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Generalized $\left(\frac{G'}{G}\right)$ - Expansion Method for Some Soliton Wave Solution of (2+1)-Dimensional Dispersive Long Wave Equation (DLWE)

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Abstract

In this article, we have found some soliton wave solutions of the (2 + 1) -dimensional dispersive long wave equation using the generalized $\left(\frac{G'}{G}\right)$ - expansion method. Recently this method is developed for searching exact traveling wave solutions of nonlinear partial differential equations. Also, these solutions might play important role in mechanics. For this equation, we obtained hyperbolic function solutions, exponential function solutions and rational function solutions. We also saw that the solutions provided the equation using Mathematica 11.2 and we showed the graphical performance of some of the solutions found.

Keywords: *(2+1)-dimensional dispersive long wave equation, Soliton wave solutions, Generalized* $\left(\frac{G'}{G}\right)$*- expansion method, Mathematica 11.2.*

1.INTRODUCTION

Nonlinear partial differential equations (NPDEs) have an important place in applied sciences [1,2]. There are some analytical methods for solving these equations in the literature [3-11]. In addition to these methods, there are many methods of solving such equations by using an auxiliary equation. By using these methods, partial differential equations are converted to ordinary differential equations and the solutions of partial differential equations are found with the help of these ordinary differential equations. These methods are given in [12-26]. Many authors have applied these and similar methods to various equations [27-47].

We used the generalized $\left(\frac{G'}{G}\right)$- Expansion Method for finding the some soliton wave solution (2+1)-dimensional (DLWE). This method is presented in[29].

2. ANALYSIS OF METHOD

Let's introduce the method briefly. Consider a general partial differential equation of two variables,

$$Q(u, u_t, u_x, u_{xx}, \dots) = 0, \tag{1}$$

Using the wave variable $u(x,t) = u(\xi),\ \xi = x - \mu t$ the Eq.(1) turns into an ordinary differential equation,

$$Q'(u', u'', u''', \dots) = 0 \tag{2}$$

here μ is constant. With this conversion, we obtain a nonlinear ordinary differential equation for $u(\xi)$. We can express the solution of Eq.(2) as below,

$$u(\xi) = \sum_{k=0}^{m} d_k \Phi(\xi)^k + \sum_{k=1}^{m} e_k \Phi(\xi)^{-k} \tag{3}$$

where m is a positive integer is found as the result of balancing the highest order linear term and the highest order nonlinear term found in the equation, the coefficients d_k and e_k are constants. $\Phi(\xi) = \left(\frac{G'}{G}\right)$ satisfies the following ordinary differential equation,

$$k_1 GG'' - k_2 GG' - k_3 (G')^2 - k_4 G^2 = 0. \tag{4}$$

Substituting solution (3) into Eq. (2) yields a set of algebraic equation for $\left(\frac{G'}{G}\right), \left(\frac{G'}{G}\right)^{-k}$, then, all coefficients of $\left(\frac{G'}{G}\right), \left(\frac{G'}{G}\right)^{-k}$, have to vanish. Then, we can found $d_k, e_k, k_1, k_2, k_3, k_4$ and μ. The special solutions of Eq. (4) are as follows,[29]

1. When $k_2 \neq 0, f = k_1 - k_3$ and $s = k_2^2 + 4k_4(k_1 - k_3) > 0$, then
$$\Phi(\xi) = \frac{k_2}{2f} + \frac{\sqrt{s}}{2f} \frac{C_1 sinh\left(\frac{\sqrt{s}}{2k_1}\xi\right) + C_2 cosh\left(\frac{\sqrt{s}}{2k_1}\xi\right)}{C_1 cosh\left(\frac{\sqrt{s}}{2k_1}\xi\right) + C_2 sinh\left(\frac{\sqrt{s}}{2k_1}\xi\right)}.$$
2. When $k_2 \neq 0, f = k_1 - k_3$ and $s = k_2^2 + 4k_4(k_1 - k_3) < 0$, then
$$\Phi(\xi) = \frac{k_2}{2f} + \frac{\sqrt{-s}}{2f} \frac{-C_1 sin\left(\frac{\sqrt{-s}}{2k_1}\xi\right) + C_2 cos\left(\frac{\sqrt{-s}}{2k_1}\xi\right)}{C_1 cos\left(\frac{\sqrt{-s}}{2k_1}\xi\right) + C_2 sin\left(\frac{\sqrt{-s}}{2k_1}\xi\right)}.$$
3. When $k_2 \neq 0, f = k_1 - k_3$ and $s = k_2^2 + 4k_4(k_1 - k_3) = 0$, then
$$\Phi(\xi) = \frac{k_2}{2f} + \frac{C_2}{C_1 + C_2\xi}.$$
4. When $k_2 = 0, f = k_1 - k_3$ and $g = fk_4 > 0$, then
$$\Phi(\xi) = \frac{\sqrt{g}}{f} \frac{C_1 sinh\left(\frac{\sqrt{g}}{k_1}\xi\right) + C_2 cosh\left(\frac{\sqrt{g}}{k_1}\xi\right)}{C_1 cosh\left(\frac{\sqrt{g}}{k_1}\xi\right) + C_2 sinh\left(\frac{\sqrt{g}}{k_1}\xi\right)}.$$
5. When $k_2 = 0, f = k_1 - k_3$ and $g = fk_4 < 0$, then
$$\Phi(\xi) = \frac{\sqrt{-g}}{f} \frac{-C_1 sin\left(\frac{\sqrt{-g}}{k_1}\xi\right) + C_2 cos\left(\frac{\sqrt{-g}}{k_1}\xi\right)}{C_1 cos\left(\frac{\sqrt{-g}}{k_1}\xi\right) + C_2 sin\left(\frac{\sqrt{-g}}{k_1}\xi\right)}.$$
6. When $k_4 = 0$ and $f = k_1 - k_3$, then
$$\Phi(\xi) = \frac{C_1 k_2^2 exp\left(\frac{-k_2}{k_1}\xi\right)}{fk_1 + C_1 k_1 k_2 exp\left(\frac{-k_2}{k_1}\xi\right)}.$$
7. When $k_2 \neq 0$ and $f = k_1 - k_3 = 0$, then

