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Equivalence of Codes over Finite Chain Ring

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Abstract

Equivalence of Codes are constructed classification on finite rings according to odd codes, even codes and recurrent codes. It is determined that the codes are odd codes or even codes and the relationship with the Hadamard codes was studied. In this study, some matrices is written with the elements on the finite rings. Codes are generated with these matrices. The relationship between the generated codes and the perfect codes is revealed.

Keywords: Odd codes, Even codes, Gray map, Hadamard codes, Special matrices.

Sonlu Zincir Halkası üzerinde Kodların Denkliği

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Özet

Tek kodlar, çift kodlar ve tekrarlı oluşum kodlarına göre sonlu halkalarda kodların denkliğinin sınıflandırılması kurulmuştur. Kodların tek kod ya da çift kod olduğu ve çalışılan Hadamard kodları ile ilişkisi tespit edilmiştir. Bu çalışmada, bileşenleri sonlu halkalarda matrisler yazılmıştır. Kodlar bu matrisler ile üretilmiştir. Üretilen kodlar ve mükemmel kodlar arasında ilişki ortaya konulmuştur.

Anahtar Kelimeler : Tek kodlar, Çift kodlar, Gray dönüşümü, Hadamard kodlar, Özel matrisler.

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1. Introduction

Binary operations (+ and .) on the ring $R_2 = IF_2 + uIF_2 + u^2 IF_2 = \{0,1,u,u^2,1+u,1+u^2,u+u^2,1+u+u^2\}$ $(|IF_2 + uIF_2 + u^2 IF_2| = 8)$ are defined as below:

+	0	1	u	u^2	$1+u$	$1+u^2$	$u+u^2$	$1+u+u^2$
0	0	1	u	u^2	$1+u$	$1+u^2$	$u+u^2$	$1+u+u^2$
1	1	0	$1+u$	$1+u^2$	u	u^2	$1+u+u^2$	$u+u^2$
u	u	$1+u$	0	$u+u^2$	1	$1+u+u^2$	u^2	$1+u^2$
u^2	u^2	$1+u^2$	$u+u^2$	0	$1+u+u^2$	1	u	$1+u$
$1+u$	$1+u$	u	1	$1+u+u^2$	0	$u+u^2$	$1+u^2$	u^2
$1+u^2$	$1+u^2$	u^2	$1+u+u^2$	1	$u+u^2$	0	$1+u$	u
$u+u^2$	$u+u^2$	$1+u+u^2$	u^2	u	$1+u^2$	$1+u$	0	1
$1+u+u^2$	$1+u+u^2$	$u+u^2$	$1+u^2$	$1+u$	u^2	u	1	0

.	0	1	u	u^2	$1+u$	$1+u^2$	$u+u^2$	$1+u+u^2$
0	0	0	0	0	0	0	0	0
1	0	1	u	u^2	$1+u$	$1+u^2$	$u+u^2$	$1+u+u^2$
u	0	u		0	$u+u^2$	u	u^2	$u+u^2$
u^2	0	u^2	0	0	u^2	u^2	0	u^2
$1+u$	0	$1+u$	$u+u^2$	u^2	$1+u^2$	$1+u+u^2$	u	1
$1+u^2$	0	$1+u^2$	u	u^2	$1+u+u^2$	1	$u+u^2$	$1+u$
$u+u^2$	0	$u+u^2$	u^2	0	u	$u+u^2$	u^2	u
$1+u+u^2$	0	$1+u+u^2$	$u+u^2$	u^2	1	$1+u$	u	$1+u^2$

The Lee weight $w_L(r)$ of $r \in R_2$ is given by

$$w_L(r) = \begin{cases} 0 & ; r = 0 \\ 4 & ; r = u^2 \\ 2 & ; \ otherwise \end{cases} \qquad (1)$$

This extends to Lee weight function in R_2^n such that $w_L(r) = \sum_{i=0}^{n-1} w_L(r_i)$ for $r = (r_o, r_1, ..., r_{n-1}) \in R_2^n$.

The Lee distance $d_L(x, y)$ between any distinct vectors $x, y \in R_2^n$ is defined to be $w_L(x - y)$. The d_L minimum Lee distance of C is defined as $d_L(C) = \min\{d_L(x, y)\}$ for any The Hamming weight of C is defined as $w_H(c) = \sum_{i=0}^{n-1} w_H\ (c_i)$
where $w_H(c_i) = 0$ if $c_i = 0$ and $w_H(c_i) = 1$ if $c_i = 1$. The minimum Hamming distance of C is called as $d_H(C) = \min\{d_H(c, c')\}$ for any $c, c' \in C, c \neq c'$.

