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THE PRINCIPAL FUNCTIONS OF THE SYSTEM OF FINITE STRUM-LIOUVILLE DIFFERENTIAL OPERATORS CORRESPONDING TO THE EIGENVALUES AND SPECTRAL SINGULARITIES

Esra KIR*
Gazi University, Faculty of Arts and Sciences , Department of Mathematics, 06500, Ankara.-Turkey, e-mail:esrakir@gazi.edu.tr

ABSTRACT

In this paper, the principal functions of the system of finite Sturm-Liouville differential operators corresponding to the eigenvalues and spectral singularities are investigated. We have proved that the principal functions of L corresponding to the eigenvalues are in $L_2(R_+, C^N)$ and the principal functions corresponding to the spectral singularities are in another Hilbert space which contains $L_2(R_+, C^N)$.

***Key Words*:** Eigen-values, spectral singularities, principal function

SONLU STURM - LIOUVILLE DİFERENSİYEL OPERATÖRLER SİSTEMİNİN ÖZDEĞERLERİNE VE SPEKTRAL TEKİLLİKLERİNE KARŞILIK GELEN ESAS FONKSİYONLAR

ÖZET

Bu çalışmada sonlu Sturm-Liouville diferensiyel operatörler sisteminin özdeğerlerine ve spektral tekilliklerine karşılık gelen esas fonksiyonlar incelenmiştir. Özdeğerlerine karşılık gelen esas fonksiyonların $L_2(R_+, C^N)$ uzayına, spektral tekilliklere karşılık gelen esas fonksiyonların $L_2(R_+, C^N)$ uzayını içeren bir başka Hilbert uzaya ait olduğu ispatlanmıştır.

Anahtar Kelimeler: Özdeğer, spektral tekillik, esas fonksiyon

1. GİRİŞ

Non-selfadjoint diferensiyel operatörlerin spektral analizi, Naimark tarafından incelenmeye başlanmıştır[2]. Operatörün kutbu olup, özdeğer olmayan, sürekli spektrumda yer alan noktalara Schwartz spektral tekillik adını vermiştir[4]. Bu noktalar operatörün spektral analizinde önemli bir rol oynar. Lyance spektral tekilliklere karşılık gelen esas fonksiyonların özelliklerini incelemiş ve spektral tekilliklerin spektral açılımındaki etkisini incelemiştir [3]. Spektral tekilliklere sahip non-selfadjoint operatörlerin spektral analizinin bazı problemleri çalışılmıştır [6]- [9].

1. INTRODUCTION

The spectral analysis of the non-selfadjoint differential operators has been investigated by Naimark [2]. He has proved that some of the poles of the resolvent are not the eigenvalues of the operator and also shown that they are on the continuous spectrum.These points are called spectral singularities by Schwartz [4]. So, in [2], it has been shown that pectral singularities have a special role in the spectral analysis of an operator. The properties of the principal functions corresponding to the spectral singularities and the effect of the spectral singularities to the spectral expansion is investigated by Lyance [3]. Some problems of the spectral analysis of non-selfadjoint operators with spectral singularities are studied in [6]- [9].

$L_2(R_+, C^N)$ uzayı

$L_2(R_+, C^N)$ is a Hilbert space of the vector functions such that

$$f = \begin{pmatrix} f_1 \\ f_2 \\ . \\ . \\ . \\ f_N \end{pmatrix}$$

biçimindeki bütün vektör fonksiyonlarının

with

$$\|f\|^2 = \int_0^\infty \sum_{n=1}^\infty |f_n(x)|^2 dx$$

normu ile birlikte bir Hilbert uzayıdır.

$l_n(y_n) := -y_n'' + q\ (x) y_n,$

$n = 1,2,...,N$, $x \in R_+ = [0,\infty)$

biçimindeki sonlu Sturm-Liouville diferensiyel ifadeler sistemini göz önüne alalım. Burada q_n, $n = 1,2,...,N$ kompleks değerli fonksiyonlardır. $L_2(R_+, C^N)$ uzayında

We consider following the system of Sturm-Liouville differential expression

$l_n(y_n) := -y_n'' + q\ (x) y_n,$

$n = 1,2,...,N$, $x \in R_+ = [0,\infty)$

Here q_n, $n = 1,2,...,N$ are complex-valued functions. Let L is generated operator by the expression

$$l(y) := \begin{pmatrix} l_1(y_1) \\ l_2(y_2) \\ . \\ . \\ . \\ l_N(y_N) \end{pmatrix}$$

ifadesi ve y(0)=0 başlangıç koşulu yardımıyla üretilen operatör L olsun. $q_n, n = 1,2,...,N$ kompleks değerli fonksiyonlar olduklarından, L operatörü non- selfadjointtir [2].

in $L_2(R_+, C^N)$ and y(0)=0 . Since $q_n, n = 1,2,...,N$ are complex-valued functions, L is non-selfadjoint [2].

