Available online: July 30, 2019

Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.
Volume 68, Number 2, Pages 2238–2257 (2019)
DOI: 10.31801/cfsuasmas.467286
ISSN 1303–5991 E-ISSN 2618-6470

http://communications.science.ankara.edu.tr/index.php?series=A1

DECISION MAKING FOR PORTFOLIO SELECTION BY FUZZY MULTI CRITERIA LINEAR PROGRAMMING

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Abstract. In daily life events, there are many complexities arising from lack of information and uncertainty. Fuzzy linear programming approach has been developed to reduce or eliminate this complexity. This approach is the process of choosing the optimum solution from among the decision alternatives to achieve a specific purpose in cases where the information is not certain. One of the fields where uncertainty or the lack of information makes it difficult to decide is financial markets. Investors who have a certain amount of accumulations aim to increase in various ways as well as protecting the value of their income. While doing this, investors face the challenge of deciding to what extent they should invest in which investment instrument. Therefore, investors use fuzzy linear programming approach to eliminate this uncertainty and to create the optimal portfolio. In the proposed methods for the portfolio selection process in the literature, the weights of the criteria are calculated by using triangular fuzzy numbers. In this study, as an alternative to the Enea and Piazza's portfolio selection model, which uses the triangular fuzzy numbers for criteria weighting, a new model that uses the trapezoidal fuzzy numbers for the same aim was proposed. With the solution of the linear programming model which is based on the determined weights, an alternative solution has been produced to the problem of which investment instrument will be invested at what proportion. The results regarding to the proposed and the existing method in the literature were compared.

1. Introduction

The decision-making process is an indispensable part of life, and this process is happening in every problem that ranges from the simplest to the most complicated. Decision making is a problem-solving process. Basically, the decisions taken to achieve a single goal have progressed towards systems aimed at realizing more than one purpose in line with developing understanding and thoughts. In such cases, Multi-Criteria Decision Making (MCDM) methods have been developed so that the

Received by the editors: October 04, 2018; Accepted: June 11, 2019.

2010 *Mathematics Subject Classification.* Primary 05C38, 15A15; Secondary 05A15, 15A18.

Key words and phrases. Multi-criteria decision making, linear programming, analytic hierarchy process, trapezoidal fuzzy numbers, portfolio selection.

correct selection can be made. In the same decision problem for each person, the level of importance of decision criteria can vary. The Analytic Hierarchy Process (AHP), one of the MCDM methods, are provided more effective decision-making in solving such these problems. In order to solve complex problems encountered in many areas in daily life, MCDM methods are used. One of these areas is the portfolio selection process. One of the most important topics of portfolio management is the modeling of the relationship between risk and return. However, the fact that financial markets are impress by political, financial and social events and the estimation of the risk / return factors that are effective in portfolio selection are cause uncertainty in the portfolio selection process. When there is such an uncertainty, a fuzzy logic approach is used. Fuzzy Logic approach was first proposed by L. A. Zadeh in 1965. Zadeh argued that classical mathematical methods were inadequate when dealing with complex systems in the real world. He stated that it is fuzzy that the vast majority of human thinking is not certain. Fuzziness means mathematically multi valuable. There is a binary value in classical logic. Fuzzy clusters are a set of inadequately defined objects that do not have sufficient criteria for membership. Zadeh proposed the identification of fuzzy clusters, which are expressed by a multi value membership function, instead of classical clusters where the properties are expressed by the binary membership function [1]. In classical logic, the truth values of variables may only be the integer values 0 or 1. Classical Logic is based on two values i.e. true and false. It is sometimes inadequate when describing human reasoning. Fuzzy logic on the other hand is employed to handle the concept of partial truth, where the truth value may range between completely true and completely false. In fuzzy logic, information can be in the form of linguistic expressions such as big, small, little, and everything is represented by a certain membership value in the range [0, 1]. A fuzzy set allows an element to belong to more than one cluster with different membership grades. Membership functions are functions that determine how much an element belongs to a set. That is, an element may be a member of more than one fuzzy set.

There are many studies in the literature that use the portfolio selection process and the fuzzy logic approach [2–6]. Xu et al. proposed a new fuzzy model for portfolio selection problem. Their proposed model also regards the elastic increment of decision-making risk, background risk, and other financial risks. They presented a modified evolutionary algorithm called modified chaos fruit fly optimization algorithm in their study [7]. Ostermark proposed a portfolio management model using the fuzzy decision-making principle, taking into account the fuzzy of objective functions and constraint functions [8]. Ramaswamy developed a fuzzy portfolio selection model [9]. Inuiguchi and Ramik compared stochastic programming and mathematical programming methods for the portfolio selection problem. They point out the advantages and disadvantages of these two methods for portfolio selection problems [10]. Wu and Liu proposed a fuzzy expectation-spread (E-S) model for the portfolio problem [3]. Sadjadi et al. addressed fuzzy linear programming method, which determines the amount of investment in different time periods. They expressed the rate of return and borrowing rates as triangular fuzzy numbers. Using fuzzy set theory, they developed a model for the cash amount and profits of investors [11]. Ghapanchi et al. noticed that when studying the literature, the majority of work is the study of the interaction between projects in deterministic environments. But they realized that work in the stochastic environment do not take project dependencies into account. They intended to fill this gap in their work. They used Data Envelopment Analysis to select the best portfolio of IS/IT projects while taking both project uncertainties and project interactions into consideration simultaneously [12]. Rahmani et al. defined decision criteria for project selection in information technology. The decision criteria were weighted using the AHP method [13]. Gupta et al. proposed a three-stage multi-criteria decision-making model for portfolio selection. They used the AHP method to compare the criteria [14]. Yue and Wang proposed a new algorithm for portfolio selection. They included various portfolio selection methods in order to evaluate the performance of the proposed algorithm [15]. Kemaloglu et al. investigated the differences between the two kinds of portfolio optimization problems which are the risk aversion portfolio optimization problem based on the classical Markowitz framework and the max-min counterpart problem based on the robust optimization framework, in their study [16]. In their work, Kim and Kim developed a new approach for the optimal Liquefied Natural Gas (LNG) import portfolio. Their approach consists of a two-step portfolio model combining the mean-variance (MV) portfolio and the linear programming (LP) model [17]. Liagkouras proposed a new algorithm for the solution of portfolio optimization problem. He tested the performance of the proposed algorithm to the optimal allocation of limited resources to a number of competing investment opportunities for optimizing the objectives [18].

