



# Optimization and Decision Making Stages for Multiple Responses: An Application of NSGA-II and FCM Clustering Algorithm

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## ABSTRACT

Optimization and decision making are closely related but two distinct fields in multi-response studies. Generally, multiple responses are aggregated in a single objective function and the optimization result is considered as a compromise solution for all the responses. However, this approach does not meet required targets of all the responses simultaneously. In this study, Non-dominated Sorting Genetic Algorithm-II (NSGA-II), a well known multi objective optimization method is preferred to optimize multiple responses and adapted with penalty function approach to handle constraints. In order to decide a compromise solution among the obtained many non-dominated solutions, two different decision making methods are used: (i) a fuzzy based clustering algorithm (fuzzy c means-FCM), and (ii) a mostly used multi criteria decision making (MCDM) method (technique for order preference by similarity to an ideal solution-TOPSIS). The selected combination of the NSGA-II with FCM and TOPSIS are performed on a real world data set given in the literature and results are discussed. The results show the applicability of the FCM for decision making in multiple responses. It can be said that the FCM makes easier the selection of a compromise solution in the non-dominated solution set by using membership degrees of each solution to the clusters without removing any non-dominated solution.

**Key words:** Multi-response optimization (MRO), NSGA-II, non-dominated solutions, FCM, TOPSIS.

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## 1. INTRODUCTION

Many process and product design problems may have more than one response which are called multi-response problems. These problems are commonly analyzed in two main stages after data collection: (i) modeling, and (ii) optimization. Before modeling of responses, dependencies among the responses should be checked and considerable effort should be made to uncover their causes [1]. In modeling stage, possible correlations among the responses should be checked in order to compose an approximating multi-response model with minimum error. If the response variables are correlated, it is seen that the seemingly unrelated regression (SUR),

originally developed by Zellner [2], produces more precise estimates of model parameters than the ordinary least squares (OLS) [3]. And also, Principle Component Analysis (PCA) is used as an alternative modeling method for correlated responses as in the studies of Liao [4], Wang [5], Šibalija and Majstorović [6].

The optimization stage is generally called multi-response optimization (MRO) in multi response studies. In the literature, many approaches have been seen for MRO such as the constrained optimization approach [7], desirability function approach [8-14], generalized distance approach [15] and loss function approach [16-19]. The general purpose of these methods is to convert

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the MRO problem into a single aggregated objective function. According to this, it is not possible to find a single solution that would optimize all the objectives simultaneously. And also, the existing MRO methods require preference parameters to incorporate the preference of decision maker (DM) for the responses, which represent the preference information on the trade-offs among the responses, such as weights of the responses. However, determining the preference parameter values is difficult process for complex problems with multiple responses.

Since the MRO problem can be viewed as a multi-objective optimization (MOO) problem as in the studies of Park and Kim [20], Pareto optimality is employed to MRO problem and infinite number of alternative solutions are generated. These alternative solutions are denoted with a vector called Pareto optimal (or non-inferior or efficient or non-dominated) set in which none of its components can be improved without deterioration to at least one of other components [21]. In order to get Pareto optimal set, the use of multi-objective evolutionary algorithms (MOEAs), which are posterior preference approaches in nature, has been motivated in recent years [22]. The posterior preference approaches have an advantage of not requiring any advance information on the DM's preference [23]. By using MOEAs, it is possible to find a representative set of non-dominated solutions without requiring any prior information from the DM. Then, a compromise solution should be selected among the representative non-dominated solutions. The process of selection a compromise solution is called decision making stage which is necessary when the MRO is achieved with a set of non-dominated solutions. Generally, the most suitable solution is chosen by using multi-criteria decision making (MCDM) methods according to the preferences of the DM. These methods are grouped as the a priori, the progressive and the a posteriori decision in Miettinen [24].

In this study, the NSGA-II is applied for MRO with modification by using penalty function approach to handle constraints with multiple responses. In decision making stage, FCM is used rather than the classical MCDM methods. The FCM prefers the most suitable solution by using the membership degrees of each non-dominated solution according to the preferences of DM. The higher the membership degree, the more satisfactory solution one.

The rest of the paper is organized as follows. In Section 2, a brief description about general multi-response model and parameter estimation of correlated responses are given. The modified version of the NSGA-II and the FCM are presented in Section 3. A data set is chosen as an application and the obtained results are presented in Section 4. In Section 5, conclusions are given.

## 2. THE GENERAL MULTI-RESPONSE MODEL AND CORRELATED RESPONSES

Simultaneous modeling of the response variables as a function of input variables with minimum error is considered one of the main aim in the presence of

multiple responses. The model associated with such a function is called multi-response model which can be defined as given below.