$$\Phi(\xi) = -\frac{k_4}{k_2} + C_1 exp\left(\frac{k_2}{k_1}\xi\right).$$

8. When $k_1 = k_3,\ k_2 = 0$ and $f = k_1 - k_3 = 0$, then

$$\Phi(\xi) = C_1 + \frac{k_4}{k_1}\xi.$$

9. When $k_3 = 2k_1,\ k_2 = 0$ and $k_4 = 0$, then

$$\Phi(\xi) = -\frac{1}{C_1 + \left(\frac{k_3}{k_1} - 1\right)\xi}.$$

3.EXAMPLE. We consider the (2+1)-dimensional DLWE [47],

$$\begin{aligned} &u_{yt} + v_{xx} + u_y u_x + u u_{xy} = 0 \\ &v_t + u_x + u_x v + u v_x + u_{xxy} = 0, \end{aligned} \tag{5}$$

Using the wave variable $u(x,y,t) = u(\xi),\ \xi = x + y - \mu t$ the equation (1) turns into an ordinary differential equation,

$$\begin{aligned} &-\mu u'' + v'' + (u')^2 + uu'' = 0 \\ &-\mu v' + u' + u'v + uv' + u''' = 0, \end{aligned} \tag{6}$$

when balancing v'' with uu'' and u''' with $u'v$ then $m = 1$ and $n = 2$ gives. The solutions are as follows:

$$\begin{aligned} &u(\xi) = d_0 + d_1\Phi(\xi) + e_1\Phi(\xi)^{-1} \\ &v(\xi) = f_0 + f_1\Phi(\xi) + f_2\Phi(\xi)^2 + g_1\Phi(\xi)^{-1} + g_2\Phi(\xi)^{-2} \end{aligned} \tag{7}$$

If Eq. (7) is substituted in Eq. (6), the following algebraic equation system is obtained for $d_0, d_1, e_1,$ $,k_1, k_2, k_3, k_4$ and μ;

$$\begin{aligned} &e_1^2 + 2g_2 + \frac{\mu e_1 k_2}{k_1} - \frac{d_0 e_1 k_2}{k_1} - \frac{g_1 k_2}{k_1} - \frac{2e_1^2 k_3}{k_1} - \frac{4g_2 k_3}{k_1} - \frac{\mu e_1 k_2 k_3}{k_1^2} + \frac{d_0 e_1 k_2 k_3}{k_1^2} + \frac{g_1 k_2 k_3}{k_1^2} + \frac{e_1^2 k_3^2}{k_1^2} + \frac{2g_2 k_3^2}{k_1^2} - \\ &\frac{\mu d_1 k_2 k_4}{k_1^2} + \frac{d_0 d_1 k_2 k_4}{k_1^2} + \frac{f_1 k_2 k_4}{k_1^2} + \frac{d_1^2 k_4^2}{k_1^2} + \frac{2f_2 k_4^2}{k_1^2} = 0, \\ &\frac{3e_1^2 k_4^2}{k_1^2} + \frac{6g_2 k_4^2}{k_1^2} = 0, \\ &\frac{5e_1^2 k_2 k_4}{k_1^2} + \frac{10g_2 k_2 k_4}{k_1^2} - \frac{2\mu e_1 k_4^2}{k_1^2} + \frac{2d_0 e_1 k_4^2}{k_1^2} + \frac{2g_1 k_4^2}{k_1^2} = 0, \\ &\frac{2e_1^2 k_2^2}{k_1^2} + \frac{4g_2 k_2^2}{k_1^2} - \frac{4e_1^2 k_4}{k_1} - \frac{8g_2 k_4}{k_1} - \frac{3\mu e_1 k_2 k_4}{k_1^2} + \frac{3d_0 e_1 k_2 k_4}{k_1^2} + \frac{3g_1 k_2 k_4}{k_1^2} + \frac{4e_1^2 k_3 k_4}{k_1^2} + \frac{8g_2 k_3 k_4}{k_1^2} = 0, \cdots \end{aligned} \tag{8}$$

If this linear algebraic equation system is solved, the coefficients are found as follows:

Case 1.