1. $l(y) = \rho^2 y$ Denkleminin Özel Çözümleri

$$-y'' + Q(x)y = \rho^2 y\ ,\quad x \in R_+ \tag{1.1}$$

denklemini göz önüne alalım. Burada

1. Special Solutions of $l(y) = \rho^2 y$:

Let us consider the following equation

$$-y'' + Q(x)y = \rho^2 y\ ,\quad x \in R_+ \tag{1.1}$$

where

$$y = \begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_N \end{pmatrix},$$

$$Q(x) == \begin{bmatrix} q_1(x) & 0 & . & . & .0 \\ 0 & q_2(x) & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & q_N(x) \end{bmatrix}$$

olarak tanımlanır. ρ spektral parametredir. Kabul edelim ki, q_n, $n = 1,2,...,N$ fonksiyonları

ρ is spectral parameter. We assume that q_n, $n = 1,2,...,N$ satisfies

$$\int_0^\infty x\left|q_n(x)\right| dx \langle \infty \;,\; n=1,2,...,N \qquad (1.2)$$

koşulunu sağlasın[1].

(1.1.) denkleminin $\lim\limits_{x\to\infty} E(x,\lambda)e^{-i\rho x}=I$ koşulunu bütün $\rho \in \overline{C_+} := \{\rho : \rho \in C, \operatorname{Im}\rho \geq 0\}$ için sağlayan $E(x,\rho)$ matris çözümünü göz önüne alalım[1]. Burada I, C^N deki birim matristir.

(1.2) şartı varsa , $E(x,\rho)$ çözümü

$$\int_0^\infty x\left|q_n(x)\right| dx \langle \infty \;,\; n=1,2,...,N \qquad (1.2)$$

[1].

$E(x,\rho)$ is a matrix solution of (1.1) equation which satisfies $\lim\limits_{x\to\infty} E(x,\lambda)e^{-i\rho x}=I$ condition $\rho \in \overline{C_+} := \{\rho : \rho \in C, \operatorname{Im}\rho \geq 0\}$ [1]. I is a unit matrix on C^N.

If (1.2) exists , $E(x,\rho)$ has following representation

$$E(x,\rho)=\begin{bmatrix} e_1(x,\rho) & 0 & . & . & .0 \\ 0 & e_2(x,\rho) & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & e_N(x,\rho) \end{bmatrix} \qquad (1.3)$$

gösterimine sahiptir [1]. Burada

$$e_n(x,\rho)=e^{i\rho x}+\int_x^\infty K_n(x,t)\, e^{i\rho t t} dt \,, \qquad n=1,2,...,N$$

biçiminde tanımlanır. $K_n(x,t)$ fonksiyonları

$$\left|K_n(x,t)\right| \leq c \int_{\frac{x+t}{2}}^\infty \left|q_n(u)\right| du \;,\; n=1,2,...,N \qquad (1.4)$$

şartını sağlar. Böylece $e_n(x,\rho),\quad n=1,2,...,N$ fonksiyonları $C_+ := \{\rho : \rho \in C, \operatorname{Im}\rho \rangle\, 0\}$ bölgesinde ρ ya göre analitiktir ve $\overline{C_+}$ da süreklidir.

Aşağıdaki cümleleri tanımlayalım:

$$A_n^1=\{\rho : \rho \in C_+, e_n(\rho)=0\}$$

$$A_n^2=\{\rho : \rho \in R, e_n(\rho)=0\}$$

$$\det E(\rho)=\prod_{n=1}^N e_n(\rho)\,,$$

$$e_n(\rho):=e_n(0,\rho)=1+\int_x^\infty K_n(0,t)e^{i\rho t t} dt$$

$\sigma_d(L)$ ve $\sigma_{ss}(L)$ L operatörünün sırasıyla özdeğerlerinin ve spektral tekilliklerinin cümlesi olsun.

Teorem 1.1[5]:

a) $\sigma_d(L)=\bigcup\limits_{n=1}^N B_n^1$ dir. Burada

$B_n^1=\left\{\lambda : \lambda=\rho^2, \rho \in A_n^1\right\}$ olarak tanımlanır.

b) $\sigma_{ss}(L)=\bigcup\limits_{n=1}^N B_n^2 \;\{0\}$

Burada $B_n^2=\left\{\lambda : \lambda=\rho^2, \rho \in A_n^2\right\}$ olarak tanımlanır.

Ayrıca (1.1) denkleminin $S(0,\rho)=0$, $S'(0,\rho)=I$ başlangıç koşullarını sağlayan çözümü $S(x,\rho)$ olsun.