In this study, a linear programming model is proposed for solution of portfolio selection process problems under fuzziness. In the proposed model, instead of the triangular fuzzy numbers used in the Constrained Fuzzy AHP method proposed by Enea and Piazza in the literature for the portfolio selection problems, the weights of the criteria were determined by defining the trapezoidal fuzzy numbers (TrFNs). In addition, a linear programming model using the obtained weights has been developed. The Constrained Fuzzy AHP method uses only the opinions in the form of triangular fuzzy numbers obtained from the decision makers. Instead of using triangular fuzzy numbers, this study focused on TrFNs to characterize fuzzy measures of linguistic values. The reason for using the TrFNs is that it is more representative to linguistic estimations in portfolio selection. TrFNs provide more flexible and successful results than triangular fuzzy numbers in terms of optimal solution coverage. The weights of stocks are determined by using the decision makers' opinions. Asset allocation is made according to determined stock weights. In this paper, in addition to decision makers' opinions, financial ratios belonging to stocks are also used. Moreover, a novel method was proposed by modifying the Constrained Fuzzy AHP method for the case where the decision makers' opinions were TrFNs. In the application part of the study, the results obtained from the existing method and the results obtained from the proposed model were compared. According to the results, asset allocation was realized. Then, monthly return rates were obtained according to the asset allocation. In order to investigate the effectiveness of the model, the monthly return rates obtained from the two methods were compared. With the proposed algorithm, an alternative method has been produced to the problem of which stock will be invested at what proportion. Thus, a more efficient asset allocation plan was achieved by using more information in the method. The organization of the paper is as follows: In Section 2, basic information about portfolio selection and the literature on portfolio selection are given. In Section 3, multi-criteria decision making, and fuzzy logic approach are included. In Section 4, proposed algorithm which uses TrFNs to weight the objective functions is briefly mentioned. In Section 5, the proposed algorithm is applied to an example existing in the literature and the results, existing in the literature and obtained with the proposed algorithm, were compared.

2. Portfolio Selection

A portfolio is a financial asset held in the hands of a specific person or group, mainly comprised of various securities such as stocks, bonds. This financial asset can consist of just one securities or it can be created by bringing together more than one securities according to the investor's attitude. The risk of portfolio; is measured by the standard deviation or average absolute deviation of the securities in the portfolio. The higher the mean absolute or the standard deviation, the higher the risk. For an investor avoiding risk, investing more than one securities is more advantageous than investing a single security. In the portfolio selection, the economic situation in the country and the investment instruments must be carefully monitored in this situation. In addition, the investor must establish an optimum balance between the available opportunities and objectives. Today's developments in the economy cause a rapid change in capital markets. With capital markets becoming operational and savings shifting to capital markets, portfolio-related issues have begun to gain importance [19]. Depending on these developments, investors have turned to various investment instruments and started to evaluate their savings in these markets. Investors need to invest in a portfolio consisting of more than one stock, instead of investing in a single stock, in order to minimize the risk or to maximize the profit. The diversity of the securities varies according to the risk level of the investor. As financial markets are affected by economic, social and political events, uncertainty arises in the portfolio selection process. Due to the fact that the risk and return information effective in portfolio selection cannot be predicted, uncertainty comes to the fore. Therefore, this situation needs to be considered in portfolio selection problems. In such cases of uncertainty, the fuzzy logic approach, which is an effective method, should be preferred. Enea and Piazza aim to select the best among multiple project options using the Fuzzy AHP method. In their work, they mentioned the shortcomings of the Extended Analysis Method in Fuzzy AHP and proposed Constrained Fuzzy AHP approach to make up this deficiency. In this approach, it is stated that the uncertainty would be reduced by decreasing the interval values of the fuzzy numbers [20]. The Constrained Fuzzy AHP method focuses on the constraints within the Fuzzy AHP method in order to take for all available information into consideration. This method is also used to calculate the weights of alternatives in the portfolio selection process. In this method, weights of alternatives are calculated using triangular fuzzy numbers. The formulas used in the calculations are given in Equations (1-3). Let $S_i = (S_{li}, S_{mi}, S_{ui})$ be the fuzzy score for the i^{th} criterion of triangular fuzzy pairwise comparison matrix, where the indices l, m and u denote its lower, medium and upper respectively. According to the Constrained Fuzzy AHP method, the center value of the fuzzy score related to i^{th} criterion (S_{mi}) is calculated by Equation (1) [20].

$$S_{mi} = \frac{\left(\prod_{j=1}^{n} m_{ij}\right)^{1/n}}{\sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} m_{kj}\right)^{1/n}\right]} \qquad (i,j,k = 1,...,n) \tag{1}$$

