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Research Article

New Exact Solutions for the (3 + 1) Dimensional B-type Kadomtsev-Petviashvili Equation

Faruk DÜŞÜNCELİ*

Faculty of Economics and Administrative Sciences, Mardin Artuklu University, Mardin, Turkey

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Abstract
In this study, we use the improved Bernoulli sub-equation function method for exact solutions to the (3 + 1) dimensional B-type Kadomtsev-Petviashvili equation. Some new solutions are successfully constructed. We carried out all the computations and the graphics plot in this paper by Wolfram Mathematica.

Keywords: Kadomtsev-Petviashvili equation ,Improved Bernoulli sub-equation function method, Exact solution.

(3 + 1) boyutlu B tipi Kadomtsev-Petviashvili denklemi için geliştirilmiş Bernoulli alt denklem fonksiyon yöntemi ile elde edilmiş tam çözümler

Öz
Bu çalışmada (3 + 1) boyutlu B tipi Kadomtsev-Petviashvili denkleminin tam çözümleri için geliştirilmiş Bernoulli alt denklem fonksiyon yöntemini kullandık. Bazı yeni çözümler elde ettik. Bütün hesaplamaları ve grafik çizimlerini Wolfram Mathematica programı yardımıyla yaptık.

Anahtar Kelimeler: Kadomtsev-Petviashvili denklemi, Geliştirilmiş Bernoulli alt denklem fonksiyon yöntemi, Tam çözüm.

1. Introduction

The purpose of this paper is to investigate solutions the first of two form the (3+1) dimensional generalized B-type Kadomstev-Petviashili (BKP) equation given by Wazwaz (2012)

$$u_{xxxy} + \alpha\left(u_x u_y\right)_x + \left(u_x + u_y + u_z\right)_t - \left(u_{xx} + u_{zz}\right) = 0, \qquad (1)$$

where α is nonzero parameter and this nonlinear wave equation in three spatial (x,y,z) an done temporal coordinate (t). The Kadomstev-Petviashili equation describes weakly dispersive and small amplitude waves propagating in quasi-two-dimensional medium (Wazwaz,2011). Abudiab and Khalique (2013) used the multiple-exp function and simplest equation method for solutions eq.(1) . Different forms of KP equation were studied by Ma et al. (2011), Shen and Tu (2011) , Wazwaz (2011) by different approachs and Baskonus et al. (2017) by Sine-Gordon expansion method. In this context, various articles were presented to the literatüre (Akin,2017;Akin and Zeren, 2017; Baskonus et.al., 2018; Modanlı and Akgül, 2017; Modanlı,2018)

This article consist of five parts. In second part, the steps of IBSEFM are identified, in third part the method is applied to BKP equation. Finally, fourth part is the result and discussion section.

* Corresponding Author: farukdusunceli@artuklu.edu.tr

2. Material and Method

In this part, we use the improved Bernoulli sub-equation function method (IBSEFM) (Baskonus and Bulut, 2015 a and b; Bulut et al.2016; Dusunceli,2018,2019) for solutions eq.(1).

Step1:Let's consider the following partial differential equation;

$$P(u, u_x, u_y, u_y, u_z, u_{xx}, u_{xy}, ..) = 0. \quad (2)$$

and take the wave transformation;

$$u(x, y, z, t) = U(\gamma), \gamma = k_1 x + k_2 y + k_3 z + k_4 t, \quad (3)$$

where $k_1 k_2, k_3$ and k_4 are nonzero constants. Substituting Eq.(3) into Eq.(2), we obtain the following nonlinear ordinary differential equation(NODE);

$$N(U, U', U'', U''', ...) = 0. \quad (4)$$

Step 2. Considering trial equation of solution in Eq.(4), it can be written as following;

$$U(\gamma) = \frac{\sum_{i=0}^{n} a_i F^i(\gamma)}{\sum_{j=0}^{m} b_j F^j(\gamma)}. \quad (5)$$

According to the Bernoulli theory, we can consider the general form of Bernoulli differential equation for as following;

$$F' = wF + dF^M, w \neq 0, d \neq 0, M \in R - \{0,1,2\}, \quad (6)$$

where F is Bernoulli differential polynomial. Substituting Eq.(5-6) into Eq.(4), it converts an equations of polynomial $\sigma(F)$ as following;

$$\sigma(F) = \rho_s F^s + \cdots + \rho_1 F + \rho_0 = 0. \quad (7)$$

According to the balance principle, we can determine the relationship between n, m and M.

Step 3. The coefficients of $\sigma(F)$ all be zero will yield us an algebraic system of equations;

$$\rho_i = 0, i = 0, \dots, s. \quad (8)$$

Solving this system, we will specify the values of $a_0, \dots, a_n$ and $b_0, \dots, b_m$.

