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Dynamic Wavelet-Based Causal Relationship between Equity Returns and Aggregate Economic Activity in G7 and E7 Countries

G7 ve E7 Ülkelerinde Borsa Getirisi ve Ekonomik Büyüme Arasında Dalgacık Bazlı Dinamik Nedensellik İlişkisi

Arş. Gör. Remzi Gök

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Öz

Bu çalışmada G7 ve E7 ülkelerine ait 1998M01-2017M08 aylık veriler kullanarak borsa getirisi ve sanayi üretimi arasındaki ilişkinin varlığı, derecesi ve yönü, zaman ve frekans bazlı teknikler yardımıyla araştırılmıştır. Kullanılan metodun gücü, kimi zaman değişkenler arasındaki ilişkiyi ortaya çıkarmada yetersiz kalabilir. Bu doğrultuda, farklı zaman periyotlarında saklı ilişkinin gerçek dinamikleri ortaya çıkarmak için dalgacıklar analizi kullanılmıştır. Yapısal kırılmalı birim kök test sonucuna göre iki ülke değişkenleri hariç, değişkenler I(1) ya da I(0) bulunmuştur. Birim köklü değişkenler arasında kısa dönemde geçerli çift yönlü ve uzun dönemlik tek yönlü nedensellik sonucuna ulaşılmıştır. Elde edilen zaman bazlı test bulgularına göre endeks getirisinden büyümeye doğru anlamlı nedensellik ilişkisi bulunmuştur. Büyüme değişkeninden endeks getirisine doğru bazı ülkeler için anlamlı sonuçlar ortaya çıkmıştır. Anlamlı ilişkilerin hangi zaman frekanslarında geçerli olduğunu ve standart metotların ortaya çıkaramadığı anlamlı ilişkiyi bulmak için dalgacıklar metoduna başvurma ihtiyacı duyulmuştur. Yapılan analiz sonuçlarına göre tüm ülkeler için geçerli çift yönlü nedensellik ilişkisine ulaşılmıştır. Diğer taraftan, dalgacık bazlı borsa varyansının sanayi üretimi varyansından daha yüksek olduğu, ayrıca, volatilitenin büyük bir çoğunluğunun kısa dönem değişmelerle açıklanabildiği görülmüştür. Son olarak, ölçek sayısı arttıkça dalgacık varyansının azaldığı, korelasyon katsayısının ise artığı gözlemlenmiştir. Elde edilen bulgular, klasik yöntemlerin yetersiz kaldığı alanda dalgacıkların belirli zaman periyotlarında, piyasa katılımcıları ve piyasa yapıcıları için, daha önemli sonuçları ortaya koyabildiğini göstermektedir.

Anahtar Kelimeler: *Dalgacıklar, Frekans, Simetrik, Granger, Dalgacık Varyansı & Korelasyonu*

Abstract

This paper studies the nexus of equity returns and industrial production growths for G7 and E7 countries, in order to identify the possible strength and/or direction of the causal and wavelets-statistics based relationships in the time and frequency domain utilizing monthly data over the period 1998M01-2017M08. Since standard methods are unable to reveal the true dynamics, we prefer to implement wavelet analysis to offer a deeper understanding. According to unit root tests with structural breaks some variables are found to be I(0) or I(1). There exist bidirectional and one-way causalities in the short and long run, respectively, between cointegrated variables. The standard and symmetric causality tests report strong evidence of one-way causal relationships running from index returns to economic activity for all countries and economic activity to index returns in some G7 countries. After implementing wavelet approach to uncover the hidden relationships,

Arş. Gör. Remzi Gök, Dicle Üniversitesi İİBF, **remzigok1984@gmail.com,** ORCID: 0000-0002-9216-5210

it turns out that there are statistically significant two-way causalities for all countries. On the other hand, the wavelet variance of equity return is found to be more volatile and their most of volatilities are explained by short-term fluctuations. Finally, the wavelet variance decreases while correlation increases as the wavelet scale increase in all countries. Overall, the results of this study have significant suggestions for policy-makers and market participants, which are not possible with standard methods, before implementing policy rate and investment decisions.

Keywords: *Wavelets, Frequency, Symmetric, Granger, Wavelet Variance* & *Correlation*

Introduction

The relationship between stock market movements and economic activity (real or nominal) has been largely investigated empirically or/and theoretically by researchers, academicians, investors, and regulators. The major questions are to find out the existence and/or strength and direction of this kind of relationship. Although there are a lot of papers that investigate this relationship with a myriad of different methods, this highly controversial subject in the literature remains inconclusive because of the obtained different results. Despite research findings vary regarding the countries under scrutiny and sample period, it is said that the relationship is mainly driven by both market fundamentals and investors sentiments.

According to economic and finance theories, there are several theoretical explanations for their relationships. The widely accepted proposition is related to the discounted-cash-flow valuation model where it is claimed that the stock price is equal to the present value of the future payments of the firm, namely stock price is a mirror of the expectations of the investors regarding dividend payments in the future. Notable, stock prices are basically determined by the three factors, as Fama (1990, s.1089) notes, changes in expected cash flows and discount/required rates and variations in predictable returns. If this reflection is accepted for a single firm, then it can be generalized that the aggregate stock market reflects overall economic activity conditions in a country. Equivalently speaking, there should be a strong relationship between the current stock prices and the future economic activity in terms of the GDP or industrial production or vice versa according to the papers of Fischer and Merton (1984, s.57), Fama (1990, s.1089), Schwert (1990, s.1237), Cheung and Ng (1998, s.281), Mauro (2003, s.129), Humpe and Macmillan (2009, s.111), Sancar et al. (2017, s.1774) and Saidi et al. (2017, s.527). Employing standard OLS regressions, Fischer and Merton (1984, s.9) find out that the forward-looking characteristic of stock prices makes the aggregate stock markets as a predictor of the future economic activity. Using annually, quarterly and monthly data for the US between 1953 and 1987, Fama (1990, s.1102) reports a strong and positive relationship between the underlying variables, in which the strength increases with the longer time period. Schwert (1990, s.1256), in addition, corroborates the results of Fama (1990, s.1089) and states that stock returns are related to future economic activity with a positive and strong connection. In the related paper, Mauro (2003, s.151) presents positive and significant correlation relationship between the lagged stock returns and economic activity in the US, England ("UK"), Japan ("JAP"), Canada ("CAN"), and Mexico ("MEX"), dictating the importance of the stock market developments in the predictability of the output. Humpe and Macmillan (2009, s.118) highlight that the US and Japan stock markets were influenced positively by industrial production index regarding normalized cointegrating coefficients. In the terms of long-run relationship investigated by cointegration approaches, Cheung and Ng (1998, s.295), Sancar et al. (2017, s.1774) and Saidi et al. (2017, s.532) revealed a significantly positive long-run relationship in the "U.S.", Germany ("GER"), "CAN", Italy ("ITA"), and "JAP"; Turkey ("TUR") and Indonesia ("INDO"), respectively.

On the other hand, Stock and Watson (1990, s.2), Choi et al. (1999, s.1771) and Binswanger (2000, s.379) find out that the relation between stock market and economic activity has been unstable over time and the predictability power of the lagged stock markets for economic activity is also included in other market fundamentals. For instance, Binswanger (2000, s.386) reports a temporal breakdown in this relationship for the US case where a predictive power of the stock market for economic activity is only confirmed for the sample period between the 1950s and

1980s. However, the author (2000, s.386) did not find out any significant results for the period after 1984, namely, it is said that equity returns ceased to lead future economic activity may be as a result of irrational exuberance or the existence of speculative bubbles or shocks in interest rates/risk premia or globalization. Morck et al. (1990, s.200) remind that the stock prices are determined by both the market fundamentals and investor sentiments. Namely, stock prices are affected by behaviors of the noise traders (irrational investors) through changing the demand of the sufficient number of investors, resulting diverges from fundamental value of the stocks.

In economics and finance literature, the majority of the papers are devoted to testing causal relationship between financial variables (stock markets) and economic activity in the countries under scrutiny, particularly in the "U.S." and G7 countries. The results, in general, are that the stock markets have a powerful ability on forecasting economic growth rate in the short or/and long-run. The most comprehensive empirical papers related to causality form are presented by Choi et al. (1999, s.1771), Hassapis and Kalyvitis (2002, s.543) and Binswanger (2004, s.237) for G7; Muradoglu et al. (2000, s.33) for 19 developing countries; Wongbangpo and Sharma (2002, s.27) for the ASEAN-5; Duca (2007, s.1) for "U.S.", "GER", "UK", "JAP", and "FRA"; Panopoulou (2009, s.1414) for the 12 EU countries; Tsouma (2009, s.668) for 22 advanced (MMs) and 19 emerging (EMs) countries; and Pradhan et al. (2015, s.98) for 34 OECD countries. Chen and Chen (2011, s.112), for example, have contended that both the underlying variables have long-run relationships and equity returns linearly Granger causes real economic activity both in the short- and long-run in the seven developing countries, including the "U.S.", "UK", "CAN", "FRA", Australia, Finland, and Swiss. They (2011) also report, however, nonlinearly unidirectional and bidirectional causality results which indicate that variables have significant information about each other during the sample period. Similar evidence is reported for the U.S. by Lee (1992, s.1591). In addition, some empirical papers supporting the existence of a bi-directional causality relationship are conducted by Wongbangpo and Sharma (2002, s.27) for "INDO"; Singh (2010, s.263) and Kumar and Puja (2012, s.1) for India ("IND"). Pradhan et al. (2015, s.109), for instance, reveal an evidence of the strong feedback causal relationship for the OECD countries over the sample period of 1960-2012. Likewise, Wongbangpo and Sharma (2002, s.44) find out that economic activity and stock prices as well as consumer price index reinforce each other in the ASEAN-5 countries, including "INDO"; Malaysia, Philippines, Singapore, and Thailand. Conversely, the papers of Mohanamani and Sivagnanasithi (2012, s.38) and Yılmaz et al. (2006, s.1), however, are in favor of non-causality in "IND" and "TUR", respectively. It is evident that the relationship is mainly analyzed by standard econometric methods where researchers presented both short- or/and long-run results for the "U.S.", G7, European or other regional countries. These approaches ignore the medium term, namely they generally tacitly disregard, according to Croux and Reusens (2013, s.94), the possibility that the strength and/or direction of the relationship observed could be different over different scales, i.e. at frequencies.

The wavelet analysis approach is an attempt to bridge this gap and it is a widely accepted effective tool and has been preferred by many researchers. This method gives an ability to study the relationship between two variables at different time horizons at the same time, namely, it decomposes a time series into different time scales and enables to see, as noted by Graps (1995, s.2), both the trees and forest simultaneously. Besides, Gallegati et al. (2017, s.7) remind that the wavelet transform gives an insight into the basic features of association between variables, thus, revealing the true interdependence which is invisible in the case of using conventional approaches on the original data. For example, the wavelet variance scale-by-scale is approximately equal to the sample variance and it enables to examine how wavelet variance components change regarding frequencies or focus on a particular component that of special interest. It is natural to see that this method is more powerful than the standard ones. For instance, in the one of the earliest paper related to wavelet analysis in finance, Kim and In (2003, s.14) report that the stock prices leads economic activity but the opposite is not true. However, after conducting the standard methods on the wavelet coefficients, they concluded that there exist bidirectional causalities between economic activity and financial variables at different time horizons. Equivalently saying, it is observed that the strength and direction of the relationships in the sense of the Granger causality are different at each wavelet scales, making this method more accurate and preferable.

The remainder part of the empirical research is structured as follows. In the second section, the underlying variables are reviewed and their basic statistics are discussed. Next, a detailed description of the empirical methodology procedure including the Fourier and wavelet analysis is outlined. In the fourth section, however, the causality test findings as well as the of unit root and cointegration results for the G7 and E7 countries both in the time and frequency domains are presented and they are compared with the earlier papers' results. In the same section, besides, the wavelet-based ANOVA statistics results are interpreted. The last Section 5 draws conclusions about the main empirical findings and offers policy implications for investors and regulators.

Data

For this paper, the monthly industrial production index and equity market index closing prices of both developed (G7) and developing countries (E7) derived from various database sources are used. The industrial production index data for economic growth are obtained drawn from the OECD (2017) database whilst the stock market closing prices are drawn from Yahoo-Finance (2017), except BIST100 Index retrieved from the CBRT database called EVDS (2017). The stock markets taken into consideration for countries are the DJIA (US), DAX (Germany), FTSE100 (UK), Nikkei225 (Japan), CAC40 (France), FTSE MIB Index (Italy), S&P/TSX Composite Index (Canada), Bist100 (Turkey), IBOVESPA (Brazil), MICEX (Russia), MXX Index (Mexico), JKSE Jakarta Composite Index (Indonesia), S&P BSE SENSEX (India) and JSE All Share Index (SAFR). The monthly dataset spans from January 1998 to August 2017, totaling 236 monthly observations.

To conduct our analysis, all dataset transformed into natural logarithms to remedy potential heteroskedasticity problems. Besides, the returns of time series are calculated as $r_t = ln(P_t/P_{t-1})$ to obtain continuously compounded returns, where P_t is the monthly closing price at (t) period. Table 1 shows the results of the basic descriptive statistics for the continuously compounded returns of the underlying countries.

