INTERNATIONAL ELECTRONIC JOURNAL OF GEOMETRY
VOLUME 11 NO. 1 PAGE 17–25 (2018)

On Timelike Rectifying Slant Helices in Minkowski 3-Space

Bülent Altunkaya*and Levent Kula

(Communicated by H. Hilmi Hacısalihoğlu)

ABSTRACT

In this work, we study timelike rectifying slant helices in E_1^3. First, we find general equations of the curvature and the torsion of timelike rectifying slant helices. After that, by solving second order linear differential equations, we obtain families of timelike rectifying slant helices that lie on cones.

Keywords: *timelike curve; rectifying curve; slant helix; cone.*

AMS Subject Classification (2010): *Primary: 51B20; 53A04; 53A35.*

1. Introduction

Helices arise in nanosprings, carbon nanotubes, DNA double and collagen triple helices. The double helix shape is commonly associated with DNA [1].

In differential geometry, a general helix in Euclidean 3-space is characterized by the property that the tangent lines make a constant angle with a fixed direction [12, 13].

Similarly, the notion of slant helix was introduced by Izuyama and Takeuchi by the property that the principal normal lines make a constant angle with a fixed direction [8, 9]. They showed that a space curve is a slant helix if and only if the geodesic curvature of the principal normal of the curve is a constant function. In [10, 11], Kula et al. studied the spherical images of slant helices.

Later, Ahmet T. Ali studied slant helices in Minkowski 3-space [1, 2].

The notion of rectifying curve has been introduced by Chen [5, 6]. Chen proposed the conditions under which the position vector of a unit speed curve lies in its rectifying plane. Besides, he stated the importance of rectifying curves in Physics.

In [3, 4], Altunkaya and Kula studied rectifying slant helices and found the position vector of these curves. They obtained unit speed families of rectifying slant helices which lie on cones.

The papers mentioned above led us to study on the notion of timelike rectifying slant helices. We begin with finding the equations of curvature and torsion of a timelike rectifying slant helix. After that, we construct second order linear differential equations to determine position vector of timelike rectifying slant helices. By solving these equations for some special cases, we find unit speed families of rectifying slant helices which lie on cones.

2. Basic Concepts

E_1^3 denote the Minkowski 3-space with the metric,

$$g = dx_1^2 + dx_2^2 - dx_3^2$$

where (x_1, x_2, x_3) is a rectangular coordinate system of E_1^3. Since g is an indefinite metric, the pseudo-norm of a vector v is given by $\|v\| = \sqrt{|g(v, v)|}$.

Received : *13-10-2017,* ***Accepted :*** *06-04-2018*
*** Corresponding author***

A vector $v \in R^3$ is called spacelike if $g(v,v) > 0$ or $v = 0$, timelike if $g(v,v) < 0$, and lightlike (null) if $g(v,v) = 0$ with $v \neq 0$ [7].

Given a curve $\alpha : I \subset R \to E_1^3$, we say that the curve α is spacelike (resp. timelike, lightlike) if $\alpha'(s)$ is spacelike (resp. timelike, lightlike) at any $s \in I$ where $\alpha'(s) = d\alpha/ds$ [7].

A non-lightlike or a lightlike curve $\alpha : I \subset R \longrightarrow E_1^3$ is said to be parametrized by the pseudo arclength parameter s if $g(\alpha'(s), \alpha'(s)) = \pm 1$ or $g(\alpha''(s), \alpha''(s)) = 1$. In both cases, we call α a unit speed curve.

In E_1^3 a unit speed timelike curve which has at least four continuous derivatives has a natural frame called Frenet Frame with the equations below [2, 7],

$$\begin{aligned} t' &= \kappa n \\ n' &= \kappa t + \tau b \\ b' &= -\tau n, \end{aligned}$$

where κ is the curvature, τ is the torsion, and $\{t, n, b\}$ is the Frenet Frame of the curve α. We denote unit timelike tangent vector field with t, unit spacelike principal normal vector field with n, and the unit spacelike binormal vector with b.

As we know, n can be considered as the normal indicatrix curve of the curve α. If n is a non-lightlike curve, we know that $\varepsilon = \operatorname{sgn}[g(n', n')] = \pm 1$. Note that when n is a timelike curve $\varepsilon = -1$.

Definition 2.1. A curve is called a slant helix if its principal normal vector field makes a constant angle with a fixed direction in E_1^3 [8].

