Secrecy Analysis of Multi-user Half/Full-Duplex Wireless Bi-directional Relaying Network

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***Abstract*—This study investigates the secrecy performance of the relay-assisted orthogonal frequency division multiplexing technique. The investigation utilizes a multi-user illegitimate half/full-duplex based bi-directional relaying, which is under the effect of a finite number of friendly jammers, system model. In reference to Monte-Carlo computer simulation results, the system model that operates in half-duplex mode achieves slightly better secrecy outage performance than the system model that operates in full-duplex mode. The results also show that the information leakage can be minimized by using friendly jammers, yet the friendly jammers degrade the system secrecy performance and result in system coding gain losses in high signal-to-noise ratios.**

***Index Terms*—Physical Layer Security, OFDM, Bi-directional Relay, Half/Full-Duplex Relay**

I. Introduction

IN RECENT years, the number of mobile users/applications have increased dramatically. This results severe mobile data traffic in the cellular coverage areas. A combined orthogonal frequency division multiplexing (OFDM) technique and relay-assited networks have made a great contribution to cover these capacity and throughput demands. However, information security demands are still an open challenge, which needs to be enhanced with cutting edge techniques. Until quite recently, the higher level encryption techniques, Rivest Shamir Adleman (RSA) [1] and advanced encryption standard (AES) [2], have been considered. However, these techniques do not provide a broad solution for the secure communication demands. In this regard, Wyner's [3] wiretap channels brought a new perspective and turn out to be a pioneering technique for the physical layer (PHY) security of wireless communications.

Recently, non-orthogonal multiple access (NOMA) strategy [4] has got much attention in the eyes of the researcher for the multi-user information exchange process. This is because, the NOMA strategy allows each user to access all sub-carrier channels, which directly affects the spectral efficiency [4]. It is also reported in [4] that the NOMA strategy achieves better performance than orthogonal multiple access strategies. However, since the NOMA technique employs the successive interference cancellation technique for the signal decoding, the last user should wait the other users' decoding process. This challenge is named as user-delay decoding process in the literature. This is indeed a big challenge especially in dense wireless/internet-of-things networks. To overcome this challenge, the OFDM technique is considered in this study.

In this regard, some of the studies in the literature that consider the OFDM/orthogonal frequency division multiple access (OFDMA) techniques to conduct information are summarized as: Reference [5] assumes that source terminal communicates with L destinations by means of a single amplify-and-forward (AF) one-way relay (OWR). [5] also assumes that each mobile station utilizes the OFDM transceiver with N subcarriers. Reference [5] considers a joint optimization of power allocation, subcarrier allocation, and subcarrier pairing to maximize the secrecy rate. Reference [6] utilizes a system that source terminal communicatates with the destination by means of a single decode-and-forward (DF) relay. [6] also considers that the communication overhears by an illegitimate terminal. [6] also considers the OFDM technique for the information exchange and also assumes three possible transmission scenarios: no communication, direct communication, and relay communication. In addition, [6] utilizes the power allocation strategy for maximizing the system sum-secrecy rate.

Reference [7] considers that M pre-assigned partner users communicate via two-way relay (TWR) terminal, which has an OFDMA technique, in the presence of an eavesdropper with/without cooperative jamming. [7] also considers that relay terminal operates in the half-duplex (HD) mode with an AF strategy. [7] also utilizes power allocation strategies for maximizing the system secrecy sum-rate. [8] considers an OFDMA downlink network, which contains a single antenna K mobile users and multiple antennas equipped M HD based DF relays and also a multiple antenna base station. [8] also assumes that system structure contains an eavesdropper with multiple antenna. [8] also considers optimization for the secure resource allocations and scheduling. [9] considers the single antenna OFDMA based multi-user and multi AF based OWR network with an eavesdropper. [9] also considers resource optimization PHY security of such a system model. Reference [10] assumes that the source terminal conducts information with the destination terminal via L available DF based OWR terminals in the presence of a single illegitimate terminal. [10] also assumes that each terminal in the system model posses an OFDM transceiver that has N sub-carriers. [10] investigates the system secrecy rates and outage performance.

The aformentioned studies consider various types of system structures and also considers that the trustworthy/untrustworthy relay terminal run in HD mode. Alternatively, [11] considers multi-user one-way information exchange traffic with an untrustworthy full-duplex (FD) relay terminal by using OFDM strategy. In addition, to mitigate the information leakage, [11] also considers that the illegitimate terminal is under the effect

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of a finite number of friendly jammers and also investigates secrecy outage probability (SOP) performance. This paper distinguishes itself from aforementioned studies in a several ways. The differences can be summarized as: First, this paper utilizes a multi-user untrustworthy AF HD/FD based TWR system structure that considers OFDM strategy. Second, this paper utilizes the SOP performance metric and investigates the secrecy performance of given system structure, which is presented in figure 1.

The rest of the paper is organized as follows: Section II provides the channel statistics and system structure details. Section III presents the analytical derivations. Section IV provides the numerical results and the paper is finalized in section V.

Notations: The $F_h(.)$ and $f_h(.)$ represent the cumulative distribution function (CDF) and the probability density function (PDF) of a random variables (RVs) h, respectively. The $\mathbb{E}[.]$ term represents the expectation, while the $\Pr(.)$ represents the probability. All $\log$ are considered base 2. The term $G^{m,n}_{p,q}$ represents the Meijer-G function [12] and the term $G^{m,0:m_1,n_1:m_2,n_2}_{q,p:p_1,q_1:p_2,q_2}$ is the extended generalized bivariate Meijer-G function (EGBMGF) [13, Eq. (13)].

II. System Model and Channel Statistics

Figure 1 plots a multi-user AF based untrustworthy two-way HD/FD based relaying network. Figure 1 also plots that to mitigate the information leakage, the illegitimate terminal is under the effect of a finite number of friendly jammers. Here, x_k and y_k, $\forall_k = 1, ..., N$, conduct information exchange, by means of k^{th} sub-carrier among N, via a single HD/FD based untrustworthy TWR terminal. In the case that the untrustworthy relay terminal in HD mode, the user terminals conduct information exchange in two phases, which are multiple access (MAC) and broadcast (BC), while it requires a single phase, which is MAC, for the FD mode. Please note that MAC and BC channels are considered as reciprocal in such a system model. x_k and y_k terminals do not have a direct-link because of the possible obstacles. It is also assumed that legitimate/illegitimate terminals posses a single omni-directional antenna in the system model.

In figure 1, h_k and g_k, $\forall_k = 1, ..., N$ represent the channel impulse responses between $x_k \rightarrow$ untrustworthy relay and $y_k \rightarrow$ untrustworthy relay, respectively. h_k are independent and identically distributed (i.i.d.) complex Gaussian RVs with zero mean and variances $\sigma^2_{h_k}$. (i.e. $h_k \sim \mathcal{CN}(0, \sigma^2_{h_k})$). Likewise, g_k are also i.i.d. and $g_k \sim \mathcal{CN}(0, \sigma^2_{g_k})$. $a_k \sim \mathcal{CN}(0, \sigma^2_{a_k})$, $b_k \sim \mathcal{CN}(0, \sigma^2_{b_k})$, and $c_k \sim \mathcal{CN}(0, \sigma^2_{c_k})$ represent the loop-interference (LI), which is caused by transmitting and receiving the signal at the same time period, at x_k, y_k, and untrustworthy relay, respectively. f_j are also i.i.d. and $f_j \sim \mathcal{CN}(0, \sigma^2_{f_j})$, $\forall_j = 1, ..., M$ is the channel impulse response, j^{th} friendly jammer $\rightarrow$ relay. Amplitudes of all channels are distributed according to the Rayleigh distributions.

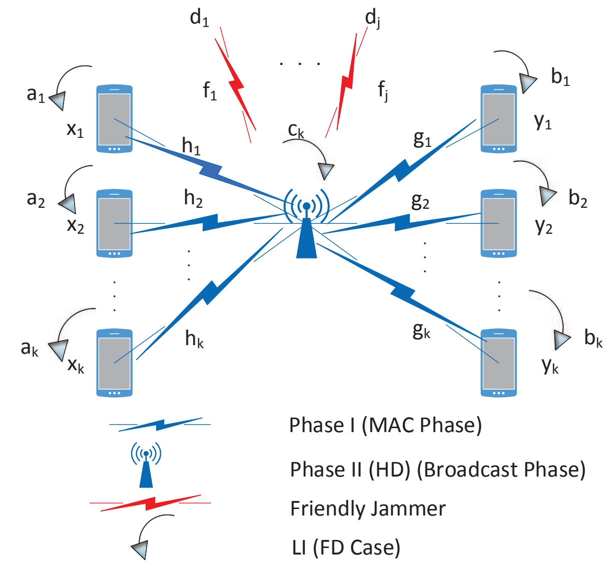

Fig. 1: The OFDM based multi-user AF HD/FD TWR.

A. The Half-Duplex Case

This subsection now turns its attention to the illegitimate terminal operates in the HD mode. The received signals at the HD based illegitimate relay terminal on k^{th} sub-carrier can be written as

$$Z^{HD}_{r_k} = \sqrt{P_s} m_{x_k} h_k + \sqrt{P_s} m_{y_k} g_k + \sum_{j=1}^{M} \sqrt{P_J} f_j d_j + n_{r_k} \tag{1}$$

