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A General Fixed Point Theorem In Complete G - Metric Spaces For Weakly Compatible Pairs

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ABSTRACT

In this paper a general fixed point theorem in complete G - metric space for weakly compatible pairs of mappings is proved, which generalize the results by Theorems 3.2 and 3.3 [18] and obtained another particular results.

Key words: complete G - metric space, fixed point, weakly compatible mappings, implicit relation.

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1. INTRODUCTION

Let (X,d) be a metric space and S, $T:(X,d)\to(X,d)$ be two mappings. In 1994, Pant [13] introduced the notion of pointwise R - weakly commuting mappings. It is proved in [14] that the notion of pointwise R - weakly commutativity is equivalent to commutativity in coincidence points. Jungck [4] defined S and T to be weakly compatible if $Sx=Tx$ implies $STx=TSx$. Thus, S and T are weakly compatible if and only if S and T are pointwise R - weakly commuting.

In [2], [3] Dhage introduced a new class of generalized metric spaces, named D - metric space. Mustafa and Sims [6], [7] proved that most of the claims concerning the fundamental topological structures on D - metric spaces are incorrect and introduced appropriate notion of generalized metric space, named G - metric space. In fact, Mustafa, Sims and other authors studied many fixed point results for self mappings in G - metric spaces under certain conditions [5] – [12], [17].

Quite recently, Srivastava et al. [18] proved two fixed point theorems for weakly compatible mappings in complete G - metric spaces.

In [15] and [16], Popa initiated the study of fixed points for mappings satisfying implicit relations.

The purpose of this paper is to prove a general fixed point theorem in G - metric spaces for weakly compatible pairs of mappings satisfying an implicit relation which generalize the results from Theorems 3.2 and 3.2 [18].

2. PRELIMINARIES

Definition 2.1 [7] *Let X be a nonempty set and $G:X^3\to\mathbf{R}_+$ be a function satisfying the following properties:*

$(G_1): G(x,y,z)=0$ if $x=y=z$,

$(G_2): 0<G(x,x,y)$ for all $x,y\in X$ with $x\neq y$,

$(G_3): G(x,x,y)\le G(x,y,z)$ for all $x,y,z\in X$ with $z\neq y$,

$(G_4): G(x,y,z)=G(y,z,x)=G(z,x,y)=...$ (symmetry in all three variables),

$(G_5): G(x,y,z)\le G(x,a,a)+G(a,y,z)$ for all $x,y,z,a\in X$ (rectangle inequality).

Then the function G is called a G - metric on X and the pair (X,G) is called a G - metric space.

Note that $G(x,y,z)=0$, then $x=y=z$.

Definition 2.2 [7] *Let (X,G) be a G - metric space. A sequence (x_n) in X is said to be*

a) G - convergent if for $\varepsilon>0$, there exists an $x\in X$ and $k\in\mathbf{N}$ such that for all $m,n\ge k$, $G(x,x_n,x_m)<\varepsilon$,

b) G - Cauchy if for each $\varepsilon>0$, there exists $k\in\mathbf{N}$ such that for all $n,m,p\ge k$, $G(x_n,x_m,x_p)<\varepsilon$, that is $G(x_n,x_m,x_p)\to 0$ as $n,m,p\to\infty$.

c) A G - metric space is said to be G - complete if every G - Cauchy sequence is G - convergent.

Lemma 2.1 [7] *Let (X,G) be a G - metric space. Then, the following properties are equivalent:*

1) (x_n) is G - convergent to x;

2) $G(x_n,x_n,x)\to 0$ as $n\to\infty$;

3) $G(x_n,x,x)\to 0$ as $n\to\infty$;

4) $G(x_n,x_m,x)\to 0$ as $m,n\to\infty$.

Lemma 2.2 [7] *If (X,G) is a G - metric space and $(x_n)\in X$, then the following properties are equivalent:*

1) (x_n) is G - Cauchy;

2) For every $\varepsilon>0$, there exists $k\in\mathbf{N}$ such that $G(x_n,x_m,x_m)<\varepsilon$ for all $n,m\ge k$.

Lemma 2.3 [7] 1 *Let (X,G) be a G - metric space, then the function $G(x,y,z)$ is jointly continuous in all three of its variables.*

Lemma 2.4 [7] *Let (X,G) be a G - metric space. Then $G(x,y,y)\le 2G(y,x,x)$ for all $x,y\in X$.*

Quite recently, the following theorems are proved in [18].