Lemma 2.1. *Let α be a unit speed timelike curve in E_1^3. Then, α is a slant helix if and only if the geodesic curvature of the spherical image of principal normal indicatrix n of α*

$$\frac{\kappa^2}{(\varepsilon\tau^2 - \varepsilon\kappa^2)^{3/2}} \left(\frac{\tau}{\kappa}\right)'$$

is constant everywhere $\tau^2 - \kappa^2$ does not vanish [2].

The curve α is called a rectifying curve when its position vector always lies in its rectifying plane [5]. So, for a rectifying curve, we can write

$$\alpha(s) = \lambda(s)\, t(s) + \mu(s)\, b(s).$$

Lemma 2.2. *Let α be a unit speed non-lightlike curve with timelike or spacelike principal normal vector field in E_1^3, then α is congruent to a rectifying curve if and only if*

$$\frac{\tau(s)}{\kappa(s)} = c_1 s + c_2$$

for some constants c_1 and c_2 with $c_1 \neq 0$ [7].

The angle between two vectors in E_1^3 is defined at [1]:

Definition 2.2. Let u and v be spacelike vectors that span a spacelike vector subspace. Then, there is a unique positive real number θ such that

$$|g(u,v)| = \|u\| \, \|v\| \cos\theta.$$

θ is called the Lorentzian spacelike angle between u and v.

Definition 2.3. Let u and v be spacelike vectors that span a timelike vector subspace. Then, there is a unique positive real number θ such that

$$|g(u,v)| = \|u\| \, \|v\| \cosh\theta.$$

θ is called the Lorentzian timelike angle between u and v.

Definition 2.4. Let u be a spacelike vector and v a positive timelike vector. Then, there is a unique positive real number θ such that

$$|g(u,v)| = \|u\| \, \|v\| \sinh\theta.$$

θ is called the Lorentzian timelike angle between u and v.

3. Curvatures of Timelike Rectifying Slant Helices in E_1^3

In E_1^3, if the position vector of a unit speed timelike slant helix always lies in its rectifying plane, we call it timelike rectifying slant helix. For the curvatures of timelike rectifying slant helices, we have the following two theorems.

Theorem 3.1. *Let α be a unit speed timelike curve which has spacelike principal normal indicatrix with the pseudo arclength parameter s in E_1^3, then α is a timelike rectifying slant helix if and only if the curvature and torsion of the curve satisfy the equations below*

$$\kappa(s) = \frac{c_3}{\left((c_1 s + c_2)^2 - 1\right)^{3/2}}, \tau(s) = \frac{c_3 (c_1 s + c_2)}{\left((c_1 s + c_2)^2 - 1\right)^{3/2}}$$

where $c_1 \neq 0, c_2 \in R$, and $c_3 \in R^+$.

Proof. Let α be a unit speed timelike rectifying slant helix with the pseudo arclength parameter s in E_1^3, then the equations in Lemma 2.1, and Lemma 2.2 exists. If we combine them, we have

$$\begin{aligned} m &= \frac{\kappa^2}{(\tau^2 - \kappa^2)^{3/2}} \left(\frac{\tau}{\kappa}\right)' \\ &= \frac{c_1}{\kappa \left((c_1 s + c_2)^2 - 1\right)^{3/2}} \end{aligned}$$

where $m \neq 0$ is a constant, we can write κ as follows

$$\kappa(s) = \frac{c_3}{\left((c_1 s + c_2)^2 - 1\right)^{3/2}},$$

then

$$\tau(s) = \frac{c_3 (c_1 s + c_2)}{\left((c_1 s + c_2)^2 - 1\right)^{3/2}},$$

where $c_3 = |c_1/m|$.

Conversely, it can easily be seen that the curvature functions as mentioned above satisfy the equations at Lemma 2.1 and Lemma 2.2. So, α is a timelike rectifying slant helix. □

Theorem 3.2. *Let α be a unit speed timelike curve which has timelike principal normal indicatrix with the pseudo arclength parameter s in E_1^3, then α is a timelike rectifying slant helix if and only if the curvature and torsion of the curve satisfy the equations below*

$$\kappa(s) = \frac{c_3}{\left(1 - (c_1 s + c_2)^2\right)^{3/2}}, \tau(s) = \frac{c_3 (c_1 s + c_2)}{\left(1 - (c_1 s + c_2)^2\right)^{3/2}}$$

where $c_1 \neq 0, c_2 \in R$, and $c_3 \in R^+$.

