and (9) we have

$$u_2 = -c\frac{k_1(s)}{k_2(s)} = m_1 sin\left(\int_0^s k_3(s)ds\right) - m_2 cos\left(\int_0^s k_3(s)ds\right) \tag{17}$$

From the equations above we find

$$m_1 = -\frac{c}{k_3(s)}\frac{d}{ds}\left(\frac{k_1(s)}{k_2(s)}\right)cos\left(\int_0^s k_3(s)ds\right) - c\frac{k_1(s)}{k_2(s)}sin\left(\int_0^s k_3(s)ds\right) \tag{18}$$

and

$$m_2 = c\frac{k_1(s)}{k_2(s)}cos\left(\int_0^s k_3(s)ds\right) - \frac{c}{k_3(s)}\frac{d}{ds}\left(\frac{k_1(s)}{k_2(s)}\right)sin\left(\int_0^s k_3(s)ds\right) \tag{19}$$

By taking $A_1 = m_1 + m_2$ and $A_2 = m_1 - m_2$, if we calculate $A_1^2 + A_2^2$ we find

$$c^2\left(\frac{k_1(s)}{k_2(s)}\right)^2 + \frac{c^2}{k_3^2(s)}\left[\frac{d}{ds}\left(\frac{k_1(s)}{k_2(s)}\right)\right]^2 = constant \tag{20}$$

or

$$\left(\frac{k_1(s)}{k_2(s)}\right)^2 + \frac{1}{k_3^2(s)}\left[\frac{d}{ds}\left(\frac{k_1(s)}{k_2(s)}\right)\right]^2 = constant. \tag{21}$$

Conversely, let us consider vector given by

$$U = \left\{T - \frac{k_1(s)}{k_2(s)}B_1 - \frac{1}{k_3(s)}\frac{d}{ds}\left(\frac{k_1(s)}{k_2(s)}\right)B_2\right\}cos\theta \tag{22}$$

Differentiating vector U and considering differential equation of (21) we obtain

$$\frac{dU}{ds} = 0 \tag{23}$$

Thus U is a constant vector and so the curve $\alpha(s)$ is an inclined curve in R^4_2. Thus we have the following theorem.

Theorem 1. *Let $\alpha = \alpha(s)$ be a spacelike curve in R^4_2 . α is an inclined curve if and only if*

$$(\frac{k_1(s)}{k_2(s)})^2 + \frac{1}{k_3^2(s)}\{\frac{d}{ds}(\frac{k_1(s)}{k_2(s)})\}^2 = constant. \tag{24}$$

Proof. It is obvious from the computations above. □

Corollary 2. *Let $\alpha = \alpha(s)$ be a spacelike curve in R^4_2. α is an inclined curve if and only if*

$$k_3(s)\frac{k_1(s)}{k_2(s)} + \frac{d}{ds}[\frac{1}{k_3(s)}\frac{d}{ds}(\frac{k_1(s)}{k_2(s)})] = 0. \tag{25}$$

Proof. If we differentiate the equation (24) respect to s we find the equation (25). □

Now let us solve the equation (25) respect to $\frac{k_1}{k_2}$. If we use exchange variable $t = \int_0^s k_3(s)ds$ in (25) we have

$$\frac{d^2}{dt^2}(\frac{k_1}{k_2}) + (\frac{k_1}{k_2}) = 0. \tag{26}$$

So we arrive

$$\frac{k_1}{k_2} = W_1 cos \int_0^s k_3(s)ds + W_2 sin \int_0^s k_3(s)ds. \tag{27}$$

where W_1 and W_2 are real numbers.