Step 4. When we solve nonlinear Bernoulli differential equation Eq.(6), we obtain the following two situations according to b and d ;

$$F(\gamma) = \left[\frac{-d}{w} + \frac{E}{e^{w(M-1)\gamma}}\right], \; w \neq d. \quad (9)$$

$$F(\gamma) = \left[\frac{(E-1)+(E+1)\tanh(w(1-M)\frac{\gamma}{2}}{1-\tanh(w(1-M)\frac{\gamma}{2}}\right]^{\frac{1}{1-M}}, \; w = d, E \in R. \quad (10)$$

3. Findings

In this section, application of the improved Bernoulli sub-equation function method to BKP equation is presented. Using the wave transformation on Eq. (1)

$$u(x, y, z, t) = U(\gamma), \gamma = k_1 x + k_2 y + k_3 z + k_4 t\,. \quad (11)$$

we get the following nonlinear ordinary differential equation:

$$k_1^3 k_2 U^{(4)} + 2\alpha k_1^2 k_2 U' U'' + \; (k_1^3 k_2 + k_1 k_4 + k_2 k_4 + k_3 k_4 - k_1^2 - k_3^2) U'' = 0. \quad (12)$$

Integrating the equation in (12), we get

$$k_1^3k_2U''' + \alpha k_1^2k_2(U')^2 + (k_1^3k_2 + k_1k_4 + k_2k_4 + k_3k_4 - k_1^2 - k_3^2)U' = 0. \quad (13)$$

Finally, If we write V instead of U', the equation (13) becomes a second order nonlinear ordinary differential equation:

$$k_1^3k_2V'' + \alpha k_1^2k_2V^2 + (k_1^3k_2 + k_1k_4 + k_2k_4 + k_3k_4 - k_1^2 - k_3^2)V = 0. \,(14)$$

Balancing Eq. (14) by considering the highest derivative(V'') and the highest power(V^2), we obtain

$$n + 2 = 2M + m.$$

Choosing $M = 3,\ m = 1$, gives $n = 5$. Thus, the trial solution to Eq. (1) takes the following form:

$$U(\gamma) = \frac{a_0+a_1F(\gamma)+a_2F^2(\gamma)+a_3F^3(\gamma)+a_4F^4(\gamma)+a_5F^5(\gamma)}{b_0+b_1F(\gamma)}. \qquad (15)$$

where $F' = wF + dF^3, w \neq 0, d \neq 0$. Substituting Eq. (15), its second derivative and power along with $F' = wF + dF^3, w \neq 0, d \neq 0$ into Eq. (14), yields a polynomial in F. Solving the system of the algebraic equations, yields the values of the parameter involved. Substituting the obtained values of the parameters into Eq. (15), yields the solutions to Eq (1).

For $w \neq d$, we can find following coefficients:

Case 1.

$$a_0 = -\frac{4\sigma^2b_0k_1}{\alpha};\ a_1 = -\frac{4\sigma^2b_1k_1}{\alpha};\ a_2 = -\frac{24w\sigma b_0k_1}{\alpha};\ a_3 = -\frac{24w\sigma b_1k_1}{\alpha};\ a_4 = -\frac{24w^2b_0k_1}{\alpha}; a_5 = -\frac{24w^2b_1k_1}{\alpha}; k_3 = \frac{1}{2}(k_4 - \sqrt{-4k_1^2(1+4\sigma^2k_1k_2) + 4(k_1+k_2)k_4 + k_4^2}) \qquad (16)$$

Case 2.

$$a_0 = \frac{a_1b_0}{b_1}; a_2 = \frac{6wa_1b_0}{\sigma b_1}; a_3 = \frac{6wa_1}{\sigma}; a_4 = \frac{6w^2a_1b_0}{\sigma^2b_1}; a_5 = \frac{6w^2a_1}{\sigma^2}; \quad k_1 = -\frac{\alpha a_1}{4\sigma^2b_1};\ \ k_4 = \frac{\alpha^2a_1^2b_1 - \alpha^3a_1^3k_2 + 16\sigma^4b_1^3k_3^2}{4\sigma^2b_1^2(-\alpha a_1 + 4\sigma^2b_1(k_2+k_3))}; \qquad (17)$$

Substituting Eq. (16) into Eq. (15), gives

$$U = -\frac{4\sigma^2k_1\left(-\frac{3h}{e^{2\sigma\left(xk_1+yk_2+tk_4+\frac{1}{2}z\left(k_4-\sqrt{-4k_1^2(1+4\sigma^2k_1k_2)+4(k_1+k_2)k_4+k_4^2}\right)\right)}w-h\sigma} + xk_1+yk_2+tk_4+\frac{1}{2}z\left(k_4-\sqrt{-4k_1^2(1+4\sigma^2k_1k_2)+4(k_1+k_2)k_4+k_4^2}\right)\right)}{\alpha}. \qquad (18)$$