Let  $N$  be the number of experimental runs and  $r$  be the number of response variables which can be measured for each setting of a group of  $k$  coded input variables  $\mathbf{x} = [x_1, x_2, \dots, x_k]'$ . Suppose that the  $i$ th response value at the  $u$ th experimental run is represented by the model

$$Y_{ui} = f_i(\mathbf{x}_u, \boldsymbol{\beta}) + \varepsilon_{ui}, \quad u = 1, 2, \dots, N, \quad i = 1, 2, \dots, r \quad (1)$$

where  $\mathbf{x}_u$  is the vector  $[x_{u1}, x_{u2}, \dots, x_{uk}]'$  with  $x_{uj}$  being the  $u$ th level of the  $j$ th coded variable,  $u = 1, 2, \dots, N$ ;  $j = 1, 2, \dots, k$ ,  $\boldsymbol{\beta}$  is a vector of unknown parameters.  $\varepsilon_{ui}$  is a random error,  $f_i$  is a function of known form for the  $i$ th response and is assumed to be continuous.

If  $f_i$  is linear in elements of  $\boldsymbol{\beta}$ , then the model in Eq. (1) is reduced to model for  $r$  responses represented as

$$\mathbf{Y} = \mathbf{Z}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (2)$$

in which  $\mathbf{Y} = [\mathbf{Y}'_1, \mathbf{Y}'_2, \dots, \mathbf{Y}'_r]'$ ,  $\boldsymbol{\beta} = [\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2, \dots, \boldsymbol{\beta}'_r]'$ ,  $\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}'_1, \boldsymbol{\varepsilon}'_2, \dots, \boldsymbol{\varepsilon}'_r]'$ , and  $\mathbf{Z}$  is the block-diagonal matrix,  $\text{diag}(\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_r)$ . The best linear unbiased estimator (BLUE) of  $\boldsymbol{\beta}$ , given in Eq. (2), is obtained below

$$\hat{\boldsymbol{\beta}} = (\mathbf{Z}'\Delta^{-1}\mathbf{Z})^{-1}\mathbf{Z}'\Delta^{-1}\mathbf{Y} \quad (3)$$

where  $\Delta$  is a covariance matrix of  $\boldsymbol{\varepsilon}$  denoted as  $\Sigma \otimes I_N$ . Eq. (3) requires knowledge of variance-covariance matrix,  $\Sigma$ . If  $\Sigma$  is unknown, the estimate  $\hat{\Sigma} = \hat{\sigma}_{ij}$  is used as

$$\hat{\sigma}_{ij} = \mathbf{Y}'_i \left[ I_N - \mathbf{Z}_i (\mathbf{Z}'_i \mathbf{Z}_i)^{-1} \mathbf{Z}'_i \right] \left[ I_N - \mathbf{Z}_j (\mathbf{Z}'_j \mathbf{Z}_j)^{-1} \mathbf{Z}'_j \right] / N,$$

$i, j = 1, 2, \dots, r$ , [25]. Using this estimate of  $\Sigma$ , the estimator of  $\hat{\boldsymbol{\beta}}^*$  is obtained as

$$\hat{\boldsymbol{\beta}}^* = (\mathbf{Z}'\Delta^{*-1}\mathbf{Z})^{-1}\mathbf{Z}'\Delta^{*-1}\mathbf{Y} \quad (4)$$

in which  $\Delta^* = \hat{\Sigma} \otimes I_N$ . The estimator given in Eq. (4) is called SUR estimator in the studies of Shah et al. [2]. In this study, the data set with multiple correlated responses is considered and the model parameters are estimated by using the estimator vector given in Eq. (4).

## 3. MULTI RESPONSE OPTIMIZATION (MRO) AND DECISION MAKING

### 3.1. MRO with Adapted NSGA-II

The main aim of the MRO is to find an optimal setting of the input variables that provides the best compromise

solution set for the multiple responses simultaneously. A MRO problem, consist of  $r$  predicted response functions, can be formulated as following

$$\begin{aligned} & \text{optimize } \{\hat{Y}_1(\mathbf{x}), \hat{Y}_2(\mathbf{x}), \dots, \hat{Y}_r(\mathbf{x})\} \\ & \text{subject to } \mathbf{x} \in S \end{aligned} \quad (5)$$

where  $\mathbf{x}$  is an input vector and  $S$  is an experimental region. The MRO problem can be viewed as a MOO problem. The general form of the MOO problem can be formulated as