[1]. Where

$$e_n(x,\rho)=e^{i\rho x}+\int_x^\infty K_n(x,t)\, e^{i\rho t t} dt \,, \qquad n=1,2,...,N$$

$K_n(x,t)$ functions satisfy

$$\left|K_n(x,t)\right| \leq c \int_{\frac{x+t}{2}}^\infty \left|q_n(u)\right| du \;,\; n=1,2,...,N \qquad (1.4)$$

Thus $e_n(x,\rho),\quad n=1,2,...,N$ vector functions are analytic with respect to ρ in $C_+ := \{\rho : \rho \in C, \operatorname{Im}\rho \rangle\, 0\}$ and continuos in $\overline{C_+}$.

Let us define following sets

$$A_n^1=\{\rho : \rho \in C_+, e_n(\rho)=0\}$$

$$A_n^2=\{\rho : \rho \in R, e_n(\rho)=0\}$$

$$\det E(\rho)=\prod_{n=1}^N e_n(\rho)\,,$$

$$e_n(\rho):=e_n(0,\rho)=1+\int_x^\infty K_n(0,t)e^{i\rho t t} dt$$

$\sigma_d(L)$ ve $\sigma_{ss}(L)$ will denote eigenvalues and spectral singularities of the operator L, respectively.

Theorem 1.1[5]:

a) $\sigma_d(L)=\bigcup\limits_{n=1}^N B_n^1$ Where

$B_n^1=\left\{\lambda : \lambda=\rho^2, \rho \in A_n^1\right\}$.

b) $\sigma_{ss}(L)=\bigcup\limits_{n=1}^N B_n^2 \;\{0\}$ where

$B_n^2=\left\{\lambda : \lambda=\rho^2, \rho \in A_n^2\right\}$.

Let $S(x,\rho)$ be the solution of the equation (1.1) subject to the initial conditions $S(0,\rho)=0$, $S'(0,\rho)=I$.

$-y'' + q_n(x)y_n = \rho^2 y_n$, $x \in R_+$ denkleminin $s_n(0,\rho)=0$ $s_n'(0,\rho)=1$ sınır şartlarını sağlayan çözümleri $s_n(x,\rho)$ olmak üzere

$s_n(x,\rho)$ are solutions of the equations $-y'' + q_n(x)y_n = \rho^2 y_n$, $x \in R_+$ which satisfies the initial conditions $s_n(0,\rho)=0$ $s_n'(0,\rho)=1$.

$$S(x,\rho) = \begin{bmatrix} s_1(x,\rho) & 0 & . & . & .0 \\ 0 & s_2(x,\rho) & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & s_N(x,\rho) \end{bmatrix}$$

biçimindedir[1].

[1].

Her $\rho \in \overline{C_+}$ ve $\det E(\rho) \neq 0$ için

$$G(x,t;\rho) = \begin{Bmatrix} S(t,\rho)E(x,\rho)E^{-1}(\rho), & 0 \le t \langle x \\ E(t,\rho)S(x,\rho)E^{-1}(\rho), & x \le t \langle \infty \end{Bmatrix}$$

olmak üzere L operatörünün resolventi $f \in L_2(R_+, C^N)$ için

$(R_\rho(L)f)(x) := \int_0^\infty G(x,t;\rho)f(t)dt$ biçimindedir [5].

For $\forall \rho \in \overline{C}_+$ and $\det E(\rho) \neq 0$

The resolvent of the operator L

$$(R_\rho(L)f)(x) := \int_0^\infty G(x,t;\rho)f(t)dt$$

where $G(x,t;\rho) = \begin{Bmatrix} S(t,\rho)E(x,\rho)E^{-1}(\rho), & 0 \le t \langle x \\ E(t,\rho)S(x,\rho)E^{-1}(\rho), & x \le t \langle \infty \end{Bmatrix}$

and $f \in L_2(R_+, C^N)$ [5].

2. Esas Fonksiyonlar

2.1. L Operatörünün Özdeğerlerine Karşılık Gelen Esas Fonksiyonlar

C_+ da $\det E(\rho)$ nun sıfırları $\rho_1^+, \rho_2^+, ..., \rho_j^+$ ve bu sıfırların katı $m_1^+, m_2^+, ..., m_j^+$ olsun. Reel eksendeki $\det E(\rho)$ nun sıfırları $\rho_1, \rho_2, ..., \rho_\alpha$ ve bu sıfırların katı $m_1, m_2, ..., m_\alpha$ olsun.