Proof. Similar to the proof of the theorem 3.1. □

Now, we give another theorem for a special case of the theorem 3.1 to determine c_3. Some parts of this theorem will be useful for us later on.

Theorem 3.3. *Let α be a unit speed timelike rectifying slant helix whose principal normal vector field makes a constant angle with a unit positive timelike vector v, then the curvature and torsion of α satisfy the equations below*

$$\kappa(s) = \frac{|c_1 \coth(\theta)|}{\left((c_1 s + c_2)^2 - 1\right)^{3/2}}, \tau(s) = \frac{|c_1 \coth(\theta)| (c_1 s + c_2)}{\left((c_1 s + c_2)^2 - 1\right)^{3/2}}$$

where $c_1 \neq 0, c_2 \in R$.

Proof. Let α be a unit speed timelike rectifying slant helix whose principal normal vector field makes a constant angle with a unit positive timelike vector v. Then, from Definition 2.4

$$g(n, v) = \sinh(\theta)$$

where $\theta \in R^{+}$. If we differentiate this equation with respect to pseudo arclength parameter s, we have

$$g(\kappa t + \tau b, v) = 0.$$

If we divide both parts of the equation with κ, we get

$$g(t + (c_1 s + c_2)b, v) = 0,$$

then

$$g(t, v) = (c_1 s + c_2)g(b, v).$$

While $\{t, n, b\}$ is a orthonormal frame, we can write

$$v = \lambda_1 t + \lambda_2 n + \lambda_3 b$$

with $-\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = -1$. If we make the neccessary calculations, we have

$$\lambda_1 = \pm\frac{(c_1 s + c_2)\cosh(\theta)}{\sqrt{(c_1 s + c_2)^2 - 1}}, \quad \lambda_2 = \sinh(\theta), \quad \lambda_3 = \pm\frac{\cosh(\theta)}{\sqrt{(c_1 s + c_2)^2 - 1}}.$$

So,

$$\pm\frac{c_1\cosh(\theta)}{\kappa\sqrt{(c_1 s + c_2)^2 - 1}} + (1 + (c_1 s + c_2)^2)\sinh(\theta) = 0.$$

Therefore,

$$\kappa(s) = \frac{|c_1\coth(\theta)|}{\left((c_1 s + c_2)^2 - 1\right)^{3/2}}$$

and

$$\tau(s) = \frac{|c_1\coth(\theta)|\,(c_1 s + c_2)}{\left((c_1 s + c_2)^2 - 1\right)^{3/2}}.$$

□

Remark 3.1. For a unit speed timelike rectifying slant helix whose principal normal vector field makes a constant angle with a unit vector v, we easily see

Case 1: If v is a spacelike unit vector and $\{n, v\}$ spans a timelike subspace, then

$$c_3 = |c_1\tanh\theta|\,.$$

Case 2: If v is a spacelike unit vector and $\{n, v\}$ spans a spacelike subspace, then

$$c_3 = |c_1\tan\theta|\,.$$

4. Position vector of timelike rectifying slant helices

For the position vector of the timelike rectifying slant helices, we have the following two theorems.

Theorem 4.1. *Let α be a unit speed timelike rectifying slant helix which has spacelike principal normal indicatrix with the pseudo arclength parameter s in E_1^3. Then, the vector h satisfies the linear vector differential equation of second order as follows*

$$h''(s) + \frac{c_3^2}{\left((c_1 s + c_2)^2 - 1\right)^2}h(s) = 0 \tag{4.1}$$

where $h = \frac{n'}{\kappa}$.