S_{li} can be evaluated using the crisp mathematical programming model:

$$S_{li} = min\left[\frac{\left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}}{\sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{1/n}\right]}\right] \qquad (i,j,k = 1,...,n) \tag{2}$$

subject to $a_{kj} \in [l_{kj}, u_{kj}],\ \forall j > k;\ a_{jk} = \dfrac{1}{a_{kj}},\ \forall j < k;\ a_{jj} = 1$ and similarly, S_{ui} can be evaluated using the crisp mathematical programming model,

$$S_{ui} = max\left[\frac{\left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}}{\sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{1/n}\right]}\right] \qquad (i,j,k = 1,...,n) \tag{3}$$

subject to $a_{kj} \in [l_{kj}, u_{kj}],\ \forall j > k;\ a_{jk} = \dfrac{1}{a_{kj}},\ \forall j < k;\ a_{jj} = 1.$

Tiryaki and Ahlatcioglu used the Fuzzy AHP method in the problem of portfolio selection. They intended to decide the content of the portfolio that will be created. To do this, they handled the Fuzzy AHP method proposed by Enea and Piazza. Then, they proposed Revised Constrained Fuzzy AHP method by revising some mistakes in this method [21]. Ahari et al. planned to allocate a limited funds among the stocks of some pharmaceutical companies in the Tehran stock market, in their study. They used two Fuzzy AHP method which proposed by Enea - Piazza and Van Laarhoven-Pedrycz [22]. Krejci et al. interested Fuzzy AHP method in their paper. The aim of their paper is to highlight the necessity of applying the concept of constrained fuzzy arithmetic in a fuzzy extension of AHP. They considered a fuzzy extension of the geometric mean method and simplified the formulas proposed by Enea and Piazza [23]. In their study, Dong and Wan developed a new method for the fuzzy linear program in which all the objective coefficients, technological coefficients and resources are TrFNs [24]. The aim of Ebrahimnejad's article is to introduce a formulation of FLP problems involving interval-valued TrFNs for the decision variables and the right-hand-side of the constraints. He proposed a new method for solving this kind of FLP problems based on comparison of interval-valued fuzzy numbers by the help of signed distance ranking [25].

3. Fuzzy Multi Criteria Linear Programming

Decision-making is the process of defining and selecting alternatives that will yield the best solution based on various factors and the expectations of decision-makers. The diversity of the set of criteria used in the evaluation of alternatives in the decision-making process leads to complexity. In many of the multi-criteria decision-making problems, the purpose functions collide with one another, and different group decision-makers can participate in the process. In order to overcome such difficulties in the decision-making process, MCDM methods are used. For example; even in the simplest individual decision-making processes such as buying a new house, a number of criteria are taken into consideration, such as price, proximity to the city center, and security. As another example, an investor must also include the risk, market share, sales rate of assets, average profit, liquidity, price/earnings and asset criteria of the investment instruments to be used in the decision process. Therefore, MCDM methods are used in decision problems that deal with more than one criterion simultaneously. The MCDM method is an analytical method that can evaluate strategically factors that can be measured or not.

The Multi-Criteria Decision-making method can be defined as the process of choosing among multiple alternatives under contradictory criteria. The main objective of the this method is to keep the decision-making process under control in case of alternative and criterion diversity and to reach the decision as quickly and easily as possible. The general mathematical structure of multi-criteria decision-making models is,

$$P_1 \begin{cases} max\ Z_1 = c\underline{x} = f_1 \\ \qquad \vdots \\ max\ Z_j = c\underline{x} = f_j \\ g_k(x) \leqslant 0,\ (k = 1, ..., K;\ j = 1, ..., J) \\ \underline{x} \geqslant 0 \end{cases} \tag{4}$$

where,

f_j : j^{th} objective function,
J : The number of objective functions,
g_k : k. constraint function,
K : The number of constraint functions,
$\underline{x}$: The decision variables vector.

In daily life events, there are many complexities arising from lack of information and uncertainty. For this reason, it is difficult to be completely objective in the decision-making process. Fuzzy linear programming model has been developed to reduce or eliminate this complexity. This model is the process of choosing the optimum solution from among the decision alternatives to achieve a specific purpose in cases where the information is not certain. One of the areas of application of fuzzy logic theory introduced by Zadeh in the mid-1960s is fuzzy linear programming. Fuzzy linear programming approach is an optimization method in which the parameters in the optimization model are not known precisely. Here, the coefficients of the objective function, the constraints, the input-output coefficients are not completely known, and some of the inequalities may have uncertain boundaries. Fuzzy linear programming approach was used as a decision model for the first time by Zimmerman [26]. For the models of fuzzy objective and fuzzy constrained linear programming, Zimmerman suggested that the decision maker could determine the amount of tolerance that he/she aims for the objective function, prior to the solution. The fuzzy linear programming problem identified by Zimmerman can be expressed as;

$$\begin{aligned} &max\ \lambda \\ &c^T x \geqslant b_0 - (1-\lambda)p_0 \\ &(Ax)_k \leqslant b_k + (1-\lambda)p_k \\ &x \geqslant 0,\ \lambda \in [0,1] \end{aligned} \tag{5}$$

In Equation (5), b_0 and b_k stand for the goal and the constraint while p_0 and p_k represent their tolerances, respectively.

There are many studies about fuzzy linear programming in the literature. Sharma and Aggarwal focused on Fully Fuzzy Multi-Objective Linear Programming (FF-MOLP) problem in which all the coefficients and decision variables are LR flat fuzzy numbers. They proposed a new algorithm for solving FFMOLP problem [27]. Nakamura solved the multi-objective linear programming models, which are represented by triangular membership functions, by transforming them into fuzzy linear programming models with partial membership functions [28]. Tanaka and Asai used objective function coefficients and right-hand side coefficients of constraints as fuzzy functions in their studies [29]. Verdegay argued that the solution of a fuzzy constrained linear programming model should be represented by a fuzzy set [30].