$E(x,\rho)$ ve $S(x,\rho)$ çözümlerinin Wronskian'ı

$W[E(x,\rho), S(x,\rho)] = E(x,\rho)S'(x,\rho) - E'(x,\rho)S(x,\rho)$ biçiminde tanımlanır[2]. İki çözümün Wronskian'ı x'ten bağımsız olduğundan

2. Principal Functions

2.1. Principal Functions Corresponding to The Eigenvalues of The Operator L:

Let $\rho_1^+, \rho_2^+, ..., \rho_j^+$ denote the zeros of $\det E(\rho)$ in C_+ with multiplicity $m_1^+, m_2^+, ..., m_j^+$, $\rho_1, \rho_2, ..., \rho_\alpha$ denote the zeros of $\det E(\rho)$ on the real axis with multiplicity $m_1, m_2, ..., m_\alpha$.

Wronskian of the solutions $E(x,\rho)$ ve $S(x,\rho)$ is defined as

$W[E(x,\rho), S(x,\rho)] = E(x,\rho)S'(x,\rho) - E'(x,\rho)S(x,\rho)$ [2]. As Wronskian of the two solutions is independent of x, it is obtained

$$\lim_{x \to o} W[E(x,\rho), S(x,\rho)] = \lim_{x \to 0} [E(x,\rho)S'(0,\rho) - E'(x,\rho)S(x,\rho)$$
$$= E(0,\rho) := E(\rho)$$

elde edilir.

It is trivial that from definition of the Wronskian

$p = 0,1,...,m_k^+ - 1$, k=1,2,…,j

$$\frac{\partial^p}{\partial \rho^p}\{W[E(x,\rho), S(x,\rho)]\}_{\rho=\rho_k^+} = \left\{\frac{\partial^p}{\partial \rho^p}\det E(\rho)\right\}_{\rho=\rho_k^+} = 0 \tag{2.1.1}$$

eşitliği elde edilir. Eğer n=0 ise

$$S(x,\rho_k^+) = a_0(\rho_k^+)E(x,\rho_k^+), \quad k=1,2,…,j \tag{2.1.2}$$

olur. Burada $a_0(\rho_k^+) \neq 0$ dır.

for $p = 0,1,...,m_k^+ - 1$. If k=1,2,…,j n=0 then

$$S(x,\rho_k^+) = a_0(\rho_k^+)E(x,\rho_k^+), \quad k=1,2,…,j \tag{2.1.2}$$

Where $a_0(\rho_k^+) \neq 0$

Teorem 2.1.1.

$p = 0,1,...,m_k^+ - 1, k = 1,2,...,j$ için

Theorem 2.1.1.

$$\left\{\frac{\partial^p}{\partial\rho^p} S(x,\rho_k)\right\}_{\rho=\rho_k^+} = \sum_{i=0}^{p}\binom{p}{i} a_{n-i}\left\{\frac{\partial^i}{\partial\rho^i} E(x,\rho_k)\right\}_{\rho=\rho_k^+} \tag{2.1.3}$$

formülü vardır. Burada $a_0, a_1,..., a_n$ sabitleri ρ_k^+ ya bağlıdır.

$p = 0,1,...,m_k^+ - 1, k = 1,2,...,j$ The constants $a_0, a_1,..., a_n$ depend on ρ_k^+ .

İspat:

İspatta tümevarım metodunu kullanacağız. n=0 için (2.1.2) den (2.1.3) elde edilir. n=1 için ispatlayalım. (2.1.1) eşitliğinden

Proof:

We use mathematical induction. For n=0 the proof is trivial from (2.1.2). Let us prove (2.1.3) for n=1. From (2.1.1)

$$\frac{\partial^p}{\partial\rho^p}\{W[E(x,\rho),S(x,\rho)]\}_{\rho=\rho_k^+} = \left\{\frac{\partial^p}{\partial\rho^p}\det E(\rho)\right\}_{\rho=\rho_k^+} = 0 \tag{2.1.4}$$

olur. $U(x,\rho)$ (1.1) denkleminin bir çözümü ise

If $U(x,\rho)$ is a solution of equation(1.1) , then

$$\left\{-\frac{d}{dx^2} + Q(x)y - \rho^2\right\}\frac{\partial}{\partial\rho}U(x,\rho) = U(x,\rho) \tag{2.1.5}$$

olur. (2.1.5) eşitliği kullanılırsa

By using (2.1.5) we obtain

$$\left\{-\frac{d}{dx^2} + Q(x)y - \rho_k{}^2\right\}\left\{\frac{\partial}{\partial\rho}S(x,\rho)\right\}_{\rho=\rho_k^+} = S(x,\rho_k^+) \tag{2.1.6}$$

$$\left\{-\frac{d}{dx^2} + Q(x)y - \rho_k{}^2\right\}\left\{\frac{\partial}{\partial\rho}E(x,\rho)\right\}_{\rho=\rho_k^+} = E(x,\rho_k^+) \tag{2.1.7}$$

elde edilir. (2.1.2), (2.1.6) ve (2.1.7) den

From (2.1.2), (2.1.6) and (2.1.7)

$$\left\{-\frac{d}{dx^2} + Q(x)y - \rho_k{}^2\right\}f_1(x,\rho_k^+) = 0$$

bulunur. Burada

Here

$$f_1(x,\rho_k^+) = \left\{\frac{\partial}{\partial\rho}S(x,\rho)\right\}_{\rho=\rho_k^+} - a_0(\rho_k^+)\left\{\frac{\partial}{\partial\rho}E(x,\rho)\right\}_{\rho=\rho_k^+}$$