It is evident from Table 1 that the average value of the industrial production (the RIP, hereafter) and stock market growth rates (the RSP, hereafter) of the all developing (E7) countries and four out of seven developed countries (G7) are positive. On the other hand, the RIP of France, England, and Italy are negative during the sample period. Notable, the only stock market index that has a negative value over the period is the FTSE MIB Index, suggesting a poor performance largely due to Eurozone developments, such as EU Debt Crisis. Noteworthy to mention that, the largest average growth rates are related to monthly equity returns of the E7 countries of "TUR_DLSE", "RUS_DLSE", "INDO_DLSE", "SAFR_DLSE", and "MEX_DLSE", starting from , , , , and , respectively, while the lowest values are observed for the RIP variables, except for "ITA_DLSE" over the tested period. The highest top ten volatility values represented by greater standard deviation are to be found only, except "INDO_DLIP", in the equity markets. Not surprisingly, the highest average and standard deviations are related to the same countries, to be more precise, "TUR", "RUS", and "INDO" are the countries that have the highest volatility and average monthly values among the countries. On the other hand, the most monthly increases and decreases are observed mostly in the stock markets during the studied period. For instance, "RUS" stock market index decreased sharply by at the end of September after deciding a new currency regime, the freely floating currency regime, during the financial crisis hit "RUS" in August 1998, resulting devaluing the national currency and also defaulting on its domestic debt. At the same time, the stock market of "BRA" and "TUR" lost nearly of their value. The other countries that experienced their highest drop in equity market value are "SAFR" [], "MEX" [], "CAN" [], and the "U.S." []. The DAX, CAC40 and FTSE MIB Index, on the other hand, had the highest monthly decrease during the Dot-com crisis of 2000-02, since 1998. Overall, the recent global financial crisis (GFC) had a huge negative impact on both stock markets and industrial production for all countries during the sample period. On the other side, the largest increases during the sample period in stock market are observed at both in "TUR" and "RUS", where the BIST100 index increased by percent to just before the local financial/banking crisis at the beginning of the 2000s while "RUS" MICEX index rose by percent to from at the end of 1998.

Regarding the third and fourth moment, the skewness and kurtosis coefficient values of the underlying data are also given in Table 1. As it can be seen that apart from "IND_DLIP", "RUS_DLIP", "TUR_DLIP", and "TUR_DLSE", the other variables have a negative skewness coefficient during the studied period, implying a left-skewed distribution, namely, the right tail is short corresponding to the left tail. Besides, the fourth moment, kurtosis, coefficient value is higher than for all variables, namely, both the RIP and the RSP variables possess a leptokurtic behavior, thus, they have fat tails and peakedness over the period under investigation. Lastly, as both the skewness and excess kurtosis coefficients show, the null hypothesis of normally-distributed is rejected for all variables according to the Jarque-Bera test results.

Methodology

In this section, we will give describe the frequency-based transform methods and time-domain econometric analysis tools. Firstly, we briefly explain the theory of the Fourier and wavelet analysis procedure used in calculating the wavelet variance, covariance, and correlations by scale. Lastly, we give detailed information about the methodology of the econometric analysis of the unit root process and causality tests.

Fourier Analysis vs. Wavelet Analysis

The major frequency-based analysis and the origin of the wavelets, the Fourier analysis's history goes back to the beginning of the 19th century. In 1807, as reported by Mallat (1989, s.689), the French mathematician J.B. Joseph Fourier presented a paper of the detailed study of trigonometric series where Fourier argues that any periodic function can be expressed by means of the sinusoids, i.e. sine and cosine functions. At first, his ideas were controversial in the 19th century due to his unsubstantiated arguments and exaggerated outcomes; however, it took nearly fifteen decades to understand the convergence of the theory.

In and Kim (2012, s.2) state that the essential idea of Fourier series is that any deterministic function of frequency $f \in L^2[-\pi,\pi]$ can be represented as an infinite or a finite sum of the two dilated sinusoids:

$$f(x) = \frac{1}{2}\lambda_0 + \sum_{j=1}^{J}(\lambda_j + \cos(jx) + \beta_j \sin(jx)) \qquad (1)$$

Gencay et al. (2002, s.97) contend that the function in Equation 1 is an example of the discretely sampled process of an *f(x)* function generated by a linear combination of the basic trigonometric sinusoids, i.e. it is a decomposition on frequency-by-frequency basis of the discrete Fourier transform (DFT, hereafter). These transform approach, however, is very appealing when the underlying data or signal is stationary.

An example of the frequency-based transform is depicted in Figure 1. This method fundamentally transforms the original data on frequency basis or the transformed data to on time domain to reveal several singularities and symmetries otherwise hidden in the underlying data. Besides, Hubbard (2005, s.193) asserts that the original data and its transformed data are the two different visages of the same information, where the transformation outcome depicts the original data only with regarding frequency basis, namely, it neglects the time information. Putting the same point in simpler terms, the Fourier transform of a musical recording reveals only the frequency information, i.e. which notes are played, but it lacks in telling when these notes are played. For a clear understanding, both transformations are depicted in the upper panels in Figure 1. Evidently, as reported by Gencay et al. (2002, s.98), the time representation of the data discloses only a full-time resolution without frequency information whereas the Fourier transform reveals a full frequency resolution without time information. Evidently, the Fourier transform has some major drawbacks. The first unsuitable feature is that after completing the transformation of periodic or non-periodic signals, the time information is lost, which makes impossible to figure out when a particular event occurred. Miner (1998, s.5) argues that this drawback proceeds from the infinite support of the basis function of the Fourier transform. Hence, due to localization in the frequency-domain, the outcome will be a global representation of the data. On the other hand, the other drawback is related to the assumption of stationarity. As mentioned above, it is assumed that the underlying data is stationary during the sample period, which is not always true for the most signals, data or time series.

In the light of all the drawbacks of aforementioned reasons, D. Gabor introduced a new transform method. To overcome the stationary assumption, as stated by Goswami and Chan (2011, s.68), one firstly should partition the original signal into several small sections via specified window function and then pick up the desired section to analyze the local frequency contents. Here, it is assumed that this partitioned section is stationary during the duration of the window function. Lastly, the Fourier transform is applied to this removed small section. The transformation process is done to the other remaining sections by shifting the window functions, changing the value of the translation parameter, along with the time axis. In literature, this transformation process is called Gabor Transform, or the Short-Time Fourier transform (STFT, hereafter) or the Windowed Fourier transform (WFT, hereafter). Notwithstanding that its effectiveness of capturing both the time and frequency information simultaneously, this method have some major drawbacks. The window function depends upon the trade-off between the stationary assumption and the desired frequency resolution is the same for all frequencies, and, as noted by Gencay et al. (2002, s.99), is fixed with the respect to frequency, leading to accept some compromises. Once a decision made, i.e. the window length is determined, it is not allowed to change it for other frequency locations. In and Kim (2012, s.4) state that both the frequency and time resolutions are fixed for all frequency and time locations, therefore, the outcome is limited due to the particular window size, which is obvious in Figure 1. The second unsuitable feature is that, as reported by Gencay et al. (2002, s.99), the STFT approach is unsuccessful at determining the events took place within the width of the window. Soman et al. (2010, s.45) assert that because both the time resolution and the frequency resolution are fixed, even though the window function is shifted along the time axis, the uncertainty box will have the same shape for all locations. The major reason behind this result proceeds from the uncertainty principle of Heisenberg where it is claimed that one cannot obtain both a good resolution in time and frequency simultaneously.

Table 1. Descriptive Statistics of Return Series

Variables	ΔLog								
	Mean	SD	Min	Max	Skewness	Kurtosis	JB		n
US_DLIP	0.0008	0.0066	-0.0440	0.0203	-1.9077	13.3311	1187.61	***	235
US_DLSE	0.0043	0.0426	-0.1641	0.1008	-0.7548	4.7610	52.68	***	235
GER_DLIP	0.0015	0.0158	-0.0828	0.0453	-0.7196	6.0719	112.68	***	235
GER_DLSE	0.0045	0.0635	-0.2933	0.1937	-0.8933	5.7649	106.11	***	235
ENG_DLIP	-0.0003	0.0092	-0.0483	0.0254	-0.8352	7.2002	200.06	***	235
ENG_DLSE	0.0015	0.0404	-0.1395	0.0830	-0.7215	3.7653	26.13	***	235
JAP_DLIP	0.0000	0.0213	-0.1720	0.0639	-2.9946	23.0747	4297.24	***	235
JAP_DLSE	0.0011	0.0571	-0.2722	0.1209	-0.7526	4.4725	43.42	***	235
FRA_DLIP	-0.0003	0.0138	-0.0510	0.0413	-0.2038	3.7209	6.72	**	235
FRA_DLSE	0.0023	0.0539	-0.1923	0.1259	-0.6020	3.7899	20.31	***	235
ITA_DLIP	-0.0006	0.0139	-0.0448	0.0380	-0.3080	3.8142	10.21	***	235
ITA_DLSE	-0.0005	0.0637	-0.1831	0.1909	-0.2313	3.6986	6.87	**	235
CAN_DLIP	0.0008	0.0102	-0.0328	0.0346	-0.1598	3.7169	6.03	**	235
CAN_DLSE	0.0035	0.0436	-0.2257	0.1119	-1.3431	7.7554	292.08	***	235
TUR_DLIP	0.0033	0.0267	-0.0996	0.1543	0.3189	8.4739	297.38	***	235
TUR_DLSE	0.0146	0.1185	-0.4949	0.5866	0.1840	7.3968	190.62	***	235
BRA_DLIP	0.0009	0.0177	-0.1364	0.0588	-2.0431	17.5838	2246.07	***	235
BRA_DLSE	0.0079	0.0846	-0.5034	0.2155	-1.1116	8.3928	333.17	***	235
RUS_DLIP	0.0025	0.0221	-0.1285	0.1522	0.3555	18.2073	2269.39	***	235
RUS_DLSE	0.0133	0.1136	-0.5826	0.4255	-0.8532	8.3323	306.92	***	235
MEX_DLIP	0.0011	0.0081	-0.0288	0.0281	-0.2953	4.6173	29.03	***	235
MEX_DLSE	0.0097	0.0618	-0.3498	0.1766	-1.0028	7.7360	259.01	***	235
INDO_DLIP	0.0028	0.0610	-0.2845	0.2528	-0.5655	11.2733	682.74	***	235
INDO_DLSE	0.0114	0.0760	-0.3846	0.2313	-0.9736	7.5433	239.24	***	235
IND_DLIP	0.0044	0.0199	-0.0610	0.0918	0.4988	5.7933	86.14	***	235
IND_DLSE	0.0093	0.0695	-0.2730	0.2489	-0.4139	4.2173	21.22	***	235
SAFR_DLIP	0.0008	0.0203	-0.0792	0.0752	-0.2877	4.9249	39.52	***	235
SAFR_DLSE	0.0097	0.0560	-0.3513	0.1319	-1.2390	9.6612	494.60	***	235

The wavelet-based analysis depends upon the Fourier analysis, i.e. the latter actually is one of the origins of the former analysis. Despite being derived from the latter approach, there are some similarities and differences between the wavelet analysis and the Fourier analysis. According to Graps (1995, s.5), the first similarity is that both transform methods (FFT and DFT) are linear operations. The second similarity is related to their reversibility features. Their inverse transform matrix is equal to their transpose of the original data, namely, to obtain the original data, their inverse method can be used. The last similarity is about their basis functions, i.e. the trigonometric series and wavelets are both localized in frequency. This last feature enables to calculate power distributions and pick out frequencies. On the other hand, the author (1995, s.6) lines up also some major differences (see Figure 1). In fact, wavelet transform method has three major advantageous over the Fourier method. The first dissimilarity is that the localization in space is true only for wavelet functions. To put it in the same way, the wavelets have the capability to break down the signal or data into a number of time scales, making possible to study the behavior of the underlying data over all frequencies and plotting them in the frequency-time plane. The second advantage is related to varying window shapes for different time scales. As it can be seen from Figure 1, the STFT method has a fixed window for all frequencies and results the same resolution levels for all locations whereas the wavelet functions uses both very long and short basis functions to obtain detailed and overall representation of the data, namely, smaller windows for high- and larger windows for low-frequency oscillations. The last advantage is that the wavelets can handle both nonstationary and stationary data where the stationarity is a necessary requirement for the Fourier analysis.