Theorem 2.1 *Let (X,G) be a complete G - metric space and let $S,T:X\to X$ be two mappings which satisfy the following conditions:*

(i) $T(X)\subset S(X)$,

(ii) $T(X)$ or $S(X)$ is G - complete, and

(iii) $G(Tx,Ty,Tz)\le\alpha G(Sx,Sy,Sz)+\beta G(Tx,Sx,Sx)+\gamma G(Ty,Sy,Sy)+\delta G(Tz,Sz,Sz)+\eta G(Tx,Sy,Sy)$,

for all $x,y,z\in X$, where $\alpha,\beta,\gamma,\delta,\eta\ge 0$ and $\alpha+2\beta+2\gamma+2\delta+2\eta<1$.

Then S and T have an unique point of coincidence in X. Moreover, if S and T are weakly compatible, then S and T have an unique common fixed point.

Theorem 2.2 *Let (X,G) be a complete G - metric space and let $S,T : X \to X$ be two mappings which satisfy the following conditions:*

(i) $T(X) \subset S(X)$,

(ii) $T(X)$ or $S(X)$ is G - complete, and

(iii) $G(Tx,Ty,Tz) \le \alpha \max\{G(Sx,Sy,Sz), G(Tx,Sx,Sx), G(Ty,Sy,Sy), G(Tz,Sz,Sz), (Tx,Sy,Sy)\}$,

for all $x,y,z \in X$, where $\alpha \in \left(0,\frac{1}{2}\right)$.

Then S and T have an unique point of coincidence in X. Moreover, if S and T are weakly compatible, then S and T have an unique common fixed point in X.

3. IMPLICIT RELATIONS

Definition 3.1 [2] *Let $\mathcal{F}_S$ be the set of all continuous functions $F(t_1,...,t_5) : \mathbf{R}_+^5 \to \mathbf{R}$ satisfying the following conditions:*

(F_1) F is nonincreasing in variables t_3 and t_4,

(F_2) There exists $h \in [0,1)$ such that for all $u,v \ge 0$, $F(u,v,2v,2u,0) \le 0$ implies $u \le hv$,

(F_3) $F(t,t,0,0,t) > 0$, $\forall t > 0$.

Example3.1
$F(t_1,...,t_5) = t_1 - at_2 - bt_3 - (c+d)t_4 - et_5$, *where* $a,b,c,d,e \ge 0$ *and* $a+2b+2c+2d+e<1$.

(F_1): Obviously.

(F_2): Let $u,v \ge 0$ be and $F(u,v,2v,2u,0) = u - av - 2bv - 2(c+d)u \le 0$. Then, $u \le hv$ where $0 \le h = \dfrac{a+2b}{1-2(c+d)}$.

(F_3): $F(t,t,0,0,t) = t(1-(a+e)) > 0, \forall t > 0$.

Example 3.2 $F(t_1,...,t_5) = t_1 - k\max\{t_2,t_3,t_4,t_5\}$, *where* $k \in \left[0,\frac{1}{2}\right)$.

(F_1): Obviously.

(F_2): Let $u,v \ge 0$ be and $F(u,v,2v,2u,0) = u - k\max\{v,2v,2u\} \le 0$. If $u > v$, then $u(1-2k) \le 0$, a contradiction. Hence $u \le v$ and $u \le hv$, where $0 \le h = 2k < 1$.

(F_3): $F(t,t,0,0,t) = t(1-k) > 0, \forall t > 0$.

Example3.3
$F(t_1,...,t_5) = t_1^2 - t_1(at_2 + bt_3 + ct_4) - dt_5^2$, *where* $a,b,c,d \ge 0$, $a+2b+2c<1$ *and* $a+d<1$.

(F_1): Obviously.

(F_2): Let $u,v \ge 0$ be and $F(u,v,2v,2u,0) = u^2 - u(av+2bv+2cu) \le 0$. If $u > 0$, then $u - av - 2bv - 2cu \le 0$ which implies $u \le hv$, where $0 \le h = \dfrac{a+2b}{1-2c} < 1$. If $u = 0$ then $u \le hv$.

(F_3): $F(t,t,0,0,t) = t^2(1-(a+d)) > 0, \forall t > 0$.

Example 3.4 $F(t_1,...,t_5) = t_1 - a\dfrac{t_2+t_3}{2} - b\dfrac{t_4+t_5}{2}$, *where* $a,b \ge 0$ *and* $3a+2b<2$.

(F_1): Obviously.

(F_2): Let $u,v \ge 0$ be and $F(u,v,2v,2u,0) = u - a\dfrac{3v}{2} - bu \le 0$. Hence $u \le hv$, where $0 \le h = \dfrac{3a}{2-2b} < 1$.