şeklindedir. (2.1.4) den

From (2.1.4)

$$W[E(x,\rho),S(x,\rho)] = \left\{\frac{\partial}{\partial\rho}W[S(x,\rho),E(x,\rho)]\right\}_{\rho=\rho_k^+} = 0$$

elde edilir. O halde öyle bir $a_1(\rho_k^+)$ sabiti vardır ki

Hence there is a constant $a_1(\rho_k^+)$ such that

$$f_1(x,\rho_k^+) = a_1(\rho_k^+)E(x,\rho_k^+) \text{ veya / or}$$

$$\left\{\frac{\partial}{\partial\rho}S(x,\rho_k)\right\}_{\rho=\rho_k^+} = a_1(\rho_k^+)E(x,\rho_k^+) + a_0(\rho_k^+)\left\{\frac{\partial}{\partial\rho}E(x,\rho)\right\}_{\rho=\rho_k^+}$$

olur. O halde n=1 için (2.1.3) formülü elde edilir.

Thus (2.1.3) holds for n=1.

(2.1.3) formülü $2 \le p_0 \le m_k^+ - 2$ için var olsun.

Let us assume that for $2 \le p_0 \le m_k^+ - 2$ the equality (2.1.3) holds; i.e.

$$\left\{\frac{\partial^{p_0}}{\partial\rho^{p_0}}S(x,\rho)\right\}_{\rho=\rho_k^+} = \sum_{i=0}^{p}\binom{p_0}{i}a_{p_0-i}(\rho_k^+)\left\{\frac{\partial^i}{\partial\rho^i}E(x,\rho)\right\}_{\rho=\rho_k^+} \tag{2.1.8}$$

(2.1.3) formülünü p_0+1 için ispatlayalım. Eğer $U(x,\rho)$, (1.1) denkleminin bir çözümü ise $\frac{\partial^p}{\partial\rho^p}U(x,\rho)$ ifadesi

Now we prove that (2.1.3) holds for p_0+1, too. If $U(x,\rho)$ is a solution of (1.1), then $\frac{\partial^p}{\partial\rho^p}U(x,\rho)$ satisfies

$$\left\{-\frac{d}{dx^2}+Q(x)y-\rho^2\right\}\frac{\partial}{\partial\rho}U(x,\rho)=p\frac{\partial^{p-1}}{\partial\rho^{p-1}}U(x,\rho) \tag{2.1.9}$$

denklemini sağlar. $S(x,\rho_k^+)$ ve $E(x,\rho_k^+)$ çözümleri için (2.1.9) denklemini yazıp (2.1.8) denklemini kullanırsak

Writting (2.1.9) for $S(x,\rho_k^+)$ ve $E(x,\rho_k^+)$, and then using , we find

$$\left\{-\frac{d}{dx^2}+Q(x)-\rho_k{}^2\right\}f_{p_0+1}(x,\rho_k{}^+)=0$$

buluruz. Burada

where

$$f_{p_0+1}(x,\rho_k^+)=\left\{\frac{\partial^{p_0+1}}{\partial\rho^{p_0+1}}S(x,\rho_k)\right\}_{\rho=\rho_k^+}-\sum_{i=0}^{p_0+1}\binom{p_0+1}{i}a_{p_0+1-i}\left\{\frac{\partial^i}{\partial\rho^i}E(x,\rho)\right\}_{\rho=\rho_k^+}$$

şeklindedir. (2.1.1) eşitliğinden

From (2.1.1) we have

$$W[f_{p_0+1}(x,\rho_k^+),E(x,\rho_k^+)]=\left\{\frac{\partial^{p_0+1}}{\partial\rho^{p_0+1}}W\left[S(x,\rho),E(x,\rho)\right]\right\}_{\rho=\rho_k^+}=0$$

elde edilir. Bir $a_{p_0+1}(\rho_k^+)$ sabiti vardır öyle ki

Hence there exists a constant $a_{p_0+1}(\rho_k^+)$ such that

$$f_{p_0+1}(x,\rho_k{}^+)=a_{p_0+1}(\rho_k{}^+)E(x,\rho_k^+)$$

şeklindedir.

This shows that (2.1.3) holds for $n=n_0+1$.