Proof. Let α be a unit speed timelike rectifying slant helix, then we can write Frenet equations as follows

$$\begin{aligned} t^{'} &= \kappa n, \\ n^{'} &= \kappa t + f\kappa b, \\ b^{'} &= -f\kappa n \end{aligned} \tag{4.2}$$

where $f(s) = c_1 s + c_2$. If we divide the second equation by κ, we have

$$\frac{n'}{\kappa} = t + fb. \tag{4.3}$$

By differentiating (4.3), we have

$$c_1 b = \left(\frac{n'}{\kappa}\right)' + \kappa(f^2 - 1)n. \tag{4.4}$$

By differentiating (4.4) and using (4.2), we have

$$\left(\frac{n'}{\kappa}\right)'' + \kappa(f^2 - 1)n' + \left[\left(\kappa(f^2 - 1)\right)' + c_1 f\kappa\right] n = 0, \tag{4.5}$$

we know

$$\kappa(s) = \frac{c_3}{\left((c_1 s + c_2)^2 - 1\right)^{3/2}}.$$

With the necessary calculations, we easily see

$$\left(\kappa(f^2 - 1)\right)' + c_1 f\kappa = 0.$$

So, (4.5) becomes

$$\left(\frac{n'}{\kappa}\right)'' + \kappa(f^2 - 1)n' = 0. \tag{4.6}$$

Let us denote $\frac{n'}{\kappa} = h$. Then, (6) becomes to

$$h''(s) + \frac{c_3^2}{\left((c_1 s + c_2)^2 - 1\right)^2} h(s) = 0.$$

This completes the proof. □

Theorem 4.2. *Let α be a unit speed timelike rectifying slant helix which has timelike principal normal indicatrix with the pseudo arclength parameter s in E_1^3. Then, the vector h satisfies the linear vector differential equation of second order as follows*

$$h''(s) - \frac{c_3^2}{\left((c_1 s + c_2)^2 - 1\right)^2} h(s) = 0 \tag{4.7}$$

where $h = \frac{n'}{\kappa}$.

Proof. Similar to the proof of the theorem 4.1. □

As we know, every component of vector $h = (h_1, h_2, h_3)$ must satisfy (4.1) or (4.7). Therefore, if we take

$$\begin{aligned} h_1(s) &= \sqrt{\left((c_1 s + c_2)^2 - 1\right)} \sin\left[\operatorname{csch}(\theta) \operatorname{arccoth}(c_1 s + c_2)\right], \\ h_2(s) &= \sqrt{\left((c_1 s + c_2)^2 - 1\right)} \cos\left[\operatorname{csch}(\theta) \operatorname{arccoth}(c_1 s + c_2)\right], \\ h_3(s) &= 0. \end{aligned} \tag{4.8}$$

We can show h satisfies (4.1).

On the other hand, if α is a unit speed timelike rectifying slant helix which has spacelike principal normal indicatrix with its principal normal vector field makes a constant angle θ with e_3. Then; from Definition 2.4, we can write

$$g(n, e_3) = \sinh(\theta).$$

From (4.8), we can write

$$\begin{aligned} n_1(s) &= \int \kappa(s)h_1(s)ds = \cosh(\theta)\cos\left[\operatorname{csch}(\theta)\operatorname{arccoth}(c_1 s + c_2)\right], \\ n_2(s) &= \int \kappa(s)h_2(s)ds = -\cosh(\theta)\sin\left[\operatorname{csch}(\theta)\operatorname{arccoth}(c_1 s + c_2)\right], \\ n_3(s) &= -\sinh(\theta) \end{aligned}$$

with

$$g(n', n') = \frac{c_1^2\coth^2(\theta)}{(c_1 s + c_2 - 1)^2(c_1 s + c_2 + 1)^2} > 0.$$

While α is a unit speed timelike curve, we have

$$\begin{aligned} \alpha_1(s) &= \int(\int \kappa(s)n_1(s)ds)ds, \\ \alpha_2(s) &= \int(\int \kappa(s)n_2(s)ds)ds, \\ \alpha_3(s) &= \int(\int \kappa(s)n_3(s)ds)ds \end{aligned}$$

and so

$$\begin{aligned} \alpha_1(s) &= -\frac{\sinh(\theta)}{c_1}\sqrt{(c_1 s + c_2)^2 - 1}\cos\left[\operatorname{csch}(\theta)\operatorname{arccoth}(c_1 s + c_2)\right], \\ \alpha_2(s) &= \frac{\sinh(\theta)}{c_1}\sqrt{(c_1 s + c_2)^2 - 1}\sin\left[\operatorname{csch}(\theta)\operatorname{arccoth}(c_1 s + c_2)\right], \\ \alpha_3(s) &= \frac{\cosh(\theta)}{c_1}\sqrt{(c_1 s + c_2)^2 - 1} \end{aligned}$$

where $\alpha = (\alpha_1, \alpha_2, \alpha_3)$.