(F_3): $F(t,t,0,0,t) = t\left(1-\dfrac{a+b}{2}\right) > 0, \forall t > 0$.

Example 3.5 $F(t_1,...,t_5) = t_1^2 - at_2^2 - b\dfrac{t_3^2+t_4^2}{1+t_5^2}$, *where* $a+8b<1$.

(F_1): Obviously.

(F_2): Let $u,v \ge 0$ be and $F(u,v,2v,2u,0) = u^2 - av^2 - (4u^2+4v^2)b \le 0$ which implies $u \le hv$, where $0 \le h = \sqrt{\dfrac{a+4b}{1-4b}}$.

(F_3): $F(t,t,0,0,t) = t^2(1-a) > 0, \forall t > 0$.

Example3.6
$F(t_1,...,t_5) = t_1 - at_2 - bt_3 - c\min\{t_4,t_5\}$, *where* $a,b,c \ge 0$ *and* $a+2b<1$.

(F_1): Obviously.

(F_2): Let $u,v \geq 0$ and $F(u,v,2v,2u,0) = u - av - 2bv \leq 0$ which implies $u \leq hv$, where $0 \leq h = a + 2b < 1$.

(F_3): $F(t,t,0,0,t) = t(1-a) > 0, \forall t > 0$.

Example 3.7 $F(t_1,...,t_5) = t_1 - c\max\{t_2, t_3, \sqrt{t_4 t_5}\}$, *where* $c \in \left(0, \frac{1}{2}\right)$.

(F_1): Obviously.

(F_2): Let $u,v \geq 0$ and $F(u,v,2v,2u,0) = u - 2cv \leq 0$, which implies $u \leq hv$, where $0 \leq h = 2c < 1$.

(F_3): $F(t,t,0,0,t) = t(1-c) > 0, \forall t > 0$.

Example3.8

$$F(t_1,...,t_5) = t_1 - k\max\left\{t_2, t_3, \frac{t_3 + 2t_4}{2}, \frac{t_4 + 2t_5}{2}\right\},$$

where $k \in \left(0, \frac{1}{4}\right)$.

(F_1): Obviously.

(F_2): Let $u,v \geq 0$ and

$$F(u,v,2v,2u,0) = u - k\max\left\{v, 2v, \frac{2v+4u}{2}, u\right\} = u - k\max\{2v, v+2u\} \leq 0$$

, which implies $u \leq 2k(u+v)$. Hence $u \leq hv$, where $0 \leq h = \frac{2k}{1-2k} < 1$.

(F_3): $F(t,t,0,0,t) = t(1-k) > 0, \forall t > 0$.

4. MAIN RESULTS

Definition 4.1 *Let S and T two self mappings of a nonempty set X. If $w = Tx = Sx$ for some $x \in X$, then x is called a coincidence point of S and T and w is called a point of coincidence of T and S.*

Lemma 4.1 [1] *Let T and S be weakly compatible self mappings of a nonempty set X. If T and S have an unique point of coincidence $w = Tx = Sx$, then w is the unique common fixed point of T and S.*

Theorem 4.1 *Let (X,G) be a G - metric space and T, S self mappings of X such that*

$$F(G(Tx,Ty,Ty), G(Sx,Sy,Sy), G(Tx,Sx,Sx), G(Ty,Sy,Sy), G(Tx,Sy,Sy)) \leq 0 \quad (4.1)$$

for all $x,y \in X$ and F satisfying property (F_3). Then T and S have at most a point of coincidence.

Proof. Suppose that $u = Tp = Sp$ and $v = Tq = Sq$ are two distinct points of coincidence. Then, by (4.1) we have successively:

$$F(G(Tq,Tp,Tp), G(Sq,Sp,Sp), G(Tq,Sq,Sq), G(Tp,Sp,Sp), G(Tq,Sp,Sp)) \leq 0,$$

$$F(G(Sq,Sp,Sp), G(Sq,Sp,Sp), 0, 0, G(Sq,Sp,Sp)) \leq 0,$$

a contradiction of (F_3) if $G(Sq,Sp,Sp) > 0$. Hence $G(Sq,Sp,Sp) = 0$, so $Sq = Sp$ which implies $u = v$.

Theorem 4.2 *Let (X,G) be a G - metric space and let $T,S:(X,G) \to (X,G)$ be two mappings such that*

2. (i) $T(X) \subset S(X)$,

(ii) $T(X)$ or $S(X)$ is G - complete,

(iii) T and S satisfy the inequality (4.1) for all $x,y \in X$ and $F \in \mathcal{F}_S$.