Teorem 2.1.2:

Özdeğerlere karşılık gelen esas fonksiyonlar $L_2(R_+,C^N)$ uzayının elemanıdır.

$p=0,1,...,m_k^+-1, k=1,2,...,j$ için

Theorem 2.1.2:

The principal functions corresponding to the eigenvalues belong $L_2(R_+,C^N)$.

$$\left\{\frac{\partial^{p_0}}{\partial\rho^{p_0}}S(x,\rho)\right\}_{\rho=\rho_k^+}\in L_2(R_+,C^N)$$

$p=0,1,...,m_k^+-1, k=1,2,...,j$

İspat:

(1.3) ten

Proof:

From (1.3) we have

$$\left\{\frac{\partial^p}{\partial\rho^p}E(x,\rho)\right\}_{\rho=\rho_k^+}=\begin{bmatrix}\left\{\frac{\partial^p}{\partial p^p}e_1(x,\rho)\right\}_{\rho=\rho_k^+} & 0 & .\ . & .0\\ 0 & \left\{\frac{\partial^p}{\partial p^p}e_2(x,\rho)\right\}_{\rho=\rho_k^+} & .\ . & 0\\ . & . & .\ . & .\\ . & . & .\ . & .\\ 0 & 0 & .\ . & \left\{\frac{\partial^p}{\partial p^p}e_N(x,\rho)\right\}_{\rho=\rho_k^+}\end{bmatrix}$$

p=0,1,…, m_k^+-1 , k=1,2,…,j için

for p=0,1,…, m_k^+-1 , k=1,2,…,j

$$\left\{\frac{\partial^p}{\partial\rho^p}E(x,\rho)\right\}_{\rho=\rho_k^+}=\begin{bmatrix}(ix)^p e^{i\rho_k^+x}+\int_x^\infty (it)^p K_1(x,t)e^{i\rho_k^+t}dt & 0 & . & .0\\ 0 & (ix)^p e^{i\rho_k^+x}+\int_x^\infty (it)^p K_2(x,t)e^{i\rho_k^+t}dt & . & 0\\ . & . & . & .\\ . & . & . & .\\ 0 & 0 & . & (ix)^p e^{i\rho_k^+x}+\int_x^\infty (it)^p K_N(x,t)e^{i\rho_k^+t}dt\end{bmatrix} \tag{2.1.10}$$

elde edilir. (2.1.10) kullanılırsa $n=1,2,...,N$ için

Using (2.1.10) we have for $n=1,2,...,N$

$$\left|\left\{\frac{\partial^p}{\partial\rho^p}e_n(x,\rho)\right\}_{\rho=\rho_k^+}\right|=\left|(ix)^p e^{i\rho_k^+x}+\int_x^\infty (it)^p K_n(x,t)e^{i\rho_k^+t}dt\right|\le x^p e^{-x\operatorname{Im}\rho_k^+}+\int_x^\infty t^p\left|K_n(x,t)\right|e^{-i\operatorname{Im}\rho_k^+}dt$$

Im $\rho_k^+ \rangle 0$ olduğundan ve ayrıca (1.2), (1.4) eşitsizliklerinden $c_1\rangle 0$ sabit olmak üzere

Since Im $\rho_k^+\rangle 0$, we have from (1.2), (1.4)

$$\left|\left\{\frac{\partial^p}{\partial\rho^p}e_n(x,\rho)\right\}_{\rho=\rho_k^+}\right|\le x^p e^{-x\operatorname{Im}\rho_k^+}+c_1$$

olur. $n=1,2,...,N$ için

for $n=1,2,...,N$ where $c_1\rangle 0$ is a constant . by using last equality we have

$$\left\|\left\{\frac{\partial^p}{\partial\rho^p}e_n(x,\rho)\right\}_{\rho=\rho_k^+}\right\|_{L_2(R_+,C^N)}=\int_0^\infty\left|\left\{\frac{\partial^p}{\partial\rho^p}e_n(x,\rho)\right\}_{\rho=\rho_k^+}\right|^2 dx\langle\infty$$

elde edilir.

or

p=0,1,…, m_k^+-1 , k=1,2,…,j için

$$\left\{\frac{\partial^p}{\partial\rho^p}E(.,\rho)\right\}_{\rho=\rho_k^+}\in L_2(R_+,C^N)$$

olur.

for p=0,1,…, m_k^+-1 , k=1,2,…,j .

Tanım 2.1.3:

Definition 2.1.3:

$$S(x,\rho_k^+),\left\{\frac{\partial}{\partial\rho}S(x,\rho)\right\}_{\rho=\rho_k^+},...,\left\{\frac{\partial^{m_k^+-1}}{\partial\rho^{m_k^+-1}}S(x,\rho)\right\}_{\rho=\rho_k^+}$$

fonksiyonlarına L operatörünün $\rho=\rho_k^+$, k=1,2,…,j özdeğerlerine bağlı esas fonksiyonları denir[2].

are called the principal functions corresponding to the eigenvalues $\rho=\rho_k^+$, k=1,2,…,j of L [2].

