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ORIGINAL ARTICLE

Some Common Coincidence Point Theorems on Weakly Increasing Mappings in Partially Ordered Cone Metric Spaces

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ABSTRACT

We study some common coincidence and common fixed point theorems for weakly increasing mappings with respect to a self maps on partially ordered cone metric spaces, where the cone is not necessarily normal. Our results generalized several well-known comparable results in the literature.

Key Words: *Common fixed Point, common coincidence point, cone metric space,ordered sets.*

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1. INTRODUCTION

The concept of cone metric space was initiated by Huang and Zhang [6]. They proved some fixed point theorems of contractive type mappings over cone etric spaces. Later, many authors generalized their fixed point theorems in different types.

In the present paper, E stands for a real Banach space. Let P be a subset of E with $\operatorname{int}(P) \neq \phi.$ Then *P* is called a cone if the following conditions are satisfied:

1. *P* is closed and $p \neq \{\theta\}$.

2. $a, b \in R^+$, $x, y \in P$ implies $ax + by \in P.$

3. $x \in P \cap -P$ implies $x = \theta.$

For a cone *P*, define a partial ordering $\leq$ with respect to *P* by $x \leq y$ if and only if

$y - x \in P$. We shall write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while $x << y$ will stand

for $y - x \in \operatorname{int}(P)$. It can be easily shown that $\lambda \operatorname{int}(P) \subseteq \operatorname{int}(P)$ for all positive scalar λ.

Definition 1.1 [6] *Let X be a nonempty set. Suppose the mapping* $d : X \times X \to E$

satisfies

1. $\theta \leq d(x, y)$ *for all* $x, y \in X$ *and* $d(x, y) = \theta$ *if and only if x = y.*

2. $d(x, y) = d(y, x)$ *for all* $x, y \in X$.

3. $d(x, y) \leq d(x, z) + d(z, y)$ *for all* $x, y, z \in X$.

Then d is called a cone metric on X, and (X, d) is called a cone metric space.

Definition 1.2 [6] *Let (X, d) be a cone metric space. Let* (x_n) *be a sequence in X and*

$x \in X$. *If for every* $c \in E$ *with* $\theta << c$ *, there is an* $k \in N$ *such that* $d(x_n, x) << c$ *for all*

$n \geq k$, *then* (x_n) *is said to be convergent and* (x_n) *converges to x and x is the limit of*

(x_n). *We denote this by* $\lim_{n \to +\infty} x_n = x$ *or* $x_n \to x$ *as* $n \to +\infty$. *If for every* $c \in E$ *with*

$\theta << c$ *there is an* $k \in N$ *such that* $d(x_n, x_m) << c$ *for all* $n, m \geq k$, *then* (x_n) *is called*

a Cauchy sequence in X. The space (X, d) is called a complete cone metric space if every

Cauchy sequence is convergent.

Let (*X*, *d*) be a cone metric space, $f : X \to X$ and $x_0 \in X$. Then *f* is said to be continuous

at x_0 if for any sequence $x_n \to x_0$, we have $fx_n \to fx_0$ [14, 24].

The cone *P* in a real Banach space *E* is called normal if there is a number *k* > 1 such

that for all $x, y \in E$,

$\theta \leq x \leq y$ implies $\| x \| \leq k \| y \|$.

The following theorem has been proved by Huang and Zhang.

Theorem 1.1 [6] *Let (X, d) be a complete cone metric space, P be a normal cone with*

normal constant k. Suppose the mapping $f : X \to X$ *satisfies the contractive condition:*

$$d(fx, fy) \leq kd(x, y)$$

for all $x, y \in X$. *If* $k \in [0,1)$, *then f has a unique fixed point in X.*

Also, Huang and Zhang [6] gave an example showing that their result is a generalization of the Banach fixed point principle. Later normality was removed by Rezapour and Hamlbarani [19].

Turkoglu et al. [25, 26] studied some results on cone metric spaces. While Karapinar [10, 11, 12] and Shatanawi [20, 21, 24] studied a coupled coincidence point in cone metric spaces.

The existence of fixed points in partially ordered set has been considered by Ran and Reuring in [17], they proved the following theorem:

Theorem 1.2 [17] *Let* $(X, \leq)$ *be a partially ordered set and suppose that there exists a*

metric d in X such that the metric space (X, d) is complete. Let $f : X \to X$ *be a continuous* *mapping with respect to* $\leq$ *. Suppose that the following two assertions hold:*

1. there exists $k \in (0,1)$ *such that* $d(fx, fy) \leq kd(x, y)$ *for each* $x, y \in X$ *with* $x \leq y$*;*

2. there exists $x_0 \in X$ *such that* $x_0 \leq fx_0$*.*

Then f has a fixed point $x^* \in X$ *.*

After then many authors generalized Ran-Reuring result in different ways [2, 5, 13, 16, 18, 23, 27].