Here, m_{x_k} and m_{y_k} represent the corresponding transmit information of x_k and y_k on k^{th} sub-carrier, respectively. Here, $\mathbb{E}[|m_{x_k}|^2] = 1$, $\mathbb{E}[|m_{y_k}|^2] = 1$, and $\mathbb{E}[|d_j|^2] = 1$. P_s represents the corresponding transmit power of x_k and y_k terminals. P_J represents the j^{th} friendly jammer's transmit power. n_{r_k} is the additive white Gaussian noise (AWGN) at the untrustworthy relay terminal on k^{th} sub-carrier. The leakage rate (LR) expressions with regard to x_k and y_k on k^{th} sub-carrier can be written as

$$LR^{HD}_{x_k} = \frac{1}{2} \log \left(1 + \frac{\gamma_x}{\gamma_y + \gamma_J + \sigma^2} \right) \tag{2}$$

$$LR^{HD}_{y_k} = \frac{1}{2} \log \left(1 + \frac{\gamma_y}{\gamma_x + \gamma_J + \sigma^2} \right) \tag{3}$$

where σ^2 is the noise variance, $\gamma_{x_k} = \frac{P_s |h_k|^2}{\sigma^2}$, $\gamma_{y_k} = \frac{P_s |g_k|^2}{\sigma^2}$, and $\gamma_J = \sum_{j=1}^{M} \frac{P_J |f_j|^2}{\sigma^2}$. Since the untrustworthy relay terminal in AF mode, the amplification factor, which is G, on k^{th} sub-carrier can be calculated as

$$G^{HD}_k = \sqrt{\frac{P_r}{P_s |h_k|^2 + P_s |g_k|^2 + \sum_{j=1}^{M} P_J |f_j|^2 + \sigma^2}} \tag{4}$$

where P_r represents the untrustworthy relay terminal's transmit power. In the second phase, the untrustworthy relay terminal broadcasts the amplified version of the received signals to the user-pairs. The received signal at x_k on k^{th} sub-carrier can be calculated as

$$Z_{x_k}^{HD} = G^{HD}\sqrt{P_s}m_{x_k}h_k^2 + G^{HD}\sqrt{P_s}m_{y_k}g_kh_k + G^{HD}\sum_{j=1}^{M}\sqrt{P_J}f_jd_jh_k + G^{HD}n_rh_k + n_{x_k} \quad (5)$$

Substituting (4) into (5), the signal-to-interference noise ratio (SINR) at x_k on k^{th} sub-carrier can be calculated as

$$\gamma_{x_k}^{HD} = \frac{\frac{\varphi\gamma_x\gamma_y}{[\gamma_J+\sigma^2]}}{\varphi\gamma_x + \frac{\varphi\gamma_x+\gamma_x+\gamma_y}{[\gamma_J+\sigma^2]} + 1} \quad (6)$$

where $\varphi = \frac{P_r}{P_s}$ [14]. Likewise, γ_{y_k} on k^{th} sub-carrier can be calculated as

$$\gamma_{y_k}^{HD} = \frac{\frac{\varphi\gamma_x\gamma_y}{[\gamma_J+\sigma^2]}}{\varphi\gamma_y + \frac{\varphi\gamma_y+\gamma_x+\gamma_y}{[\gamma_J+\sigma^2]} + 1} \quad (7)$$

B. Full-Duplex Case

The subsection assumes that the illegitimate terminal runs in the FD mode. In this regard, the received signal at the illegitimate terminal on k^{th} sub-carrier can be written as

$$Z_{r_k}^{FD} = \sqrt{P_s}m_{x_k}h_k + \sqrt{P_s}m_{y_k}g_k + \sum_{j=1}^{M}\sqrt{P_J}f_jd_j + \sqrt{P_r}c_k + n_r \quad (8)$$

By using (8), the LR expressions with respect to x_k and y_k on k^{th} sub-carrier can be calculated as

$$LR_{x_k}^{FD} = \log\left(1 + \frac{\gamma_x}{\gamma_y + \gamma_J + \gamma_{c_k} + \sigma^2}\right) \quad (9)$$

$$LR_{y_k}^{FD} = \log\left(1 + \frac{\gamma_y}{\gamma_x + \gamma_J + \gamma_{c_k} + \sigma^2}\right) \quad (10)$$

Here, $\gamma_{c_k} = \frac{P_r|c_k|^2}{\sigma^2}$. The G amplification factor on k^{th} sub-carrier for the FD case can be re-calculated as

$$G_k^{FD} = \sqrt{\frac{P_r}{P_s|h_k|^2 + P_s|g_k|^2 + \sum_{j=1}^{M} P_J|f_j|^2 + P_r|c_k|^2 + \sigma^2}} \quad (11)$$

The received signal at x_k on k^{th} sub-carrier can be calculated as

$$Z_{x_k}^{FD} = G^{FD}\sqrt{P_s}m_{x_k}h_k^2 + G^{FD}\sqrt{P_s}m_{y_k}g_kh_k + G^{FD}\sum_{j=1}^{M}\sqrt{P_J}f_jd_jh_k + G^{FD}\sqrt{P_r}c_kh_k + G^{FD}n_rh_k + \sqrt{P_s}a_k + n_{x_k} \quad (12)$$

Substituting (11) into (12) and doing some mathematical manipulations, the received SINR at x_k on k^{th} sub-carrier can be calculated as in (13). Likewise, the received SINR at y_k on k^{th} sub-carrier can be calculated as in (14).

III. Performance Analysis

This section gives analytical derivations related to the secrecy of the multi-user HD/FD based relay-assisted TWR with OFDM strategy. In this regard, the SOP is considered as a performance metric and the details are presented in following subsection.

A. The Secrecy Outage Probability

The SOP is defined as the secrecy achievable rate, which is based on subtracting the LR expression from the system total achievable rate, cannot support R in bps/Hz, which is a pre-defined target rate. From the analytical perspective, by using the logarithm properties, the SOP can also be defined as the CDF of the secrecy achievable rate's received SINR evaluated at target threshold rate, γ_{th}. The total secrecy rate expression at X line users with respect to HD and FD strategies are given as follows:

$$R_X^{HD} = \frac{1}{2}\left[\sum_{k=1}^{N}\log\left(1+\gamma_{x_k}^{HD}\right) - \log\left(1+\gamma_{x_k,R}^{HD}\right)\right]^+ \quad (15)$$

$$R_X^{FD} = \left[\sum_{k=1}^{N}\log\left(1+\gamma_{x_k}^{FD}\right) - \log\left(1+\gamma_{x_k,R}^{FD}\right)\right]^+ \quad (16)$$

where $[\mathrm{x}]^+ = \max(0, \mathrm{x})$. The end-to-end (e2e) SOP for HD and FD cases can be written as

$$R_{e2e}^{HD} = \Pr\left(\min\left(\mathrm{R_X^{HD}}, \mathrm{R_Y^{HD}}\right) \leq \mathrm{R}\right) \quad (17)$$

$$R_{e2e}^{FD} = \Pr\left(\min\left(\mathrm{R_X^{FD}}, \mathrm{R_Y^{FD}}\right) \leq \mathrm{R}\right) \quad (18)$$

Here, R_Y^{HD} and R_Y^{FD} are the symmetry of R_X^{HD} and R_X^{FD}, respectively. The CDF expressions of (17) and (18) can be calculated as in the proposition 1 and proposition 2, respectively.

IV. Numerical Results

This section provides numerical results, which is based on monte-carlo simulations, regarding the system e2e secrecy outage performance. The friendly jammers are located by using the Euclidean distance formulation, which is d^{-v} [15], in the system model. The d term represents the distance and v term represents the path-loss exponent, which takes values between 2-6 [15]. In simulation setup, the d and v terms are set to 10 and 2, respectively. In this regard, the friendly jammers' transmit power, P_J, is chosen relatively low, which is $P_J \ll P_T$, $P_J = P_T/100$, in comparison to user-pairs' total transmit powers, which is $P_T = 2NP_s + P_r$. In an equal interference and user-pairs' transmit power case, the external interference severely degrades the system performance [16]. The number of the friendly jammer, which is M, is set to 1 and the friendly jammers' channel variances, which is $\sigma_{f_j}^2$ are set to 10^{-2}.

$$\gamma_{x_k}^{FD} = \frac{\frac{\varphi\gamma_x\gamma_y}{[\gamma_J+\sigma^2][\gamma_c+\sigma^2]\left[\gamma_{a_k}+\sigma^2\right]}}{\left[\frac{\varphi\gamma_x}{[\gamma_c+\sigma^2]\left[\gamma_{a_k}+\sigma^2\right]} + \frac{\gamma_x(\varphi+1)+\gamma_y}{[\gamma_J+\sigma^2]\left[\gamma_{a_k}+\sigma^2\right][\gamma_c+\sigma^2]} + \frac{\varphi\gamma_x}{[\gamma_J+\sigma^2]\left[\gamma_{a_k}+\sigma^2\right]} + \frac{\gamma_x+\gamma_y}{[\gamma_J+\sigma^2][\gamma_c+\sigma^2]} + 1\right]} \tag{13}$$

$$\gamma_{y_k}^{FD} = \frac{\frac{\varphi\gamma_x\gamma_y}{[\gamma_J+\sigma^2][\gamma_c+\sigma^2]\left[\gamma_{b_k}+\sigma^2\right]}}{\left[\frac{\varphi\gamma_y}{[\gamma_c+\sigma^2]\left[\gamma_{b_k}+\sigma^2\right]} + \frac{\gamma_y(\varphi+1)+\gamma_x}{[\gamma_J+\sigma^2]\left[\gamma_{b_k}+\sigma^2\right][\gamma_c+\sigma^2]} + \frac{\varphi\gamma_y}{[\gamma_J+\sigma^2]\left[\gamma_{b_k}+\sigma^2\right]} + \frac{\gamma_x+\gamma_y}{[\gamma_J+\sigma^2][\gamma_c+\sigma^2]} + 1\right]} \tag{14}$$

By using the signal processing strategies and special antenna design, the LI effects can be minimized. In this regard, the LI variances at x_k and y_k, which are $\sigma_{a_k}^2$ and $\sigma_{c_k}^2$, and at untrustworthy relay terminal, which is $\sigma_{b_k}^2$, are modeled as: $P_s^{\lambda-1}$ and $P_r^{\lambda-1}$, respectively. The λ term takes values between $0 \leq \lambda \leq 1$ [17]. The λ term is set to 0.1 in the simulation setup. The number of the mobile terminal, which is N, is set to 2, 4, 6, 8, and 10 in the system model setup. The x_k and y_k have P_s transmit power and the untrustworthy relay terminal has P_r transmit power. In this regard, as earlier mentioned the system total transmit power is equal to $P_T = 2NP_s + P_r$. Two different target rates, which are $R = 1$ bps/Hz and $R = 3$ bps/Hz, are considered in the performance analysis. Figure 2 and figure 3 utilize $R = 1$ bps/Hz and $R = 3$ bps/Hz target rates, respectively. Commenting the figure 2 and 3 based on the aforementioned system model configurations, following results can be obtained.

Figure 2 presents the e2e secrecy outage performance comparison of the HD and FD based system model configurations. According to figure 2, a large number of user achieve better e2e secrecy outage performance than a small number of user in low and high SNR regimes. This is because a large number of sub-carrier allow more users to conduct information exchange compared to a small number of users. This also means that a large number of users posses more total transmit powers, which is directly related to $P_T = 2NP_s + P_r$. Results also show that the secrecy outage performance curves tend to saturate in high SNR regimes. This is because the friendly jammers' negative effects on the system secrecy performance. In addition, the system model that operates in HD mode achieves slightly better outage performance than FD mode. This is because the LI effect on the FD mode. A large number of user reach the 10^{-5} outage levels while a small number of users saturate in high outage regimes, which is around 10^{-2} and 10^{-4}.