2.2. L operatörünün Spektral Tekilliklerine Karşılık Gelen Esas Fonksiyonlar

2.2. Principal Functions Corresponding to The Spectral Singularities of The Operator L

$\rho_1,\rho_2,...,\rho_\alpha$ L operatörünün spektral tekillikleri ise $p=0,1,...,m_k-1$, $k=1,2,...,\alpha$ için

If $\rho_1,\rho_2,...,\rho_\alpha$ are spectral singularities of L, then we have for $p=0,1,...,m_k-1$, $k=1,2,...,\alpha$

$$\frac{\partial^p}{\partial\rho^p}\{W[E(x,\rho),S(x,\rho)]\}_{\rho=\rho_k}=\left\{\frac{\partial^p}{\partial\rho^p}\det E(\rho)\right\}_{\rho=\rho_k}=0$$

eşitliği vardır.

Smilarly to the proof of Theorem 2.1.1, we have the following remark.

Sonuç 2.2.1:

$p = 0,1,...,m_k - 1$, $k = 1,2,...,\alpha$ için

Remark 2.2.1:

$$\left\{ \frac{\partial^p}{\partial \rho^p} S(x,\rho) \right\}_{\rho=\rho_k} = \sum_{i=0}^{n} \binom{n}{i} b(\rho_k) \left\{ \frac{\partial^i}{\partial \rho^i} E(x,\rho) \right\}_{\rho=\rho_k} \quad (2.2.1)$$

eşitliği vardır. Teorem 2.1.1 in ispatına benzer olarak yapılır.

$p = 0,1,...,m_k - 1$, $k = 1,2,...,\alpha$ (2.2.1)

Lemma 2.2.2:

$p = 0,1,...,m_k - 1$, $k = 1,2,...,\alpha$

Lemma 2.2.2:

$$\left\{ \frac{\partial^p}{\partial \rho^p} S(.,\lambda) \right\}_{\rho=\rho_k} \notin L_2(R_+, C^N)$$

dir.

Lemmanın ispatı (2.2.1) ve (2.1.10) dan kolayca gösterilir.

$p = 0,1,...,m_k - 1$, $k = 1,2,...,\alpha$

The proof of the lemma may be easily obtained from (2.2.1) and (2.1.10)

Şimdi aşağıdaki Hilbert uzaylarını tanımlayalım:

Now let us introduce the Hilbert spaces:

$$H(R_+, C^N, m) = \left\{ f : f = \begin{pmatrix} f_1 \\ . \\ f_n \end{pmatrix}, \int_o^\infty (1+x)^{2m} \left\{ |f_1(x)|^2 + ... + |f_n(x)|^2 \right\} dx \langle \infty \right\},$$

$$H(R_+, C^N, -m) = \left\{ g : g = \begin{pmatrix} g_1 \\ . \\ g_n \end{pmatrix}, \int_o^\infty (1+x)^{-2m} \left\{ |g_1(x)|^2 + ... + |g_n(x)|^2 \right\} dx \langle \infty \right\},$$

$H(R_+, C^N, m)$ uzayının normu

with

$$\|f\|^2_{H(R_+,C^N,m)} = \int_o^\infty (1+x)^{2m} \left\{ |f_1(x)|^2 + ... + |f_n(x)|^2 \right\} dx$$

$H(R_+, C^N, -m)$ uzayının normu

$$\|g\|^2_{H(R_+,C^N,-m)} = \int_o^\infty (1+x)^{-2m} \left\{ |g_1(x)|^2 + ... + |g_n(x)|^2 \right\} dx$$

biçiminde tanımlıdır [9]. Bu tanımların sonucunda $m = 1,2,...$ için

$H(R_+, C^N, 0) = L_2(R_+, C^N)$ ve

$H(R_+, C^N, m) \underset{\neq}{\subset} L_2(R^+, C^N) \underset{\neq}{\subset} H(R_+, C^N, -m)$

bulunur.

respectively [9]. It is evident that

$H(R_+, C^N, 0) = L_2(R_+, C^N)$ and

$H(R_+, C^N, m) \underset{\neq}{\subset} L_2(R^+, C^N) \underset{\neq}{\subset} H(R_+, C^N, -m)$

$m = 1,2,...$

Teorem 2.2.3

Spektral tekilliklere karşılık gelen esas fonksiyonlar Hilbert uzayına aittir.