$$\begin{aligned} & \text{min/ max } \mathbf{f}(\mathbf{x}) \\ & \text{subject to } g_j(\mathbf{x}) \geq 0, \quad j = 1, 2, \dots, J \\ & \quad h_t(\mathbf{x}) = 0, \quad t = 1, 2, \dots, T \\ & \quad x_s^{(L)} \leq x_s \leq x_s^{(U)}, \quad s = 1, 2, \dots, k \end{aligned} \quad (6)$$

in which  $\mathbf{f}(\mathbf{x})$  is an objective function vector defined as  $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_r(\mathbf{x})]^T$ . It is well-known that the main aim in a MOO framework is to find out a set of feasible and non-dominated alternative solutions which form Pareto optimal set.

The NSGA-II, a very often used optimization method in multi-objective studies, is an efficient algorithm to find well distributed Pareto optimal or near Pareto optimal solutions as many as possible [22]. The NSGA-II finds a set of non-dominated solutions without requiring any preference information from the DM in a single run so the algorithm is called posterior preference articulation approach. The principle of this algorithm is to use a fast non-dominated sorting mechanism and a crowding distance to construct the population fronts that dominate each other in a non-dominated order. In order to optimize the constrained MOO problem given in Eq. (6) through NSGA-II, the algorithm should be adapted by using constraint handling strategy such as given in the study of Kaveh et al. [26]. In this study, penalty function approach is applied as constraint handling strategy to NSGA-II and called adapted NSGA-II. Before constraint violation is calculated, all constraints are normalized which means  $g_j(\mathbf{x}^{(u)}) \geq 0$ ,  $j = 1, 2, \dots, J$ ,  $u = 1, 2, \dots, N$  and all the objective functions are assumed to be minimized. For each solution  $\mathbf{x}^{(u)}$ , the constraint violation for each constraint is calculated as follows

$$w_j(\mathbf{x}^{(u)}) = \begin{cases} |g_j(\mathbf{x}^{(u)})|, & g_j(\mathbf{x}^{(u)}) < 0 \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

Thereafter, all constraint violations are added together as  $\Omega(\mathbf{x}^{(u)}) = \sum_{j=1}^J w_j(\mathbf{x}^{(u)})$  to get overall constraint violation. This constraint violation is then multiplied with a penalty parameter denoted as  $R_i$  which makes the terms to have same magnitude and the product is

added to each of the objective function values as given below

$$F_i(\mathbf{x}^{(u)}) = f_i(\mathbf{x}^{(u)}) + R_i \Omega(\mathbf{x}^{(u)}), \quad i = 1, 2, \dots, r \quad (8)$$

in which the functions  $F_i$ ,  $i = 1, 2, \dots, r$  takes into account the constraint violations. In this study, the objective functions given in Eq. (8) are considered as objective functions for the NSGA-II and called violated objective functions. Since the original objective functions may have different magnitude, the penalty parameter may also vary from one objective function to another. If an appropriate penalty parameter is chosen, the NSGA-II will work well [27]. The proposed algorithmic steps of the adapted NSGA-II are given in Appendix A.

### 3.2. Decision Making in the Pareto Optimal Set

The main aim of decision making in multi-response studies is determining the best feasible solution according to the interested multiple responses. Since many problems are characterized by several conflicting responses, there may be no solution satisfying all responses simultaneously. Thus, the compromise solution can be a set of non-dominated solutions or can be only a solution according to the DM's preferences. If the DM agrees with a set of non-dominated solutions, then this is possible thanks to the cluster analysis in which each cluster is homogeneous or compact with respect to certain objective criterions and each cluster is different from other clusters with respect to some objective criterions. In multi-response studies, the clustering analysis are applied to non-dominated set to organize the solutions based on their objective functions without removing any elements of non-dominated set before presenting the solutions to the DM. Yang et al. [28], Zio and Bazzo [29], and Buditjahjanto [30] used clustering analysis based approach for MRO. The cluster analysis can be classified in two categories: hard c-means (HCM) clustering and FCM clustering. HCM clustering methods are based on classical set theory, and require that a non-dominated solution either does or does not belong to a cluster. And also, non-dominated solutions have equal importance degree in a cluster. If the DM needs only a solution as a compromise solution, it is necessary a method that makes easier for the DM to select the best feasible one in the cluster.