Figure 3 also plots the e2e secrecy outage performance comparison of the HD and FD based system model. Differently from figure 2, the figure 3 utilizes the $R = 3$ bps/Hz. As in figure 2, a large number of users achieve better secrecy outage performance than a small number of users. In addition, the HD based system model achieves slightly better performance than FD mode. However, the outage performance gap between two modes become less then $R = 1$ bps/Hz. This is because the pre-log factor differences as described in (15) and (16). In addition, by definition of the secrecy outage probability, which is described in section III A, the outage performance curves slightly move to the high SNR regimes.

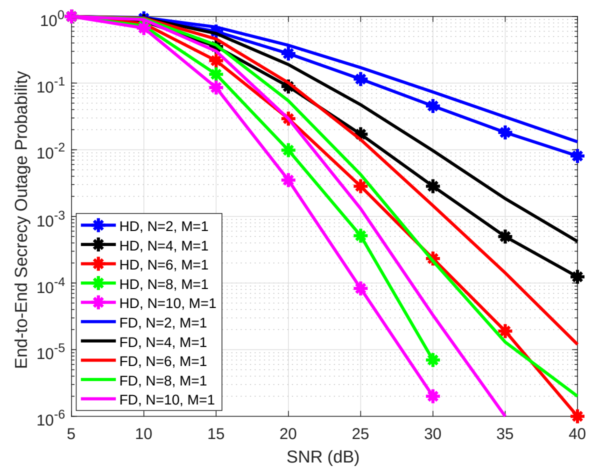

Fig. 2: The e2e secrecy outage performance comparison of the OFDM based multi-user HD/FD based TWR network with $N = 2, 4, 6, 8, 10$, $M = 1$, and $R = 1$ bps/Hz.

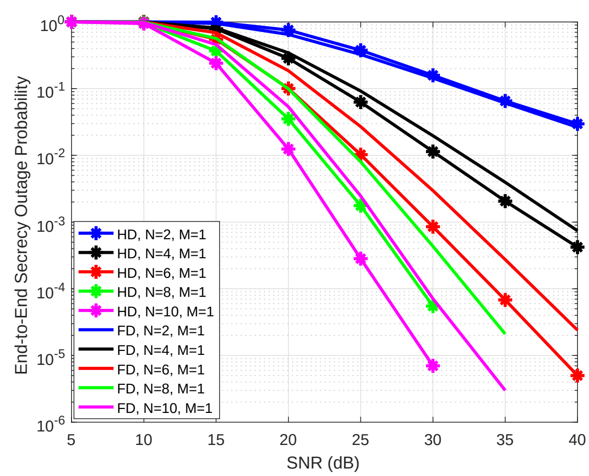

Fig. 3: The e2e secrecy outage performance comparison of the OFDM based multi-user HD/FD based TWR network with $N = 2, 4, 6, 8, 10$, $M = 1$, and $R = 3$ bps/Hz.

Proposition 1: The CDF expression of the R_{e2e}^{HD} can be re-written as

$$
\begin{aligned}
F^{\mathrm{up}}_{R_{e2e}^{HD}}\left(\gamma_{th}^{HD}\right) = 1 - \Bigg[& \frac{P_s\Omega_{g_k}}{\Gamma(2)\Gamma(M)} G^{1,0:1,1:1,1}_{1,0:1,1:1,1}\left(\begin{array}{c|c|c|c} 1 & -1 & 1-M & \\ - & 0 & 0 & P_s\Omega_{g_k}, P_J\Omega_{f_j}\end{array}\right) \\
& + M\frac{P_J\Omega_{f_j}}{\Gamma(M+1)} G^{1,0:1,1:1,1}_{1,0:1,1:1,1}\left(\begin{array}{c|c|c|c} 1 & 0 & -M & \\ - & 0 & 0 & P_s\Omega_{g_k}, P_J\Omega_{f_j}\end{array}\right) + \frac{1}{\Gamma(M)} G^{1,0:1,1:1,1}_{1,0:1,1:1,1}\left(\begin{array}{c|c|c|c} 1 & 0 & 1-M & \\ - & 0 & 0 & P_s\Omega_{g_k}, P_J\Omega_{f_j}\end{array}\right) \\
& - \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)\Theta^{-1}\sum_{i=1}^{M}\frac{A_i}{\Gamma(i)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}\right)}{\Theta}\,\middle|\, \begin{array}{c} 1-i,0 \\ 0,- \end{array}\right) - \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)\Theta^{-1}\frac{V}{\Gamma(2)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)}{\Theta}\,\middle|\, \begin{array}{c} -1,0 \\ 0,- \end{array}\right) \\
& - \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)\Theta^{-1}\sum_{l=1}^{M}\frac{C_l}{\Gamma(l)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}\right)}{\Theta}\,\middle|\, \begin{array}{c} 1-l,0 \\ 0,- \end{array}\right) - M\left(\frac{P_j\Omega_{f_k}}{P_s\Omega_{h_k}}\right)\Theta^{-1}\sum_{aa=1}^{M}\frac{D_{aa}}{\Gamma(aa)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}\right)}{\Theta}\,\middle|\, \begin{array}{c} 1-aa,0 \\ 0,- \end{array}\right) \\
& - M\left(\frac{P_j\Omega_{f_k}}{P_s\Omega_{h_k}}\right)E\Theta^{-1} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)}{\Theta}\,\middle|\, \begin{array}{c} 0,0 \\ 0,- \end{array}\right) - M\left(\frac{P_j\Omega_{f_k}}{P_s\Omega_{h_k}}\right)\Theta^{-1}\sum_{bb=1}^{M+1}\frac{F_{bb}}{\Gamma(bb)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}\right)}{\Theta}\,\middle|\, \begin{array}{c} 1-bb,0 \\ 0,- \end{array}\right) \\
& - \left(\frac{1}{P_s\Omega_{h_k}}\right)\Theta^{-1}\sum_{cc=1}^{M}\frac{G_{cc}}{\Gamma(cc)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}\right)}{\Theta}\,\middle|\, \begin{array}{c} 1-cc,0 \\ 0,- \end{array}\right) - \left(\frac{1}{P_s\Omega_{h_k}}\right)\Theta^{-1}\frac{H}{\Gamma(1)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)}{\Theta}\,\middle|\, \begin{array}{c} 0,0 \\ 0,- \end{array}\right) \\
& - \left(\frac{1}{P_s\Omega_{h_k}}\right)\Theta^{-1}\sum_{bb=1}^{M}\frac{I_{dd}}{\Gamma(dd)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}\right)}{\Theta}\,\middle|\, \begin{array}{c} 1-dd,0 \\ 0,- \end{array}\right)\Bigg] \times \Bigg[\frac{P_s\Omega_{h_k}}{\Gamma(2)\Gamma(M)} G^{1,0:1,1:1,1}_{1,0:1,1:1,1}\left(\begin{array}{c|c|c|c} 1 & -1 & 1-M & \\ - & 0 & 0 & P_s\Omega_{h_k}, P_J\Omega_{f_j}\end{array}\right) \\
& + M\frac{P_J\Omega_{f_j}}{\Gamma(M+1)} G^{1,0:1,1:1,1}_{1,0:1,1:1,1}\left(\begin{array}{c|c|c|c} 1 & 0 & -M & \\ - & 0 & 0 & P_s\Omega_{h_k}, P_J\Omega_{f_j}\end{array}\right) + \frac{1}{\Gamma(M)} G^{1,0:1,1:1,1}_{1,0:1,1:1,1}\left(\begin{array}{c|c|c|c} 1 & 0 & 1-M & \\ - & 0 & 0 & P_s\Omega_{h_k}, P_J\Omega_{f_j}\end{array}\right) \\
& - \left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\right)\Lambda^{-1}\sum_{ee=1}^{M}\frac{J_{ee}}{\Gamma(ee)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{h_k}}\right)}{\Lambda}\,\middle|\, \begin{array}{c} 1-ee,0 \\ 0,- \end{array}\right) - \left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\right)\Lambda^{-1}\frac{K}{\Gamma(2)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\right)}{\Theta}\,\middle|\, \begin{array}{c} -1,0 \\ 0,- \end{array}\right) \\
& - \left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\right)\Lambda^{-1}\sum_{l=1}^{M}\frac{L_{ff}}{\Gamma(ff)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{g_k}}\right)}{\Theta}\,\middle|\, \begin{array}{c} 1-ff,0 \\ 0,- \end{array}\right) - M\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\right)\Lambda^{-1}\sum_{gg=1}^{M}\frac{M_{gg}}{\Gamma(gg)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{h_k}}\right)}{\Lambda}\,\middle|\, \begin{array}{c} 1-gg,0 \\ 0,- \end{array}\right) \\
& - M\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\right)N^{***}\Lambda^{-1} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\right)}{\Lambda}\,\middle|\, \begin{array}{c} 0,0 \\ 0,- \end{array}\right) - M\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\right)\Lambda^{-1}\sum_{hh=1}^{M+1}\frac{O_{hh}}{\Gamma(hh)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{g_k}}\right)}{\Lambda}\,\middle|\, \begin{array}{c} 1-hh,0 \\ 0,- \end{array}\right) \\
& - \left(\frac{1}{P_s\Omega_{g_k}}\right)\Lambda^{-1}\sum_{ii=1}^{M}\frac{P_{ii}}{\Gamma(ii)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{h_k}}\right)}{\Lambda}\,\middle|\, \begin{array}{c} 1-ii,0 \\ 0,- \end{array}\right) - R\left(\frac{1}{P_s\Omega_{g_k}}\right)\Lambda^{-1} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\right)}{\Lambda}\,\middle|\, \begin{array}{c} 0,0 \\ 0,- \end{array}\right) \\
& - \left(\frac{1}{P_s\Omega_{g_k}}\right)\Lambda^{-1}\sum_{jj=1}^{M}\frac{S_{jj}}{\Gamma(jj)} G^{1,2}_{2,1}\left(\frac{\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{g_k}}\right)}{\Lambda}\,\middle|\, \begin{array}{c} 1-jj,0 \\ 0,- \end{array}\right)\Bigg] \qquad (19)
\end{aligned}
$$

where $\Theta = \left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}} + \frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}} + \frac{1}{P_s\Omega_{h_k}}\right)$ and $\Lambda = \left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{g_k}} + \frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{h_k}} + \frac{1}{P_s\Omega_{g_k}}\right)$.