Remark 4.1. For a unit speed timelike rectifying slant helix whose principal normal vector field makes a constant angle with a unit vector v, we have

Case 1: If $v = e_2$ and $g(n', n') > 0$, then

$$\begin{aligned} \beta_1(s) &= \frac{\cosh(\theta)}{c_1}\sqrt{(c_1 s + c_2)^2 - 1}\sinh\left[\operatorname{sech}(\theta)\operatorname{arccoth}(c_1 s + c_2)\right], \\ \beta_2(s) &= -\frac{\sinh(\theta)}{c_1}\sqrt{(c_1 s + c_2)^2 - 1}, \\ \beta_3(s) &= -\frac{\cosh(\theta)}{c_1}\sqrt{(c_1 s + c_2)^2 - 1}\cosh\left[\operatorname{sech}(\theta)\operatorname{arccoth}(c_1 s + c_2)\right] \end{aligned}$$

where $\beta = (\beta_1, \beta_2, \beta_3)$.

Case 2: If $v = e_1$ and $g(n', n') < 0$, then

$$\begin{aligned} \gamma_1(s) &= -\frac{1}{c_1}\sqrt{1 - (c_1 s + c_2)^2}\sin(\theta), \\ \gamma_2(s) &= \frac{\cos(\theta)}{c_1}\sqrt{1 - (c_1 s + c_2)^2}\cosh\left[\sec(\theta)\operatorname{arctanh}(c_1 s + c_2)\right], \\ \gamma_3(s) &= \frac{\cos(\theta)}{c_1}\sqrt{1 - (c_1 s + c_2)^2}\sinh\left[\sec(\theta)\operatorname{arctanh}(c_1 s + c_2)\right] \end{aligned}$$

where $\gamma = (\gamma_1, \gamma_2, \gamma_3)$. Now, we can write new lemmas.

Lemma 4.1. *Let α be a space curve in E_1^3 below,*

$$\begin{aligned} \alpha(s) = \frac{\sqrt{(c_1 s + c_2)^2 - 1}}{c_1}\Big(&-\sinh(\theta)\cos\left[\operatorname{csch}(\theta)\operatorname{arccoth}(c_1 s + c_2)\right], \\ &\sinh(\theta)\sin\left[\operatorname{csch}(\theta)\operatorname{arccoth}(c_1 s + c_2)\right], \\ &\cosh(\theta)\Big) \end{aligned} \tag{4.9}$$

where $\theta \in R^{+}$, $c_1 \neq 0$, and $c_2 \in R$. Then, α is a unit speed timelike rectifying slant helix which lies on the cone

$$z^2 = \coth^2(\theta)\left(x^2 + y^2\right).$$

Proof. With direct calculations, we have $g(\alpha', \alpha') = -1$, $g(n, n) = 1$, $g(n', n') > 0$, and the curvature functions of α as

$$\kappa(s) = \frac{|c_1 \coth(\theta)|}{\left((c_1 s + c_2)^2 - 1\right)^{3/2}}, \quad \tau(s) = \frac{|c_1 \coth(\theta)|\,(c_1 s + c_2)}{\left((c_1 s + c_2)^2 - 1\right)^{3/2}}$$

with

$$\frac{\kappa^2(s)}{\left(\kappa^2(s) + \tau^2(s)\right)^{3/2}}\left(\frac{\tau(s)}{\kappa(s)}\right)' = \tanh(\theta)$$

and

$$\frac{\tau(s)}{\kappa(s)} = c_1 s + c_2.$$

So, α is a unit speed timelike rectifying slant helix. We have

$$\coth^2(\theta)\left({\alpha_1}^2(s) + {\alpha_2}^2(s)\right) - {\alpha_3}^2(s) = 0,$$

then α lies on the cone above.

□

Lemma 4.2. *Let β be a space curve in E_1^3 with the equation below,*

$$\begin{aligned}\beta(s) = \frac{\sqrt{(c_1 s + c_2)^2 - 1}}{c_1}\Big(&\cosh(\theta)\sinh\left[\operatorname{sech}(\theta)\operatorname{arccoth}(c_1 s + c_2)\right], \\ &-\sinh(\theta), \\ &-\cosh(\theta)\cosh\left[\operatorname{sech}(\theta)\operatorname{arccoth}(c_1 s + c_2)\right]\Big)\end{aligned} \tag{4.10}$$