Now we will give a different characterization for inclined curves. Let α be an inclined curve in R^4_2. By differentiating (24) with respect to s we get

$$(\frac{k_1}{k_2})(\frac{k_1}{k_2})' + \frac{1}{k_3}(\frac{k_1}{k_2})'[(\frac{1}{k_3})(\frac{k_1}{k_2})']' = 0 \tag{28}$$

and hence

$$\frac{1}{k_3}(\frac{k_1}{k_2})' = -\frac{(\frac{k_1}{k_2})(\frac{k_1}{k_2})'}{[(\frac{1}{k_3})(\frac{k_1}{k_2})']'} \tag{29}$$

If we define a function $f(s)$ as

$$f(s) = -\frac{(\frac{k_1}{k_2})(\frac{k_1}{k_2})'}{[(\frac{1}{k_3})(\frac{k_1}{k_2})']'} \tag{30}$$

then

$$f(s) = -\frac{1}{k_3(s)}(\frac{k_1}{k_2})' = W_1 sin \int_0^s k_3(s)ds - W_2 cos \int_0^s k_3(s)ds. \tag{31}$$

By using (28) and (31) we have

$$f'(s) = -\frac{k_1 k_3}{k_2}. \tag{32}$$

Conversely, consider the function

$$f(s) = -\frac{1}{k_3}(\frac{k_1}{k_2})' = W_1 sin \int_0^s k_3(s)ds - W_2 cos \int_0^s k_3(s)ds$$

and assume that $f'(s) = -\frac{k_1 k_3}{k_2}$. We compute

$$\frac{d}{ds}[(\frac{k_1(s)}{k_2(s)})^2 + \frac{1}{k_3^2(s)}\{(\frac{k_1(s)}{k_2(s)})'^2] = \frac{d}{ds}[\frac{1}{k_3^2}(f'^2 + f^2(s)] := \varphi(s) \tag{33}$$

As $f(s)f'(s) = -(\frac{k_1}{k_2})(\frac{k_1}{k_2})'$ and $f''(s) = -k_3'(\frac{k_1}{k_2}) - k_3(\frac{k_1}{k_2})'$ we obtain

$$f'(s)f''(s) = k_3 k_3'(\frac{k_1}{k_2})^2 + k_3^2(\frac{k_1}{k_2})(\frac{k_1}{k_2})'. \tag{34}$$

As consequence of above computations

$$\varphi(s) = 2(ff' + \frac{f'f''}{k_3^2} - \frac{(f'^2 k_3'}{k_3^3}) = 0 \tag{35}$$

that is the function $(\frac{k_1(s)}{k_2(s)})^2 + \frac{1}{k_3^2(s)}\{(\frac{k_1(s)}{k_2(s)})'^2$ is constant. Therefore we have the following theorem.

Theorem 3. *Let α be a unit speed spacelike curve in R_2^4. Then α is an inclined curve if and only if the function $f(s) = -\frac{1}{k_3(s)}(\frac{k_1}{k_2})' = W_1 sin \int_0^s k_3(s)ds - W_2 cos \int_0^s k_3(s)ds$ satisfies $f'(s) = -\frac{k_1 k_3}{k_2}$ where k_1, k_2 and k_3 are the curvatures of α.*

Proof. The proof can be completed from the computations above. □

Now let $\alpha(s)$ be a spacelike curve in R_2^4 and let $\{T, N, B_1, B_2\}$ denotes the Frenet frame of the curve $\alpha(s)$. We call $\alpha(s)$ as spacelike B_2-slant helix if its second binormal vector makes a constant angle with a fixed direction in a vector U. From the definition of the B_2-slant helix we can write

$$B_2.U = cos\vartheta \tag{36}$$

where U is a spacelike constant vector. Differentiating both sides of this equations we have

$$-k_3 B_1.U = 0 \tag{37}$$

Since $k_3 \neq 0$ we arrive $B_1 \perp U$. Considering this we can compose U as

$$U = u_1 T + u_2 N + u_3 B_2 \tag{38}$$

where u_i, $1 \leq i \leq 3$ are arbitrary functions. Differentiating (38) and considering Frenet equations, we have

$$0 = (u_1' - u_2 k_1)T + (u_1 k_1(s) + u_2')N + (u_2 k_2(s) - u_3 k_3(s))B_1 + u_3' B_2 \tag{39}$$

From (39) we find the equations

$$\begin{cases} u_1' - u_2k_1 = 0 \\ u_1k_1(s) + u_2' = 0 \\ u_2k_2(s) - u_3k_3(s) = 0 \\ u_3' = 0 \end{cases} \tag{40}$$