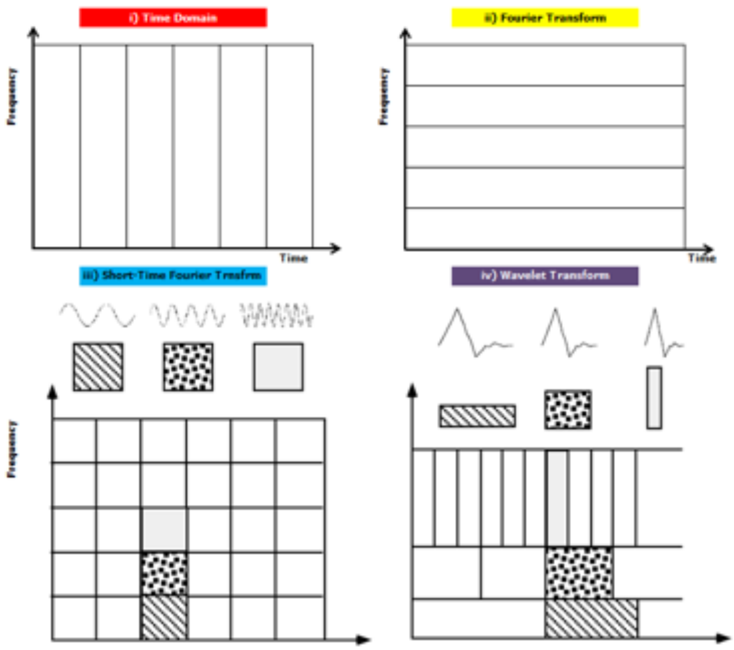

Figure 1. Fourier vs. Wavelet Transforms (Source: Gencay et al., 2002, s.98)

Discrete (DWT) and Maximal Overlap Discrete Wavelet Transforms (MODWT)

To overcome the drawbacks of the STFT approach, a different method is introduced by researchers: wavelet transform. Cascio (2007, s.3) reports that the wavelet term is first mentioned by Grossman and Morlet in 1984. Literally, wavelet indicates "small waves" due to having finite length and oscillatory behavior for a limited time period, and, besides, its average value is equal to zero. Expressed differently, they grow for short time duration and then die out, unlike the Fourier trigonometric series. This is the reason of, as Crowley (2007, s.209) remarks, wavelets are not homogenous and they have compact support during the sample period. In the case of the long sample period, the wavelet functions having compact support are strung together and they are indexed by location.

As evident from its name, Soman et al. (2010, s.31) state that wavelet-based analysis is related to analyze the interested signal/data or time series with short duration and finite energy functions. In transforming process, rather than using the pure functions as in the case of Fourier, the dilated/compressed or shifted version of the basic wavelet function, the mother wavelets are used. The outcome is described as wavelet transform, in continuous or discrete form, where it provides the time-frequency representation of the underlying data at the same time.

For the transforming a given data by wavelets, as noted by Kiermeier (2014, s.135) in the related paper, there are two manipulation ways: translation and dilation of the mother wavelet. First of all, the basis function, i.e., wavelet, can be widened (dilated) or squeezed to capture the frequency information of the data. Secondly, this function can be translated (shifted) to the left or to the right direction along the time axis to capture the time information of a specific event that occurred. After dilated and translated the basis function, the result will be both a time-domain and frequency-domain representation of the underlying data. Actually, the success of the transformation totally depends upon the local matching of the basis function with the tested data.

Putting the same subject in simpler terms, if the wavelet basis function and the shape of data match well at a certain point and scale, then the transform value will be large, according to Soman et al. (2010, s.34). On the other hand, if they do not correlate together, namely if their shape does not match well at a certain scale and point, however, the transform value will be low. If the transform is computed for all data locations and wavelet scales at continues steps, the outcome will be called as continuous wavelet transform, hereafter (CWT), but, on the other side, the outcome will be called as discrete wavelet transform (hereafter, DWT), in the case of a process at discrete steps.

Crowley (2007, s.209) says, in wavelet literature, there are two basic wavelets genders/functions denoted in Greek alphabets as follows:

$$\int \psi(\mathrm{t})\, dt = 0 \quad \& \quad \int \phi(\mathrm{t})\, dt = 1 \tag{2}$$

where, ψ (psi) and ϕ (phi) represents mother and father wavelets, respectively.

Ramsey and Lampart (1998, s.54) document that the father wavelet (or scaling function), $\phi(\mathrm{t})$, integrates to one whereas the mother wavelet function, $\psi(\mathrm{t})$, integrates to zero. The scaling functions (smooth components) are utilized for the lower-frequency to capture the long-term trend of the data and the mother wavelet functions (detailed components) are utilized for the higher-frequency to capture fluctuations from the trend. Reconstruction of a data or signal is related to, Ramsey (2014, s.9) says, the scaling function, $\phi(.)$ and this function represents averaging as opposed to wavelet function which signifies weighted "differences" at each scale. With the aid of these functions, the approximating wavelet functions $\phi_{ba}(t)$ and $\psi_{ba}(\mathrm{t})$ can be generated as:

$$\psi_{b,a}(\mathrm{t}) = \frac{1}{\sqrt{b}}\psi\left(\frac{t-a}{b}\right)$$

&

$$\phi_{b,a}(\mathrm{t}) = \frac{1}{\sqrt{b}}\phi\left(\frac{t-a}{b}\right)$$

The first component is a translation parameter whereas the second component b is a sequence of scales or dilation/scaling parameter. Crowley (2007, s.210) denotes that b controls the length of the window and a is a measure of the location. In addition, $1/\sqrt{b}$

parameter guarantees that the norm of $\psi(.)$ is equivalent to (one) and the energy is intensified in a neighborhood of the translation parameter, *a*, with size proportional to the scaling parameter, *b*. The length of support is positively related to the scaling parameter, *b*, namely the support increases if the dilation parameter increases. Gallegati (2008, s.3065) asserts that the basis function, mother wavelet, is squeezed (or stretched) and translated along the time axis to capture information features which are local in frequency and local in time through the scaling/dilation parameter. The result representation is depicted in Figure 1, where the narrower windows in low frequency locations yield good frequency resolutions but poor time resolutions, whereas the wider windows in high frequency locations yield good time resolutions but poor frequency resolutions. Gencay et al. (2002, s.106) remark that this transformation process is not violating the uncertainty principle as mentioned above, but it adjusts itself for each frequency and time locations. The outcome of the transform, wavelet coefficients in the case of the CWT, is actually a function of the translation and dilation parameters. Unfortunately, the CWT yields redundant information, but, since the underlying time series is observed at regular intervals, the following section is restricted only the theory of the DWT. Through critical sampling of the CWT, the DWT can be obtained with the aid of denotations of the parameters of $a=2^jk$ and $b=2^j$ where *j* and *k* integers represent the discrete translation and dilations, respectively, observed at $j=1, \ldots ,J$. It does not matter which transform method is preferred, the outcome will be a time-frequency or a time-scale representation of the original data.

Lindsay et al. (1996, s.777) note that in wavelet literature, there exist several wavelets and scaling filters in different forms, such as discrete wavelets (the first family members, Haar wavelets with compact support), symmetric wavelets (the Mexican Hat, Symmlets, and Coiflets), and asymmetric (Daublets). These wavelet families differ by their filter features and filter lengths. Besides, trade-offs between the localization and the approximation of high-pass filters degrees differ with the different wavelet families. On the other hand, Daubechies (1992, s.194) introduced a new wavelet family, the least asymmetric (LA[*L*] hereafter), with compact support and two different filter sets identified by the number of vanishing moments, calculated by , instead of the filter length.

Ramsey (2014, s.10) documents that wavelets are generated by the combination of two filter bank members. The high-pass filter, i.e. the linear time-invariant operator, produces moving differences whereas the low-pass filter generates a moving average. These two filters together decompose a data into frequency bands. For any set of filters must satisfy the following three basic features in Equation (5):

$$g_l = (-1)^{1+l} h_{L-(1+l)} \ \& \ h_l = (-1)^l g_{L-(1+l)} \tag{4}$$

$$\text{(a)} \sum_{l=0}^{L-1} h_l = 0 \qquad \text{(b)} \sum_{l=0}^{L-1} h_l^2 = 1 \qquad \text{(c)} \sum_{l=0}^{L-1} h_l h_{l+2n} = 0 \tag{5}$$

where the scaling filter coefficients in Equation (4) corresponds to the low-pass filter (the father wavelet) and the wavelet filter coefficient h_l stands for the high-pass filter (the mother wavelet) observed at $l=0,1, \ldots, L-1$, and both are related to each other, as noted by Gallegati (2008, s.3065), through a quadrature mirror filter association. Besides, Crowley (2007, s.209) documents, in Equation (5), the finite length discrete wavelet filter (a) integrates to zero, i.e. have zero-sum value, (b) has unit energy, and (c) is orthogonal to its even shifts for all non-zero integers. The orthonormality conditions of which last two out of three properties describes the father wavelet in filter terms.

After giving some detailed information about the CWT and DWT, now it is time to provide brief information about the Maximal Overlap DWT (MODWT, hereafter) which is a non-decimated version of the DWT. In literature, the widely used term MO is firstly used by Percival and Guttorp (1994, s.3) to determine the relationship between the wavelet estimators and Allan variance. Before delving into the related theory, it is required to mention the main advantageous over the DWT, which are summarized by Percival and Mofjeld (1997, s.871) in their paper as follows: First of all, (i) the most challenging problem to contend with is that, in the case of the DWT, the sample size *N* must be dyadic; on the other hand, any sample size N (≥ the length of L_1 of the underlying wavelet filter actually) is appropriate for the MODWT. Besides, the computational cost of the MODWT is higher, namely, it takes $0(N\log_2 N)$ multiplications for the MODWT, which is equal to the number of multiplications in the case of FFT, while it is $0(N)$ for the DWT. Secondly,

(ii) contrary to the DWT, and the MODWT is a shift-invariant method. Equivalently speaking, when one circularly shifts a signal by an integer value to the left or right direction, the MODWT wavelet and scaling coefficients and multiresolution analysis do not change which is not true for the DWT. Thirdly, (iii) meaningfully line up the features in MRD with the raw data under study is possible in the MODWT decomposition because of the relation to zero-phase filters. In other saying, the number of MODWT coefficients is the same with at each scale at the cost of losing the orthogonality due to not decimating the related coefficients during the transformation process. Besides, the MODWT is over-sampled at coarse decomposition levels. Lastly, (iv) both transformation methods are capable for variance estimation. However, under a stationarity assumption, as dictated by Crowley (2007, s.227), the MODWT transformation method provides a more statistically efficient estimation.

Let assume a data X with N dimensional vector (Gencay et al., 2002, s.135). The MODWT coefficients vector $\tilde{w} = \left[\tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \ldots, \tilde{w}_J, \tilde{v}_J\right]^T$ with $J+1$ vectors is equal to $\widetilde{W}X$. Here $\widetilde{W}$ is a $(j+1)\ NxN$ matrix, while the vector $\tilde{w}_J$ and $\tilde{v}_J$ include the MODWT-based wavelet coefficient associated with changes at scale $\gamma_j = 2^{j-1}$ and the scaling coefficient associated with averages at scale $2\gamma_J = 2^J$, respectively. The associated wavelet and scaling coefficients with rescaled wavelet filters of the DWT are given as:

$$\tilde{v}_{j,t} = \frac{1}{\sqrt[2]{2^j}} \sum_{l=0}^{L-1} \tilde{g}_{j,l} X_{t-l}$$

&

$$\tilde{w}_{j,t} = \frac{1}{\sqrt[2]{2^j}} \sum_{l=0}^{L-1} \tilde{h}_{j,l} X_{t-l} \tag{6}$$

where X_t is equal to $X_{t\text{mod}t-1}$ in the case of $t<0$ condition and $\tilde{g}_{j,l} = g_{j,l}/\sqrt[2]{2^j}$ and $\tilde{h}_{j,l} = h_{j,l}/\sqrt[2]{2^j}$ are the jth level scaling and wavelet filters of the DWT. In contrast to the DWT, the MODWT filters have half energy as given in Equation (5).

Gencay et al. (2002, s.136) report that a pyramid algorithm introduced by Mallat (1989, s.683) is utilized for both DWT and MODWT-based transform process to obtain the wavelet $\tilde{w}_l$ and scaling coefficients $\tilde{v}_l$. First of all, at the 1th level, the wavelet and scaling filters are applied to the data vector X. After the first level, computation is completed, at the second iteration the required data is only the scaling coefficients to obtain the wavelet $\tilde{w}_2$ and scaling coefficients $\tilde{v}_2$. The out come is $\tilde{w} = [\tilde{w}_1 \tilde{w}_2 \tilde{v}_2]^T$ where the length o each set of coefficients is equal to N, the length of original data decomposition. At the third iteration, the filtering operating is applied again to only scaling coefficients $\tilde{v}_2$ obtained from the previous level and the result is $\tilde{w} = [\tilde{w}_1 \tilde{w}_2 \tilde{w}_3 \tilde{v}_3]^T$. The decomposition procedure, however, can be repeated up to the optimal resolution level j times, which is calculated as $\leq log_2 N$. It should be noted that in contrast to the MODWT, the DWT does use downsampling factor at each resolution level. The observation number for each scale, hence, is equal to $N/2^j$.

Gencay et al. (2002, s.144) remark that time series has generally a finite length sampled intervals at a discrete form. When one end of the vector is encountered during wavelet application process, there exists a need for a method that capable of computing the remaining coefficients. In wavelet terminology, one of the methods to deal with the problem is called brick wall boundary condition where it is assumed that the underlying series is "periodic". The periodicity of the series is accomplished by taking the observations from the other end, however, to complete computations. The other approach, as noted by Cornish et al. (2006, s.343), is the "reflection" or "mirror" boundary condition, namely, it is extending the underlying series to $2N$ length observations. Put differently, the reflected part, $X_{N-1}, X_{N-2}, \ldots, X_2, X_1, X_0$, is added to the end of the original series and the result is $X_0, X_1, X_2, \ldots, X_{N-2}, X_{N-1}, X_{N-1}, X_{N-2}, \ldots, X_2, X_1, X_0$.

Multiresolution Decomposition by Scale

In the related paper, Mallat (1989, s.674) demonstrates that a multiresolution decomposition (MRD, hereafter) gives a chance to obtain a scale-invariant interpretation of the underlying data or image. In other saying, the distance between the optical center of the camera and the scene determines the scale of data or image under investigation. If one gets to obtain a bigger (smaller) scene, then it is required to get the camera by a resolution step times closer (further) to the scene, in other words, zooming in (zooming out), then the detail of the scene gets larger (smaller) multiplied by for each step. Putting differently, Hubbard (2005, s.137) reports that the MRD tool enables researchers to simultaneously attain both the details and the overall picture of the data which seems as if the camera is come closer and then moved away.