#### 3.2.1 Compromise solution selection with FCM algorithm

FCM clustering algorithm was presented by Bezdek [31] as the extension of HCM with the advantage of fuzzy set theory. The FCM allows a solution belonging to one or more clusters utilizing membership value concept with the restriction that the sum of all membership values for a single solution in all of the clusters has to be unity. Therefore, the non-dominated solution partially belongs to each cluster. While in HCM clustering membership degrees have two values as 0 (does not belong to cluster) and 1 (belong to cluster), non-dominated solutions can take all values between 0 and 1 in FCM, as well. Basically, the

algorithm calculates fuzzy partition matrix to group some of non-dominated solutions into  $c$  clusters. The FCM starts with initial cluster centers and the aim of the algorithm is to cluster centers that minimize the following objective function

$$J(\mathbf{S}, \mathbf{V}, \mathbf{U}) = \sum_{i=1}^c \sum_{k=1}^{n_{pop}} \mu_{ik}^m d_{ik}^2(S_k, V_i) \quad (9)$$

where  $c$  is the number of clusters,  $n_{pop}$  is the size of Pareto optimal set,  $\mathbf{V} = [V_1, V_2, \dots, V_c]$  is a vector of cluster centers,  $\mathbf{U}$  is a membership degrees matrix (fuzzy partition matrix),  $m$  is fuzziness index (weighting parameter) ( $m \in [1, \infty)$ ),  $\mu_{ik}$  is the membership value of the  $k$  th solution to the  $i$  th cluster,  $S_k$  is a  $k$  th non-dominated solution in the Pareto optimal set. And also,  $d(S_k, V_i)$  is Euclidean distance between non-dominated solutions and cluster centers calculated by

$$d_{ik}(S_k, V_i) = \left\| S_k^{(i)} - V_i \right\|, \quad i=1, \dots, c, \quad k=1, \dots, n_{pop} \quad (10)$$

where  $S_k^{(i)}$  is the  $k$  th solution belonging to the  $i$  th cluster. Here, the cluster centers are calculated using following formulation

$$V_{ij} = \frac{\sum_{k=1}^{n_{pop}} \mu_{ik}^m x_{kj}}{\sum_{k=1}^{n_{pop}} \mu_{ik}^m}, \quad i=1, 2, \dots, c, \quad j=1, 2, \dots, r \quad (11)$$

in which  $r$  is the number of features and  $x_{kj}$  is the  $k$  th solution for the  $j$  th feature. In this study, the features are considered as objective functions which wanted to be optimized. The algorithm keeps reassigning the solutions to clusters until the predefined accuracy criterion for cluster centers is satisfied which is defined as

$$\left\| \mathbf{V}^{(t)} - \mathbf{V}^{(t-1)} \right\| < \varepsilon, \quad t=1, 2, \dots \quad (12)$$

in which  $t$  is the iteration number and  $\varepsilon$  is a tolerance level for termination which varies between 0 and 1.

The FCM is an iterative algorithm. The algorithmic steps of the FCM are summarized in Appendix B. It should be noted here that the differences of measuring units in the data may effect the clustering and the final results would be deviated from the essence of data. To eliminate such nagging effect, it is important to work on dimensionless transformation, e.g. normalization methods [32].

### 3.2.2 TOPSIS for compromise solution selection

TOPSIS method is presented in Chen and Hwang [33], with reference to Hwang and Yoon [34]. It is based on an aggregating function representing closeness to the reference points. TOPSIS method uses two reference points: (i) ideal solution, and (ii) negative-ideal solution. The basic principle of the TOPSIS is that the chosen alternative solution should have the “shortest distance” from the ideal solution and the “farthest distance” from the negative ideal solution. In this study, the TOPSIS procedure given in the studies of Opricovic and Tzeng [35] is used.

## 4. APPLICATION

In this section, the suggested approach is illustrated to a MRO problem originally presented in the work of Tseo et al. [36] and used in the studies of Shah et al. [2]. The data set is about washing treatments for quality improvement of minced mullet flesh. The effects of three input variables; washing temperature ( $X_1$ ), washing time ( $X_2$ ), and washing ratio ( $X_3$ ) on  $r=4$  response variables are investigated. The response variables include springiness ( $Y_1$ ), thiobarbituric acid (TBA) number ( $Y_2$ ), percent cooking loss ( $Y_3$ ), and whiteness index ( $Y_4$ ). The goal is to find the values of input variables that satisfy the following conditions on the response variables simultaneously:

$$\begin{aligned} \max Y_1 & \{ \geq 1.7 \} \\ \min Y_2 & \{ \leq 21 \} \\ \min Y_3 & \{ \leq 20 \} \\ \max Y_4 & \{ \geq 45 \}. \end{aligned} \quad (13)$$

The input settings in the original variables with coded values and the multi-response data in central composite design (CCD) are given in Table 1.