Proof: See Appendix A. ■

V. CONCLUSION

This study has investigated the secrecy performance of the multi-user HD/FD based relay-assisted TWR with OFDM strategy. In reference to Monte-Carlo computer simulations, the HD based untrustworthy relaying system achieves better secrecy outage performance in comparison to FD case. Results have also showed that the friendly jammers beside minimizing the information leakage also result in system coding gain losses in high SNR regimes. In addition, in the case that the target rate increases the system secrecy performance gets worse.

Proposition 2: The CDF expression of R_{e2e}^{FD} can be calculated as

$$\begin{aligned}
F^{\text{up}}_{R^{FD}_{e2e}}\left(\gamma_{th}^{FD}\right) = 1 - \Bigg[& \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right) A^* \frac{P_s\Omega_{h_k}}{\Gamma(2)} G_{2,1}^{1,2}\left(P_s\Omega_{g_k} \,\middle|\, \begin{matrix} -1,0 \\ 0,- \end{matrix} \right) + \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right) \sum_{a^3=1}^{M} B^*_{a^3} \frac{P_s\Omega_{h_k}}{\Gamma(a^3)} G_{2,1}^{1,2}\left(P_j\Omega_{f_j} \,\middle|\, \begin{matrix} 1-a^3,0 \\ 0,- \end{matrix} \right) \\
& + \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right) C^* P_s\Omega_{h_k} G_{2,1}^{1,2}\left(P_r\Omega_{c_k} \,\middle|\, \begin{matrix} 0,0 \\ 0,- \end{matrix} \right) + D^* P_s\Omega_{h_k} G_{2,1}^{1,2}\left(P_s\Omega_{g_k} \,\middle|\, \begin{matrix} 0,0 \\ 0,- \end{matrix} \right) \\
& + \sum_{b^3=1}^{M+1} E^*_{b^3} \frac{P_s\Omega_{h_k}}{\Gamma(b^3)} G_{2,1}^{1,2}\left(P_j\Omega_{f_j} \,\middle|\, \begin{matrix} 1-b^3,0 \\ 0,- \end{matrix} \right) + F^* P_s\Omega_{h_k} G_{2,1}^{1,2}\left(P_r\Omega_{c_k} \,\middle|\, \begin{matrix} 0,0 \\ 0,- \end{matrix} \right) \\
& + G^* P_s\Omega_{h_k} G_{2,1}^{1,2}\left(P_s\Omega_{g_k} \,\middle|\, \begin{matrix} 0,0 \\ 0,- \end{matrix} \right) + \sum_{c^3=1}^{M} H^*_{c^3} \frac{P_s\Omega_{h_k}}{\Gamma(c^3)} G_{2,1}^{1,2}\left(P_j\Omega_{f_j} \,\middle|\, \begin{matrix} 1-c^3,0 \\ 0,- \end{matrix} \right) \\
& + I^* \frac{P_s\Omega_{h_k}}{\Gamma(2)} G_{2,1}^{1,2}\left(P_r\Omega_{c_k} \,\middle|\, \begin{matrix} -1,0 \\ 0,- \end{matrix} \right) + J^* P_s\Omega_{h_k} G_{2,1}^{1,2}\left(P_s\Omega_{g_k} \,\middle|\, \begin{matrix} 0,0 \\ 0,- \end{matrix} \right) \\
& + \sum_{d^3=1}^{M} K^*_{d^3} \frac{P_s\Omega_{h_k}}{\Gamma(d^3)} G_{2,1}^{1,2}\left(P_j\Omega_{f_j} \,\middle|\, \begin{matrix} 1-d^3,0 \\ 0,- \end{matrix} \right) + L^* \frac{P_s\Omega_{h_k}}{\Gamma(2)} G_{2,1}^{1,2}\left(P_r\Omega_{c_k} \,\middle|\, \begin{matrix} 0,0 \\ 0,- \end{matrix} \right) \\
& - \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right) \delta^{-1} \sum_{e^3=1}^{M} \frac{M^*_{e^3}}{\Gamma(e^3)} G_{2,1}^{1,2}\left(\frac{\frac{P_J\Omega_{f_j}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}}{\delta} \,\middle|\, \begin{matrix} 1-e^3,0 \\ 0,- \end{matrix} \right) - \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right) \delta^{-1} N^* G_{2,1}^{1,2}\left(\frac{\frac{P_r\Omega_{c_k}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}}{\delta} \,\middle|\, \begin{matrix} 0,0 \\ 0,- \end{matrix} \right) \\
& - \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right) \delta^{-1} O^* G_{2,1}^{1,2}\left(\frac{\left(\frac{\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{g_k}\gamma_{th}^{FD} + \varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{h_k}\gamma_{th}^{FD}}{P_s\Omega_{h_k}P_s\Omega_{g_k}}\right)}{\delta} \,\middle|\, \begin{matrix} 0,0 \\ 0,- \end{matrix} \right) - \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right) \delta^{-1} \frac{P^*}{\Gamma(2)} G_{2,1}^{1,2}\left(\frac{\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}}{\delta} \,\middle|\, \begin{matrix} -1,0 \\ 0,- \end{matrix} \right) \\
& - \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right) \delta^{-1} \sum_{f^3=1}^{M} \frac{R^*_{f^3}}{\Gamma(f^3)} G_{2,1}^{1,2}\left(\frac{\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}}{\delta} \,\middle|\, \begin{matrix} 1-f^3,0 \\ 0,- \end{matrix} \right) - \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right) \delta^{-1} S^* G_{2,1}^{1,2}\left(\frac{\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}}{\delta} \,\middle|\, \begin{matrix} 0,0 \\ 0,- \end{matrix} \right) \Bigg]
\end{aligned} \tag{20}$$

where $\delta = \left(\frac{2\varphi^{-1}\gamma_{th}^{FD}}{P_s\Omega_{h_k}} + \frac{2\varphi^{-1}+3}{P_s\Omega_{g_k}} + \frac{\gamma_{th}^{FD}}{P_s\Omega_{h_k}}\right)$

Proof: See Appendix B. ■

PROOF OF PROPOSITION 1

Considering (15) and utilizing the definition of the logarithm properties and Appendix II of [16] and also Appendix A of [11] , R_X^{HD} can be re-written as

$$\begin{aligned}
R_X^{HD} &= \frac{1}{2}\left[\sum_{k=1}^{N}\left(\log_2\left(1+\gamma_{x_k}^{HD}\right) - \log_2\left(1+\gamma_{x_k,R}^{HD}\right)\right)\right]^{+} \leq R \\
&\approx \frac{\gamma_{x_k}^{HD}}{\gamma_{x_k,R}^{HD}} \leq \underbrace{2^{\frac{2R}{N}}}_{\gamma_{th}^{HD}}
\end{aligned} \tag{21}$$

Following the same procedures as in (21), R_Y^{HD} can be written as $\frac{\gamma_{y_k}^{HD}}{\gamma_{y_k,R}^{HD}} \leq \underbrace{2^{\frac{2R}{N}}}_{\gamma_{th}^{HD}}$. With the help of these expressions, (17) can be written as

$$\begin{aligned}
F_{R_{e2e}^{HD}}\left(\gamma_{th}^{HD}\right) &= \mathrm{P_r}\left(\min\left(\mathrm{R_X^{HD}}, \mathrm{R_Y^{HD}}\right) \leq \mathrm{R}\right) \\
&= \mathrm{P_r}\left(\min\left(\frac{\gamma_{\mathrm{x_k}}^{\mathrm{HD}}}{\gamma_{\mathrm{x_k,R}}^{\mathrm{HD}}}, \frac{\gamma_{\mathrm{y_k}}^{\mathrm{HD}}}{\gamma_{\mathrm{y_k,R}}^{\mathrm{HD}}}\right) \leq \gamma_{\mathrm{th}}^{\mathrm{HD}}\right) \\
&= 1 - \mathrm{Pr}\left(\frac{\gamma_{\mathrm{x_k}}^{\mathrm{HD}}}{\gamma_{\mathrm{x_k,R}}^{\mathrm{HD}}} \geq \gamma_{\mathrm{th}}^{\mathrm{HD}}, \frac{\gamma_{\mathrm{y_k}}^{\mathrm{HD}}}{\gamma_{\mathrm{y_k,R}}^{\mathrm{HD}}} \geq \gamma_{\mathrm{th}}^{\mathrm{HD}}\right) \\
&= 1 - \mathrm{Pr}\left(\gamma_{\mathrm{x_k}}^{\mathrm{HD}} \geq \gamma_{\mathrm{th}}^{\mathrm{HD}}\gamma_{\mathrm{x_k,R}}^{\mathrm{HD}}, \gamma_{\mathrm{y_k}}^{\mathrm{HD}} \geq \gamma_{\mathrm{th}}^{\mathrm{HD}}\gamma_{\mathrm{y_k,R}}^{\mathrm{HD}}\right) \\
&= 1 - \Bigg(\Bigg(\underbrace{\int_0^\infty F_{\gamma_{x_k}^{HD}}\left(\gamma_{th}^{HD}\gamma_{x_k,R}^{HD}\right) f_{\gamma_{x_k,R}}(\gamma_{x_k,R}^{HD}) d\gamma_{x_k,R}^{HD}}_{\Delta} \Bigg) \\
&\quad \times \Bigg(\underbrace{\int_0^\infty F_{\gamma_{y_k}}\left(\gamma_{th}^{HD}\gamma_{y_k,R}^{HD}\right) f_{\gamma_{y_k,R}^{HD}}(\gamma_{y_k,R}^{HD}) d\gamma_{y_k,R}^{HD}}_{\Upsilon} \Bigg) \Bigg)
\end{aligned} \tag{22}$$

In order to continue the analysis, (22) requires $F_{\gamma_{x_k}}$ and $F_{\gamma_{y_k}}$ expressions. Before starting these derivations, because of the intractable form of (6) and (7), this paper upper-bound these expressions by using the $\frac{\mathrm{XY}}{\mathrm{X+Y}} \leq \min(\mathrm{X,Y})$ as in (23) and

(24), respectively.