In MRD analysis, the transformation process is performed via the pyramid (cascade) algorithm introduced by Mallat (1989, s.685) which enables to zoom in on scaling function (Daubechies, 1992, s.3). Therefore, the multiresolution approximation of an underlying *X(t)* variable, as reported by Ramsey (2014, s.12), is formulated as follows:

$$S_J(\mathrm{t}) = \sum_k S_{J,k}\, \phi_{J,k}(\mathrm{t}) \qquad \& \qquad D_j(\mathrm{t}) = \sum_k d_{j,k}\psi_{j,k}(\mathrm{t})$$

$$X(t) = S_J(\mathrm{t}) + \sum_{j=1}^{J} D_j(\mathrm{t}) \quad \text{or} \quad X(t) = D_1(t) + \cdots + D_{J-2}(t) + D_{J-1}(t) + D_J(t) + S_J(t) \tag{7}$$

where $D_j(t)$, observed at $j=1, \ldots ,J$, parameters include the detail components at an increasingly finer resolution level, while the $S_j(t)$ parameter includes the smooth component of the series under investigation. Gallegati et al., (2017, s.8) remark that the detail component denotes the scale deviation from the smooth process, namely, it is the degree of difference of the observations of the series at each individual resolution level, while the smooth component provides the smooth long-term behavior of the underlying series. Therefore, due to the feature of additive decomposition, it is easy to reconstruct the original series, *X(t)*, by summing up the detail and smooth components.

To be more precise, let us give an example of the MRD process with six different *j= 2,...,7* resolution levels based on MODWT approach. For the purpose of brevity, only the MRD of the first observation, , of "US_DLIP" variable is given in Table ***2***. It is evident that the details coefficients remain the same, while the smooth coefficient is decomposed into two parts at each resolution level due to orthogonality property of the wavelet filter. Equivalently saying, the decomposition process focuses only on the nonstationary coefficients of the series until the optimal resolution level is reached. Therefore, the sum of the detail and the smooth coefficient of scale d3, for example, will be equal to the smooth coefficient of s2. At each scale, if one adds these two coefficients, then, the result will be the first observation . This is also true for all other observations, namely, if one sums their details and smooth coefficients, for example at scale d5, the result will yield the original time series as US_DLIP= d1+d2+d3+d4+d5+s5.

Table 2. Multiresolution decomposition (MRD) with different resolution levels

J	d_1	d_2	d_3	d_4	d_5	d_6	d_7	s_j
2	0.00294979	-0.00072004					$[s_2]$	-0.00105176
3	0.00294979	-0.00072004	-0.00182575				$[s_3]$	0.00077399
4	0.00294979	-0.00072004	-0.00182575	-0.0011589			$[s_4]$	0.00193288
5	0.00294979	-0.00072004	-0.00182575	-0.0011589	-0.00013675		$[s_5]$	0.00206963
6	0.00294979	-0.00072004	-0.00182575	-0.0011589	-0.00013675	0.00091154	$[s_6]$	0.00115809
7	0.00294979	-0.00072004	-0.00182575	-0.0011589	-0.00013675	0.00091154	0.00010809	0.00105

Source: Author's calculation

Wavelet Variance, Covariance, and Correlations by Scale

In this section, we will give brief information about the MODWT based wavelet variance, covariance, correlation and cross-correlation measure.

The first concept is the wavelet-based variance measure. Percival and Walden (2000, s.296) demonstrate that wavelet variance is a practical concept for both stationary process and nonstationary process having stationary backward differences for the underlying sample variance. On the other hand, Percival (1995, s.621) describes the wavelet variances as a particular part of total sample variance, namely, the sample variance is broken down into several components associated with scales. Equivalently speaking, Lindsay et al. (1996, s.778) say that using wavelets one can straightforwardly decompose the sample variance into a scale-by-scale basis, leading to both the notion of the scale-dependent wavelet variance estimation and determination of the locations of the events contributing to the total sample variance at each time horizon or frequencies.

In wavelet literature, it is widely known that the wavelet variance is treated as energy decomposition. Because each wavelet detail coefficients have a zero-mean, as noted by Crowley (2007, s.228), the variance analysis is regarded as energy decomposition. Due to the energy preserving property of the MODWT transformation, as Gallegati and Ramsey (2013, s.187) report, the sum of the energies of the two wavelet and scaling coefficients is equal to the total energy of the original time series. Percival and Mofjeld (1997, s.872) provide the energy decomposition of wavelet analysis as given

$$\|X\|^2 = \sum_{j=1}^{J} \left\|\tilde{w}_j\right\|^2 + \left\|\tilde{v}_j\right\|^2; \quad for\ j \geq 1; \tag{8}$$

where $\tilde{w}_j$ and $\tilde{v}_j$ denotes wavelet and scaling coefficients derived from MODWT transform process, respectively. The wavelet coefficients detain the variations of the underlying time series from its long-run average value at each scale.

Regarding statistically significant results, Whitcher (1998, s.107) provides the unbiased estimator of the (time-independent) wavelet variance based upon the MODWT for each scale, $\lambda_j = 2^{j-1}$ in Equation (9) with the following conditions for a stationary or nonstationary variable

$$\tilde{\sigma}_X^2(\lambda_j) = \frac{1}{\tilde{N}_j} \sum_{t=L_j-1}^{N-1} \tilde{w}_{j,t}^2 \tag{9}$$

where $L_j[= (L-1)*(2^j - 1) + 1]$ and $\tilde{N}_j[= N - L_j + 1]$ denote the wavelet filter length and the number of coefficients unaffected by the boundary conditions for each scale λ_j decomposition of the underlying variable. However, Gencay et al. (2002, s.137) remark that since the related wavelet filter is a rescaled version of the DWT wavelet filter, a normalization factor $2\lambda_j$ is not necessary in the case of the MOWT.

It is noteworthy that the number of the MODWT() and MRA() coefficient is equivalent to the length of the time series in the case of MODWT on the contrary to DWT approach. However, the number of the MODWT() coefficients are scale-dependent due to boundary conditions, namely, as scale increases the number of useful coefficients decreases.

It is easy to derive the wavelet covariance scale-by-scale between two time series of interest after wavelet variances are calculated. Cornish et al. (2006, s.363) define the wavelet covariance as a measure of the degree of simultaneous correlation between two wavelet crystals for each scale. Gencay et al. (2001, s.254) state that the unbiased estimator of the wavelet covariance based upon MODWT() can be described as the covariance between the wavelet scale of *X* and *Y* as follows

$$\gamma_{X,Y}(\lambda_i) = \frac{1}{\tilde{N}_j} \sum_{t=L_j-1}^{N-1} \tilde{W}_{X,j,t} \tilde{W}_{Y,j,t} \tag{10}$$

Providing a unique technique for attributing levels of relationship with different time horizons, as noted by Gencay et al. (2001, s.255), the wavelet covariance gives an ability to determine which wavelet scale (or time horizon) are significantly contributing to the covariance relationship between the underlying time series during the sample period.

It is widely known that the covariance measure does not take into consideration, as dictated by In and Kim (2012, s.32) of the strength of the association. Hence, it is required to turn our attention to the wavelet correlation terminology. Despite indicating a co-movement between wavelet scales of two time series up to some extent, the wavelet correlation would be a more suitable analyzing tool with regard to wavelet covariance. Whitcher (1998, s.115) introduces the wavelet correlation measure between the wavelet scale, λ_i, based upon unbiased MODWT() coefficients of the underlying bivariate time series as

$$\hat{\rho}_{X,Y}(\lambda_i) = \frac{\gamma_{X,Y}(\lambda_i)}{\sigma_X(\lambda_j) * \sigma_Y(\lambda_j)} \qquad (11)$$

where $\gamma_{X,Y}(\lambda_i)$ given in the numerator Equation (11) is covariance between two time series and $\sigma_X(\lambda_j)$ and $\sigma_Y(\lambda_j)$ given in the denominator are the square root of the wavelet variance of *X* and *Y*, respectively.

Because of intrinsic non-normality of the correlation measure in the case of small-sized samples, Gencay et al. (2001, s.256) document that, it is sometimes required to use a nonlinear transformation, i.e., Fisher's *z* transform, to build a confidence interval for the estimated wavelet correlation, $\hat{\rho}$, for each scale decomposition as given in Equation (12)

$$\text{h}(\hat{\rho}) = \frac{1}{2} log\left(\frac{1+\tilde{\rho}}{1-\hat{\rho}}\right) = \tan h^{-1}(\hat{\rho}) \qquad (12)$$

$$\tan h\left\{h[\hat{\rho}_{XY}(\lambda_j)] - \frac{\Phi^{-1}(1-p)}{\sqrt{\hat{N}_j - 3}}\right\},$$

$$\tan h\left\{h[\hat{\rho}_{XY}(\lambda_j)] + \frac{\Phi^{-1}(1-p)}{\sqrt{\hat{N}_j - 3}}\right\} \qquad (13)$$

where the transformed and unbiased estimated correlation coefficient is based on independent samples. Whitcher et al. (2000, s.14947) state that $\sqrt{N-3}[h(\hat{\rho}) - h(\rho)]$ has approximately a *N(0,1)* distribution. For a better approximation of the distribution, however, the factor $\sqrt{N-3}$ is used in calculation of the confidence intervals given in Equation (13) and the transformation factor. According to Gencay et al. (2002, s.241) it maps the confidence interval back to between [-1] and [+1] to generate an approximate *100(1-2p)%* confidence interval based upon DWT coefficients. The reason behind using DWT instead of MODWT is to obtain more realistic confidence intervals. Gencay et al. (2001, s.256) remark that the CIs do not exploit any information about whether it is distributed by Gaussian or non-Gaussian condition.

After calculating wavelet correlation, it is natural to derive the wavelet cross-correlation coefficients for each wavelet scale. Whitcher (1998, s.115) introduce MODWT based cross-correlation measure for each scale λ at lag τ as

$$\hat{\rho}_{\tau,X,Y}(\lambda_i) = \frac{\gamma_{\tau,X,Y}(\lambda_i)}{\sigma_{\tau,X}(\lambda_j) * \sigma_{\tau,Y}(\lambda_j)} \qquad (14)$$

Gencay et al. (2002, s.258) report that this analysis method, just as in the case of standard cross-correlation, can be used to measure lead/lag relationships between two time series. Besides, at zero lag, $\tau=0$, the cross-correlation measure will be equal to basic wavelet correlation coefficient. Whitcher (1998, s.122) reminds that the magnitude of the wavelet correlation and cross-correlation coefficients are bounded $|\hat{\rho}_{\tau,X,Y}(\lambda_i)| \leq 1$.

Econometric Methodology

In line with the main objective, the cointegration and causal relationships are measured by both the standard and frequency-based analyzing tools. The first challenging step is examining whether the underlying variables are stationary or not. The stationarity is defined by Gujarati and Porter (2004, s.797) as two main moments, mean and variance, are unchanged during the sample period and the covariance depends only upon the time intervals or lags not the actual time. If a time series is not stationary, then it is said that it follows a random walk or it has a unit root. The first major consequence of the using nonstationary variable is that, as noted by Brooks (2014, s.354), the model result may be spurious, namely, they cannot be trusted. In addition, the standard assumption for asymptotic analysis is invalid, i.e. the t-ratios and F-statistics do not follow t-distribution and F-distribution, respectively. After determining the integration order of the variable, the next step is to investigate the cointegration orders.

The cointegration or long-run relationship framework is introduced by Granger (1981, s.128) where it is defined in terms of reduction of the order of integration. If two series are both found to be stationary at the same integration order, , then their linear combination is stationary as well. The result of cointegrating vector leads to examine the possible relationship by including an error correction term in vector autoregressive (VAR) model. Thus, the existence of cointegrating vector(s) implies at least one-way causal relationship in the long run. If not, then the causality is tested by a standard VAR model. The causal relationship is introduced by Granger (1969, s.429) as to test whether the lagged information of stationary X variable is helpful in predicting the current values of stationary Y or not.

Lee and Strazizich (2003) Unit Root Test

Brooks (2014, s.365) states that if a test does not take into account possibility of structural breaks, in case of a larger break and a small-sized sample, test power, therefore, will reduce; namely, it might tend to reject the null hypothesis straightforwardly when actually it is correct. In econometric literature, the first test that takes the structural breaks into account is the test of Perron (1989) where the test permits a one-time change in level or trend. In this test, the change point, however, is determined exogenously by the researchers. After the Perron (1989) test, several unit root tests with structural breaks are introduced by different researchers. However, some important problem occurs when interpreting test results with structural breaks, as dictated by Lee and Strazizich (2003) in their paper, because these tests do not allow for breaks both in the null and alternative hypothesis. They (2003, s.1082) report that if a test does not suppose break(s) under the null, the test statistic might diverge and may cause significant rejections of the null hypothesis. Therefore, they (2003) offer a unit root test where it permits for two change points and the rejection of the null entails that the time series under study is trend-stationary. They (2003, s.1083) establish two different models: "Model A" includes two shifts in level and "Model C" allows two shifts in level and trend. For the unit root testing, Lee and Strazizich (2003, s.1083) suggest using the regression of $X_t = \vartheta' W_t + \epsilon_t$ where $\epsilon_t = \beta\epsilon_{t-1} + \upsilon_t$ and error term is $\upsilon_t \sim \text{iid} N(0, \sigma^2)$. The essential regression to calculate the two-breakpoint LM test statistic is given as

$$\Delta X_t = \vartheta' \Delta W_t + \emptyset \tilde{R}_{t-1} + u_t \tag{15}$$

where $\tilde{R}_t = X_t - \widetilde{\omega}_y - W_t\tilde{\vartheta}$, observed at $t = 2,3, \dots T$. Hence, the null hypothesis is determined as testing $\emptyset = 0$.