Initially, it should be checked that if the multiple responses are correlated or not. Therefore, the Pearson correlation coefficients matrix is calculated as given below

$$\rho(Y_1, Y_2, Y_3, Y_4) = \begin{bmatrix} 1 & & & \\ -0.932 & 1 & & \\ (0.000) & & & \\ -0.673 & 0.685 & & \\ (0.003) & (0.002) & 1 & \\ 0.024 & -0.168 & -0.118 & \\ (0.924) & (0.519) & (0.652) & 1 \end{bmatrix} \quad (14)$$

where the correlation coefficients of three pairs such as  $\rho(Y_1, Y_2)$ ,  $\rho(Y_1, Y_3)$ , and  $\rho(Y_2, Y_3)$  are significant at  $\alpha=0.05$ . Note that the number in parenthesis are the  $p$  values in the matrix given in Eq. (14).

Table 1. Experimental CCD and response values (Khuri and Cornell [25])

No	Original Input Variables			Coded Input Variables	Responses					
	Temp (°C)	Washing Time (min)	Washing Ratio		Springiness (mm)	TBA No.	Cooking Loss (%)	Whiteness Index		
	$X_1$	$X_2$	$X_3$	$x_1$	$x_2$	$x_3$	$Y_1$	$Y_2$	$Y_3$	$Y_4$
1	26	2.8	18	-1	-1	-1	1.83	29.31	29.5	50.36
2	40	2.8	18	1	-1	-1	1.73	39.32	19.4	48.16
3	26	8.2	18	-1	1	-1	1.85	25.16	25.7	50.72
4	40	8.2	18	1	1	-1	1.67	40.81	27.1	49.69
5	26	2.8	27	-1	-1	1	1.86	29.82	21.4	50.09
6	40	2.8	27	1	-1	1	1.77	32.20	24.0	50.61
7	26	8.2	27	-1	1	1	1.88	22.01	19.6	50.36
8	40	8.2	27	1	1	1	1.66	40.02	25.1	50.42
9	21.2	5.5	22.5	-1.68	0	0	1.81	33.00	24.2	29.31
10	44.8	5.5	22.5	1.68	0	0	1.37	51.59	30.6	50.67
11	33	1	22.5	0	-1.68	0	1.85	20.35	20.9	48.75
12	33	10	22.5	0	1.68	0	1.92	20.53	18.9	52.70
13	33	5.5	14.9	0	0	-1.68	1.88	23.85	23.0	50.19
14	33	5.5	30.1	0	0	1.68	1.90	20.16	21.2	50.86
15	33	5.5	22.5	0	0	0	1.89	21.72	18.5	50.84
16	33	5.5	22.5	0	0	0	1.88	21.21	18.6	50.93
17	33	5.5	22.5	0	0	0	1.87	21.55	16.8	50.98

The predicted functions of responses are obtained by using SUR estimators as below:

$$\hat{Y}_1 = 1.8846 - 0.0974x_1 - 0.1039x_1^2; \quad R^2 = 0.9211, \quad MSE = 0.0015$$

$$\hat{Y}_2 = 22.6488 + 5.6148x_1 - 0.3411x_2 + 7.8304x_1^2 + 2.6819x_1x_2; \quad R^2 = 0.9328, \quad MSE = 6.2258$$

$$\hat{Y}_3 = 18.9564 + 0.7444x_1 - 0.2075x_2 - 1.3311x_3 + 3.2227x_1^2 + 1.3925x_3^2 + 1.5874x_1x_2 + 1.8049x_1x_3; \quad R^2 = 0.8309, \quad MSE = 2.91$$

$$\hat{Y}_4 = 51.91 + 2.4364x_1 - 3.4287x_1^2; \quad R^2 = 0.5407, \quad MSE = 13.2703.$$

Here,  $R^2$ , called coefficient of determination, indicates that how well data points fit a statistical model. And also,  $MSE$  is the mean squared error of an estimator measures the average of the squares of the errors. The individual optimal input values  $\mathbf{x}^* = [x_1 \ x_2 \ x_3]$  and optimal values of predicted responses,  $\mathbf{f}^*$ , are given in Table 2. Here, these optimal values are considered as ideal solutions.