$$\gamma_{x_k}^{HD} = \frac{\frac{\varphi\gamma_{s_1}\gamma_{s_2}}{(\varphi\gamma_A+\varphi+1)}}{\gamma_{s_1} + \frac{\gamma_{s_2}}{(\varphi\gamma_A+\varphi+1)}} = \varphi\frac{AB}{A+B} \le \gamma_{\mathrm{x_k}}^{\mathrm{HD(up)}} = \varphi\mathrm{min(A,B)} \quad (23)$$

$$\gamma_{y_k}^{HD} = \frac{\frac{\varphi\gamma_{s_1}\gamma_{s_2}}{(\varphi\gamma_A+\varphi+1)}}{\frac{\gamma_{s_1}}{(\varphi\gamma_A+\varphi+1)} + \gamma_{s_2}} = \varphi\frac{CD}{C+D} \le \gamma_{\mathrm{y_k}}^{\mathrm{HD(up)}} = \varphi\mathrm{min(C,D)} \quad (24)$$

where $A=\gamma_{S_1}$, $B=\frac{\gamma_{s_2}}{(\varphi\gamma_A+\varphi+1)}$, $C=\frac{\gamma_{s_1}}{(\varphi\gamma_A+\varphi+1)}$, $D=\gamma_{s_2}$, and $\gamma_A=\gamma_J+1$. The CDF expression of (23) can be calculated as in (25).

$$\begin{aligned} F_{\gamma_{x_k}^{HD}} &= Pr\left(\varphi\mathrm{min(A,B)} \le \gamma_{\mathrm{th}}^{\mathrm{HD}}\right) \\ &= \mathrm{Pr}\left(\varphi\mathrm{min}\left(\gamma_{\mathrm{s_1}}, \frac{\gamma_{\mathrm{s_2}}}{(\varphi\gamma_{\mathrm{A}}+\varphi+1)}\right) \le \gamma_{\mathrm{th}}^{\mathrm{HD}}\right) \\ &= 1 - \mathrm{Pr}\Big(\gamma_{\mathrm{s_1}} \ge \gamma_{\mathrm{th}}^{\mathrm{HD}}\varphi^{-1}, \\ &\times \gamma_{s_2} \ge \gamma_{th}^{\mathrm{HD}}\varphi^{-1}\left(\varphi\gamma_A+\varphi+1\right)\Big) \\ &= 1 - \left(1 - F_{\gamma_{s_1}}\left(\gamma_{th}^{\mathrm{HD}}\varphi^{-1}\right)\right) \\ &\times \left(1 - F_{\gamma_{s_2}}\left(\gamma_{th}^{\mathrm{HD}}\varphi^{-1}\left(\varphi\gamma_A+\varphi+1\right)\right)\right) \end{aligned} \quad (25)$$

Since the amplitude of all channels are distributed according to the Rayleigh distribution, the PDF expressions of γ_{s_1} and γ_{s_2} can be written as: $f_{\gamma_{s_1}}(\gamma_{th}) = \frac{1}{P_s\Omega_h}e^{-\frac{\gamma_{th}}{P_s\Omega_h}}$ and $f_{\gamma_{s_2}}(\gamma_{th}) = \frac{1}{P_s\Omega_g}e^{-\frac{\gamma_{th}}{P_s\Omega_g}}$, respectively [18]. where Ω_h=$\mathbb{E}\left[|h|^2\right]$ and Ω_g=$\mathbb{E}\left[|g|^2\right]$. In light of all these information, $F_{\gamma_{s_1}}\left(\gamma_{th}\varphi^{-1}\right)$ and $F_{\gamma_{s_2}}\left(\gamma_{th}\varphi^{-1}\left(\varphi\gamma_A+\varphi+1\right)\right)$ can be computed as:

$$\begin{aligned} F_{\gamma_{x_k}^{HD}} &= 1 - \mathbb{E}_{\gamma_J}\left[e^{-\gamma_{th}^{\mathrm{HD}}\left(\frac{\varphi^{-1}}{P_s\Omega_{h_k}} + \frac{\varphi^{-1}(\varphi\gamma_J+2\varphi+1)}{P_s\Omega_{g_k}}\right)}\middle|\gamma_J\right] \\ &= 1 - \Bigg[e^{-\gamma_{th}^{\mathrm{HD}}\left(\frac{\varphi^{-1}}{P_s\Omega_{h_k}} + \frac{\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}\right)} \\ &\times \int_0^\infty e^{-\gamma_J\left(\frac{\gamma_{th}^{\mathrm{HD}}}{P_s\Omega_{g_k}}\right)} f_{\gamma_J}\left(\gamma_J\right)d_{\gamma_J}\Bigg] \end{aligned} \quad (26)$$

Sum of *M* i.i.d. Rayleigh distribution become a Gamma distribution [18]. Within this scope, the PDF expression of γ_J can be expressed as: $f_{\gamma_J}(\gamma) = \frac{\gamma^{M-1}}{(P_J\Omega_f)^M(M-1)!}e^{-\frac{\gamma}{P_J\Omega_f}}$ [18], where Ω_f=$\mathbb{E}\left[|f|^2\right]$. Substituting $f_{\gamma_J}(\gamma)$ into (26) and utilizing [19, Eq. (3.351^3)] for the integral expression, the final expression can be computed as

$$F_{\gamma_{x_k}^{HD}}\left(\gamma_{th}^{\mathrm{HD}}\right) = 1 - \left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-M} \times e^{-\gamma_{th}^{\mathrm{HD}}\left(\frac{\varphi^{-1}}{P_s\Omega_{h_k}} + \frac{\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}\right)} \quad (27)$$

Following the same procedures, $F_{\gamma_{y_k}^{HD}}\left(\gamma_{th}^{\mathrm{HD}}\right)$ can be computed as

$$F_{\gamma_{y_k}^{HD}}\left(\gamma_{th}^{HD}\right) = 1 - \left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\gamma_{th}^{HD}+1\right)^{-M} \times e^{-\gamma_{th}^{HD}\left(\frac{\varphi^{-1}(2\varphi+1)}{P_s\Omega_{h_k}} + \frac{\varphi^{-1}}{P_s\Omega_{g_k}}\right)} \quad (28)$$

To continue the analysis, (22) also requires the $f_{\gamma_{x_k,R}}(\gamma)$ and $f_{\gamma_{y_k,R}}(\gamma)$ derivations. Starting with (2), following expressions can be obtained.

$$\begin{aligned} F_{\gamma_{x_k,R}}\left(\gamma_{th}^{\mathrm{HD}}\right) &= \mathrm{Pr}\left(\frac{\gamma_{\mathrm{x}}}{\gamma_{\mathrm{y}}+\gamma_{\mathrm{J}}+1} \le \gamma_{\mathrm{th}}^{\mathrm{HD}}\right) \\ &= \int_0^\infty\int_0^\infty\int_0^{(\gamma_y+\gamma_J+1)} f_{\gamma_x}(\gamma_x)f_{\gamma_y}(\gamma_y)f_{\gamma_J}(\gamma_J)d_{\gamma_x}d_{\gamma_y}d_{\gamma_J} \\ &= 1 - \int_0^\infty\int_0^\infty e^{-\gamma_{th}^{\mathrm{HD}}\left(\frac{\gamma_y+\gamma_J+1)}{P_s\Omega_{h_k}}\right)} f_{\gamma_y}(\gamma_y)f_{\gamma_J}(\gamma_J)d_{\gamma_y}d_{\gamma_J} \\ &= 1 - e^{-\left(\frac{\gamma_{th}^{\mathrm{HD}}}{P_s\Omega_{h_k}}\right)}\int_0^\infty e^{-\gamma_y\left(\frac{\gamma_{th}^{\mathrm{HD}}}{P_s\Omega_{h_k}}\right)} f_{\gamma_y}(\gamma_y)d_{\gamma_y} \\ &\times \int_0^\infty e^{-\gamma_J\left(\frac{\gamma_{th}^{\mathrm{HD}}}{P_s\Omega_{h_k}}\right)} f_{\gamma_J}(\gamma_J)d_{\gamma_J} \end{aligned} \quad (29)$$

Substituting $f_{\gamma_y}(\gamma)$ and $f_{\gamma_J}(\gamma)$ into (29) and also solving the integral expressions by using [19, Eq. $(3.310^{11}, 3.351^3)$], the final CDF expression can be achieved as in (30)

$$F_{\gamma_{x_k,R}}\left(\gamma_{th}^{\mathrm{HD}}\right) = 1 - \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-1} \times \left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-M} e^{-\left(\frac{\gamma_{th}^{\mathrm{HD}}}{P_s\Omega_{h_k}}\right)} \quad (30)$$

The derivative of (30) yields the $f_{\gamma_{x_k,R}}(\gamma)$ as

$$\begin{aligned} f_{\gamma_{x_k,R}}\left(\gamma_{th}^{\mathrm{HD}}\right) &= e^{-\left(\frac{\gamma_{th}^{\mathrm{HD}}}{P_s\Omega_{h_k}}\right)}\Bigg[\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-2}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-M} \\ &+ \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-1} M\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-M-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\right) \\ &+ \left(\frac{1}{P_s\Omega_{h_k}}\right)\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-M}\Bigg] \end{aligned} \quad (31)$$

Likewise, $f_{\gamma_{y_k,R}}(\gamma)$ computed as

$$\begin{aligned} f_{\gamma_{y_k,R}}\left(\gamma_{th}^{\mathrm{HD}}\right) &= e^{-\left(\frac{\gamma_{th}^{\mathrm{HD}}}{P_s\Omega_{g_k}}\right)}\Bigg[\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-2}\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\right)\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-M} \\ &+ \left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-1} M\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-M-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\right) \\ &+ \left(\frac{1}{P_s\Omega_{g_k}}\right)\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\gamma_{th}^{\mathrm{HD}}+1\right)^{-M}\Bigg] \end{aligned} \quad (32)$$

Substituting (27) and (31) into (22), the Δ term in (22) can be re-written as in (33).