Then T and S have an unique point of coincidence. Moreover, if T and S are weakly compatible, then T and S have an unique common fixed point.

Proof. Let $x_0 \in X$ be an arbitrary point. Then, there exists $x_1 \in X$ such that $Tx_0 = Sx_1$. In this way we defined a sequence $\{Sx_n\}$ with $Tx_{n-1} = Sx_n$ for $n = 1,2,...$. Then by (4.1) we have successively:

$$F(G(Tx_{n-1},Tx_n,Tx_n), G(Sx_{n-1},Sx_n,Sx_n), G(Tx_{n-1},Sx_{n-1},Sx_{n-1}), G(Tx_n,Sx_n,Sx_n), G(Tx_{n-1},Sx_n,Sx_n)) \leq 0,$$

$$F(G(Sx_n,Sx_{n+1},Sx_{n+1}), G(Sx_{n-1},Sx_n,Sx_n), G(Sx_n,Sx_{n-1},Sx_{n-1}), G(Sx_{n+1},Sx_n,Sx_n), 0) \leq 0.$$

By Lemma 2.4

$G(Sx_{n+1},Sx_n,Sx_n) \leq 2G(Sx_n,Sx_{n+1},Sx_{n+1})$ and

$G(Sx_n, Sx_{n-1}, Sx_{n-1}) \le 2G(Sx_{n-1}, Sx_n, Sx_n)$. By (F_1) we obtain:

$$F(G(Sx_n, Sx_{n+1}, Sx_{n+1}), G(Sx_{n-1}, Sx_n, Sx_n), 2G(Sx_{n-1}, Sx_n, Sx_n), 2G(Sx_n, Sx_{n+1}, Sx_{n+1}), 0) \le 0$$

which implies by (F_2) that

$$G(Sx_n, Sx_{n+1}, Sx_{n+1}) \le hG(Sx_{n-1}, Sx_n, Sx_n).$$

By repeated application of the above inequality, we have

$$G(Sx_n, Sx_{n+1}, Sx_{n+1}) \le h^n G(Sx_0, Sx_1, Sx_1).$$

Then for $n, m \in \mathbf{N}$, $n < m$, we have by rectangle inequality that

$$\begin{aligned} G(Sx_n, Sx_m, Sx_m) &\le G(Sx_n, Sx_{n+1}, Sx_{n+1}) + G(Sx_{n+1}, Sx_{n+2}, Sx_{n+2}) + \\ &\quad + \ldots + G(Sx_{m-1}, Sx_m, Sx_m) \\ &\le (h^n + h^{n+1} + \ldots + h^{m-1}) G(Sx_0, Sx_1, Sx_1) \\ &\le \frac{h^n}{1-h} G(Sx_0, Sx_1, Sx_1). \end{aligned}$$

Taking limit as $n, m \to \infty$, we get $\lim_{n,m\to\infty} G(Sx_n, Sx_m, Sx_m) = 0$. Hence $\{Sx_n\}$ is a G - Cauchy sequence. Now, since $S(X)$ is G - complete, there exists a point $q \in S(X)$ such that $Sx_n \to q$ as $n \to \infty$. Consequently, we can find a point $p \in X$ such that $Sp = q$.

If $T(X)$ is G - complete, there exists $q \in T(X)$ such that $Sx_n \to q$ as $T(X) \subset S(X)$, we have $q \in Sx$. Then, there exists $p \in X$ such that $Sp = q$.

We prove that p is a coincidence point for T and S. By (4.1) we have successively:

$$F(G(Tx_{n-1}, Tp, Tp), G(Sx_{n-1}, Sp, Sp), G(Tx_{n-1}, Sx_{n-1}, Sx_{n-1}), G(Tp, Sp, Sp), G(Tx_{n-1}, Sp, Sp)) \le 0,$$

$$F(G(Sx_n, Tp, Tp), G(Sx_{n-1}, Sp, Sp), G(Sx_n, Sx_{n-1}, Sx_{n-1}), G(Tp, Sp, Sp), G(Sx_n, Sp, Sp)) \le 0.$$

Letting n tend to infinity, we obtain

$$F(G(Sp, Tp, Tp), 0, 0, G(Tp, Sp, Sp), 0) \le 0.$$

By Lemma 2.4, $G(Tp, Sp, Sp) \le 2G(Sp, Tp, Tp)$.

By (F_1) we obtain $F(G(Sp, Tp, Tp), 0, 0, 2G(Sp, Tp, Tp), 0) \le 0$.