By using the equations above we have $u_3 = c = cons$,

$$u_2 = c\frac{k_3(s)}{k_2(s)} = \frac{1}{k_1(s)}\frac{du_1}{ds} \tag{41}$$

and

$$u_1 = -\frac{c}{k_1(s)}\frac{d}{ds}\frac{k_3(s)}{k_2(s)} \tag{42}$$

From the equation $u_1' - u_2k_1(s) = 0$ we have

$$\frac{du_1}{ds} = k_1(s)u_2 \tag{43}$$

Differentiating u_1 we have

$$\frac{d}{ds}(-\frac{1}{k_1(s)}\frac{du_2}{ds}) = k_1(s)u_2. \tag{44}$$

By a direct computation we have the differential equation

$$\frac{d}{ds}(\frac{1}{k_1(s)}\frac{du_2}{ds}) + k_1(s)u_2 = 0 \tag{45}$$

By using exchange variable $t = \int_0^s k_1(s)ds$ in (45) we find

$$\frac{d^2u_2}{dt^2} + u_2 = 0 \tag{46}$$

The general solution of (46) is

$$u_2 = m_1cost + m_2sint \tag{47}$$

where $m_1, m_2 \in R$. Replacing variable $t = \int_0^s k_1(s)ds$ in (47) we have

$$u_2 = c\frac{k_3(s)}{k_2(s)} = m_1cos(\int_0^s k_1(s)ds) + m_2sin(\int_0^s k_1(s)ds) \tag{48}$$

Considering equation (48) we have

$$u_1 = -\frac{c}{k_1(s)}\frac{d}{ds}(\frac{k_3(s)}{k_2(s)}) = -m_1sin(\int_0^s k_1(s)ds) + m_2cos(\int_0^s k_1(s)ds) \tag{49}$$

From the equations above we find

$$m_1 = -\frac{c}{k_1(s)}\frac{d}{ds}(\frac{k_3(s)}{k_2(s)})cos(\int_0^s k_1(s)ds) + c\frac{k_3(s)}{k_2(s)}sin(\int_0^s k_1(s)ds) \tag{50}$$

and

$$m_2 = c\frac{k_3(s)}{k_2(s)}cos(\int_0^s k_1(s)ds) - \frac{c}{k_1(s)}\frac{d}{ds}(\frac{k_3(s)}{k_2(s)})sin(\int_0^s k_1(s)ds) \tag{51}$$

By taking $B_1 = m_1 + m_2$ and $B_2 = m_1 - m_2$, if we calculate $B_1^2 + B_2^2$ we find

$$c^2(\frac{k_3(s)}{k_2(s)})^2 + \frac{c^2}{k_1^2(s)}[\frac{d}{ds}(\frac{k_3(s)}{k_2(s)})]^2 = constant \tag{52}$$

or

$$(\frac{k_3(s)}{k_2(s)})^2 + \frac{1}{k_1^2(s)}[\frac{d}{ds}(\frac{k_3(s)}{k_2(s)})]^2 = constant. \tag{53}$$

Conversely, let us consider vector given by

$$U = \{-\frac{1}{k_1(s)}\frac{d}{ds}(\frac{k_3(s)}{k_2(s)})T + \frac{k_3(s)}{k_2(s)}N + B_2\}cos\vartheta \tag{54}$$

Differentiating vector U and considering differential equation of (53) we obtain

$$\frac{dU}{ds} = 0 \tag{55}$$

Thus U is a constant vector and so the curve $\alpha(s)$ is a spacelike B_2 slant helix in R_2^4. As a result we can give the following theorem.

Theorem 4. *Let $\alpha = \alpha(s)$ be a spacelike curve in R_2^4. α is a spacelike B_2 slant helix if and only if*

$$(\frac{k_3(s)}{k_2(s)})^2 + \frac{1}{k_1^2(s)}\{\frac{d}{ds}(\frac{k_3(s)}{k_2(s)})\}^2 = constant. \tag{56}$$

Proof. The proof can easily seen from the computations above. □

Corollary 5. *Let $\alpha = \alpha(s)$ be a spacelike curve in R_2^4. α is a B_2-slant helix if and only if*

$$k_1(s)\frac{k_3(s)}{k_2(s)} - \frac{d}{ds}[\frac{1}{k_1(s)}\frac{d}{ds}(\frac{k_3(s)}{k_2(s)})] = 0. \tag{57}$$