Theorem 2.2.3

The principal functions corresponding to the spectral singularities belong Hilbert space .

$$\left\{ \frac{\partial^p}{\partial \rho^p} S(.,\lambda) \right\}_{\rho=\rho_k} \in H(R_+, C^N, -(n+1)) \quad p = 0,1,...,m_k - 1 \ , \ k = 1,2,...,\alpha \quad (2.2.3)$$

İspat:

(2.1.10) dan $p = 0,1,...,m_k - 1$, $k = 1,2,...,\alpha$ için

Proof:

From (2.1.10) we have

$$\left\{ \frac{\partial^p}{\partial \rho^p} E(x,\rho) \right\}_{\rho=\rho_k} \le x^p + \int_x^\infty t^p |K_n(x,t)| dt \quad (2.2.4)$$

elde edilir.

$H(R_+, C^N, -(n+1))$ uzayının tanımından ve (2.2.4) ten (2.2.3) elde edilir.

Tanım 2.2.4.

$$S(x,\rho_k), \left\{ \frac{\partial}{\partial\rho} S(x,\rho) \right\}_{\rho=\rho_k}, ..., \left\{ \frac{\partial^{m_k-1}}{\partial\rho^{m_k-1}} S(x,\rho) \right\}_{\rho=\rho_k}$$

fonksiyonlarına sırasıyla L operatörünün $\rho = \rho_k$, $k = 1,2,...,\alpha$ spektral tekilliklerine bağlı esas fonksiyonları denir [9].

$m_0 = \max\{m_1, m_2, ..., m_\alpha\}$ olmak üzere

$H_{m_0} = H(R_+, C^N, m_0 + 1)$, $H_{-m_0} = H(R_+, C^N, -(m_0 + 1))$

alınırsa

$H_{m_0} \underset{\neq}{\subset} L_2(R_+, C^N) \underset{\neq}{\subset} H_{-m_0}$

olur.

Sonuç 2.2.5

Spektral tekilliklere bağlı esas fonksiyonlar H_{-m_0} uzayına aittir.

$p = 0,1,..., m_k - 1$, $k = 1,2,...,\alpha$

(2.2.3) is obtained from the definition of the space $H(R_+, C^N, -(n+1))$ and (2.2.4).

Definition 2.2.4.

are called the principal functions corresponding to the spectral singularities $\rho = \rho_k$, $k = 1,2,...,\alpha$ of L [9].

Let us choose m_0 so that

$m_0 = \max\{m_1, m_2, ..., m_\alpha\}$.

$H_{m_0} = H(R_+, C^N, m_0 + 1)$, $H_{-m_0} = H(R_+, C^N, -(m_0 + 1))$

then

$H_{m_0} \underset{\neq}{\subset} L_2(R_+, C^N) \underset{\neq}{\subset} H_{-m_0}$

Remark 2.2.5:

The principal functions corresponding to the spectral singularities belong H_{-m_0} .

$$\left\{ \frac{\partial^p}{\partial\rho^p} S(.,\lambda) \right\}_{\rho=\rho_k} \in H_{-m_0}, \quad p = 0,1,...,m_k - 1 \ , \quad k = 1,2,...,\alpha$$

elde edilir.

KAYNAKLAR/ REFERENCES

1. Z.S. Agranovich and Marchenko V.A., *The Inverse Problem of Scattering Theory*, (1963).
2. M.A. Naimark., *Linear Differential Operators I,II,* Ungar, ***New-York***, (1968).
3. V.E. Lyance, "A Differential Operator with Spectral Singularities I,II", ***AMS Translations*** 2(60), 185-225, 227-283 (1960).
4. J.T. Schwartz, " Some Non-selfadjoint Operators", ***Comm.Pure and Appl.Math 13,*** 609-639 (1960).
5. E. Bayramov and E. Kır," Spectral Properties of a Finite System of Sturm-Liouville Differential Operators" ***Indian J. Pure and Appl.Math***.35(2), 249-256. 2004
6. E. Bairamov, Ö. Çakar and A.O. Çelebi, " Quadratic Pencil of Schrödinger Operators with Spectral Singularities: Discrete Spectrum and Principal Functions", ***J.Math.Anal.Appl***. 216, 303-320 (1997).
7. E. Bairamov, Ö. Çakar, and A.M.Krall, " Spectrum anad Spectral Singularities of a Quadratic Pencil of a Schrödinger Operator with a General Boundary Condition" ***J. Differential Equations*** 151, 252-267 (1999).
8. E.Bairamov, Ö. Çakar and C. Yanık, " Spectral Singularities of the Klein-Gordon s- wave Equation" ***Indian J. Pure appl. Math***,32, 851-857 (2001).
9. Yu. M., Brezanski, Expansion in Eigenfunctions of Selfadjoint Operators. ***Amer. Math. Soc.***, Providence R.I. (1968).

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