Financial markets are one of the areas where lack of information or uncertainty makes it difficult to make decisions. Investors with a certain accumulation aim to increase the value of their income in a variety of ways. While doing this, investors trying to create a portfolio from various securities, encounter the problem of deciding to which investment vehicle they need to invest in what extent. A fuzzy linear programming approach is used to eliminate this uncertainty and to create the optimal portfolio.

4. An Algorithm for Portfolio Selection with Linear Programming

The Analytic Hierarchy Process (AHP) was originally developed by Thomas L. Saaty in the 1970s. This method is one of the most criticized decision-making techniques used in the analysis of complex decision problems [31]. AHP is a method used to select from a large number of alternatives and in which more than one decision maker can take part in the process. In the selection process, decision makers' experience and knowledge are incorporated into the decision-making process through the AHP method. The elements are compared with matrices according to the criteria determined in the hierarchy and thus weights are obtained. This weighting process is transformed into a broad eigenvector problem and results in a normalized weight vector. These weights help to prioritize the distribution of resources [32].

Although AHP's aim is to reveal the knowledge of decision makers, conventional AHP still do not reflect the human thought accurately. In addition, the AHP is criticized for its inability to address uncertainty and indecision in the binary comparison process. For these reasons, the proposed Fuzzy AHP method is different from the AHP in which the exact values are used, in that the decision-maker opinions are in a range of values. This situation makes easier to overcome uncertainty in the decision-making process [33].

In this study, as an alternative to the Constrained Fuzzy AHP approach, which uses the triangular fuzzy numbers for criteria weighting, a new algorithm that uses the TrFNs for the same aim was proposed. In proposed algorithm, the weights of the criteria were obtained by using TrFNs. The obtained weights were used in the objective function of the proposed linear programming model by carried out the following algorithm steps.

Step 1: n being the number of criteria and p being the number of decision makers involved, comparison values (b_{ijp}) relative to each criterion are determined by the decision makers as in Table 1.

TABLE 1. General form of the comparison matrix of each criterion with each other

	C_1	C_2	$\cdots$	C_n
C_1	(1 1 1 1) $\vdots$ (1 1 1 1)	b_{121} $\vdots$ b_{12P}	$\vdots$	b_{1n1} $\vdots$ b_{1nP}
C_2	b_{211} $\vdots$ b_{21P}	(1 1 1 1) $\vdots$ (1 1 1 1)	$\vdots$	b_{2n1} $\vdots$ b_{2nP}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	b_{n11} $\vdots$ b_{n1P}	b_{n21} $\vdots$ b_{n2P}	$\vdots$	(1 1 1 1) $\vdots$ (1 1 1 1)

A TrFN consists of four parameters indicated as b^1, b^2, b^3, b^4 and is expressed as follows:

$$b_{ijp} = \begin{pmatrix} b_{ijp}^1 & b_{ijp}^2 & b_{ijp}^3 & b_{ijp}^4 \end{pmatrix} \tag{6}$$

b_{ijp} : The importance value of m^{th} criteria corresonding to n^{th} criteria, according to p^{th} decision maker,
i, j, k : The number of criteria $(i = 1, \cdots, n; j = 1, \cdots, n)$,
p : The number of decision maker$(p = 1, \cdots, P)$.

Step 2: New TrFNs are obtained with pairwise comparisons of n criteria shown in Table 1. For the comparison of C_i and C_j criteria, $b_{ijp} = \begin{pmatrix} b_{ijp}^1 & b_{ijp}^2 & b_{ijp}^3 & b_{ijp}^4 \end{pmatrix}$, where

$$\begin{gathered} a_{ij}^1 = \begin{pmatrix} b_{ij1}^1 & b_{ij2}^1 & \cdots & b_{ijp}^1 \end{pmatrix}^{1/p} \\ \vdots \\ a_{ij}^4 = \begin{pmatrix} b_{ij1}^4 & b_{ij2}^4 & \cdots & b_{ijp}^4 \end{pmatrix}^{1/p} \end{gathered} \tag{7}$$

$$A_{ij} = \begin{bmatrix} a_{ij}^1 & a_{ij}^2 & a_{ij}^3 & a_{ij}^4 \end{bmatrix} \tag{8}$$

The new TrFNs, which express the decision makers' opinions, are obtained by repeating the same process n times for each paired comparison, using geometric mean [34]. The obtained new TrFNs are given in Table 2.

TABLE 2. The comparison matrix of each criterion, which consists of the new trapezoidal fuzzy numbers obtained

	C_1	C_2	$\cdots$	C_n
C_1	(1 1 1 1)	a_{12}	$\cdots$	a_{1n}
C_2	a_{21}	(1 1 1 1)	$\cdots$	a_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
C_n	a_{n1}	a_{n2}	$\cdots$	(1 1 1 1)

Step 3: The weights of the alternatives or criteria are calculated with the proposed method using TrFNs. The formulas used in the calculations are given in Equations (9-12). Let $S_i = (S_{li}, S_{m_1 i}, S_{m_2 i}, S_{ui})$ be the fuzzy score for the i^{th} criterion of trapezoidal fuzzy pairwise comparison matrix, where the indices l, m_1, m_2 and u denote its lower, medium1,medium2 and upper respectively.