Hacker and Hatemi (2006) Symmetric Causality Test

In the case of different integration orders of variables, researchers have commonly preferred implementing the Toda and Yamamoto (1995) approach since last two decades due to its simplicity and being free of conduction cointegration tests. However, as pointed out by Hacker and Hatemi-J (2006, s.1490), the Toda and Yamamoto (1995) approach is sensitive for the assumption of normality and the presence of the ARCH effects in the case of small-sized samples. On the other hand, Al Janabi et al. (2010) stated that it is accepted that the financial time series are generally not normally distributed. Hence, Hacker and Hatemi-J (2006, s.1492) remark that the Wald test statistics generated by Monte Carlo simulation results can be biased. They (2006) offer a modified version of the Toda and Yamamoto (1995) approach where the critical values of MWALD test are generated by leveraged bootstrap simulation technique to remedy the problems mentioned before.

Hacker and Hatemi-J (2006, s.1490) propose using the same augmented VAR *(p+d)* model that Toda and Yamamoto (1995) offer for using test of causality

$$Y_t = \gamma + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \cdots + \beta_p Y_{t-p} + \cdots + \beta_{p+d} Y_{t-p-d} + u_t \tag{16}$$

where p denotes the optimal lag order of the model determined beforehand and d represents the maximum integration order of the variables under study. If the th parameter of dependent variable of Y_t does not lead the kth parameter of dependent variable of Y_t, then the null hypothesis H_0 cannot be rejected, where H_0 is that the row k column j parameter in β_z is equivalent to zero with the condition of $z=1, \dots, p$. As noted by Toda and Yamamoto (1995, s.227), adding the parameters for the extra lag(s) in causality test is to guarantee the use of asymptotical distribution theory.

The required VAR *(p+d)* model including a constant term is described as $Y = \widehat{M}W + \hat{u}$, where

$\widehat{M} := (\gamma, \beta_1, \dots, \beta_p, \dots, \beta_{p+d})$ is an $(n \text{ x } (1 + n(k+d)))$ matrix. They (2006, s.1491) suggest using the following modified Wald (MWALD) test statistic

$$\mathrm{MWALD_{HH}} = (C\vartheta)'[C((W'W)^{-1} \oplus + S_U)C']^{-1}(C\vartheta) \sim \chi_p^2 \quad (17)$$

where $\oplus$ and C denote the Kronecker product and a $(p \text{ x } n(1 + n(p+d)))$ matrix, respectively. Besides, in Equation (17) ϑ is equal to *vec(M)* and if the related parameter in ϑ is zero, then C is equal to one or zero otherwise. The null hypothesis of non-causality is formulated as $H_0: C\vartheta = 0$. However, it is convenient at this point to remark that, the MWALD test statistic used for this approach is assumed to be asymptotically X^2 distributed where the number of degrees of freedom is equal to the number of restrictions, p.

Table 3. Unit Root Test Results with the Standard Methods

Variable	Level (C+T)			Difference (C+T)		
	ADF	KPSS	PP	ADF	KPSS	PP
US_LIP	-3.4361 **	0.108	-2.4800	-3.8969 **	0.0549	-14.0945 ***
US_LSE	-1.8355	0.255 ***	-1.9949	-14.5025 ***	0.0577	-14.4853 ***
GER_LIP	-3.4723 **	0.0637	-3.0043	-6.0547 ***	0.0337	-17.3118 ***
GER_LSE	-2.0476	0.2285 ***	-2.0476	-13.9916 ***	0.0481	-13.9916 ***
ENG_LIP	-2.2662	0.1693 **	-2.0741	-18.1581 ***	0.1085	-18.1975 ***
ENG_LSE	-1.9348	0.1996 **	-2.1218	-15.1729 ***	0.047	-15.1785 ***
JAP_LIP	-3.3409 *	0.1538 **	-3.0137	-13.5031 ***	0.0344	-13.5880 ***
JAP_LSE	-1.3944	0.2154 **	-1.7056	-13.5457 ***	0.0518	-13.6035 ***
FRA_LIP	-1.9203	0.1406 *	-2.6256	-21.7076 ***	0.0779	-20.9190 ***
FRA_LSE	-2.2936	0.0931	-2.6388	-13.5107 ***	0.0827	-13.5183 ***
ITA_LIP	-2.7107	0.2064 **	-2.3556	-6.4969 ***	0.075	-17.7270 ***
ITA_LSE	-2.6079	0.1089	-2.8702	-14.5528 ***	0.0848	-14.5781 ***
CAN_LIP	-1.9531	0.1805 **	-2.3218	-13.9521 ***	0.1204 *	-14.3050 ***
CAN_LSE	-2.9566	0.1111	-2.9003	-12.1332 ***	0.0304	-12.1496 ***
TUR_LIP	-2.5929	0.1709 **	-2.8005	-19.4879 ***	0.0763	-19.4354 ***
TUR_LSE	-2.6874	0.2944 ***	-2.7985	-15.5559 ***	0.0262	-15.5562 ***
BRA_LIP	-1.1493	0.4032 ***	-1.1493	-16.4468 ***	0.0452	-16.4468 ***
BRA_LSE	-1.7832	0.3604 ***	-1.8813	-14.7195 ***	0.0511	-14.7185 ***
RUS_LIP	-1.7401	0.3892 ***	-1.9286	-18.6728 ***	0.043	-18.7693 ***
RUS_LSE	-2.0508	0.4285 ***	-1.8474	-13.0636 ***	0.0324	-13.0770 ***
MEX_LIP	-2.5274	0.0703	-2.7785	-17.6884 ***	0.0464	-17.4794 ***
MEX_LSE	-1.6769	0.326 ***	-1.8027	-15.1441 ***	0.0773	-15.1447 ***
INDO_LIP	-1.4578	0.3548 ***	-13.0883 ***	-7.9090 ***	0.1509 **	-93.0774 ***
INDO_LSE	-2.8812	0.1832 **	-2.3956	-12.4649 ***	0.0582	-12.4649 ***
IND_LIP	-0.4727	0.3804 ***	-1.1235	-15.1634 ***	0.1141	-24.1820 ***
IND_LSE	-2.1727	0.1699 **	-2.4799	-14.5803 ***	0.0605	-14.6561 ***
SAFR_LIP	-2.2905	0.2458 ***	-2.8131	-21.7656 ***	0.0364	-22.0717 ***
SAFR_LSE	-2.4283	0.1656 **	-2.5455	-16.0876 ***	0.0397	-16.0877 ***

Empirical Results

This section is divided into two groups where the first part includes the econometric test results both in the time and frequency domain. The second part comprises the wavelet-based statistics.

Wavelet-Based Econometric Test Results

The first step in the econometric analysis is the unit root testing of variables. In this paper, at first we test the nonstationarity with three conventional unit root tests of the ADF, PP, and KPSS and report it in Table 3. Note that, the null hypothesis of the ADF and PP test is different from the null hypothesis of the KPSS test. Table 3 shows that all variables, with the exception of the "US_LIP", "GER_LIP", "JAP_LIP", and "INDO_LIP", have unit root in log-level according to the ADF and PP test. To robustness check, the KPSS test is implemented as well. The null hypothesis of stationary for the KPSS test is not rejected for six out of the twenty-eight variables, where the ADF and PP test results of the "US_LIP" and "GER_LIP" are confirmed. Note that all stock markets are found to be nonstationary, according to the ADF and PP, indicating that they are efficient in the weak-form. On the other hand, all variables become stationary at the first-differenced, except the "CAN_LIP" and "INDO_LIP" regarding the KPSS. Consequently, it can be said that all variables with two exceptions are integrated of order one, .

Evidently, the different tests without structural breaks yield different results. However, not including structural breaks might lead to spurious/biased results. Hence, the modern L&S (2003) unit root test with two unknown structural breaks is conducted to obtain the final results, reported in Table 4.

Table 4 shows that the twelve out of the twenty-eight variables are found to be stationary with two breaks in log-levels. Note that the "CAN_LIP" and "INDO_LIP" variables are stationary at significance level in the first log-differenced. These results imply that only the E7 countries of the "TUR_LSE", "BRA_LSE", "RUS_LSE", "MEX_LSE", and "IND_LSE" stock markets are not weak-form efficient, namely, they do not follow a random walk. Therefore, the cointegration process is required for only "ITA" and "CAN" country variables.

Table 4. Unit Root Tests with the L&S (2003) Approach

G7 Countries	Model C				E7 Countries	Model C			
	LM test		BP1	BP2		LM test		BP1	BP2
US_LIP	-5.758	**	2008-Jun	2011-Feb	TUR_LIP	-4.519		2002-Oct	2008-Aug
US_LSE	-4.677		2003-Feb	2008-Sep	TUR_LSE	-6.717	***	2001-Dec	2005-Dec
GER_LIP	-5.798	**	2004-Nov	2008-Sep	BRA_LIP	-5.126		2006-Sep	2015-Jan
GER_LSE	-4.500		2002-Mar	2006-May	BRA_LSE	-6.090	**	2002-May	2008-May
ENG_LIP	-6.963	***	2008-Sep	2014-Sep	RUS_LIP	-4.491		2001-Apr	2006-May
ENG_LSE	-4.037		2002-Sep	2008-Aug	RUS_LSE	-6.282	**	2005-Oct	2008-Aug
JAP_LIP	-6.648	***	2003-Jul	2008-Oct	MEX_LIP	-4.055		2001-Jun	2005-Jun
JAP_LSE	-3.223		2005-May	2009-Feb	MEX_LSE	-5.596	*	2003-Mar	2008-Jul
FRA_LIP	-6.672	***	2008-Aug	2010-Sep	INDO_LIP	-7.674	***	2002-Nov	2005-Nov
FRA_LSE	-3.917		2002-Mar	2014-Jul	INDO_LSE	-4.838		2001-Sep	2006-Oct
ITA_LIP	-5.232		2008-Jul	2011-Aug	IND_LIP	-4.988		2003-Sep	2008-Aug
ITA_LSE	-3.565		2010-Apr	2013-Jun	IND_LSE	-6.141	**	2003-May	2008-Jun
CAN_LIP	-5.232		2008-Jul	2011-Aug	SAFR_LIP	-5.393	*	2008-Mar	2010-Oct
CAN_LSE	-3.565		2010-Apr	2013-Jun	SAFR_LSE	-4.613		2002-Nov	2006-Feb

For cointegration analysis, the Hatemi-J (2008) approach with two unknown structural breaks is preferred. To take into account the effect of two regime shifts allowed endogenously both in the intercept and the slope parameters during the time period of study, the required model C/S is formulated as given

$$Y_t = c_0 + c_1 D_{1t} + c_2 D_{2t} + \beta_0' X_t + \beta_1' D_{1t} X_t + \beta_2' D_{2t} X_t + e_t \tag{18}$$

$$D_{1t} = \begin{cases} 0 & \text{if } t \leq [n\tau_1] \\ 1 & \text{if } t > [n\tau_1] \end{cases}$$

&

$$D_{2t} = \begin{cases} 0 & \text{if } t \leq [s\tau_2] \\ 1 & \text{if } t > [s\tau_2] \end{cases}$$

where the dummy variables D_{1t} and D_{2t} with the unknown parameters τ_1 and τ_2 defined represent the relative timing of structural break points. The test statistics of the modified ADF^* and modified Philips tests, Z_α^* and Z_t^*, are described as

$$ADF^* = \inf_{(\tau_1,\tau_2)\in T} ADF(\tau_1,\tau_2)\,;\; Z_a^* = \inf_{(\tau_1,\tau_2)\in T} Z_a(\tau_1,\tau_2)\,;\; Z_t^* = \inf_{(\tau_1,\tau_2)\in T} Z_t(\tau_1,\tau_2) \quad (19)$$

where T is defined as $0.15n$, $0.85n$ with $\tau_1 \in T_1 = (0.15, 0.70)$ and $\tau_2 \in T_2 = (0.15 + \tau_1, 0.85)$.

The Hatemi-J cointegration test (2008) results are reported in Table 5. Obviously, there exists a long-run relationship between the variables of "CAN", indicating that "CAN_LIP" and "CAN_LSE" move in tandem in the long run, requiring of studying a causal relationship in VECM. In addition, there are not any cointegrating vectors for "ITA" variables according to the and Z_α^* test statistic results

Under the cointegration results, VECM based causal relationship test results are given in Table 6. The results represent a negative significant unidirectional long-run relationship from "CAN_LSE" to "CAN_LIP". The speed of the adjustment parameter -0.027 reveals that the disequilibrium in the "CAN_LIP" is corrected in 37 months. Besides, a bidirectional Granger causal link is obtained between the "CAN_LIP" and "CAN_LSE" in the short-run at different, and , significance level. This finding partially reinforce the results of Choi et al. (1999) where one-way causalities running from quarterly stock returns to industrial production growth rate are detected in the G7 countries of the "U.S.", "GER", "UK", "JAP", "FRA", and "CAN" in the short-run.