Altun and Durmaz [3] extended the contraction Banach principle to a partially ordered cone metric space, where *P* is assumed to be normal. Also, they gave an example [3] to show that their result is more general than Theorem 2.1 and Theorem 2.2.

Altun et al. [4] introduced the concept of weakly increasing maps as follows:

Definition 1.3 [4] *Let* $(X, \leq)$ *be a partially ordered set. Two mappings* $f, g : X \to X$ *are*

said to be weakly increasing if $fx \leq g(fx)$ *and* $gx \leq f(gx)$ *for all* $x \in X$ *.*

Altun et al. [4] proved the following theorem:

Theorem 1.3 [4] *Let* $(X, \leq)$ *be a partially ordered set and suppose that there exists a cone*

metric d in X such that the metric space (X, d) is complete. Let $f, g : X \to X$ *be two*

weakly increasing maps with respect to $\leq$*. Suppose that the following two assertions hold:*

1. *there exist* $\alpha, \beta, \gamma \geq 0$ *with* $\alpha + 2\beta + 2\gamma < 1$ *such that*

$$d(fx, gy) \leq \alpha d(x, y) + \beta(d(x, fx) + d(y, gy)) + \gamma(d(x, gy) + d(y, fx))$$

for each $x, y \in X$ *with* $x \leq y$*;*

2. *f or g is continuous.*

Then f and g have a common fixed point $x^* \in X$ *.*

Theorem 1.3 is a generalization of several well known results in literature [1, 3].

Jungck [8] introduced the concept of the notion compatible maps in metric space. While Jungck and Rhoades [9] introduced the notion of weakly compatible maps in metric space.

S. Janković et al. [7] extended the notion of compatible and weakly compatible maps to cone metric space.

Definition 1.4 [8] *Let f and g be self maps of a set X. If w = fx = gx for some x in X,*

then x is called a coincidence point of f and g, and w is called a point of coincidence of

f and g. The two maps f and g are said to be weakly compatible if they commute at their

coincidence points, that is, if x is a coincidence point of f and g, then fgx = gfx.

Definition 1.5 [7] *Let (X, d) be a cone metric space and* $f, g : X \to X$ *be two self maps.*

The pair {f, g} is said to be compatible if and only if

$$\lim_{n\to+\infty} d(fgx_n, gfx_n) = \theta$$

whenever (x_n) is a sequence in *X* such that

$$\lim_{n\to+\infty} fx_n = \lim_{n\to+\infty} gx_n = t$$

for some $t \in X$.

Lemma 1.1 [7] *If the pair {f, g} of self-maps on the cone metric space (X, d) is compatible,*

then it is weakly compatible. The converse is not true.

The aim of this paper is to study some common coincidence and common fixed point theorems for three self maps in cone metric space, where the cone is not necessarily normal.

Our results generalized Theorem 1.3 and other several well-known results in the literature.

2. MAIN RESULTS

Recently, Nashine and Samet introduced the concept of weakly increasing mappings with respect to a self map as a generalization of weakly increasing mappings as follows:

Definition 2.1 [15] *Let* $(X, \leq)$ *be a partially ordered set and* $f, g, T : X \to X$ *be three*

maps. Then we say that f and g are weakly increasing with respect to T if for all $x \in X$ *,*

we have

$$fx \leq gy \quad \forall\ y \in T^{-1}(fx),$$

and

$$gx \leq fy \quad \forall\ y \in T^{-1}(gx).$$

Remark 1 *Note that if T =* i_X *, then f and g are weakly increasing.*

Example 2.1 *Let X = [0, 1]. Define* $f, g, T : X \to X$ *by fx = x, gx =* $\sqrt{x}$ *, and*

Tx = x^2*, then it is clear that f and g are weakly increasing with respect to T.*

Theorem 2.1 *Let* $(X, \leq)$ *be a partially ordered set and (X, d) be a complete cone metric*

space. Let $f, g, T : X \to X$ *be three maps such that*

$$\begin{aligned} &d(fx, gy) \leq ad(Tx, Ty) \\ &+ b\big(d(Tx, fx) + d(Ty, gy)\big) \\ &+ c\big(d(Tx, gy) + d(Ty, fx)\big) \qquad (1) \end{aligned}$$

for all $x, y \in X$ *with* $Tx \leq Ty$*. Assume that f, g and T satisfy the following conditions:*

1. f and g are weakly increasing with respect to T.