S_{li} and S_{ui} can be evaluated using the crisp mathematical programming model,

$$S_{li} = min\left[\frac{\left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}}{\sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{1/n}\right]}\right] \qquad (i,j,k=1,...,n) \tag{9}$$

$$S_{ui} = max\left[\frac{\left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}}{\sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} a_{kj}\right)^{1/n}\right]}\right] \qquad (i,j,k=1,...,n) \tag{10}$$

subject to $a_{in} \in [l_{in}, u_{in}],\ \forall n > i;\ a_{ni} = \dfrac{1}{a_{in}},\ \forall n < i;\ a_{nn} = 1$. $S_{m_1 i}$ and $S_{m_2 i}$ are calculated by Equation (11,12).

$$S_{m_1 i} = \frac{\left(\prod_{j=1}^{n} m_{1ij}\right)^{1/n}}{\sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} m_{1kj}\right)^{1/n}\right]} \qquad (i,j,k=1,...,n) \tag{11}$$

$$S_{m_2i} = \frac{\left(\prod_{j=1}^{n} m_{2ij}\right)^{1/n}}{\sum_{k=1}^{n}\left[\left(\prod_{j=1}^{n} m_{2kj}\right)^{1/n}\right]} \qquad (i,j,k = 1,...,n) \tag{12}$$

The weights obtained by the proposed method are shown in Table 3.

TABLE 3. Fuzzy weights of criteria / alternatives

Criteria	Weights (TrFNs)
C_1	$(S_{l1}\ S_{m_11}\ S_{m_21}\ S_{u1})$
$\vdots$	$\vdots$
C_n	$(S_{li}\ S_{m_1i}\ S_{m_2i}\ S_{ui})$

Step 4: A linear programming model which is proposed for portfolio allocation, is composed by defuzzification of the fuzzy weights for the alternatives obtained as a result of the solution process started by using the fuzzy importance degree.

Defuzzification can be called the inverse of the fuzzification process. Defuzzification operations are performed using membership functions of the fuzzy scores resulting from the fuzzy operations [35]. Equation (13) is used to defuzzification the fuzzy weights obtained for each alternative. To evaluate a crisp weight for each stock, one can use the defuzzification method to replace the fuzzy numbers by crisp numbers. A ranking method which uses the defuzzification function is as follows:

$$F(A) = 1/2 \int_0^1 [{}^a\underline{a}\ +{}^a\ \overline{a}]\, da \tag{13}$$

where $\underline{a}$ and $\overline{a}$ are the infimum and supremum of α-cut of the fuzzy number A defined for $x \in R$, respectively [22]. The exact weights after defuzzification are given in Table 4.

TABLE 4. The exact weights of criteria / alternatives

Criteria	C_1	C_2	$\cdots$	C_n
The exact weights	w_1	w_2	$\cdots$	w_n

Step 5: By utilizing w_l obtained by TrFNs, corresponding to opinion of decision makers, the linear programming problem given by Equation (14) is modeled.

$$
\begin{aligned}
&max \sum_{l=1}^{L} w_l \lambda_l + \sum_{t=1}^{T} \beta_t \gamma_t \\
&\lambda_l \leqslant \mu_{z_l}(x),\ l = 1, ..., L \qquad (for \quad all \quad objective \quad functions), \\
&\gamma_t \leqslant \mu_{g_t}(x),\ t = 1, ..., T \qquad (for \quad fuzzy \quad constraints), \\
&\sum_{l=1}^{L} w_l + \sum_{t=1}^{T} \beta_t = 1 \\
&g_k(x) \leqslant b_k,\ k = 1, ..., K \qquad (for \quad deterministic \quad constraints), \\
&\lambda_l, \gamma_t \ \in [0,1];\ w_l, \beta_t \geqslant 0;\ x_m \geqslant 0,\ (m = 1, ..., M).
\end{aligned} \tag{14}
$$

In Equation (14), w_l, β_t are the weights coefficients that present the relative importance among the fuzzy goals and fuzzy constraints. λ_l and γ_t represents the constraint and the fuzzy constraint parameters. The flow chart of the proposed method is illustrated in Fig. 1.

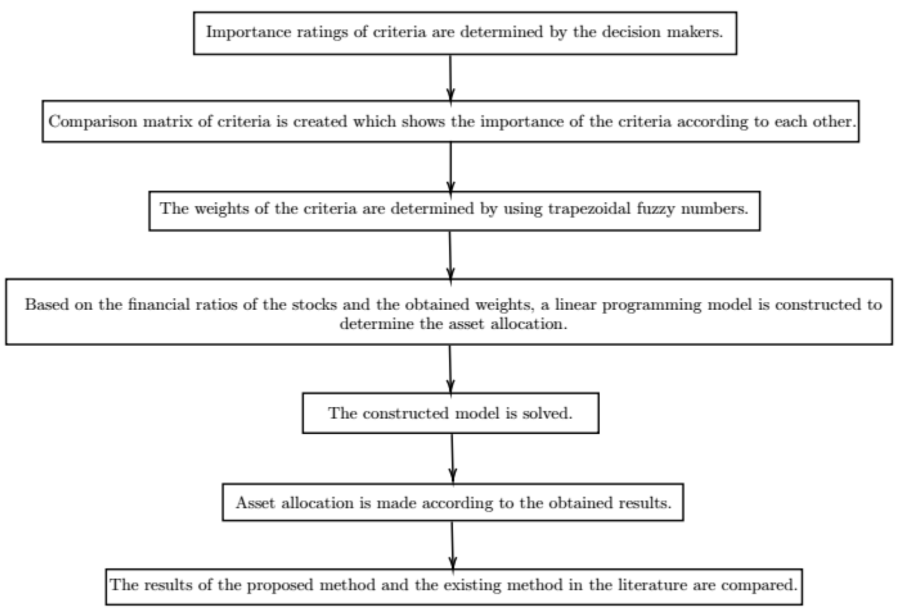

FIGURE 1. Flow chart of the proposed method

5. Application

In this section, the method based on triangular fuzzy numbers proposed in the literature and a portfolio selection problem solved with this method are addressed. This problem solved by proposed method in this study. The hierarchical structure created for the application is given in Figure 2 [36]. The constructed hierarchy consists of seven most important criteria which are: Price/Earnings (P/E), Net Profit/Stockholder's Equity (NP/SE), Net Debt/Marketing Value (ND/MV), Current Ratio (CR), Marketing Value/Carrying Amount (MV/CA), Net Profit/Sales Revenue (NP/SR) and Net Profit/Total Assets (NP/TA). In the study, 5 stocks were determined using past price movements obtained from BIST; Anadolu Cam (ANACM), Trakya Cam (TRKCM), Mardin Cimento (MRDIN), Eregli Demir Celik (EREGL) and Izmir Demir Celik (IZMDC).