Table 5. Hatemi-J Cointegration Test (2008) Results

Model (Dependent ~ Independent)	ADF* Test Stat		ADF* BP1	ADF* BP2	Philips Za* Test Stat		Philips Za* BP1	Philips Za* BP2
ITA_LIP ~ ITA_LSE	-5.024		2006-11-30	2011-01-31	-45.023		2007-10-31	2008-07-31
CAN_LIP ~ CAN_LSE	-5.983	*	2001-10-31	2006-02-28	-62.178	*	2001-11-30	2006-03-31
ITA_LSE ~ ITA_LIP	-5.186		2000-12-22	2005-05-31	-41.459		2000-12-22	2005-05-31
CAN_LSE ~ CAN_LIP	-6.503	**	2002-02-28	2003-08-29	-74.432	*	2002-02-28	2003-10-31

Table 7 documents the causality test results using the "MSBVAR" package developed by Brandt (2009) both in the time and in frequency domain using well-specified VAR models. Evidently, there exist bidirectional causal relationships between the underlying variables for the "U.S.", "UK", "JAP", "FRA", "RUS", and "SAFR" countries, indicating that both variables have a great impact on each other during the sampled period. The one-way causal links running from stock markets to industrial production index, in addition, in time domain suggest that the stock markets of "GER", "ITA", "TUR", "BRA", "MEX", "INDO", and "IND" countries are semi-strong efficient according to both unit root and VAR causality test results.

Table 6. VECM Granger Causality Test

Model	SE $\nRightarrow$ IP: χ^2 statistics		SE $\nRightarrow$ IP: ECT_{t-1}		Model	IP $\nRightarrow$ SE: χ^2 statistics		IP $\nRightarrow$ SE: ECT_{t-1}
CAN_LIP~CAN_LSE	23.508	***	-0.027	**	CAN_LSE~CAN_LIP	12.771	**	-0.001

In order to test whether there exists a Granger-causality between the stock markets and industrial production at different investment horizon, the MODWT MRA() coefficients are utilized. The wavelet scales obtained via the LA(8) wavelet filters and "periodic condition" are the scales d1, d2, d3, d4, and d5 related to [2–4), [4–8), [8–16), [16–32), and [32–64) monthly time periods, respectively.

The last five column in Table 7 shows that there are feedback relationships between two variables for all countries, where the rejection of the null hypothesis is scale-dependent. For example, the "TUR_LIP" does not Granger causes the "TUR_LSE" regarding the time domain results, however, it is observed that time-dependent causal relationships are hidden between the wavelet scales of d2 and d5, namely the "TUR_LIP" does Granger causes the "TUR_LSE" after 4 months. In addition, the stock market return Granger causes the industrial production growth rate up to scale d4 in "ITA", however, the reversal link starts at scale d2.

The test results of both methods for the "U.S" case are partly in line with the paper of Kim and In (2003). They (2003, s.14) state that, at first they failed to find out a feedback causal relationship in the time domain. However, they report time-dependent results in the wavelet approach. With the aid of the wavelets, it is found that the lagged industrial production growth rate had a significant effect on the stock returns, which is not observable in the standard method. Overall, they (2003) showed that there exist bidirectional relationships at short and long-term, corresponding to scale d1, d2, d5, and s6.

In addition to standard causality test, we also conducted the symmetric causality test introduced by Hacker and Hacker and Hatemi-J (2006). Notable, the test is conducted for all countries regardless of the existence of cointegration results. To calculate the critical values, 5.000 bootstrap simulations are performed; the optimal lag length is determined by the Hatemi-J (2003) information criterion for the models depending upon the first-differenced data. The symmetric causality test results according to the time and frequency domains are documented in Table 8.

In the time domain, it is evident that there are two-way causal relationships for the "U.S.", "GER", and "JAP", suggesting a non-efficient stock market for these countries. Remarkably, the number of the efficient markets was seven for the standard Granger test; however, with the symmetric test it is observed that "FRA", "ITA", "CAN", "TUR", "BRA", "RUS", "MEX", and "SAFR" stock markets are informationally efficient in the semi-strong form. However, the wavelet-based test shows different results from the conventional Granger test. For instance, using standard test a bidirectional causal relationship was found between the scale d3, d4 and d5 for "TUR" during the time period, but, the two-way relationship is not valid anymore at scale d5 in the case of the Hacker and Hatemi-J (2006) approach. Likewise, there exist strong evidence of the bidirectional causal relationships at scale d1, d3, d4, and d5 for "U.S."; between scales of scale d2-d5 for "UK" and "MEX"; at scale d2, d3, and d5 for "RUS"; at scale d4 and d5 for "BRA"; and at scale d2, i.e. [4-8) month periods for "ITA". In addition, thanks to the wavelet approach it is explored that actually the industrial growth rates had a significantly great impact on stock market returns between 4 and 64 monthly periods in "ITA", "RUS", "MEX", "INDO" and "IND" countries, while it was true for "FRA" and "CAN" countries in the time period of [8-64) months. Hence, it can be said that with the wavelets, it is easy to uncover the true relationship that is hidden among the different time oscillations.

The papers that fully corroborate our findings regarding bidirectional causalities in the short and/or long-run are prepared by Hassapis and Kalyvitis (2002) and Tsouma (2009) for the "U.S." and "UK"; and Ratanapakorn and Sharma (2007) for the "U.S.". The partially confirmed papers, however, are documented by Wongbangpo and Sharma (2002) for "INDO"; Singh (2010) and Kumar and Puja (2012) for "IND", suggesting a feedback causal effects on each other. On the other hand, regarding unidirectional causality from "SE" to "IP" is fully confirmed by Hassapis and Kalyvitis (2002) and Tsouma (2009) for all countries with exceptions for the "U.S.", "UK", "RUS", and "IND" countries. Besides, Duca (2007) for the "U.S.", "UK", "JAP", and "FRA"; Kaplan (2008), Büyüksalvarci and Abdioglu (2010), and Özer et al. (2011) for "TUR"; Panopoulou (2009) for "GER", "FRA", and "ITA"; Muradoglu et al. (2000) for "MEX" and "IND" report a significantly causal impact on industrial production growth rate over the sample period, indicating that stock market return improves predictability of the output for the mentioned countries.

In addition, our findings are in line with the results obtained by Şentürk et al. (2014) for "TUR" where equity return is found to be a significant predictor for industrial growth rate at high-frequency point $[\omega = 2.5]$ and the opposite is true at both medium and high frequency points, $[\omega = 1.0, 2.5]$. In addition, Tiwari et al. (2015) also report the same outcome, one-way causality from stock market performance to industrial growth rate, for "IND" country at low frequency ranges $[0.01 \leq \omega \leq 0.4]$ corresponding to a wave length of more than 15.7 months over the time period of 1993M04-2011M01 after conditioning the VAR model, suggesting to implement economic policies focused on the stock market development. Besides, Croux and Reusens (2013, s.99) corroborated these findings for the G7 countries, using quarterly data for the period 1991Q1–2010Q2. It is observed that the slowly fluctuating components of the equity returns have significant and great predictive power for the future output growth at significance level in all countries. Namely, the null hypothesis is rejected only at low frequency intervals $[0.01 \leq \omega \leq 1.5]$ corresponding to a wave length of more than 1 year and the average incremental R^2 of *8%* at the slowly fluctuating component suggests that approximately *8%* volatility in the output can be explained by the lagged equity returns.

Wavelet-Based Test Results

Next, we will discuss the wavelet-based ANOVA statistics test results. For the paper, the monthly return data is decomposed into several wavelet scales via MODWT() approach. Notable, the wavelet computation based on the Daubechies LA(8) wavelet filter is performed with the aid of the "Waveslim" R package introduced by Whitcher (2005). The optimal integer decomposition level is chosen as $5 \leq \log_2 (235)$, although, the maximum level is 7.

Table 7. VAR Model Granger Causality Test Results

Model	"**Stock Return**" does not Granger cause "**IPI Growth Rate**"					
	Return	d1 [2-4)	d2 [4-8)	d3 [8-16)	d4 [16-32)	d5 [32-64)
US_DL	10.608 ***	2.105 **	3.958 ***	2.11 **	1.022	6.907 ***
GER_DL	2.998 **	1.338	2.149 **	1.008	1.703 *	10.655 ***
ENG_DL	4.77 ***	1.472	1.595 *	2.673 ***	5.092 ***	7.235 ***
JAP_DL	1.913 **	2.668 ***	3.137 ***	3.471 ***	3.751 ***	3.686 ***
FRA_DL	4.944 ***	1.557	2.036 **	1.95 **	2.114 **	4.698 ***
ITA_DL	5.806 ***	2.233 **	2.15 **	0.761	4.328 ***	1.46
CAN_DL	NA	NA	NA	NA	NA	NA
TUR_DL	6.65 ***	2.235 **	1.516	2.095 **	5.501 ***	4.506 ***
BRA_DL	6.57 ***	2.387 ***	2.255 **	1.774 *	3.811 ***	3.429 ***
RUS_DL	2.918 **	4.218 ***	4.679 ***	1.497	1.904 **	2.862 ***
MEX_DL	3.607 ***	1.393	2.219 **	1.928 **	3.733 ***	2.462 ***
INDO_DL	2.361 *	1.461	2.26 **	2.559 ***	5.163 ***	5.579 ***
IND_DL	3.498 **	2.048 **	1.98 **	1.532	3.044 ***	2.668 ***
SAFR_DL	4.579 ***	1.238	1.223	0.848	4.847 ***	9.725 ***
Model	"**IPI Growth Rate**" does not Granger cause "**Stock Return**"					
	Return	d1 [2-4)	d2 [4-8)	d3 [8-16)	d4 [16-32)	d5 [32-64)
US_DL	2.53 ***	2.029 **	1.757 *	2.309 ***	4.607 ***	6.37 ***
GER_DL	1.617	0.601	0.675	1.679 *	1.328	8.921 ***
ENG_DL	2.073 *	1.638 *	1.466	2.824 ***	4.477 ***	7.187 ***
JAP_DL	2.668 **	0.963	1.592 *	1.613 *	3.529 ***	4.374 ***
FRA_DL	2.2 *	1.042	0.526	1.744 *	5.194 ***	6.124 ***
ITA_DL	1.336	1.038	1.879 **	2.253 **	4.949 ***	4.841 ***
CAN_DL	NA	NA	NA	NA	NA	NA
TUR_DL	0.522	0.611	2.24 **	1.886 **	4.303 ***	5.097 ***
BRA_DL	1.367	1.497	1.79 *	2.438 ***	4.014 ***	4.223 ***
RUS_DL	2.317 *	1.294	1.897 **	2.552 ***	3.114 ***	5.777 ***
MEX_DL	0.945	1.345	1.824 **	2.96 ***	2.878 ***	6.27 ***
INDO_DL	1.107	1.01	2.008 **	2.752 ***	3.325 ***	5.532 ***
IND_DL	1.269	1.388	1.189	2.144 **	2.915 ***	5.303 ***
SAFR_DL	1.824 *	0.473	0.667	1.93 **	4.686 ***	2.823 ***

Table 8. Hacker and Hatemi-J Symmetric Causality Test (2006)

"**Stock Return**" does not Granger Cause "**IPI Growth Rate**"					
Return	**d1 [2-4)**	**d2 [4-8)**	**d3 [8-16)**	**d4 [16-32)**	**d5 [32-64)**
30.648 ***	30.494 **	48.324 ***	25.421 **	25.976 *	90.333 ***
3.408 *	21.170	24.822 *	24.054 *	14.398	68.095 ***
1.728	14.208	26.597 **	30.413 **	48.761 ***	49.714 ***
3.757 *	39.751 ***	30.292 **	38.990 ***	44.254 ***	61.638 ***
5.602 **	16.357	33.344 **	23.327 *	52.670 ***	38.139 ***
16.878 ***	23.330 *	23.273 *	18.269	22.406	15.024
15.841 ***	19.057	20.916	50.003 ***	50.356 ***	17.714 **
11.317 ***	27.900 **	21.818	34.431 ***	71.144 ***	13.129
16.284 ***	25.062 *	20.197	20.947	38.170 ***	22.870 **
5.219 **	57.650 ***	71.642 ***	38.835 ***	14.095	26.118 ***
4.778 **	16.023	35.528 ***	34.679 ***	29.272 **	37.784 ***
4.303	13.647	21.055	31.147 **	48.187 ***	24.757
2.350	31.092 **	24.162 *	25.240 *	17.910	30.148 **
5.832 **	12.303	14.907	10.416	53.129 ***	66.562 ***
"**IPI Growth Rate**" does not Granger Cause "**Stock Return**"					
Return	**d1 [2-4)**	**d2 [4-8)**	**d3 [8-16)**	**d4 [16-32)**	**d5 [32-64)**
17.946 ***	24.559 *	19.937	27.654 **	62.972 ***	52.331 ***
3.473 *	8.122	6.260	26.846 **	14.774	43.589 ***
4.211 **	17.263	26.570 **	33.713 ***	66.892 ***	52.792 ***
7.775 ***	18.223	19.140	19.301	30.471 **	41.234 ***
2.086	7.641	16.297	21.039 *	48.508 ***	57.685 ***
0.081	15.156	27.986 **	29.105 **	30.344 **	40.120 ***
0.001	9.338	4.266	29.867 **	28.718 **	18.597 **
0.115	17.835	28.513 **	28.039 **	35.650 ***	18.655
1.989	12.275	13.617	27.675 **	35.298 ***	22.876 **
1.186	20.532	33.806 ***	29.642 **	28.345 **	36.968 ***
1.102	17.372	31.474 **	34.579 ***	22.908 *	30.084 **
0.549	16.877	30.241 **	29.067 **	29.502 **	73.148 ***
0.862	20.900	23.186 *	28.084 **	43.300 ***	30.619 **
1.377	6.010	4.442	22.365 *	66.810 ***	20.073

The first wavelet coefficients-based ANOVA statistic is the wavelet variance where the coefficients are obtained via the "reflection/mirror condition". Note that with this method none of the two moment values do not change because of the duplication of the coefficients for one time. Notable, the wavelet variance computation is done with the non-boundary "wavelet coefficients", where the observation number is 463 at scale d1, 449 at scale d2, 421 at scale d3, 365 at scale d4, and 253 at scale d5 thanks to the "reflection/mirror condition". Note that, if we had preferred the "periodic condition", however, the observation number at scale d5 would have been 18 indeed.