2. The pairs {f, T} and {g, T} are compatible.

3. f and T are continuous or g and T are continuous

4. $fX \subseteq TX$ *and* $gX \subseteq TX$*.*

If a, b and c are nonnegative real numbers with a+2b+2c∈ [0, 1), then f, g and T have a

common coincidence point.

Proof. Let $x_0 \in X$. Since $fX \subseteq TX$, we choose $x_1 \in X$ such that $fx_0 = Tx_1$. Also, since

$gX \subseteq TX.$, we choose $x_2 \in X$ such that $gx_1 = Tx_2$. Continuing this process, we can construct a sequences (x_n) in X such that $Tx_{2n+1} = fx_{2n}$ and $Tx_{2n+2} = gx_{2n+1}$. Since

$x_1 \in T^{-1}(fx_0)$, and $x_2 \in T^{-1}(gx_1)$, , then by using the assumption that *f* and *g* are weakly

increasing with respect to *T* we have

$$Tx_1 = fx_0 \leq gx_1 = Tx_2 \leq fx_2.$$

Thus by induction, we can show that

$$Tx_{2n+1} \leq Tx_{2n+2}\ \forall\, n \in N \cup \{0\}.$$

By inequality (1), we have

$$\begin{aligned} &d(Tx_{2n+1}, Tx_{2n+2}) = d(fx_{2n}, gx_{2n+1}) \\ &\leq ad(Tx_{2n}, Tx_{2n+1}) + \\ &b\big(d(Tx_{2n}, fx_{2n}) + d(Tx_{2n+1}, gx_{2n+1})\big) \\ &+ \mathrm{c}\big(d(Tx_{2n}, gx_{2n+1}) + d(Tx_{2n+1}, fx_{2n})\big) \\ &\leq ad(Tx_{2n}, Tx_{2n+1}) + \\ &b\big(d(Tx_{2n}, Tx_{2n+1}) + d(Tx_{2n+1}, Tx_{2n+2})\big) \\ &+ \mathrm{c}\big(d(Tx_{2n}, Tx_{2n+2}) + d(Tx_{2n+1}, Tx_{2n+1})\big). \end{aligned}$$

Using the fact that

$$\begin{aligned} &d(Tx_{2n}, Tx_{2n+2}) \leq d(Tx_{2n}, Tx_{2n+1}) \\ &+ d(Tx_{2n+1}, Tx_{2n+2}), \end{aligned}$$

then the above inequalities become

$$\begin{aligned} &d(Tx_{2n+1}, Tx_{2n+2}) \leq ad(Tx_{2n}, Tx_{2n+1}) + \\ &b\big(d(Tx_{2n}, Tx_{2n+1}) + d(Tx_{2n+1}, Tx_{2n+2})\big) \\ &+ \mathrm{c}\big(d(Tx_{2n}, Tx_{2n+1}) + d(Tx_{2n+1}, Tx_{2n+2})\big). \end{aligned}$$

Hence

$$d(Tx_{2n+1}, Tx_{2n+2}) \leq \frac{a+b+c}{1-b-c} d(Tx_{2n}, Tx_{2n+1}).$$

Put

$$k = \frac{a+b+c}{1-b-c}.$$

Then we have

$$d(Tx_{2n+1}, Tx_{2n+2}) \leq kd(Tx_{2n}, Tx_{2n+1}). \quad (2)$$

Similarly, we may show that

$$d(Tx_{2n}, Tx_{2n+1}) \leq kd(Tx_{2n-1}, Tx_{2n}). \qquad (3)$$

Thus from inequalities (2) and (3), we have

$$d(Tx_n, Tx_{n+1}) \leq kd(Tx_{n-1}, Tx_n)\ \forall\, n \in N \quad (4)$$

For $n \in N$, we have

$$\begin{aligned} d(Tx_n, Tx_{n+1}) &\leq kd(Tx_{n-1}, Tx_n) \\ &\leq k^2 d(Tx_{n-2}, Tx_{n-1}) \\ &\ \vdots \\ &\leq k^{n+1} d(Tx_0, Tx_1). \end{aligned}$$

Let $n, m \in N$ with $m > n$. Then

$$d(Tx_n, Tx_m) \le \sum_{i=n}^{m-1} d(Tx_i, Tx_{i+1})$$
$$\le \sum_{i=n}^{m-1} k^i d(Tx_0, Tx_1).$$