Proof. If we differentiate the equation (56) respect to s we have the equation (57). □

Now let us solve the equation (57) respect to $\frac{k_3}{k_2}$. If we use exchange variable $t = \int_0^s k_1(s)ds$ in (57) we have

$$\frac{d^2}{dt^2}(\frac{k_3}{k_2}) + (\frac{k_3}{k_2}) = 0. \tag{58}$$

So we arrive

$$\frac{k_3}{k_2} = L_1cos\int_0^s k_1(s)ds + L_2sin\int_0^s k_1(s)ds. \tag{59}$$

where L_1 and L_2 are real numbers.

Now we will give a different characterization for B_2-slant helices. Let α be a spacelike B_2-slant helix in R_2^4. By differentiaing (56) with respect to s we get

$$(\frac{k_3}{k_2})(\frac{k_3}{k_2})' + \frac{1}{k_1}(\frac{k_3}{k_2})'[(\frac{1}{k_1})(\frac{k_3}{k_2})']' = 0 \tag{60}$$

and hence

$$\frac{1}{k_1}(\frac{k_3}{k_2})' = -\frac{(\frac{k_3}{k_2})(\frac{k_3}{k_2})'}{[(\frac{1}{k_1})(\frac{k_3}{k_2})']'} \tag{61}$$

If we define a function $f(s)$ as

$$f(s) = -\frac{(\frac{k_3}{k_2})(\frac{k_3}{k_2})'}{[(\frac{1}{k_1})(\frac{k_3}{k_2})']'} \tag{62}$$

then

$$f(s) = -\frac{1}{k_1(s)}(\frac{k_3}{k_2})' = L_1 sin\int_0^s k_1(s)ds - L_2 cos\int_0^s k_1(s)ds. \tag{63}$$

By using (60) and (63) we have

$$f'(s) = -\frac{k_1 k_3}{k_2}. \tag{64}$$

Conversely, consider the function

$$f(s) = -\frac{1}{k_1}(\frac{k_3}{k_2})' = L_1 sin\int_0^s k_1(s)ds - L_2 cos\int_0^s k_1(s)ds \tag{65}$$

and assume that $f'(s) = -\frac{k_1 k_3}{k_2}$. We compute

$$\frac{d}{ds}[(\frac{k_3(s)}{k_2(s)})^2 + \frac{1}{k_1^2(s)}\{(\frac{k_3(s)}{k_2(s)})'^2] = \frac{d}{ds}[\frac{1}{k_1^2}(f'^2 + f^2(s)] := \varphi(s) \tag{66}$$

From $f(s)f'(s) = -(\frac{k_3}{k_2})(\frac{k_3}{k_2})'$ and $f''(s) = -k_1'(\frac{k_3}{k_2}) - k_1(\frac{k_3}{k_2})'$ we obtain

$$f'(s)f''(s) = k_1 k_1'(\frac{k_3}{k_2})^2 + k_1^2(\frac{k_3}{k_2})(\frac{k_3}{k_2})'. \tag{67}$$

As a consequence of above computations

$$\varphi(s) = 2(ff' + \frac{f'f''}{k_1^2} - \frac{(f'^2 k_1'}{k_1^3}) = 0 \tag{68}$$

that is the function $(\frac{k_3(s)}{k_2(s)})^2 + \frac{1}{k_1^2(s)}\{(\frac{k_3(s)}{k_2(s)})'^2$ is constant. Therefore we have the following theorem.

Theorem 6. *Let α be a unit speed spacelike curve in R_2^4. Then α is a B_2-slant helix if and only if the function $f(s) = -\frac{1}{k_1(s)}(\frac{k_3}{k_2})' = L_1 sin\int_0^s k_1(s)ds - L_2 cos\int_0^s k_1(s)ds$ satisfies $f'(s) = -\frac{k_1 k_3}{k_2}$, where k_1, k_2 and k_3 are the curvatures of α.*

Proof. It is obvious from the above computations. □

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