Table 2. Location of individual optimum for the predicted responses

Responses	Location of individual optimum $\mathbf{x}^* = [x_1 \ x_2 \ x_3]$	Individual optimum $\mathbf{f}^*$
$\hat{Y}_1$	[-0.4687 0 0]	1.9074
$\hat{Y}_2$	[-0.6466 1.682 0]	18.8016
$\hat{Y}_3$	[-0.8106 1.682 1.0034]	16.5555
$\hat{Y}_4$	[0.3553 0 0]	52.3428

After modeling of correlated multiple responses, it is aimed to achieve the MRO. The constrained MRO problem can be given below:

$$\begin{aligned}
 & \max \hat{Y}_1 \\
 & \min \hat{Y}_2 \\
 & \min \hat{Y}_3 \\
 & \max \hat{Y}_4 \\
 \text{subject to } & \hat{Y}_1 \geq 1.7 \\
 & \hat{Y}_2 \leq 21 \\
 & \hat{Y}_3 \leq 20 \\
 & \hat{Y}_4 \geq 45 \\
 & -1.682 \leq x_i \leq 1.682, \quad i = 1, 2, 3.
 \end{aligned} \tag{15}$$

By taking into account the correlation structure of the responses, the uncorrelated response/responses can be negligible in the objective space which helps to reduce the number of objectives. Thus, the MRO becomes easier with the dimension reduction of objective space. It can be easily said from Eq. (14) that the fourth response ( $Y_4$ ) is uncorrelated with all other three responses,  $\{Y_1, Y_2, Y_3\}$ . In this case,  $Y_4$  may be negligible in the objective space.

The constrained MRO problem given in Eq. (15) is transformed to following constrained MOO problem, by omitting uncorrelated response  $Y_4$ ,

$$\begin{aligned}
 & \max f_1(\mathbf{x}) = \hat{Y}_1 \\
 & \min f_2(\mathbf{x}) = \hat{Y}_2 \\
 & \min f_3(\mathbf{x}) = \hat{Y}_3 \\
 \text{subject to } & \hat{Y}_1 \geq 1.7 \\
 & \hat{Y}_2 \leq 21 \\
 & \hat{Y}_3 \leq 20 \\
 & \hat{Y}_4 \geq 45 \\
 & -1.682 \leq x_i \leq 1.682, \quad i = 1, 2, 3.
 \end{aligned} \tag{16}$$

The adapted NSGA-II is applied to problem given in Eq. (16) with the predefined parameter specification. The Pareto optimal set is obtained by using the adapted NSGA-II with the tunable parameter values given in Table 3. The obtained 50 non-dominated solutions are presented in Fig.1 with respect to  $f_1$ ,  $f_2$  and  $f_3$ .

Table3. The tunable parameters of the adapted NSGA-II

Algorithm parameters	
Number of input variables ( $\nu$ )	3
Population size ( $n_{pop}$ )	50
Selection operator	Tournament
Crossover operator	SBX
Mutation operator	Polynomial
Crossover probability ( $Pr_{cr}$ )	0.90
Mutation probability ( $Pr_{mut} = 1/\nu$ )	1/3
Crossover index ( $\eta_c$ )	20
Mutation index ( $\eta_m$ )	20
Number of generations ( $n_{gen}$ )	100
Penalty parameter ( $R$ )	10

In decision making stage, compromise solution should be selected among the many non-dominated solutions according to the DM's preference. Seven arbitrary decision scenarios, given in Table 4, are prepared for the problem. In this study, it is assumed that all the objective functions have equal importance. So, the decision scenario "G" is considered. The FCM clustering algorithm and TOPSIS method are used to select the compromise solution. In order to evaluate the closeness of the selected best compromise solution vector,  $\mathbf{f} = [f_1 \ f_2 \dots f_r]$ , to the individual optimal vector,  $\mathbf{f}^* = [f_1^* \ f_2^* \dots f_r^*]$ , given in Table 2, the following two criteria, root mean square error ( $RMSE$ ) and arithmetic mean absolute error ( $MAE$ ), are used.