where $\theta \in R^{+}$, $c_1 \neq 0$, and $c_2 \in R$. Then, β is a unit speed timelike rectifying slant helix which lies on the cone

$$z^2 = \coth^2(\theta) y^2 + x^2.$$

Lemma 4.3. *Let γ be a space curve in E_1^3 below,*

$$\begin{aligned}\gamma(s) = \frac{\sqrt{1 - (c_1 s + c_2)^2}}{c_1}\Big(&-\sin(\theta), \\ &\cos(\theta)\cosh\left[\sec(\theta)\operatorname{arctanh}(c_1 s + c_2)\right], \\ &\cos(\theta)\sinh\left[\sec(\theta)\operatorname{arctanh}(c_1 s + c_2)\right]\Big)\end{aligned} \tag{4.11}$$

where $\theta \in R^{+}$, $c_1 \neq 0$, and $c_2 \in R$. Then, γ is a unit speed timelike rectifying slant helix which lies on cone

$$y^2 = \cot^2(\theta) x^2 + z^2.$$

Example 4.1. If we take $c_1 = 1, c_2 = 0$, and $\theta = 1/2$ in (4.9); then, we have

$$\begin{aligned}\alpha(s) = \sqrt{s^2 - 1}\Bigg(&-\sinh\left(\frac{1}{2}\right)\cos\left[\operatorname{csch}\left(\frac{1}{2}\right)\operatorname{arccoth}(s)\right], \\ &\sinh\left(\frac{1}{2}\right)\sin\left[\operatorname{csch}\left(\frac{1}{2}\right)\operatorname{arccoth}(s)\right], \\ &\cosh\left(\frac{1}{2}\right)\Bigg)\end{aligned}$$

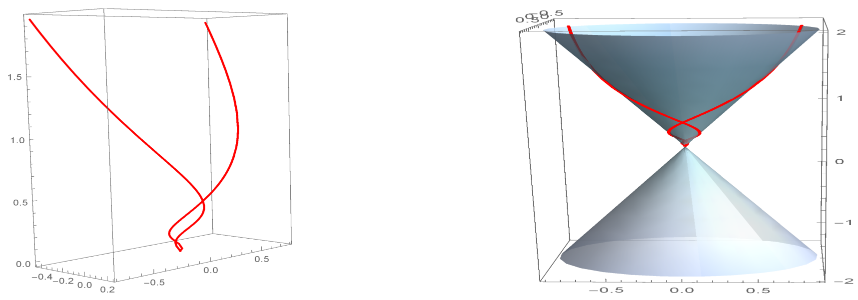

Figure 1. Timelike rectifying slant helix α lies on the cone; $\coth^2(\frac{1}{2})\left(x^2+y^2\right)=z^2$

with the curvatures

$$\kappa(s)=\frac{\coth\left(\frac{1}{2}\right)}{\left(s^2-1\right)^{3/2}},\quad \tau(s)=\frac{s\coth\left(\frac{1}{2}\right)}{\left(s^2-1\right)^{3/2}}$$

that lies on the cone (see Fig. 1)

$$\coth^2(\frac{1}{2})\left(x^2+y^2\right)=z^2.$$

We plot the spherical indicatrices of the curve α in Fig. 2.

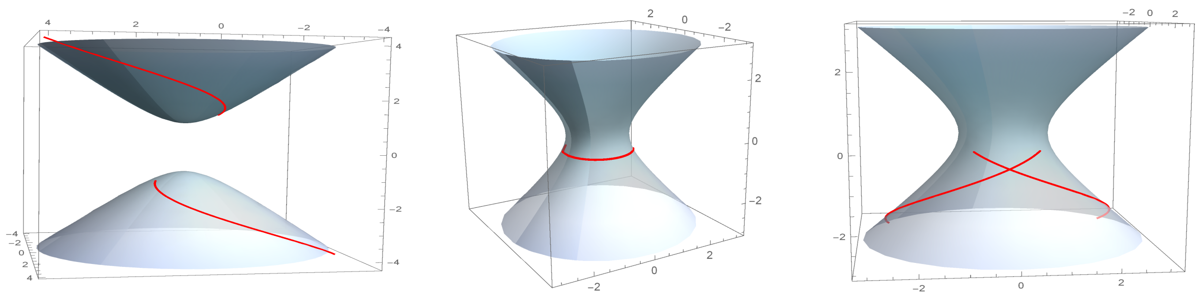

Figure 2. Tangent, Normal, and Binormal indicatrices of α

Example 4.2. If we take $c_1=1, c_2=0$, and $\theta=3$ in (4.10); then, we have

$$\begin{aligned}\beta(s)=\sqrt{s^2-1}\Big(&\cosh(3)\sinh\left[\operatorname{sech}(3)\operatorname{arccoth}(s)\right],\\ &-\sinh(3),\\ &-\cosh(3)\cosh\left[\operatorname{sech}(3)\operatorname{arccoth}(s)\right]\Big)\end{aligned}$$