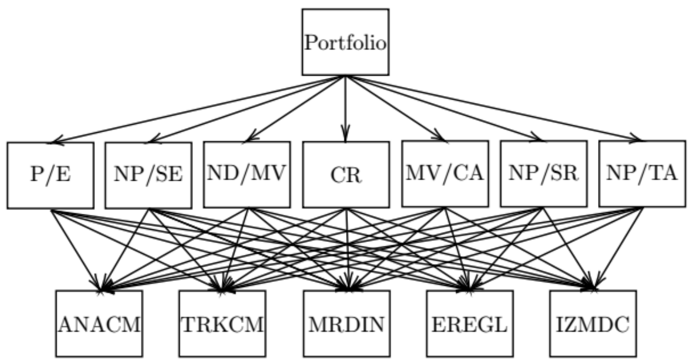

Figure 2. Hierarchy of the problem

Price/Earnings, Net Profit/Stockholder's Equity, Net Debt/Marketing Value, Current Ratio, Marketing Value/Carrying Amount, Net Profit/Sales Revenue and Net Profit/Total Assets criteria are represented by $C_1, C_2, C_3, C_4, C_5, C_6$ andC_7 respectively. While the current ratio is different in each sector, it is generally considered to be sufficient if this ratio is 2. In this study, the current ratio is used as a fuzzy constraint between 1.4 and 2.5. The financial ratios of ANACM, TRKCM, MRDIN, EREGL and IZMDC are given in Table 5.

In the aforementioned study by Ahlatcioglu, four decision makers used the linguistic variables shown in Table 6 to assess the importance degree of criteria [36].

In this study, the importance scale given as a triangular fuzzy number is transformed into TrFNs as shown in Fig. 3. The triangular fuzzy number on the existing importance scale in the literature has been converted into TrFNs by keeping the

TABLE 5. Stocks and financial ratio

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
ANACM	10.7	11.5	10.0	1.3	12.6	7.6	2.04
TRKCM	8.9	12.5	0.2	1.2	18.3	8.6	2.17
MRDIN	7.1	23.4	-26.2	1.9	34.9	20.7	5.83
EREGL	3.6	18.1	-9.0	0.7	18.0	12.7	1.86
IZMDC	4.1	30.8	2.8	1.3	9.8	19.9	1.55

TABLE 6. Importance ratings of criteria determined by the four decision makers

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Decision Maker 1	(5 7 9)	(7 9 10)	(5 7 9)	(5 7 9)	(7 9 10)	(5 7 9)	(3 5 7)
Decision Maker 2	(7 9 10)	(5 7 9)	(5 7 9)	(7 9 10)	(7 9 10)	(5 7 9)	(3 5 7)
Decision Maker 3	(9 10 10)	(7 9 10)	(5 7 9)	(9 10 10)	(7 9 10)	(5 7 9)	(5 7 9)
Decision Maker 4	(5 7 9)	(9 10 10)	(5 7 9)	(7 9 10)	(9 10 10)	(5 7 9)	(3 5 7)

upper and lower bounds of the fuzzy numbers constant and spreading the center to a certain range.

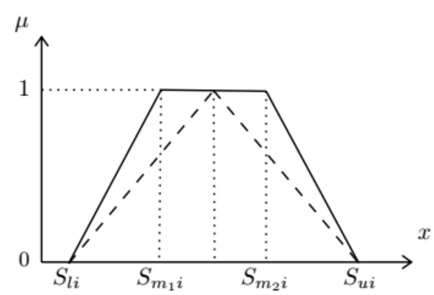

FIGURE 3. Transform from triangular fuzzy number to trapezoidal fuzzy number

The importance degree of criteria given in Table 6 are transformed into the trapezoid fuzzy number and Table 7 is obtained by operating the proposed algorithm.

The seven criteria are compared with respect to the goal "portfolio selection", and the corresponding fuzzy pairwise comparison matrix is solicited from the decision makers and presented in Table 8.

The fuzzy weight of each criterion is calculated by applying the Constrained Fuzzy AHP method which is formed by using TrFNs for the portfolio selection. The fuzzy weights for each criterion are summarized in Table 9.