$$(i) \quad RMSE = \left( (1/r) \sum_{i=1}^r (f_i - f_i^*)^2 \right)^{1/2},$$

$$(ii) \quad MAE = (1/r) \sum_{i=1}^r |f_i - f_i^*|.$$

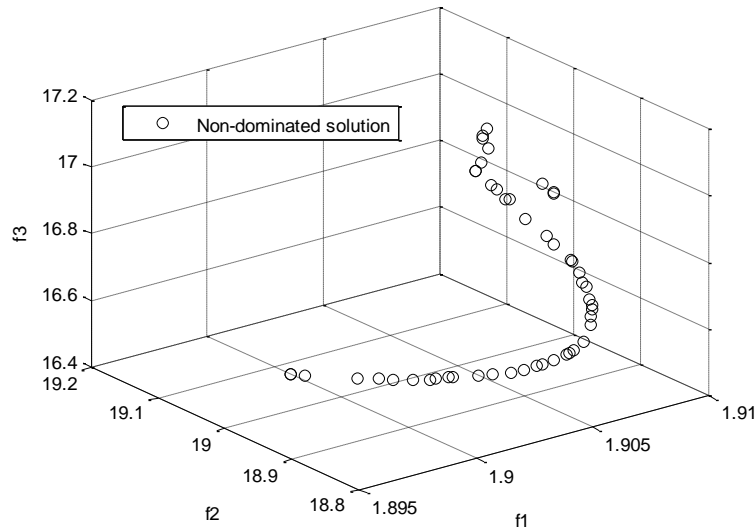


Figure 1. Pareto optimal set with 50 non-dominated solutions for the three responses with respect to  $f_1 = \hat{Y}_1$ ,  $f_2 = \hat{Y}_2$  and  $f_3 = \hat{Y}_3$

Before applying FCM to the Pareto optimal set, given in Fig.1, the solution set is normalized by using min-max normalization due to the differences of measuring units. The fuzziness index and error level are supposed to be  $m = 2$  and  $\varepsilon = 10^{-5}$ , respectively. The number of clusters,  $C$ , is chosen equal to 7. The common decision scenarios are presented in Table 4 where each scenario represents a cluster. In FCM, a non-dominated solution may belong to several clusters at the same time with different membership degrees. This is an important feature for multi-response decision making studies to get the most satisfying solution which belongs to related

cluster with the greatest membership degree. In this study, all the objective functions are assumed to have equal importance. Therefore, the decision scenario G (the 7th cluster) is considered. According to the obtained results, the 28th non-dominated solution of the Pareto optimal set is selected as a compromise solution with the highest membership degree,  $\mu_{7,28} = 0.9838$ . And also, the 4th non-dominated solution is chosen as a compromise solution by using TOPSIS method. The selected compromise solutions are presented in Fig.2 for both decision making methods.

Table 4. Decision scenarios for the problem

Cluster Number	Decision Scenarios	Objective function ranking
1	A	$(f_1 \succ f_2 \succ f_3)$
2	B	$(f_1 \succ f_3 \succ f_2)$
3	C	$(f_2 \succ f_1 \succ f_3)$
4	D	$(f_2 \succ f_3 \succ f_1)$
5	E	$(f_3 \succ f_1 \succ f_2)$
6	F	$(f_3 \succ f_2 \succ f_1)$
7	G	$(f_1 \approx f_2 \approx f_3)$

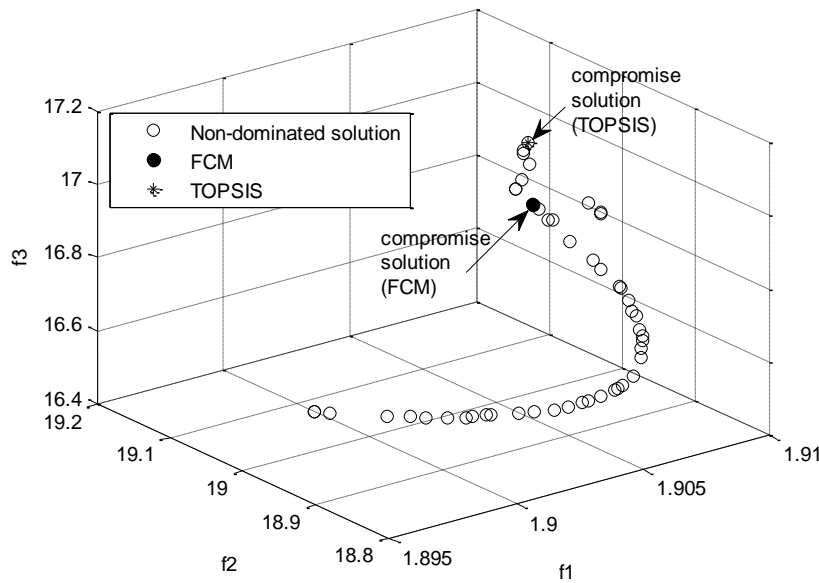


Figure 2. Compromise solutions obtained by FCM and TOPSIS in the Pareto optimal set

Table 5 gives the distance metric values between the selected compromise solutions and individual optimal vector. And also, the RMSE and MAE values for desirability function approach, given in the study of Shah et al. [3], are presented in Table 5. It should be noted here that the desirability function approach gives a single solution as an optimization result which is also considered as a compromise solution. It can be easily seen from the Table 5 that the compromise solution, obtained by FCM, gives the smallest distance values to

the individual optimal response vector,  $\mathbf{f}^*$ . And also, the compromise solution of FCM have more responses closer to their target, given in Table 2, than the compromise solutions of other decision methods. The compromise values of temperature ( $^{\circ}C$ ), washing time (min), washing ratio are obtained as -0.4761, 1.68, 0.681 corresponding to original input values 29.67, 10, and 25.56, respectively.