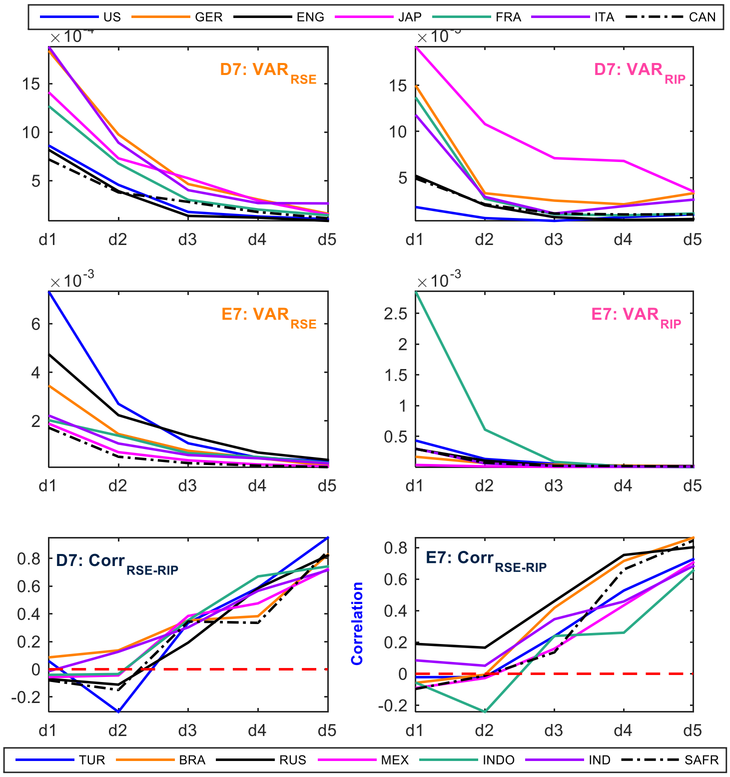

Figure 2. Wavelet Variance and Correlation Coefficients by Scale Decomposition

The unbiased wavelet estimations of the underlying variables are depicted in Figure 2. It is evident that all wavelet variance decreases as wavelet scale increases. First of all, it is not surprising that the volatility of the stock markets is found to be higher than the industrial production in both D7 and E7 countries regardless of the wavelet scales, except at scale d1 in "INDO". The highest volatility among the equity market returns is observed in Turkey at scale d1. In terms of the energy decomposition, the most contribution to total energy is concentrated in the first finest scales. For example, approximately of the stock market variance is observed in "SAFR" and "TUR" at first scale d1, however, it exceeds at the first two scales, indicating that the most volatility is explained by the short-run fluctuations, d1+d2, in the aforementioned countries.

On the other side, "INDO" has the highest volatility of the industrial production growth observed at scale d1 among the countries. The energy decomposition reaches to at the first two scales in "INDO", in "IND", and in "SAFR", suggesting that the short-term fluctuations dominate the long-term fluctuations in the case of the energy distribution. These results reinforce the findings of Kim and In (2003, s.13) for the stock markets of G7 where they (2003) suggest taking an action to every price changes in the markets provided that of being a short-time investor.

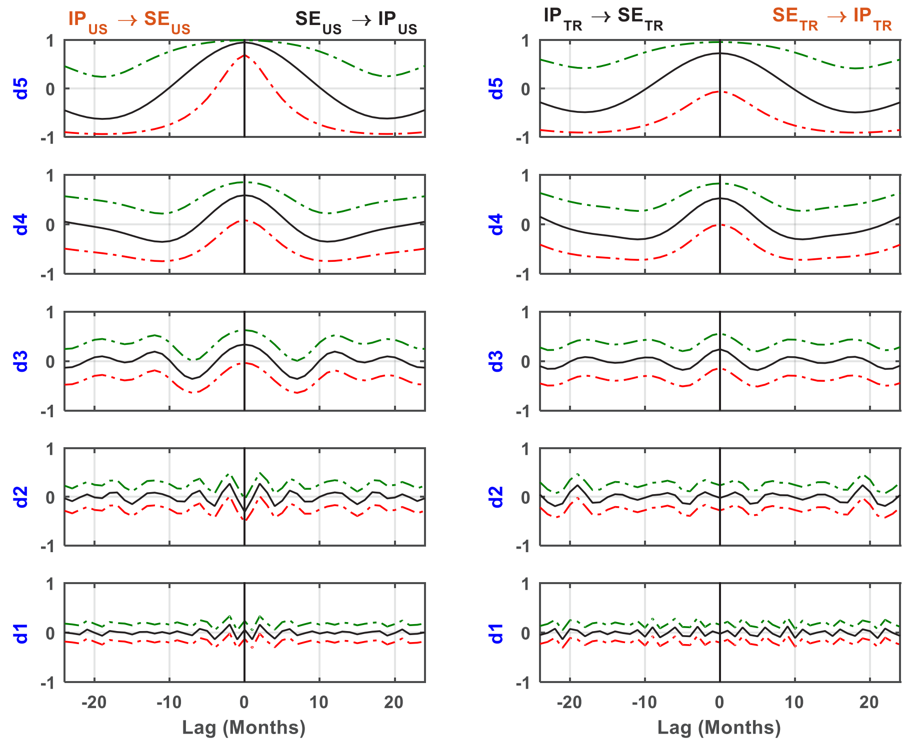

Figure 3. Wavelet cross-correlation of return series by scale-by-scale

At the bottom panel in Figure 2, the wavelet correlation using the MODWT() non-boundary "wavelet coefficients" scale is depicted. It is evident that both variables do not move together in the first two scales, except for the "U.S.", "GER", "ITA", "RUS", and "IND". However, the direction becomes reversed with the scale d3, namely, between the monthly investment horizons [8-64) there exist a positive correlation between the underlying variables in all countries. However, when the significance test is conducted via the "Brainwaver" R package introduced by Achard (2012), it is observed that the correlation coefficients become significant after scale d2. For example, the wavelet correlation coefficient is insignificant at the first three scales in "TUR", but, it turns out that the coefficient becomes significant at level at scale d4 and d5. The wavelet correlation degree decreases after scale d1 in the six out of the fourteen countries, however, it steadily increases with the scale d3 in all countries over all scales. It should be pointed out that the highest correlation coefficient between the original data is in "RUS", however, the magnitude reaches to at scale d3, to at scale d4, and to at scale d5. Besides, the DJIA return and the industrial production growth rate has a significant but negative correlation relationship at scale d2, -, but after this point, it becomes significantly positive at the latest three scales. Notable, the relationship degree reaches the peak value among all variables at scale d5, .

The cross-correlation relationship between the underlying variables for "U.S." and "TUR" by wavelet scales is illustrated in Figure ***3*** along with their 95% approximate confidence intervals. With this method, one can examine the correlation relationship at both contemporaneous and different negative and positive lags at the same time. This method enables to examine the causal relationship in the sense of Granger method. Hence, we can check the validity of the Granger causality test findings with this method. Evidently, the magnitude of the link is by and large close to zero at the finest scales of d1 and d2 in both two countries. As scale increases, the cross-correlation degree also increases over the time-scale. The largest coefficients are observed in the coarsest scale, d4 and d5 corresponding to [16-64) month periods in both countries and at both leads and lags, indicating that there exists a feedback relationship between the two variables. For instance, at a lag of period of 1 month for scales d4 and d5, two variables significantly Granger cause each other with and correlation magnitude in the "U.S." and with and correlation magnitude in "TUR". The significant effect, however, decreases as the lag increases at the same scales. Overall, it can be said that these two variables, regardless of the country, they have symmetric lagged correlation relationship at the coarsest scales. The wavelet cross-correlation results are line with the paper of Gallegati (2008, s.3072) where the DJIA and industrial growth rate variables have a strong and positive leading relationship running from the former to the latter at the coarsest scale.

Concluding Remarks

In the present empirical paper, we have analyzed dynamic relationship between equity returns and economic activity for the G7 and E7 countries, "TUR", "BRA", "RUS", "MEX", "INDO", "IND", and "SAFR", using monthly data covering the period between 1998-01 and 2017-08. The existence and/or strength and direction of the potential relationships are investigated by different econometric approaches both in the time and frequency domain. By conducting wavelet analysis, we aimed to offer a deeper understanding of the relationship since it enables to reveal dynamics which are hidden on different time periods. In the first step, it is observed that the majority of data is found to be stationary in levels according to unit root with structural breaks, indicating that the 12 out of 25 variables are integrated zero, $I(0)$. Besides, the all G7 and two of the E7 countries', "INDO" and "SAFR", stock markets are informationally efficient in the weak-form, indicating that they follow a random walk. The cointegration analysis revealed that the economic activity and stock markets have a bidirectional in the short and a unidirectional relationship running from "SE" to "IP" in the long-run in "CAN". On the basis of the causality tests, the VAR model revealed a Granger causality in the "U.S.", "UK", "JAP", "FRA", "RUS", and "SAFR" countries while the symmetric causality test partially confirmed these findings for the "U.S.", "GER", and "JAP". The general argument that the lagged stock market prices enhance the predictability of the future output, with the exception of the "UK", "INDO", and "IND", is supported according to the Hacker and Hatemi-J (2006) approach. Surprisingly, the only country that has a unidirectional causality from "IP" to "SE" is the "UK", implying that the past information of the output improves the forecasting of the current stock price information. Besides the standard approaches, we also investigate the causality relationship using the wavelet coefficients generated by MODWT MRA() process due to the fact that this approach takes into account the possibility that the strength and/or direction of the relationship could change over different time horizons. To facilitate comparison, we presented both the findings in the same tables. First of all, this new approach corroborated our time-domain based test findings regarding bidirectional and unidirectional causalities. Furthermore, it also corrected our one-way causal findings in favor of feedback relationship at different wavelet scales, mainly concentrated on at the latest time horizons corresponding to "d3", "d4", and "d5". These results offer a more enriched insight on the existence of the relationship, namely, it suggests that stock prices can be used a leading factor for future economic activity and vice versa after approximately 16 months provided that an efficient and effective regulatory framework is in force. In addition to causality tests, we also analyzed this relationship in terms of wavelet variance, correlation and cross-correlation. Test results reveal that the wavelet variances decrease by wavelet scale and the stock market is found to be more volatile than economic activity in all countries, with "INDO" as the single exception at wavelet scale "d1". The most volatility is, in general, explained by short-term fluctuations for both variables, indicating to respond to every variation in asset prices in the short-run. On the other hand, wavelet correlation decreases at scale "d1" in almost every country but then increase as wavelet scale increases, namely, it is negative but insignificant in the time horizon of [2-4) months, but it turns out to be significantly positive after scale "d3" up to "d5". Lastly, the cross-correlation results reveal that both variables lead each other at lags of the period of 10 months at the last wavelet scales, confirming our Granger test results. Overall, our test results show that policy-makers and market participants should take into account the time-dependent relationships, which is not possible with standard methods, before implementing policy rate and investment decisions and should be patient for their consequences to secure the resiliency and durability of the financial system and portfolio management, respectively.

References

Achard, S. (2012). R-Package Brainwaver: Basic Wavelet Analysis of Multivariate Time Series with A Visualisation and Parametrisation Using Graph Theory. R Package Version, 1.6.

Al Janabi, M. A., Hatemi-J, A., Irandoust, M. (2010). An Empirical Investigation of the Informational Efficiency of the GCC Equity Markets: Evidence From Bootstrap Simulation. *International Review of Financial Analysis*, *19*(1), 47-54.

Binswanger, M. (2000). Stock Returns and Real Activity: Is There Still A Connection?. *Applied Financial Economics*, *10*(4), 379-387.

Binswanger, M. (2004). Stock Returns and Real Activity in the G-7 Countries: Did the Relationship Change During the 1980s?. *The quarterly review of economics and finance, 44*(2), 237-252.

Brandt, P. (2009). Markov-Switching Bayesian Vector Autoregression Models Package (MSBVAR). Available at http://cran.r-project.org/web/packages/MSBVAR/index.html, Accessed 2018-01-18.

Brooks, C. (2014). *Introductory Econometrics for Finance* (3th Edition). Cambridge: Cambridge University Press.