Table 7. The importance degrees of criteria given by decision makers transformed into TrFNs

Criteria	Decision Maker
C_1	(6.30 7.69 8.49 9.49)
C_2	(6.85 8.22 8.95 9.74)
C_3	(5 6.5 7.5 9)
C_4	(6.85 8.22 8.95 9.74)
C_5	(7.45 8.79 9.43 10)
C_6	(5 6.5 7.5 9)
C_7	(3.41 4.93 5.94 7.45)

Table 8. Fuzzy pairwise comparison matrix for criteria with respect to goal "portfolio selection"

Goal	C_1	C_2	C_3	C_4
C_1	(1 1 1 1)	(0.65 0.86 1.03 1.39)	(0.70 1.03 1.31 1.90)	(0.65 0.86 1.03 1.39)
C_2	(0.72 0.97 1.16 1.54)	(1 1 1 1)	(0.76 1.10 1.38 1.95)	(0.70 1.03 1.31 1.90)
C_3	(0.53 0.76 0.97 1.43)	(0.51 0.72 0.91 1.32)	(1 1 1 1)	(0.51 0.73 0.91 1.31)
C_4	(0.72 0.97 1.16 1.54)	(0.70 0.92 1.09 1.43)	(0.76 1.10 1.37 1.96)	(1 1 1 1)
C_5	(0.79 1.03 1.22 1.59)	(0.76 0.98 1.15 1.45)	(0.83 1.18 1.45 2.00)	(0.76 0.98 1.14 1.45)
C_6	(0.53 0.76 0.97 1.43)	(0.51 0.72 0.91 1.32)	(0.56 0.87 1.15 1.79)	(0.51 0.72 0.91 1.32)
C_7	(0.36 0.58 0.78 1.18)	(0.35 0.55 0.72 1.09)	(0.38 0.66 0.92 1.49)	(0.35 0.55 0.72 1.09)

Goal	C_5	C_6	C_7
C_1	(0.63 0,82 0,97 1,27)	(0,70 1,03 1,31 1,90)	(0,85 1,29 1,72 2,78)
C_2	(0.69 0,87 1,02 1,31)	(0,76 1,10 1,38 1,95)	(0,92 1,38 1,82 2,86)
C_3	(0.50 0.69 0.85 1.21)	(0.56 0.87 1.15 1.80)	(0.67 1.09 1.52 2.64)
C_4	(0.69 0,87 1,02 1,31)	(0.76 1.10 1.38 1.95)	(0.92 1.38 1.82 2.86)
C_5	(1 1 1 1)	(0.83 1.17 1.45 2.00)	(1.00 1.48 1.91 2.93)
C_6	(0.50 0.69 0.85 1.20)	(1 1 1 1)	(0.67 1.09 1.52 2.64)
C_7	(0.34 0.52 0.68 1.00)	(0.38 0.66 0.92 1.49)	(1 1 1 1)

Table 9. Fuzzy weights of criteria

Criteria	Weights (TrFNs)
C_1	(0.09 0.14 0.16 0.21)
C_2	(0.10 0.15 0.17 0.23)
C_3	(0.09 0.12 0.14 0.21)
C_4	(0.11 0.15 0.17 0.23)
C_5	(0.11 0.16 0.18 0.23)
C_6	(0.09 0.12 0.14 0.21)
C_7	(0.05 0.09 0.11 0.17)

The asset allocation is realized by defuzzification of fuzzy weights related to criteria obtained as a result of the solution process started with using fuzzy importance degree. The defuzzification can be called the inverse of the fuzzification process. Defuzzification are performed using membership functions for the fuzzy scores obtained as a result of the fuzzy operations. The certain weights obtained for the criteria after the defuzzification process are given in Table 10.

TABLE 10. Evaluation of exact weights of alternatives

	P/E	NP/SE	ND/MV	MV/CA	NP/SR	NP/TA	CR
Weights of criteria	0.148	0.158	0.133	0.159	0.167	0.132	0.103

The multi-objective linear formulation of numerical example is given as P_1.

$$
\begin{aligned}
P_1:\quad & Z_{P/E(min)} = 10.7x_1 + 8.9x_2 + 7.1x_3 + 3.6x_4 + 4.1x_5 \\
& Z_{NP/SE(max)} = 11.5x_1 + 12.5x_2 + 23.4x_3 + 18.1x_4 + 30.8x_5 \\
& Z_{ND/MV(min)} = 10.0x_1 + 0.2x_2 + 26.2x_3 - 9.0x_4 + 2.8x_5 \\
& Z_{MV/CA(max)} = 1.3x_1 + 1.2x_2 + 1.9x_3 + 0.7x_4 + 1.3x_5 \\
& Z_{NP/SR(max)} = 12.6x_1 + 18.3x_2 + 34.9x_3 + 18.0x_4 + 19.9x_5 \\
& Z_{NP/TA(max)} = 7.6x_1 + 8.6x_2 + 20.7x_3 + 12.7x_4 + 19.9x_5 \\
& Z_{CR(1.4 \leqslant CR \leqslant 2.5)} = 2.04x_1 + 2.17x_2 + 5.83x_3 + 1.86x_4 + 1.55x_5 \\
& x_1 + x_2 + x_3 + x_4 + x_5 = 1 \\
& x_1, x_2, x_3, x_4, x_5 \geqslant 0
\end{aligned}
$$

The seven objective functions $Z_{P/E}$, $Z_{NP/SE}$, $Z_{ND/MV}$, $Z_{MV/CA}$, $Z_{NP/SR}$, $Z_{NP/TA}$ and Z_{CR} are respectively P/E, Net profit / Stockholder's Equity, Net Debt / Marketing Value, Marketing Value / Carrying Amount, Net profit / Sales Revenue, Net Profit/Total Assets and Current Ratio goals, x_i is the percentage of the i^{th} stock to be invested.

The maximum and minimum values for each objective function were determined under the constraints of the model by using WinQSB software. Solutions for each of 6 objective functions are given in Table 11.