$e_1 = 0, f_0 = -1,\ f_2 = -\frac{d_1^2}{2},\ g_1 = 0,\ g_2 = 0, d_1 \neq 0, k_2 = \frac{f_1 k_1}{d_1}, k_2 \neq 0, k_3 = \frac{k_1(-f_1 k_1 + 2k_2)}{2k_2},$
$k_4 = 0, -k_1 + k_3 \neq 0, \mu = \frac{-2d_0 k_1 - d_1 k_2 + 2d_0 k_3}{2(-k_1 + k_3)}.$

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Solution 1.

$$u(x,y,t)=d_0+\frac{f_1}{d_1}+\frac{\left(C_1\mathrm{Sinh}\left[\frac{\left(x+y-t\left(d_0+\frac{f_1}{d_1}\right)\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{2k_1}\right]+C_2\mathrm{Cosh}\left[\frac{\left(x+y-t\left(d_0+\frac{f_1}{d_1}\right)\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{2k_1}\right]\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{\left(C_1\mathrm{Cosh}\left[\frac{\left(x+y-t\left(d_0+\frac{f_1}{d_1}\right)\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{2k_1}\right]+C_2\mathrm{Sinh}\left[\frac{\left(x+y-t\left(d_0+\frac{f_1}{d_1}\right)\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{2k_1}\right]\right)k_1} \qquad (9)$$

$$v(x,y,t)$$
$$=-\frac{2\mathrm{Sinh}\left[\frac{\left(x+y-t\left(d_0+\frac{f_1}{d_1}\right)\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{k_1}\right]C_1C_2d_1^2+C_1^2\left(\left(1+\mathrm{Cosh}\left[\frac{\left(x+y-t\left(d_0+\frac{f_1}{d_1}\right)\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{k_1}\right]\right)d_1^2-f_1^2\right)}{2\left(\mathrm{Cosh}\left[\frac{\left(x+y-t\left(d_0+\frac{f_1}{d_1}\right)\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{k_1}\right]C_1+\mathrm{Sinh}\left[\frac{\left(x+y-t\left(d_0+\frac{f_1}{d_1}\right)\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{k_1}\right]C_2\right)^2d_1^2}$$
$$+\frac{C_2^2\left(\left(-1+\mathrm{Cosh}\left[\frac{\left(x+y-t\left(d_0+\frac{f_1}{d_1}\right)\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{k_1}\right]\right)d_1^2+f_1^2\right)}{2\left(\mathrm{Cosh}\left[\frac{\left(x+y-t\left(d_0+\frac{f_1}{d_1}\right)\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{k_1}\right]C_1+\mathrm{Sinh}\left[\frac{\left(x+y-t\left(d_0+\frac{f_1}{d_1}\right)\right)\sqrt{\frac{f_1^2k_1^2}{d_1^2}}}{k_1}\right]C_2\right)^2d_1^2} \qquad (10)$$

Case 2.

$d_1=0,\ f_0=-1,\ f_2=0,\ g_1=0,\ g_2=-\frac{e_1^2}{2},\ k_2=0,\ k_3=k_1,\ k_4=\frac{e_1k_1}{2},\ \mu=d_0,\ e_1f_1k_1\neq 0.$

Solution 2.

$$u(x,y,t)=d_0+\frac{e_1}{C_1+\frac{1}{2}(x+y-td_0)e_1} \qquad (11)$$

$$v(x,y,t)=-1-\frac{2e_1^2}{(2C_1+(x+y-td_0)e_1)^2}+\left(C_1+\frac{1}{2}(x+y-td_0)e_1\right)f_1 \qquad (12)$$

Case 3.