By (F_2), $G(Sp, Tp, Tp) = 0$ which implies $w = Tp = Sp$ and p is a coincidence point of T and S. By Theorem 4.1, w is the unique point of coincidence of T and S. Moreover, if T and S are weakly compatible, by Lemma 4.1 w is the unique common fixed point of T and S.

If $S(X)$ is complete, the proof it follows by $T(X) \subset S(X)$.

Corollary 4.1 *Let T and S be self mappings of a G - metric space satisfying the following conditions:*

(i) $T(X) \subset S(X)$,

(ii) $S(X)$ *or* $T(X)$ *is* G *- complete,*

(iii) One of the following inequalities hold for all $x, y \in X$ *(1)*

$$G(Tx, Ty, Ty) \le aG(Sx, Sy, Sy) + bG(Tx, Sx, Sx) + (c+d)G(Ty, Sy, Sy) + eG(Tx, Sy, Sy), \text{ (3)}$$

where $a, b, c, d, e \ge 0$ *and* $a + 2b + 2c + 2d + e < 1$. *(2)*

$$G(Tx, Ty, Ty) \le k \max\{G(Sx, Sy, Sy), G(Tx, Sx, Sx), G(Ty, Sy, Sy), G(Tx, Sy, Sy)\},$$

where $k \in \left(0, \frac{1}{2}\right)$*.(3)*

$$G^2(Tx,Ty,Ty) \le G(Tx,Ty,Ty)[aG(Sx,Sy,Sy) + bG(Tx,Sx,Sx) + cG(Ty,Sy,Sy)] + dG^2(Tx,Sy,Sy),$$

where $a,b,c,d \ge 0$ *,* $a + 2b + 2c < 1$ *and* $a + d < 1$*. (4)*

$$G(Tx,Ty,Ty) \le a\frac{G(Sx,Sy,Sy) + G(Tx,Sx,Sx)}{2} + b\frac{G(Ty,Sy,Sy) + G(Tx,Sy,Sy)}{2},$$

where $a,b \ge 0$ *and* $3a + 2b < 2$*. (5)*

$$G^2(Tx,Ty,Ty) \le aG^2(Sx,Sy,Sy) + b\frac{G^2(Tx,Sx,Sx) + G^2(Ty,Sy,Sy)}{1 + G^2(Tx,Sy,Sy)},$$

where $a,b \ge 0$ *and* $a + 8b < 1$*.(6)*

$$G(Tx,Ty,Ty) \le aG(Sx,Sy,Sy) + bG(Tx,Sx,Sx) + c\min\{G(Ty,Sy,Sy), G(Tx,Sy,Sy)\},$$

where $a,b,c \ge 0$ *and* $a + 2b < 1$*.(7)*

$$G(Tx,Ty,Ty) \le c\max\{G(Sx,Sy,Sy), G(Tx,Sx,Sx), [G(Ty,Sy,Sy)\cdot G(Tx,Sy,Sy)]^{1/2}\},$$

where $c \in \left(0, \frac{1}{2}\right)$*.(8)*

$$G(Tx,Ty,Ty) \le k\max\{G(Sx,Sy,Sy), G(Tx,Sx,Sx), \frac{1}{2}[G(Tx,Sx,Sx) + 2G(Ty,Sy,Sy)], \frac{1}{2}[G(Ty,Sy,Sy) + 2G(Tx,Sy,Sy)]\},$$

where $k \in \left(0, \frac{1}{4}\right)$.

If S and T are weakly compatible, then S and T have an unique common fixed point.

Proof. The proof follows by Theorem 4.2 and Examples 3.1 – 3.8.

Remark 4.1 *Because in Theorem 2.1 and* $a + 2b + 2c + 2d + 2e < 1$*, for* $y = z$ *we obtain*

$$G(Tx,Ty,Ty) \le aG(Sx,Sy,Sy) + bG(Tx,Sx,Sx) + (c+d)G(Ty,Sy,Sy) + eG(Tx,Sy,Sy)$$

and $a + 2b + 2c + 2d + e < 1$*, Theorem 2.1 follows from Corollary 4.1 (iii) (1).*

Remark 4.2 *Because in Theorem 2.2 for* $y = z$ *we obtain*

$$G(Tx,Ty,Ty) \le k\max\{G(Sx,Sy,Sy), G(Tx,Sx,Sx), G(Ty,Sy,Sy), G(Tx,Sy,Sy)\},$$

and Theorem 2.2 follows from Corollary 4.1 (iii) (2).

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