$$\Delta = \int_0^\infty \left(1 - \left(\frac{P_j\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}y+1\right)^{-M} e^{-y\left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}} + \frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}\right)}\right)$$

$$\Delta = \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-\left(\frac{y}{P_s\Omega_{h_k}}\right)}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^{-2}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{-M}dy$$
$$+M\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-\left(\frac{y}{P_s\Omega_{h_k}}\right)}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{-M-1}dy$$
$$+\left(\frac{1}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-\left(\frac{y}{P_s\Omega_{h_k}}\right)}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^{-1}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{-M}dy$$
$$-\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-y\left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}+\frac{1}{P_s\Omega_{h_k}}\right)}\left(\frac{P_j\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}y+1\right)^{-M}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^{-2}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{-M}dy$$
$$-M\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-y\left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}+\frac{1}{P_s\Omega_{h_k}}\right)}\left(\frac{P_j\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}y+1\right)^{-M}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{-M-1}dy$$
$$-\left(\frac{1}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-y\left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}+\frac{1}{P_s\Omega_{h_k}}\right)}\left(\frac{P_j\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}y+1\right)^{-M}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^{-1}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{-M}dy \quad (34)$$

$$\times\left(e^{-\left(\frac{y}{P_s\Omega_{h_k}}\right)}\left[\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^{-2}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{-M}\right.\right.$$
$$+\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^{-1}M\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{-M-1}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}\right)$$
$$\left.+\left(\frac{1}{P_s\Omega_{h_k}}\right)\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^{-1}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{-M}\right]d_y \quad (33)$$

By using the distributive property, (33) can be written as in (34). The first, second, and third integrals in (34) can be solved by using [20, Eq. (10, 11)] and [13, Eq. (13)] as in (35).

$$\frac{P_s\Omega_{g_k}}{\Gamma(2)\Gamma(M)}G_{1,0:1,1:1,1}^{1,0:1,1:1,1}\left(\begin{array}{c|c|c|c} 1 & -1 & 1-M & \\ - & 0 & 0 & \end{array} P_s\Omega_{g_k}, P_J\Omega_{f_j}\right)$$
$$+M\frac{P_J\Omega_{f_j}}{\Gamma(M+1)}G_{1,0:1,1:1,1}^{1,0:1,1:1,1}\left(\begin{array}{c|c|c|c} 1 & 0 & -M & \\ - & 0 & 0 & \end{array} P_s\Omega_{g_k}, P_J\Omega_{f_j}\right)$$
$$+\frac{1}{\Gamma(M)}G_{1,0:1,1:1,1}^{1,0:1,1:1,1}\left(\begin{array}{c|c|c|c} 1 & 0 & 1-M & \\ - & 0 & 0 & \end{array} P_s\Omega_{g_k}, P_J\Omega_{f_j}\right) \quad (35)$$

The α term in [13, Eq. (13)] is set to 1. Utilizing the partial fraction decomposition technique, fourth integral term in (34) can be written as in (36).

$$\int_0^\infty e^{-y\left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}+\frac{1}{P_s\Omega_{h_k}}\right)}\times\left[\sum_{i=1}^{M}\frac{A_i}{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}y+1\right)^i}+\frac{V}{\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^2}+\sum_{l=1}^{M}\frac{C_l}{\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^l}\right]dy \quad (36)$$

$A_i = \lim_{y\to-\frac{P_s\Omega_{g_k}}{\gamma_{th}^{HD}P_J\Omega_{f_j}}}\frac{\partial^{M-i}}{(M-i)!\partial y^{M-i}}\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}y+1\right)^M[\Phi]$, $V = \lim_{y\to-\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}}\frac{\partial}{\partial y}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^2[\Phi]$, $C_l = \lim_{y\to-\frac{P_s\Omega_{h_k}}{P_J\Omega_{f_j}}}\frac{\partial^{M-l}}{(M-l)!\partial y^{M-l}}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^M[\Phi]$, and $\Phi = \frac{1}{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}y+1\right)^M\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)^2\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_j}}y+1\right)^M}$. By using the distributed properties and [20, Eq. (10, 11)] and also [20, Eq. (21)] for solving the integral expression, (36) can be obtained as in (37).

$$\left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}+\frac{1}{P_s\Omega_{h_k}}\right)^{-1}\sum_{i=1}^{M}\frac{A_i}{\Gamma(i)}\times G_{2,1}^{1,2}\left(\frac{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}\right)}{\left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}+\frac{1}{P_s\Omega_{h_k}}\right)}\left|\begin{array}{c|c} 1-i & 0 \\ 0 & - \end{array}\right.\right)$$
$$+\left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}+\frac{1}{P_s\Omega_{h_k}}\right)^{-1}\frac{B}{\Gamma(2)}\times G_{2,1}^{1,2}\left(\frac{\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)}{\left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}+\frac{1}{P_s\Omega_{h_k}}\right)}\left|\begin{array}{c|c} -1 & 0 \\ 0 & - \end{array}\right.\right)$$
$$+\left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}+\frac{1}{P_s\Omega_{h_k}}\right)^{-1}\sum_{l=1}^{M}\frac{C_l}{\Gamma(l)}\times G_{2,1}^{1,2}\left(\frac{\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}\right)}{\left(\frac{\gamma_{th}^{HD}\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{\gamma_{th}^{HD}\varphi^{-1}(2\varphi+1)}{P_s\Omega_{g_k}}+\frac{1}{P_s\Omega_{h_k}}\right)}\left|\begin{array}{c|c} 1-l & 0 \\ 0 & - \end{array}\right.\right) \quad (37)$$

Likewise, considering the same methodologies as in (37), the fifth and sixth integrals in (34) can be solved as in the fourth and seventh lines of (19). Please note that; $D_{aa} = \lim_{y\to -\frac{P_s\Omega_{g_k}}{\gamma_{th}^{HD}P_J\Omega_{f_j}}} \frac{\partial^{M-aa}}{(M-aa)!\partial y^{M-aa}} \left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}y+1\right)^M [A^+]$, $E = \lim_{y\to -\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}} \frac{\partial}{\partial y}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)[A^+]$, $F_{bb} = \lim_{y\to -\frac{P_s\Omega_{h_k}}{P_J\Omega_{f_j}}} \frac{\partial^{M+1-bb}}{(M+1-bb)!\partial y^{M+1-bb}}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{M+1}[A^+]$, and $A^+ = \frac{1}{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}y+1\right)^M \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_j}}y+1\right)^{M+1}}$.

$G_{cc} = \lim_{y\to -\frac{P_s\Omega_{g_k}}{\gamma_{th}^{HD}P_J\Omega_{f_j}}} \frac{\partial^{M-cc}}{(M-cc)!\partial y^{M-cc}} \times \left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}y+1\right)^M [B^+]$, $H = \lim_{y\to -\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}} \frac{\partial}{\partial y}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)[B^+]$, $I_{dd} = \lim_{y\to -\frac{P_s\Omega_{h_k}}{P_J\Omega_{f_j}}} \frac{\partial^{M-dd}}{(M-dd)!\partial y^{M-dd}}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{M}[B^+]$, and $B^+ = \frac{1}{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{g_k}}y+1\right)^M \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}y+1\right)\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}y+1\right)^{M}}$.

Likewise, following the same procedures, Υ term in (22), can be obtained as in the second part of (19). Please note that, $J_{ee} = \lim_{y\to -\frac{P_s\Omega_{h_k}}{\gamma_{th}^{HD}P_J\Omega_{f_j}}} \frac{\partial^{M-ee}}{(M-ee)!\partial y^{M-ee}} \left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{h_k}}y+1\right)^M [C^+]$, $K = \lim_{y\to -\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}} \frac{\partial}{\partial y}\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}y+1\right)^2[C^+]$, $L_{ff} = \lim_{y\to -\frac{P_s\Omega_{g_k}}{P_J\Omega_{f_j}}} \frac{\partial^{M-ff}}{(M-ff)!\partial y^{M-ff}}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{g_k}}y+1\right)^{M}[C^+]$, and $C^+ = \frac{1}{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{h_k}}y+1\right)^M \left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}y+1\right)^2\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{g_k}}y+1\right)^{M}}$. $M_{gg} = \lim_{y\to -\frac{P_s\Omega_{h_k}}{\gamma_{th}^{HD}P_J\Omega_{f_j}}} \frac{\partial^{M-gg}}{(M-gg)!\partial y^{M-gg}} \left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{h_k}}y+1\right)^M [D^+]$, $N^{***} = \lim_{y\to -\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}} \frac{\partial}{\partial y}\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}y+1\right)^2[D^+]$, $O_{hh} = \lim_{y\to -\frac{P_s\Omega_{g_k}}{P_J\Omega_{f_j}}} \frac{\partial^{M+1-hh}}{(M+1-hh)!\partial y^{M+1-hh}}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{g_k}}y+1\right)^{M+1}[D^+]$, and $D^+ = \frac{1}{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{h_k}}y+1\right)^M \left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}y+1\right)^2\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{g_j}}y+1\right)^{M+1}}$.

$P_{ii} = \lim_{y\to -\frac{P_s\Omega_{h_k}}{\gamma_{th}^{HD}P_J\Omega_{f_j}}} \frac{\partial^{M-ii}}{(M-ii)!\partial y^{M-ii}} \times \left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{h_k}}y+1\right)^M [E^+]$, $R = \lim_{y\to -\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}} \frac{\partial}{\partial y}\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}y+1\right)[E^+]$, $S_{jj} = \lim_{y\to -\frac{P_s\Omega_{g_k}}{P_J\Omega_{f_j}}} \frac{\partial^{M-jj}}{(M-jj)!\partial y^{M-jj}}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{g_k}}y+1\right)^{M}[E^+]$, and $E^+ = \frac{1}{\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{HD}}{P_s\Omega_{h_k}}y+1\right)^M \left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}y+1\right)\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{g_k}}y+1\right)^{M}}$.