Since $k \in [0,1)$, we have

$$d(Tx_n, Tx_m) \le \frac{k^n}{1-k} d(Tx_0, Tx_1). \quad (5)$$

To show that (Tx_n) is a Cauchy sequence in (X, d). Let $c >> \theta$ be arbitrary. Since $c \in \text{int}(P)$, there exists a neighborhood of θ

$$N_\delta(\theta) = \{y \in E : \| y \| < \delta\}, \ \delta > 0$$

such that $c + N_\delta(\theta) \subseteq \text{int}(P)$. Choose a natural number N_1 such that

$$\left\| \frac{-k^{N_1}}{1-k} d(Tx_0, Tx_1) \right\| < \delta.$$

Then

$$\frac{-k^n}{1-k} d(Tx_0, Tx_1) \in N_\delta(\theta), \forall n \ge N_1.$$

Hence

$$c - \frac{k^n}{1-k} d(Tx_0, Tx_1) \in c + N_\delta(\theta) \subseteq \text{int}(P).$$

Thus, we have

$$\frac{k^n}{1-k} d(Tx_0, Tx_1) << c, \forall n \ge N_1. \quad (6)$$

By inequality (5) and inequality (6), we have

$$d(Tx_n, Tx_m) << c, \forall n \ge N_1.$$

Thus (Tx_n) is a Cauchy sequence of X. By the completeness of X, there is $u \in X$ such

$$Tx_n \to u \ \ as \ n \to +\infty.$$

Now, suppose that f and T are continuous, we have $T(Tx_{2n+1}) \to Tu$ as as $n \to +\infty$, and

$f(Tx_{2n}) \to fu$ as as $n \to +\infty$. By the triangular inequality, we have

$$d(Tu, fu) \le d(Tu, T(Tx_{2n+1})) +$$
$$d(T(fx_{2n}), f(Tx_{2n})) + d(f(Tx_{2n}), fu). \quad (7)$$

Noting that $fx_{2n} = Tx_{2n+1} \to u$ as $n \to +\infty$ and $Tx_{2n} \to u$ as $n \to +\infty$. Since $\{T, f\}$ is compatible, we conclude that

$$\lim_{n \to +\infty} d(T(fx_{2n}), f(Tx_{2n})) = \theta.$$

Let $\theta << c$ be given. Choose $k_1, k_2, k_3 \in N$ such that

$$d(Tu, T(Tx_{2n+1})) << \frac{c}{3}, \forall n \ge k_1,$$

$$d(T(fx_{2n}), f(Tx_{2n})) << \frac{c}{3}, \forall n \ge k_2,$$

and

$$d(f(Tx_{2n}), fu) << \frac{c}{3}, \forall n \ge k_3.$$

Let $k_0 = \max\{k_1, k_2, k_3\}$. By inequality (7), we have $d(Tu, fu) << c$. Since c is arbitrary,

we get

$$d(Tu, fu) << \frac{c}{m} \ \forall m \in N.$$

By noting that $\frac{c}{m} \to \theta$ as $m \to 0$, we conclude

$$\frac{c}{m} - d(Tu, fu) \to -d(Tu, fu)$$

as $m \to 0$. Because P is closed, we get $-d(Tu, fu) \in P$. Thus $d(Tu, fu) \in P \cap -P$. Hence

$d(Tu, fu) = \theta$. Therefore $Tu = fu$. Since $Tu \le Tu$, by inequality (1), we get

$$d(fu,gu) \le ad(Tu,Tu) + b\big(d(Tu,fu) + d(Tu,gu)\big) + c\big(d(Tu,gu) + d(Tu,fu)\big).$$

Hence

$$d(Tu,gu) = d(fu,gu) \le (b+c)d(Tu,gu).$$

Since $b + c < 1$, we conclude that $d(Tu,gu) = \theta$. Thus we get that $gu = Tu = fu$. Hence

u is a common coincidence point of g, f and T. Similarly we show that if g and T are

continuous, then g, f and T have a common coincidence point.

Theorem 2.2 *In additional to the hypotheses of Theorem 2.1 suppose that if* $x, y \in X$,

then Tx and Ty are comparable. Then f, g and T have a unique common coincidence

point. Moreover, f, g and T have a unique common fixed point.