Substituting Eq. (17) into Eq. (15), gives

$$U=\frac{a_1\left(\dfrac{3h}{-e^{2\sigma\left(-\frac{x\alpha a_1}{4\sigma^2 b_1}+yk_2+zk_3+\frac{t(\alpha^2 a_1^2 b_1-\alpha^3 a_1^3 k_2+16\sigma^4 b_1^3 k_3^2)}{4\sigma^2 b_1^2(-\alpha a_1+4\sigma^2 b_1(k_2+k_3))}\right)}w+h\sigma}-\dfrac{x\alpha a_1}{4\sigma^2 b_1}+yk_2+zk_3+\dfrac{t(\alpha^2 a_1^2 b_1-\alpha^3 a_1^3 k_2+16\sigma^4 b_1^3 k_3^2)}{4\sigma^2 b_1^2(-\alpha a_1+4\sigma^2 b_1(k_2+k_3))}\right)}{b_1} \tag{19}$$

Choosing the suitable values of parameters, we performed the numerical simulations of the obtained solutions for (18-19) equation by plotting their 2D and 3D.

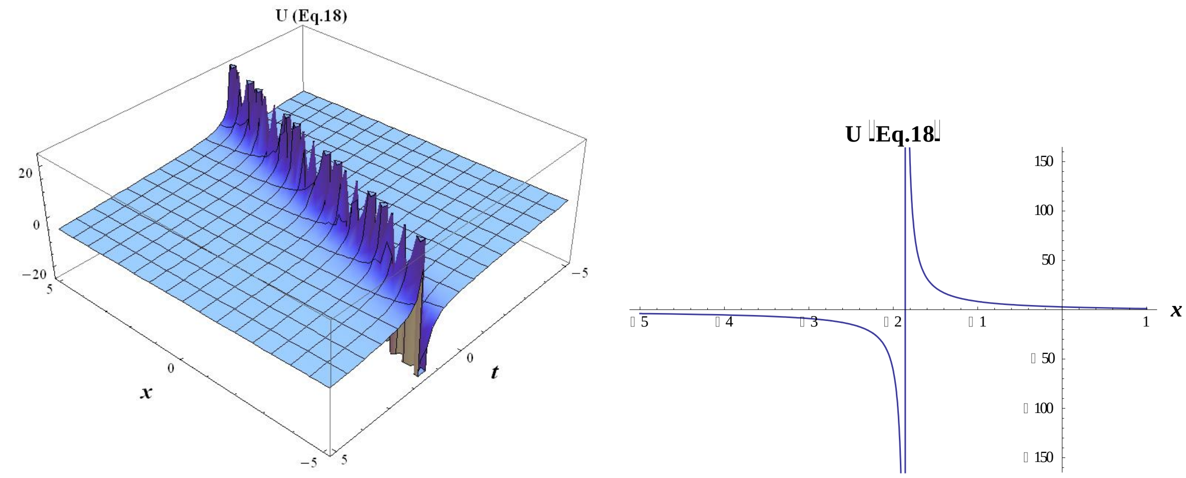

Figure-1 The 3D and 2Dsurfaces of the solution Eq.(18) for suitable values

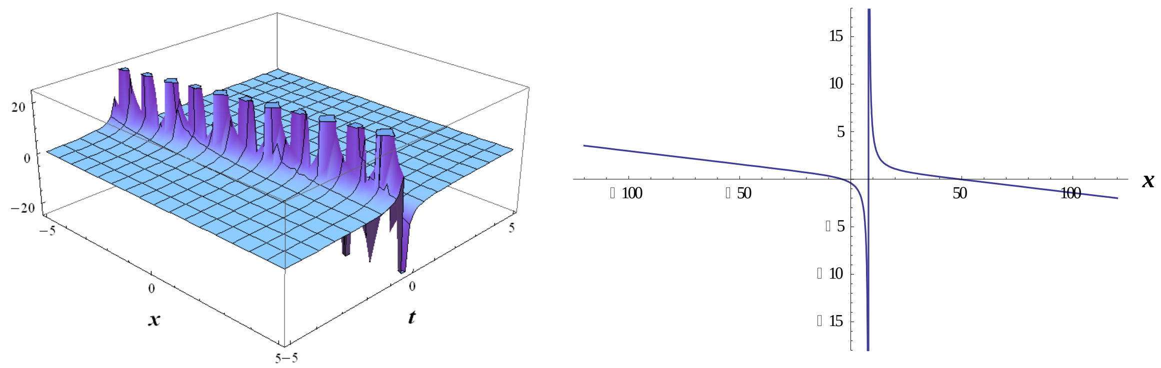

Figure-2 The 3D and 2Dsurfaces of the solution Eq.(19) for suitable values

4. Result and Discussion

In this article, new solutions are obtained for the B type KD equation using the IBSEFM method. We have seen that the results we obtained are new solutions when we compare them with previous ones. The results may be useful to explain the physical effects of various nonlinear models in non-linear sciences. IBSEFM is a powerful and efficient mathematical tool that can be used to process various nonlinear mathematical models.

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