Table 5. Comparison of compromise solutions

Method	RMSE	MAE	$\mathbf{x}^* = [x_1 \ x_2 \ x_3]$	$\mathbf{f} = [f_1 \ f_2 \ f_3 \ f_4]$
FCM	<b>1.2008</b>	<b>0.7284</b>	[-0.4761 1.68 0.681]	[1.9074 19.0324 16.8683 49.9728]
TOPSIS	1.2083	0.7672	[-0.474 1.6783 0.4365]	[1.9074 19.0407 17.0274 49.9848]
Desirability	1.2484	1.0672	[-0.5 1 0.79]	[1.9073 20.1169 17.4932 49.8346]

### 5. CONCLUSIONS

In this study, correlation structure of responses is considered during the modeling and optimization processes of multiple responses. In modeling stage, the SUR estimators are used. Then, MRO is achieved by ignoring the uncorrelated response. Hence, computational complexity is reduced by dimension reduction of objective space. Besides, NSGA-II is adapted with penalty function approach to get Pareto solution set for constrained multi-response problem. Hence, many non-dominated solutions are obtained in a single run without aggregating of objective functions. In order to get a compromise solution among the many non-dominated solutions in Pareto set, FCM and TOPSIS methods are used for decision making stage. It is well-known that clustering algorithms group the non-

dominated solutions into smaller sets which show a certain degree of similarity. In this case, the non-dominated solutions are used as alternative solutions and DM gets a set of compromise solutions instead of a compromise solution. Then, it seems necessary to use additional decision making method for obtaining a compromise solution. However, in FCM, the DM provides an opportunity to select a compromise solution among the many alternative non-dominated solutions by using membership degrees in a single application. It is seen from the results that the FCM can be used as a decision making tool to decide a compromise solution than a classical decision making method, e.g. TOPSIS.



## Appendix A

The proposed algorithmic steps of the adapted NSGA-II are given in the following:

*Initial:* Initialize the tunable parameters

Define the number of input variables ( $\nu$ ), population size ( $n_{pop}$ ), number of generations ( $n_{gen}$ ), selection operator, crossover operator, mutation operator, crossover probability ( $Pr_{cr}$ ), mutation probability ( $Pr_{mut} = 1/\nu$ ), distribution index for crossover ( $\eta_c$ ), distribution index for mutation ( $\eta_m$ ), penalty parameter ( $R$ ).

*Step 1:* Generate a random initial population  $P$  and create an offspring population  $Q$  of size  $n_{pop}$  by using constraint handling rule given in Eq.(8). Set  $n_{gen} = 0$ .

*Step 2:* Combine the parent and offspring population,  $S = P \cup Q$ .

*Step 3:* Generate all non-dominated fronts,  $F = (F_1, F_2, \dots)$  of  $S$ .

*Step 4:* Sort the non-dominated fronts by using non-dominated sorting and crowding distance operator.

*Step 5:* Choose the best solution needed to fill the population.

*Step 6:* Create an offspring solution  $Q$  from  $P$  by applying the selection operator, crossover operator with probability  $Pr_{cr}$ , and mutation operator with probability  $Pr_{mut} = 1/\nu$  with  $\eta_c$  and  $\eta_m$  indexes. Set  $n_{gen} = n_{gen} + 1$ .

*Step 7:* If the  $n_{gen}$  is not reached, go to *Step 2*, else go to *Step 8*.

*Step 8:* End

## Appendix B

The steps of the FCM clustering algorithm are given below:

*Initial:* Initialize the tunable parameters

Number of clusters ( $c$ ), fuzziness index ( $m$ ), tolerance level for termination ( $\varepsilon$ ), generate randomly cluster centers ( $\mathbf{V} = [V_1, V_2, \dots, V_c]$ ).

*Step 1:* Compute membership degree of non-dominated solution to each cluster and compose the fuzzy partition matrix,  $\mathbf{U}$ .

*Step 2:* Compute cluster centers by using Eq.(11).

*Step 3:* If  $\|\mathbf{V}^{(t)} - \mathbf{V}^{(t-1)}\| < \varepsilon$ ,  $t = 1, 2, \dots$  then stop the algorithm, otherwise go to Step 1.  
(Here,  $t$  is defined as iteration counter)

## CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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