Büyüksalvarci, A., Abdioglu, H. (2010). The Causal Relationship Between Stock Prices and Macroeconomic Variables: A Case Study for Turkey. *International Journal of Economic Perspectives, 4*(4), 601.

Cascio, I. L. (2007). *Wavelet Analysis and Denoising: New Tools for Economists* (No. 600). Working Paper, Department of Economics, Queen Mary, University of London.

Chen, S. W., Chen, H. W. (2011). Nonlinear Casual Nexus between Stock Prices and Real Activity: Evidence From the Developed Countries. *International Review of Accounting, Banking Finance, 3*(4), 93-121.

Cheung, Y. W., Ng, L. K. (1998). International Evidence on the Stock Market and Aggregate Economic Activity. *Journal of Empirical Finance, 5*(3), 281-296.

Choi, J. J., Hauser, S., Kopecky, K. J. (1999). Does the Stock Market Predict Real Activity? Time Series Evidence from the G-7 Countries. *Journal of Banking Finance, 23*(12), 1771-1792.

Cornish, C. R., Bretherton, C. S., Percival, D. B. (2006). Maximal Overlap Wavelet Statistical Analysis with Application to Atmospheric Turbulence. *Boundary-Layer Meteorology, 119*(2), 339-374.

Croux, C., Reusens, P. (2013). Do Stock Prices Contain Predictive Power for the Future Economic Activity? A Granger Causality Analysis in the Frequency Domain. *Journal of Macroeconomics, 35*, 93-103.

Crowley, P. M. (2007). A Guide to Wavelets for Economists. *Journal of Economic Surveys, 21*(2), 207-267.

Daubechies, I. (1992). Ten Lectures on Wavelets. *Society For Industrial And Applied Mathematics, 61*, 198-202.

Duca, G. (2007). The Relationship Between the Stock Market and the Economy: Experience from International Financial Markets. *Bank of Valletta Review, 36*(3), 1-12.

EVDS Central Bank of Turkey (CBRT). (2017). https://evds2.tcmb.gov.tr/index.php?/evds/serieMarket, Borsa Istanbul (BIST) Trading Volume (Price Indices) BIST-100, According to Closing Price (January 1986=1)

Fama, E. F. (1990). Stock Returns, Expected Returns, and Real Activity. *The Journal of Finance, 45*(4), 1089-1108.

Fischer, S., Merton, R. C. (1984). Macroeconomics and Finance: The Role of the Stock Market. *Carnegie-Rochester Conference Series on Public Policy*, 21, 57-108.

Gallegati, M. (2008). Wavelet Analysis of Stock Returns and Aggregate Economic Activity. *Computational Statistics Data Analysis, 52*(6), 3061-3074.

Gallegati, M., Gallegati, M., Ramsey, J. B., Semmler, W. (2017). Long Waves in Prices: New Evidence from Wavelet Analysis. *Cliometrica, 11*(1), 127-151.

Gallegati, M., Ramsey, J. B. (2013). Structural Change and Phase Variation: A Re-Examination of the Q-Model Using Wavelet Exploratory Analysis. *Structural Change and Economic Dynamics, 25*, 60-73.

Gencay, R., Selçuk, F., Whitcher, B. J. (2001). Scaling Properties of Foreign Exchange Volatility. *Physica A: Statistical mechanics and its applications, 289*(1), 249-266.

Gencay, R., Selçuk, F., Whitcher, B. J. (2002). *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics*. New York, Academic Press.

Goswami, J. C., Chan, A. K. (2011). *Fundamentals of Wavelets: Theory, Algorithms, and Applications* (2nd Edition). New York: John Wiley Sons.

Granger, C. W. (1969). Investigating Causal Relations by Econometric Models and Cross-Spectral Methods. *Econometrica: Journal of the Econometric Society*, 424-438.

Granger, C. W. (1981). Some Properties of Time Series Data and Their Use in Econometric Model Specification. *Journal of Econometrics, 16*(1), 121-130.

Graps, A. (1995). An Introduction to Wavelets. *IEEE Computational Science and Engineering, 2*(2), 50-61.

Gujarati, D. N., Porter, D. C. (2004). *Basic Econometrics* (4th Edition). New York: McGraw-Hill.

Hacker, R. S., Hatemi-J, A. (2006). Tests for Causality between Integrated Variables Using Asymptotic and Bootstrap Distributions: Theory and Application. *Applied Economics, 38*(13), 1489-1500.

Hassapis, C., Kalyvitis, S. (2002). Investigating the Links between Growth and Real Stock Price Changes with Empirical Evidence from the G-7 Economies. *The Quarterly Review of Economics and Finance, 42*(3), 543-575.

Hatemi-J, A. (2008). Tests for Cointegration with Two Unknown Regime Shifts with An Application to Financial Market Integration. *Empirical Economics, 35*(3), 497-505.

Hubbard, B. B. (2005). *The World According to Wavelets: The Story of a Mathematical Technique in the Making*. Massachusetts: A.K. Peters.

Humpe, A. Macmillan, P. (2009) Can Macroeconomic Variables Explain Long-Term Stock Market Movements? A Comparison of the US and Japan. *Applied Financial Economics, 19(2)*, 111-119, DOI: https://doi.org/10.1080/09603100701748956.

In, F., Kim, S. (2012). *An Introduction to Wavelet Theory in Finance: A Wavelet Multiscale Approach*. Singapore: World Scientific Publishing Co. Pte. Ltd.

Kaplan, M. (2008). The impact of Stock Market on Real Economic Activity: Evidence from Turkey. *Journal of Applied Sciences, 8*(2), 374-378.

Kiermeier, M. M. (2014). Wavelet Analysis and the Forward Premium Anomaly. Gallegati Semmler (Ed.), In *Wavelet Applications in Economics and Finance* (s. 131-142). New York: Springer International Publishing.

Kim, S., In, F. H. (2003). The Relationship Between Financial Variables and Real Economic Activity: Evidence from Spectral and Wavelet Analyses. *Studies in Nonlinear Dynamics Econometrics, 7*(4). 1-16.

Kumar, N.P. Puja, P. (2012). The Impact of Macroeconomic Fundamentals on Stock Prices Revisited: An Evidence from Indian Data. *MPRA Paper* (No. 38980), 1-24, available at http://mpra.ub.uni-muenchen.de/38980/

Lee, B. S. (1992). Causal Relations among Stock Returns, Interest Rates, Real Activity, and Inflation. *The Journal of Finance, 47*(4), 1591-1603.

Lee, J., Strazizich, M. C. (2003). Minimum Lagrange Multiplier Unit Root Test with Two Structural Breaks. *The Review of Economics and Statistics, 85*(4), 1082-1089.

Lindsay, R. W., Percival, D. B., Rothrock, D. (1996). The Discrete Wavelet Transform and the Scale Analysis of the Surface Properties of Sea Ice. *IEEE Transactions on Geoscience and Remote Sensing, 34*(3), 771-787.

Mallat, S. G. (1989). A Theory for Multiresolution Signal Decomposition: the Wavelet Representation. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 11*(7), 674-693.

Mauro, P. (2003). Stock Returns and Output Growth in Emerging and Advanced Economies. *Journal of Development Economics, 71*(1), 129-153.

Miner, N. E. (1998). An Introduction to Wavelet Theory and Analysis (No. SAND98-2265). Sandia National Laboratories, Albuquerque, NM, and Livermore, CA.

Mohanamani, P., Sivagnanasithi, T. (2012). Indian Stock Market and Aggregates Macroeconomic Variables: Time Series Analysis. *IOSR Journal of Economics Finance, 3*(6), 68-74.

Morck, R., Shleifer, A., Vishny, R. W., Shapiro, M., Poterba, J. M. (1990). The Stock Market and Investment: Is the Market a Sideshow?. *Brookings Papers on Economic Activity, 1990*(2), 157-215.

Muradoglu, G., Taşkın, F., Bigan, I. (2000). Causality between Stock Returns and Macroeconomic Variables in Emerging Markets. *Russian East European Finance and Trade, 36*(6), 33-53.

OECD (2017) Industrial Production (Indicator). doi:10.1787/39121c55-en (Accessed on 28 June 2017), https://data.oecd.org/industry/industrial-production.htm

Özer, A., Kaya, A., Özer, N. (2011). Hisse Senedi Fiyatlari ile Makroekonomik Değişkenlerin Etkileşimi. *Dokuz Eylül Üniversitesi İİBF Dergisi, 26*(1), 163-182.

Panopoulou, E. (2009). Financial Variables and Euro Area Growth: A Non-parametric Causality Analysis. *Economic Modelling, 26*(6), 1414-1419.

Percival, D. B., & Guttorp, P. (1994). Long-memory processes, the Allan variance, and wavelets. In *Wavelet Analysis and its Applications* (Vol. 4, s.325-344). New York: Academic Press.

Percival, D. B., Mofjeld, H. O. (1997). Analysis of Subtidal Coastal Sea Level Fluctuations Using Wavelets. *Journal of the American Statistical Association, 92*(439), 868-880.

Percival, D. B., Walden, A. T. (2000). *Wavelet Methods for Time Series Analysis* (4th Edition). UK: Cambridge University Press.

Percival, D. P. (1995). On Estimation of the Wavelet Variance. *Biometrika, 82*(3), 619-631.

Perron, P. (1989). The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis. *Econometrica: Journal of the Econometric Society*, 1361-1401.

Pradhan, R. P., Arvin, M. B., Bahmani, S. (2015). Causal Nexus between Economic Growth, Inflation, and Stock Market Development: The Case of OECD Countries. *Global Finance Journal, 27*, 98-111.

Ramsey, J. B. (2014). Functional Representation, Approximation, Bases, and Wavelets. Gallegati Semmler (Ed.), In *Wavelet Applications in Economics and Finance* (s. 1-20). New York: Springer International Publishing.

Ramsey, J. B., Lampart, C. (1998). The Decomposition of Economic Relationships by Time Scale Using Wavelets: Expenditure and Income. *Studies in Nonlinear Dynamics Econometrics, 3*(1), 49-71.

Ratanapakorn, O., Sharma, S. C. (2007). Dynamic Analysis between the US Stock Returns and the Macroeconomic Variables. *Applied Financial Economics, 17*(5), 369-377.

Saidi, L., Adam, P., Saenong, Z., Balaka, M. Y. (2017). The Effect of Stock Prices and Exchange Rates on Economic Growth in Indonesia. *International Journal of Economics and Financial Issues, 7*(3), 527-533.

Sancar, C., Uğur, A., Akbaş, Y. E. (2017). Hisse Senedi Fiyat Endeksi ile Makroekonomik Değişkenler Arasındaki Ilişkinin Analizi: Türkiye Örneği. *International Journal of Social Sciences and Education Research, 3*(5), 1774-1786.

Schwert, G. W. (1990). Stock Returns and Real Activity: A Century of Evidence. *The Journal of Finance, 45*(4), 1237-1257.

Şentürk, M., Özkan, G. S., Akbaş, Y. E. (2014). The Relationship between Economic Growth and Stock Returns: Evidence from Turkey. *Doğuş Üniversitesi Dergisi, 15*(2), 155-164.

Singh, D. (2010). Causal Relationship between Macro-Economic Variables and Stock Market: A Case Study for India. *Pakistan Journal of Social Sciences (PJSS), 30*(2), 263-274.

Soman, K. P., Ramachandran, K. I., Resmi, N.G. (2010). *Insight into Wavelets: From Theory to Practice*. New Delhi, PHI Learning Pvt. Ltd.

Stock, J. H., Watson, M. W. (1990). Business Cycle Properties of Selected US Economic Time Series, 1959-1988. National Bureau of Economic Research, No. w3376.

Tiwari, A. K., Mutascu, M. I., Albulescu, C. T., Kyophilavong, P. (2015). Frequency Domain Causality Analysis of Stock Market and Economic Activity in India. *International Review of Economics Finance, 39*, 224-238.

Toda, H. Y., Yamamoto, T. (1995). Statistical Inference in Vector Auto-regressions with Possibly Integrated Processes. *Journal of Econometrics, 66*(1), 225-250.

Tsouma, E. (2009). Stock Returns and Economic Activity in Mature and Emerging Markets. *The Quarterly Review of Economics and Finance, 49*(2), 668-685.

Whitcher, B. J. (1998). *Assessing Non-Stationary Time Series Using Wavelets*. (Ph.D. Thesis). University of Washington Press, Washington.

Whitcher, B. J. (2005). Waveslim: Basic Wavelet Routines for One-, Two-and Three-Dimensional Signal Processing. R Package Version, 1(3).

Whitcher, B., Guttorp, P., Percival, D. B. (2000). Wavelet Analysis of Covariance with Application to Atmospheric Time Series. *Journal of Geophysical Research: Atmospheres (1984–2012), 105*(D11), 14941-14962.

Wongbangpo, P., Sharma, S. C. (2002). Stock Market and Macroeconomic Fundamental Dynamic Interactions: ASEAN-5 Countries. *Journal of Asian Economics, 13*(1), 27-51.

Yahoo-Finance (2017) https://finance.yahoo.com [January, 2018].

Yilmaz, Ö., Gungor, B., Kaya, V. (2006). Hisse Senedi Fiyatları ve Makro Ekonomik Değişkenler Arasındaki Eşbütünleşme ve Nedensellik. *İMKB Dergisi, 34*, 1- 16.