PROOF OF PROPOSITION 2

Utilizing (16) and considering the same procedures as in (21), the $R_X^{FD} \approx \frac{\gamma_{x_k}^{FD}}{\gamma_{x_k,R}^{FD}} \leq \underbrace{2^{\frac{R}{N}}}_{\gamma_{th}^{FD}}$ can be obtained. Likewise, $R_Y^{FD} \approx \frac{\gamma_{y_k}^{FD}}{\gamma_{y_k,R}^{FD}} \leq \underbrace{2^{\frac{R}{N}}}_{\gamma_{th}^{FD}}$ can be obtained. Utilizing these two expressions in (18) and following the same procedures as in (22), following expressions can be obtained.

$$
\begin{aligned}
F_{R_{e2e}^{FD}}\left(\gamma_{th}^{FD}\right) &= \mathrm{P_r}\left(\min\left(\mathrm{R_X^{FD}}, \mathrm{R_Y^{FD}}\right) \leq \mathrm{R}\right) \\
&= 1 - \left(\left(\underbrace{\int_0^\infty F_{\gamma_{x_k}^{FD}}\left(\gamma_{th}^{FD}\gamma_{x_k,R}^{FD}\right) f_{\gamma_{x_k,R}^{FD}}(\gamma_{x_k,R}^{FD}) d\gamma_{x_k,R}^{FD}}_{\Psi}\right)\right. \\
&\quad \times \left.\left(\underbrace{\int_0^\infty F_{\gamma_{y_k}^{FD}}\left(\gamma_{th}^{FD}\gamma_{y_k,R}^{FD}\right) f_{\gamma_{y_k,R}^{FD}}(\gamma_{y_k,R}^{FD}) d\gamma_{y_k,R}^{FD}}_{\zeta}\right)\right)
\end{aligned} \tag{38}
$$

Because of the intractable form of (13) and (14), these expressions can be upper-bounded by using $\frac{\mathrm{XY}}{\mathrm{X+Y}} \leq \min(\mathrm{X},\mathrm{Y})$ as in (39) and (40), respectively.

$$
\begin{aligned}
\gamma_{x_k}^{FD} &= \frac{\frac{\varphi\gamma_x\gamma_y}{(\varphi\gamma_A+\varphi\gamma_B+\gamma_C+\varphi+1)(\gamma_C+1)}}{\frac{\gamma_x}{\gamma_C+1}+\frac{\gamma_y}{(\varphi\gamma_A+\varphi\gamma_B+\gamma_C+\varphi+1)}} = \varphi\frac{WZ}{W+Z} \\
&\leq \gamma_{x_k}^{\mathrm{FD(up)}} = \varphi\mathrm{min(W,Z)}
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
\gamma_{y_k}^{FD} &= \frac{\frac{\varphi\gamma_x\gamma_y}{(\varphi\gamma_A+\varphi\gamma_B+\gamma_D+\varphi+1)(\gamma_D+1)}}{\frac{\gamma_y}{\gamma_D+1}+\frac{\gamma_x}{(\varphi\gamma_A+\varphi\gamma_B+\gamma_D+\varphi+1)}} = \varphi\frac{TU}{T+U} \\
&\leq \gamma_{y_k}^{\mathrm{FD(up)}} = \varphi\mathrm{min(T,U)}
\end{aligned} \tag{40}
$$

where $W = \frac{\gamma_x}{\gamma_C+1}$, $Z = \frac{\gamma_y}{(\varphi\gamma_A+\varphi\gamma_B+\gamma_C+\varphi+1)}$, $T = \frac{\gamma_y}{\gamma_D+1}$, $U = \frac{\gamma_x}{(\varphi\gamma_A+\varphi\gamma_B+\gamma_D+\varphi+1)}$, $\gamma_B = \gamma_{c_k}+1$, $\gamma_C = \gamma_{a_k}+1$, and $\gamma_D = \gamma_{b_k}+1$. Considering the same methodology as in (25), the CDF expression of (39) can be achieved as

$$
\begin{aligned}
F^{\mathrm{up}}_{\gamma^{FD}_{x_k}} &= \Pr\left(\varphi \min(\mathrm{W},\mathrm{Z}) \leq \gamma^{\mathrm{FD}}_{\mathrm{th}}\right) \\
&= \Pr\Bigg(\varphi \min\Bigg(\frac{\gamma_{\mathrm{x}}}{\gamma_{\mathrm{C}}+1}, \\
&\quad \frac{\gamma_y}{(\varphi\gamma_A+\varphi\gamma_B+\gamma_C+\varphi+1)}\Bigg) \leq \gamma^{\mathrm{FD}}_{th}\Bigg) \\
&= 1-\Pr\Bigg(\gamma_{\mathrm{x}} \geq \gamma^{\mathrm{FD}}_{\mathrm{th}}\left(\gamma_{\mathrm{C}}+1\right)\varphi^{-1} \\
&\quad ,\gamma_y \geq \gamma^{\mathrm{FD}}_{th}\varphi^{-1}\left(\varphi\gamma_A+\varphi\gamma_B+\gamma_C+\varphi+1\right)\Bigg) \\
&= 1-\left(1-F_{\gamma_x}\left(\gamma^{\mathrm{FD}}_{th}\left(\gamma_C+1\right)\varphi^{-1}\right)\right) \\
&\quad \times\left(1-F_{\gamma_y}\left(\gamma^{\mathrm{FD}}_{th}\varphi^{-1}\left(\varphi\gamma_A+\varphi\gamma_B+\gamma_C+\varphi+1\right)\right)\right) \\
&= 1-\mathbb{E}_{\gamma_J}\Bigg[e^{-\gamma^{\mathrm{FD}}_{th}\Bigg(\frac{\left(\gamma_{a_k}+2\right)\varphi^{-1}}{P_s\Omega_{h_k}}} \\
&\quad +\frac{\left(\gamma_J+\gamma_c+\varphi^{-1}\gamma_{a_k}+2\varphi^{-1}+3\right)}{P_s\Omega_{g_k}}\Bigg)\Bigg|\gamma_{a_k},\gamma_J,\gamma_c\Bigg] \\
&= 1-\Bigg[e^{-\gamma^{\mathrm{FD}}_{th}\left(\frac{2\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{2\varphi^{-1}+3}{P_s\Omega_{g_k}}\right)} \\
&\quad \times\int_0^\infty e^{-\gamma_J\left(\frac{\gamma^{\mathrm{FD}}_{th}}{P_s\Omega_{g_k}}\right)} f_{\gamma_J}\left(\gamma_J\right)d_{\gamma_J} \\
&\quad \times\int_0^\infty e^{-\gamma_c\left(\frac{\gamma^{\mathrm{FD}}_{th}}{P_s\Omega_{g_k}}\right)} f_{\gamma_c}\left(\gamma_c\right)d_{\gamma_c} \\
&\quad \times\int_0^\infty e^{-\gamma_{a_k}\left(\frac{\gamma^{\mathrm{FD}}_{th}\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{\gamma^{\mathrm{FD}}_{th}\varphi^{-1}}{P_s\Omega_{g_k}}\right)} f_{\gamma_{a_k}}\left(\gamma_{a_k}\right)d_{\gamma_{a_k}}\Bigg]
\end{aligned} \quad (41)
$$

Substituting $f_{\gamma_J}(\gamma)$, $f_{\gamma_c}(\gamma) = \frac{1}{P_r\Omega_c}e^{-\frac{\gamma}{P_r\Omega_c}}$ and $f_{\gamma_{a_k}}(\gamma) = \frac{1}{P_s\Omega_{a_k}}e^{-\frac{\gamma}{P_s\Omega_{a_k}}}$ [18], where Ω_c=$\mathbb{E}\left[|c|^2\right]$ and Ω_{a_k}=$\mathbb{E}\left[|a_k|^2\right]$, into (41) and solving the integral expressions by using [19, Eq. (3.310[11])] and [19, Eq. (3.351[3])], the final result can be computed as

$$
\begin{aligned}
F_{\gamma^{FD}_{x_k}}\left(\gamma^{\mathrm{FD}}_{th}\right) &= 1-\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-M}\left(\frac{P_r\Omega_c}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1} \\
&\quad \times\left(\frac{\varphi^{-1}P_s\Omega_{a_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+\frac{\varphi^{-1}P_s\Omega_{a_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1} \\
&\quad \times e^{-\gamma^{\mathrm{FD}}_{th}\left(\frac{2\varphi^{-1}}{P_s\Omega_{h_k}}+\frac{2\varphi^{-1}+3}{P_s\Omega_{g_k}}\right)}
\end{aligned} \quad (42)
$$

Likewise, considering the same methodologies as in (42), the $F_{\gamma^{FD}_{y_k}}\left(\gamma^{\mathrm{FD}}_{th}\right)$ can be obtained as

$$
\begin{aligned}
F_{\gamma^{FD}_{y_k}}\left(\gamma^{\mathrm{FD}}_{th}\right) &= 1-\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-M}\left(\frac{P_r\Omega_c}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1} \\
&\quad \times\left(\frac{\varphi^{-1}P_s\Omega_{b_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+\frac{\varphi^{-1}P_s\Omega_{b_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1} \\
&\quad \times e^{-\gamma^{\mathrm{FD}}_{th}\left(\frac{2\varphi^{-1}}{P_s\Omega_{g_k}}+\frac{2\varphi^{-1}+3}{P_s\Omega_{h_k}}\right)}
\end{aligned} \quad (43)
$$

This subsection now focuses on the CDF derivation of LR expressions. By using (9) and (10) and also considering the similar methodologies as in (29), the LR expressions with respect to x_k and y_k can be obtained as in (44) and (45), respectively.

$$
\begin{aligned}
F_{\gamma_{x_k},R}\left(\gamma^{\mathrm{FD}}_{th}\right) &= 1-\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-M} \\
&\quad \times\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1} e^{-\left(\frac{\gamma^{\mathrm{FD}}_{th}}{P_s\Omega_{h_k}}\right)}
\end{aligned} \quad (44)
$$

$$
\begin{aligned}
F_{\gamma_{y_k},R}\left(\gamma^{\mathrm{FD}}_{th}\right) &= 1-\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-M} \\
&\quad \times\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1} e^{-\left(\frac{\gamma^{\mathrm{FD}}_{th}}{P_s\Omega_{g_k}}\right)}
\end{aligned} \quad (45)
$$

The derivative of (44) and (45) yields the $f_{\gamma_{x_k},R}\left(\gamma^{\mathrm{FD}}_{th}\right)$ and $f_{\gamma_{y_k},R}\left(\gamma^{\mathrm{FD}}_{th}\right)$ as in (46) and (47), respectively.

$$
\begin{aligned}
f_{\gamma_{x_k},R}\left(\gamma^{\mathrm{FD}}_{th}\right) &= e^{-\left(\frac{\gamma^{\mathrm{FD}}_{th}}{P_s\Omega_{h_k}}\right)}\Bigg[\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-2}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\gamma^{\mathrm{HD}}_{th}+1\right)^{-M} \\
&\quad \times\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}+M\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-M-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\right) \\
&\quad \times\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}+\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-M} \\
&\quad \times\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}\right)\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-2}+\left(\frac{1}{P_s\Omega_h}\right)\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-M} \\
&\quad \times\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}\Bigg]
\end{aligned} \quad (46)
$$

$$
\begin{aligned}
f_{\gamma_{y_k},R}\left(\gamma^{\mathrm{FD}}_{th}\right) &= e^{-\left(\frac{\gamma^{\mathrm{FD}}_{th}}{P_s\Omega_{g_k}}\right)}\Bigg[\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-2}\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\right)\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-M} \\
&\quad \times\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}+M\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-M-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\right) \\
&\quad \times\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}+\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-M} \\
&\quad \times\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{g_k}}\right)\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-2}+\left(\frac{1}{P_s\Omega_g}\right)\left(\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-M} \\
&\quad \times\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{g_k}}\gamma^{\mathrm{FD}}_{th}+1\right)^{-1}\Bigg]
\end{aligned} \quad (47)
$$