with the curvatures

$$\kappa(s)=\frac{\tanh(3)}{\left(s^2-1\right)^{3/2}},\quad \tau(s)=\frac{s\tanh(3)}{\left(s^2-1\right)^{3/2}}$$

that lies on the cone

$$z^2=\coth^2(3)y^2+x^2.$$

Example 4.3. If we take $c_1=1, c_2=0$, and $\theta=\pi/3$ in (4.11); then, we have

$$\gamma(s)=\frac{\sqrt{1-s^2}}{2}\left(-\sqrt{3},\cosh\left[2\operatorname{arctanh}(s)\right],\sinh\left[2\operatorname{arctanh}(s)\right]\right)$$

with the curvatures

$$\kappa(s)=\frac{\sqrt{3}}{\left(1-s^2\right)^{3/2}},\quad \tau(s)=\frac{s\sqrt{3}}{\left(1-s^2\right)^{3/2}}$$

that lies on the cone

$$y^2=\frac{x^2}{3}+z^2.$$

References

[1] Ali, T.A., Position vector of a timelike slant helix in Minkowski 3-space. *Journal of the Mathematical Analysis and Applications.* **365** (2010), no. 2, 559-569.
[2] Ali, T.A. and Lopez, R., Slant helices in Minkowski 3-space. *Journal of Korean Math. Soc.* **40** (2011), no. 1, 159-167.
[3] Altunkaya, B. and Kula, L., On rectifying slant helices in Euclidean 3-space. *Konuralp J. Math.* **4** (2016), no. 2, 17-24.
[4] Altunkaya, B. and Kula, L., On spacelike rectifying slant helices in in Minkowski 3-space. *Turk. Journal Math.* (accepted).
[5] Chen B.Y., When does the position vector of a space curve always lie in its rectifying plane? *Amer. Math. Monthly.* **110** (2003), 147-152.
[6] Chen B.Y. and Dillen, F., Rectifying curves as centrodes and extremal curves. *Bull. Inst. Math. Academia Sinica.* **33** (2005), no. 2, 77-90.
[7] Ilarslan K, Nešović E, Petrović-Torgašev M Some Characterizations of Rectifying Curves in the Minkowski 3-space. *Novi Sad. Journal Math.* **33**(2002), no. 2, 23-32.
[8] Izumiya, S. and Takeuchi, N., New special curves and developable surfaces. *Turk. Journal Math.* **28**(2004), 153-163.
[9] Izumiya, S. and Takeuchi, N., Generic properties of helices and Bertrand curves. *Journal of Geometry.* **74** (2002), no. 1, 97-109.
[10] Kula, L. and Yayli, Y., On slant helix and its spherical indicatrix. *Applied Mathematics and Computation.* **169** (2005), 600-607.
[11] Kula, L., Ekmekci, N., Yayli, Y. and Ilarslan, K., Characterizations of slant helices in Euclidean 3-space. *Turk. Journal Math.* **34** (2010), no. 2, 261-273.
[12] O'Neill, B., Elementary Differential Geometry. Academic Press, New York, 2010.
[13] Struik, D.J., Lectures on Classical Differential Geometry. Dover Publications, New York, 1961.

Affiliations

BÜLENT ALTUNKAYA
ADDRESS: University of Ahi Evran,
Department of Mathematics,
Faculty of Education,
Kırşehir, Turkey.
E-MAIL: bulent.altunkaya@ahievran.edu.tr
ORCID ID : orcid.org/0000-0002-3186-5643

LEVENT KULA
ADDRESS: University of Ahi Evran,
Department of Mathematics,
Faculty of Arts and Sciences,
Kırşehir, Turkey.
E-MAIL: lkula@ahievran.edu.tr
ORCID ID : orcid.org/0000-0002-8208-9728