Proof. As in Theorem 2.1, the maps f, g and T have a common coincidence point $u \in X$;

that is, $fu = gu = Tu = u_1$. Let v be any other common coincidence point of f, g and T;

that is, $fv = gv = Tv = v_1$. Since Tu and Tv are comparable, we have

$$\begin{aligned} d(u_1,v_1) &= d(fu,gv) \\ &\le ad(Tu,Tv) + \\ &b\big(d(Tu,fu) + d(Tv,gv)\big) + \\ &c\big(d(Tu,gv) + d(Tv,fu)\big) \\ &= (a+2c)d(u_1,v_1). \end{aligned}$$

Since $a + 2c \in [0, 1)$ and P is cone, then we can see that $d(u_1,v_1) \in P \cap -P$. Thus

$d(u_1,v_1) = \theta$. Hence $u_1 = v_1$. So u_1 is the unique common point of coincidence of f, g and

T. By Lemma 1.1, we have $\{f, T\}$ and $\{g, T\}$ are weakly compatible. Thus

$Tu_1 = T(fu) = f(Tu) = f(u_1)$

and

$Tu_1 = T(gu) = g(Tu) = g(u_1)$.

Hence $u_2 = fu_1 = gu_1 = Tu_1$; that is, u_2 is a common point of coincidence of f, g and

T. By the uniqueness of the point of coincidence, we have $u_1 = u_2$. Thus u_2 is the unique

common fixed point of f, g and T.

The following results follow from Theorem 2.1. and Theorem 2.2.

Corollary 2.1 *Let* $(X,\le)$ *be a partially ordered set and (X, d) be a complete cone metric*

space. Let $f,g,T : X \to X$ *be three maps such that*

$$d(fx,gy) \le ad(Tx,Ty) + b\big(d(Tx,fx) + d(Ty,gy)\big)$$

for all $x, y \in X$ *with* $Tx \le Ty$ *. Assume that f, g and T satisfy the following conditions:*

1. f and g are weakly increasing with respect to T.

2. The pairs {f, T} and {g, T} are compatible.

3. f and T are continuous or g and T are continuous.

4. $fX \subseteq TX$ *and* $gX \subseteq TX$.

If a and b are nonnegative real numbers with $a+2b \in [0, 1)$*, then f, g and T have a*

common coincidence point.

Corollary 2.2 *In additional to the hypotheses of Corollary 2.1 suppose that if* $x, y \in X$,

then Tx and Ty are comparable. Then f, g and T have a unique common point of

coincidence. Moreover, f, g and T have a unique common fixed point.

Corollary 2.3 *Let* $(X,\le)$ *be a partially ordered set and (X, d) be a complete cone metric*

space. Let $f,g,T : X \to X$ *be three maps such that*

$$d(fx,gy) \le ad(Tx,Ty) + c\big(d(Tx,gy) + d(Ty,fx)\big)$$

for all $x, y \in X$ *with* $Tx \le Ty$ *. Assume that f, g and T satisfy the following conditions:*

1. f and g are weakly increasing with respect to T.

2. The pairs {f, T} and {g, T} are compatible.

3. f and T are continuous or g and T are continuous.

4. $fX \subseteq TX$ *and* $gX \subseteq TX$.

If a and c are nonnegative real numbers with a+2c$\in$*[0, 1), then f, g and T have a common coincidence point.*

Corollary 2.4 *In additional to the hypotheses of Corollary 2.3 suppose that if* $x, y \in X$,

then Tx and Ty are comparable. Then f, g and T have a unique common point of

coincidence. Moreover, f, g and T have a unique common fixed point.

Corollary 2.5 *Let* $(X, \leq)$ *be a partially ordered set and (X, d) be a complete cone metric*

space. Let $f, g : X \to X$ *be two maps such that*

$$d(fx, gy) \leq ad(x, y) + b\big(d(x, fx) + d(y, gy)\big) + c\big(d(x, gy) + d(y, fx)\big)$$

for all $x, y \in X$ *with* $x \leq y$. *Assume that f and g satisfy the following conditions:*

1. f and g are weakly increasing with respect to $\leq$.

2. f or g is continuous.

If a, b and c are nonnegative real numbers with a+2b+2c$\in$*[0, 1), then f and g have a common fixed point.*

Corollary 2.6 *In additional to the hypotheses of Corollary 2.5 suppose that if* $x, y \in X$,

then x and y are comparable. Then f and g have a unique common fixed point.

Remark 2 1.

1. *Theorem 12 of [4] is a special case of Theorem 2.1.*

2. *Theorem 18 of [4] is a special case of Theorem 2.1.*

3. *Theorem 2.1 of [1] is a special case of Theorem 2.1.*

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