Substituting (42) and (46) into (38) and utilizing the distributive property, (48) can be obtained. Considering the partial fraction decomposition technique and also utilizing [20, Eq. (10, 11)] and also [20, Eq. (21)] for solving the integral expression, first, second, third, and fourth integrals in (48) can be solved as in the first part of (20). Please note that following the same procedures the fifth integral expression in (48) can be solved as in (20). The other integral expressions, which are sixth, seventh, and eighth, in (48) can be solved by following the same procedures. In addition, the ζ term in (38) can also be obtained by following the same procedures as the Ψ term in (38). However, because of the space limitation these derivation details and the results are omitted. Please also note that $A^{*} = \lim_{x \to -\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}} \frac{\partial}{\partial x}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^2 [A^{\phi}]$, $B^{*}_{a^3} =$

$$
\begin{aligned}
\Psi = & \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-\left(\frac{x}{P_s\Omega_{h_k}}\right)}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^{-2}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^{-M}\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)^{-1}dx \\
& + M\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-\left(\frac{x}{P_s\Omega_{h_k}}\right)}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^{-M-1}\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)^{-1}dx \\
& + \left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-\left(\frac{x}{P_s\Omega_{h_k}}\right)}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^{-M}\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)^{-2}dx \\
& + \left(\frac{1}{P_s\Omega_{h}}\right)\int_0^\infty e^{-\left(\frac{x}{P_s\Omega_{h_k}}\right)}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^{-1}\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^{-M}\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)^{-1}dx \\
& - \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-x\left(\frac{2\varphi^{-1}\gamma_{th}^{FD}}{P_s\Omega_{h_k}}+\frac{2\varphi^{-1}+3}{P_s\Omega_{g_k}}+\frac{\gamma_{th}^{FD}}{P_s\Omega_{h_k}}\right)}\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right)^{-M}\left(\frac{P_r\Omega_{c_k}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right)^{-1} \\
& \times\left(\left(\frac{\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{g_k}\gamma_{th}^{FD}+\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{h_k}\gamma_{th}^{FD}}{P_s\Omega_{h_k}P_s\Omega_{g_k}}\right)x+1\right)^{-1}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^{-2} \\
& \times\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^{-M}\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)^{-1}dx \\
& - M\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-x\left(\frac{2\varphi^{-1}\gamma_{th}^{FD}}{P_s\Omega_{h_k}}+\frac{2\varphi^{-1}+3}{P_s\Omega_{g_k}}+\frac{\gamma_{th}^{FD}}{P_s\Omega_{h_k}}\right)}\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right)^{-M}\left(\frac{P_r\Omega_{c_k}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right)^{-1} \\
& \times\left(\left(\frac{\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{g_k}\gamma_{th}^{FD}+\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{h_k}\gamma_{th}^{FD}}{P_s\Omega_{h_k}P_s\Omega_{g_k}}\right)x+1\right)^{-1}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^{-1} \\
& \times\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^{-M-1}\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)^{-1}dx \\
& - \left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}\right)\int_0^\infty e^{-x\left(\frac{2\varphi^{-1}\gamma_{th}^{FD}}{P_s\Omega_{h_k}}+\frac{2\varphi^{-1}+3}{P_s\Omega_{g_k}}+\frac{\gamma_{th}^{FD}}{P_s\Omega_{h_k}}\right)}\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right)^{-M}\left(\frac{P_r\Omega_{c_k}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right)^{-1} \\
& \times\left(\left(\frac{\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{g_k}\gamma_{th}^{FD}+\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{h_k}\gamma_{th}^{FD}}{P_s\Omega_{h_k}P_s\Omega_{g_k}}\right)x+1\right)^{-1}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^{-1} \\
& \times\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^{-M}\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)^{-2}dx \\
& - \left(\frac{1}{P_s\Omega_{h}}\right)\int_0^\infty e^{-x\left(\frac{2\varphi^{-1}\gamma_{th}^{FD}}{P_s\Omega_{h_k}}+\frac{2\varphi^{-1}+3}{P_s\Omega_{g_k}}+\frac{\gamma_{th}^{FD}}{P_s\Omega_{h_k}}\right)}\left(\frac{P_J\Omega_{f_j}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right)^{-M}\left(\frac{P_r\Omega_{c_k}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right)^{-1} \\
& \times\left(\left(\frac{\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{g_k}\gamma_{th}^{FD}+\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{h_k}\gamma_{th}^{FD}}{P_s\Omega_{h_k}P_s\Omega_{g_k}}\right)x+1\right)^{-1}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^{-1} \\
& \times\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^{-M}\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)^{-1}dx
\end{aligned}
\tag{48}
$$

$\lim\limits_{x\to-\frac{P_s\Omega_{h_k}}{P_J\Omega_{f_j}}}\frac{\partial^{M-a^3}}{(M-a^3)!\partial x^{M-a^3}}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^{M}[A^{\phi}]$,

$C^* = \lim\limits_{x\to-\frac{P_s\Omega_{h_k}}{P_r\Omega_{c_k}}}\frac{\partial}{\partial x}\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)[A^{\phi}]$, and

$A^{\phi} = \frac{1}{\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^2\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^M\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)}$.

$D^* = \lim\limits_{x\to-\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}}\frac{\partial}{\partial x}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)[B^{\phi}]$, $E_{b^3}^* =$

$\lim\limits_{x\to-\frac{P_s\Omega_{h_k}}{P_J\Omega_{f_j}}}\frac{\partial^{M+1-b^3}}{(M+1-b^3)!\partial x^{M+1-b^3}}\left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^{M+1}[B^{\phi}]$,

$F^* = \lim\limits_{x\to-\frac{P_s\Omega_{h_k}}{P_r\Omega_{c_k}}}\frac{\partial}{\partial x}\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)[B^{\phi}]$, and

$B^{\phi} = \frac{1}{\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^{M+1}\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)}$.

$G^* = \lim\limits_{x\to-\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}}\frac{\partial}{\partial x}\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)[C^{\phi}]$, $H_{c^3}^* =$

$$\lim_{x\to -\frac{P_s\Omega_{h_k}}{P_J\Omega_{f_j}}} \frac{\partial^{M-c^3}}{(M-c^3)!\partial x^{M-c^3}} \left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^M [C^{\phi}],$$

$$I^* = \lim_{x\to -\frac{P_s\Omega_{h_k}}{P_r\Omega_{c_k}}} \frac{\partial}{\partial x} \left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)^2 [C^{\phi}], \quad \text{and}$$

$$C^{\phi} = \frac{1}{\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^M\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)^2}.$$

$$J^* = \lim_{x\to -\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}} \frac{\partial}{\partial x} \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right) [D^{\phi}], \quad K^*_{d^3} =$$

$$\lim_{x\to -\frac{P_s\Omega_{h_k}}{P_J\Omega_{f_j}}} \frac{\partial^{M-d^3}}{(M-d^3)!\partial x^{M-d^3}} \left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^M [D^{\phi}],$$

$$L^* = \lim_{x\to -\frac{P_s\Omega_{h_k}}{P_r\Omega_{c_k}}} \frac{\partial}{\partial x} \left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right) [D^{\phi}], \quad \text{and}$$

$$D^{\phi} = \frac{1}{\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^M\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)}.$$

$$M^* = \lim_{x\to -\frac{P_s\Omega_{g_k}}{P_J\Omega_{f_j}\gamma_{th}^{FD}}} \frac{\partial^{M-e^3}}{(M-e^3)!\partial x^{M-e^3}} \left(\frac{P_J\Omega_{f_j}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right) [E^{\phi}],$$

$$N^* = \lim_{x\to -\frac{P_s\Omega_{g_k}}{P_r\Omega_{c_k}\gamma_{th}^{FD}}} \frac{\partial}{\partial x} \left(\frac{P_r\Omega_{c_k}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right) [E^{\phi}],$$

$$O^* = \lim_{x\to -\left(\frac{\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{g_k}\gamma_{th}^{FD}+\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{h_k}\gamma_{th}^{FD}}{P_s\Omega_{h_k}P_s\Omega_{g_k}}\right)}$$
$$\times \frac{\partial}{\partial x}\left(\left(\frac{\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{g_k}\gamma_{th}^{FD}+\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{h_k}\gamma_{th}^{FD}}{P_s\Omega_{h_k}P_s\Omega_{g_k}}\right)x+1\right)[E^{\phi}],$$

$$P^* = \lim_{x\to -\frac{P_s\Omega_{h_k}}{P_s\Omega_{g_k}}} \frac{\partial}{\partial x} \left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^2 [E^{\phi}], \quad R^* =$$

$$\lim_{x\to -\frac{P_s\Omega_{h_k}}{P_J\Omega_{f_j}}} \frac{\partial^{M-f^3}}{(M-f^3)!\partial x^{M-f^3}} \left(\frac{P_J\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^M [E^{\phi}],$$

$$S^* = \lim_{x\to -\frac{P_s\Omega_{h_k}}{P_r\Omega_{c_k}}} \frac{\partial}{\partial x} \left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right) [E^{\phi}], \quad \text{and}$$

$$E^{\phi} = \left[\frac{1}{\left(\frac{P_j\Omega_{f_j}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right)^M\left(\frac{P_r\Omega_{c_k}\gamma_{th}^{FD}}{P_s\Omega_{g_k}}x+1\right)}\right.$$
$$\times \frac{1}{\left(\frac{\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{g_k}\gamma_{th}^{FD}+\varphi^{-1}P_s\Omega_{a_k}P_s\Omega_{h_k}\gamma_{th}^{FD}}{P_s\Omega_{h_k}P_s\Omega_{g_k}}\right)\left(\frac{P_s\Omega_{g_k}}{P_s\Omega_{h_k}}x+1\right)^2}$$
$$\left.\times \frac{1}{\left(\frac{P_j\Omega_{f_j}}{P_s\Omega_{h_k}}x+1\right)^M\left(\frac{P_r\Omega_{c_k}}{P_s\Omega_{h_k}}x+1\right)}\right].$$

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