$d_1=0,\ f_0=-1,\ f_1=0,\ f_2=0,\ g_2=-\frac{e_1^2}{2},\ e_1\neq 0,\ k_2=\frac{g_1k_1}{e_1},\ k_3=k_1,\ k_2\neq 0,\ k_4=-\frac{g_1k_1^2}{2k_2},$
$k_4\neq 0,\ \mu=\frac{-e_1k_2+2d_0k_4}{2k_4}$

Solution 3.

$$u(x,y,t) = d_0 + \frac{2e_1 g_1}{e_1^2 + 2e^{\frac{g_1\left(x+y-t\left(d_0+\frac{g_1}{e_1}\right)\right)}{e_1}} C_1 g_1} \tag{13}$$

$$v(x,y,t) = -1 - \frac{2e_1^2 g_1^2}{\left(e_1^2 + 2e^{\frac{g_1\left(x+y-t\left(d_0+\frac{g_1}{e_1}\right)\right)}{e_1}} C_1 g_1\right)^2} + \frac{2g_1^2}{e_1^2 + 2e^{\frac{g_1\left(x+y-t\left(d_0+\frac{g_1}{e_1}\right)\right)}{e_1}} C_1 g_1} \tag{14}$$

4. EXPLANATIONS AND GRAPHICAL REPRESENTATIONS OF THE OBTAINED SOME SOLUTIONS

The graphs of Eq. (9) and Eq. (10) are shown in Figs. 1-2 respectively, within the interval $-20 \leq x \leq 20, -5 \leq t \leq 5$.

a) b)

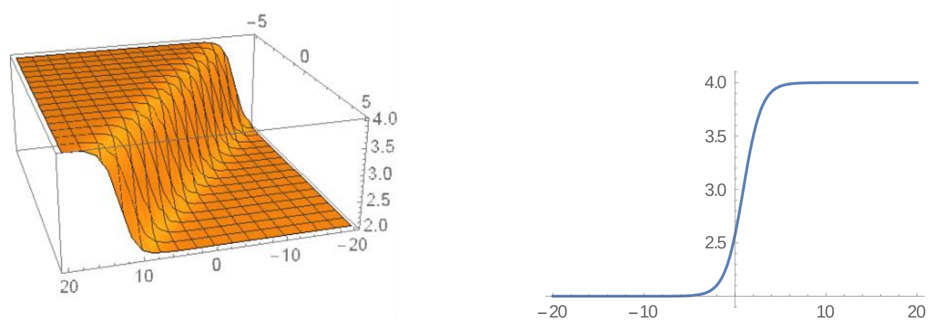

Figure 1. a) The 3 Dimensional surfaces of Eq. (9) for $f_1 = 1, d_1 = 1, d_0 = 2,\ C_1 = 2,\ k_1 = 3, C_2 = 1, y = 1$. b) The 2 Dimensional surfaces of Eq. (9) for $f_1 = 1, d_1 = 1, d_0 = 2,\ C_1 = 2,\ k_1 = 3, C_2 = 1, y = 1$ and $t = 1$.

a) b)

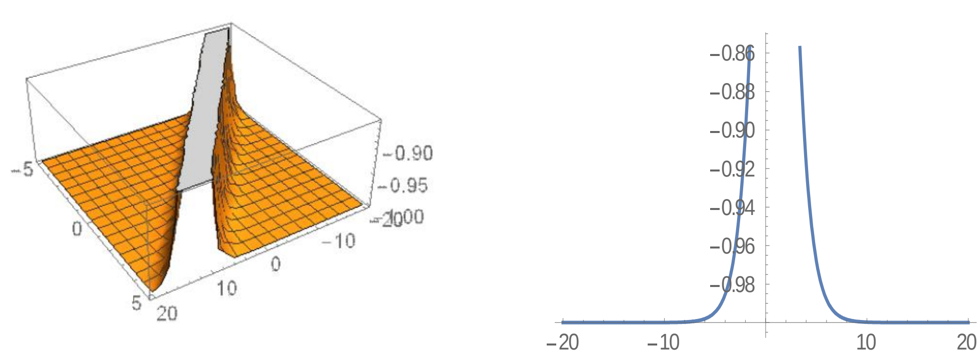

Figure 2.a)The 3 Dimensional surfaces of Eq. (10) for $f_1 = 1, d_1 = 1, d_0 = 2,\ C_1 = 2,\ k_1 = 3, C_2 = 1, y = 1$. b) The 2 Dimensional surfaces of Eq. (10) for $f_1 = 1, d_1 = 1, d_0 = 2,\ C_1 = 2,\ k_1 = 3, C_2 = 1, y = 1$ and $t = 1$.

5. CONCLUSIONS

We used the Generalized $\left(\frac{G'}{G}\right)$- Expansion Method for some soliton wave solution (2+1)-dimensional DLWE. Some nonlinear partial differential equations were solved by this method. For this equation, we obtained hyperbolic function solutions, exponential function solutions and rational function solutions. We also saw that the solutions provided the equation using Mathematica 11.2 and we showed the graphical performance of some of the solutions found. Moreover, this method is also computerizable, which lets us to perform confused and oppressive algebraic calculation on a computer by the aid of symbolic programs such as Mathematica, Maple, Matlab, and so on. It can be solved similarly in a number of nonlinear partial differential equations.

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