TABLE 11. The maximum and minimum values of the objective functions

Objective fuction	$\mu = 0$	$\mu = 1$	$\mu = 0 - \mu = 1$
$Z_{P/E(min)}$	10.7	3.6	7.1
$Z_{NP/SE(max)}$	30.8	11.5	19.3
$Z_{ND/MV(min)}$	10	-26.2	36.2
$Z_{MV/CA(max)}$	1.9	0.7	1.2
$Z_{NP/SR(max)}$	34.9	9.8	25.1
$Z_{NP/TA(max)}$	20.7	7.6	13.1

The structure of the new fuzzy multi-objective linear model created using the final weights obtained from the proposed algorithm is given as P_2.

$$
\begin{aligned}
P_2 : max \quad & 0.148\lambda_1 + 0.158\lambda_2 + 0.133\lambda_3 + 0.159\lambda_4 + 0.167\lambda_5 + 0.132\lambda_6 \\
& +0.103\lambda_7 \\
& \lambda_1 \leqslant \frac{10.7 - (10.7x_1 + 8.9x_2 + 7.1x_3 + 3.6x_4 + 4.1x_5)}{7.1} \\
& \lambda_2 \leqslant \frac{(11.5x_1 + 12.5x_2 + 23.4x_3 + 18.1x_4 + 30.8x_5) - 11.5}{19.3} \\
& \lambda_3 \leqslant \frac{10.0 - (10.0x_1 + 0.2x_2 - 26.2x_3 - 9.0x_4 + 2.8x_5)}{36.2} \\
& \lambda_4 \leqslant \frac{(1.3x_1 + 1.2x_2 + 1.9x_3 + 0.7x_4 + 1.3x_5) - 0.7}{1.2} \\
& \lambda_5 \leqslant \frac{(12.6x_1 + 18.3x_2 + 34.9x_3 + 18.0x_4 + 9.8x_5) - 9.8}{25.1} \\
& \lambda_6 \leqslant \frac{(7.6x_1 + 8.6x_2 + 20.7x_3 + 12.7x_4 + 19.9x_5) - 7.6}{13.1} \\
& \lambda_7 \leqslant \frac{2.5 - (2.04x_1 + 2.17x_2 + 5.83x_3 + 1.86x_4 + 1.55x_5)}{0.5} \\
& \lambda_7 \leqslant \frac{(2.04x_1 + 2.17x_2 + 5.83x_3 + 1.86x_4 + 1.55x_5) - 1.4}{0.6} \\
& x_1 + x_2 + x_3 + x_4 + x_5 = 1 \\
& x_1, x_2, x_3, x_4, x_5 \geqslant 0
\end{aligned}
$$

WinQSB is used to solve the problem. The optimal solution for the model is obtained as follows: 5.557 and 5.476 are the results that are obtained from the solution of the Price/Earnings and Net Debt/Marketing Value objective functions respectively, which are required to be minimized. 24.618; 1.322; 21.652 and 17.047 are the results that are obtained from the solution of the Net Profit/Stockholder's Equity, Marketing Value/Carrying Amount, Net Profit/Sales Revenue and Net Profit/Total Assets objective functions respectively, which are required to be maximized. Finally, 2.423 is the result that is obtained from the solution of the Current Ratio objective function. In the view of these results, the decision variables of the model are obtained as $x_1 = 0.05$, $x_2 = 0.14$, $x_3 = 0.17$, $x_4 = 0.11$ and $x_5 = 0.53$.

6. Conclusion

The results regarding to the proposed and the existing method (Constrained Fuzzy AHP method using triangular fuzzy numbers) (see also [20]) and the returns on assets ratios of 5 stocks between the dates February-May 2018 are presented together in Table 12. The table also includes the return investments for the months covering the dates February-May 2018. For both proposed and existing methods, the return investments were calculated by multiplying the asset allocation results with the monthly returns on assets ratios. The fact is that financial markets are impressive in terms of portfolio selection. In the case of uncertainty, the fact that the investment is not planned can be encountered with unexpected losses to the investor. It is aimed to help investors who invest in uncertainty with the proposed model for the most appropriate portfolio selection. For the existing method, the calculated return investments for February, March, April, and May were 12.76%,-2.37%, 12.03% and -1.52% respectively, whereas return the investments of the proposed method were 13.41%, 1.19%, 11.92% and -0.48% respectively. Based on this, it can be concluded that the proposed method manages to supply better asset allocation.

Table 12. The solutions obtained from the application of different methods

	Stocks	**Asset allocation**	**Return on assets ratio (Monthly)**				**Return on investment (Asset allocation x Return on assets ratio)**			
			Feb.08	**Mar.08**	**Apr.08**	**May.08**	**Feb.08**	**Mar.08**	**Apr.08**	**May.08**
The results of the proposed algorithm	ANACM	0.05	7.12	-11.99	7.65	-4.25	0.36	-0.60	0.38	-0.21
	TRKCM	0.14	7.15	-29.98	13.57	-7.25	1.00	-4.20	1.90	-1.02
	MRDIN	0.17	12.5	1.14	21.38	0.05	2.13	0.19	3.63	0.01
	EREGL	0.11	19.24	7.47	20.86	-3.12	2.12	0.82	2.29	-0.34
	IZMDC	0.53	14.72	9.39	7.01	2.03	7.80	4.98	3.72	1.08
	Total						**13.41**	**1.19**	**11.92**	**-0.48**
The results existing in the literature	ANACM	0.09	7.12	-11.99	7.65	-4.25	0.64	-1.08	0.69	-0.38
	TRKCM	0.22	7.15	-29.98	13.57	-7.25	1.57	-6.60	2.99	-1.60
	MRDIN	0.11	12.5	1.14	21.38	0.05	1.38	0.13	2.35	0.01
	EREGL	0.14	19.24	7.47	20.86	-3.12	2.69	1.05	2.92	-0.44
	IZMDC	0.44	14.72	9.39	7.01	2.03	6.48	4.13	3.08	0.89
	Total						**12.76**	**-2.37**	**12.03**	**-1.52**

In the later study, the proposed algorithm can be extended using more objective function, constraint function and decision variables to solution of multi-criteria decision problems like portfolio selection problems.

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