Generally the Gray map is defined as :

$$\Phi : R_2^n \longrightarrow IF_2^{4n}$$

$$\begin{aligned} (r_1, r_2, ..., r_n) \text{ a } \ \Phi(r_1, r_2, ..., r_n) = (c_1, c_2, ..., c_n, a_1 + c_1, \\ a_2 + c_2, ..., a_n + c_n, b_1 + c_1, b_2 + c_2, ..., b_n + c_n, \\ a_1 + b_1 + c_1, a_2 + b_2 + c_2, ..., a_n + b_n + c_n) \end{aligned} \qquad (2)$$

where $r_i = a_i + b_i.u + c_i.u^2 \in R_2$ for $1 \leq i \leq n$.

2. Materials and Methods

Certain examples for the matrix M^{α_1,α_2} constructed above are given below :

$$M^{0,0} = [1]_{1x1} \ , \ M^{0,1} = \begin{bmatrix} 1 & 1 \\ 0 & u^2 \end{bmatrix}_{2x2} \ , \ M^{0,2} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & u^2 & u^2 \\ 0 & u^2 & 0 & u^2 \end{bmatrix}_{3x4} \ ,$$

$$M^{1,0} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & u & u^2 & 1+u & 1+u^2 & u+u^2 & 1+u+u^2 \end{bmatrix}_{2x8}$$

. This matrix constructed is a special matrix which has $\alpha_1 + \alpha_2 + 1$ rows.

Define the code $C^{\alpha_1,\alpha_2} = \{ (c_1, c_2).M^{\alpha_1,\alpha_2} \mid c_1 \in R_2^{\alpha_1+1}, c_2 \in IF_2^{\alpha_2} \}$ which has a generator matrix M^{α_1,α_2} where α_1, α_2 are inregers such that $\alpha_1, \alpha_2 \geq 0$.

Definition 2.1 : Let $C^{\alpha_1,\alpha_2} \subseteq R_2^{\,n}$ be a code. $even(C^{\alpha_1,\alpha_2}) = \{ (c_1, c_3, ..., c_{n-1}) \in R_2^{\frac{n}{2}} \mid (c_1, c_3, ..., c_{n-1}) \in C^{\alpha_1,\alpha_2} \}$ is called an even code over $R_2^{\frac{n}{2}}$. $odd(C^{\alpha_1,\alpha_2}) = \{ (c_2, c_4, ..., c_n) \in R_2^{\frac{n}{2}} \mid (c_2, c_4, ..., c_n) \in C^{\alpha_1,\alpha_2} \}$ is called an odd code over $R_2^{\frac{n}{2}}$. Even codes and odd codes are defined over the field F_2 writing F_2 instead of R_2.

3. Results

Preposition 3.1 : Let C^{α_1,α_2} be a code.

i) $even(C^{\alpha_1,\alpha_2}) = odd(C^{\alpha_1,\alpha_2}) = C^{\alpha_1-1,2}$ is satisfied if $\alpha_1 \geq 1,\ \alpha_2 = 0$.

ii) $even(C^{\alpha_1,\alpha_2}) \approx odd(C^{\alpha_1,\alpha_2}) = C^{\alpha_1,\alpha_2-1}$ is satisfied if $\alpha_1 \geq 0,\ \alpha_2 \geq 1$.

Preposition 3.2 : $even(\Phi(C^{\alpha_1,\alpha_2})) = \Phi(even(C^{\alpha_1,\alpha_2}))$ is satisfied.

Proof : Let $r = (r_1, r_2, ..., r_n) \in C^{\alpha_1,\alpha_2}$ where $r_i = a_i + b_i.u + c_i.u^2 \in R_2$ for $1 \leq i \leq n$.

If $\Phi(r) = \Phi(r_1, r_2, ..., r_n) = \Phi(a_1 + b_1.u + c_1.u^2, a_2 + b_2.u + c_2.u^2, ..., a_n + b_n.u + c_n.u^2)$

$$= (c_1, c_2, ..., c_n, a_1 + c_1, a_2 + c_2, ..., a_n + c_n, b_1 + c_1, b_2 + c_2, ..., b_n + c_n, a_1 + b_1 + c_1, a_2 + b_2 + c_2, ..., a_n + b_n + c_n) \in \Phi(C^{\alpha_1,\alpha_2})$$

then
$$(c_1, c_3, ..., c_{n-1}, a_1 + c_1, a_3 + c_3, ..., a_{n-1} + c_{n-1}, b_1 + c_1, b_3 + c_3, ..., b_{n-1} + c_{n-1}, a_1 + b_1 + c_1, a_3 + b_3 + c_3, ..., a_{n-1} + b_{n-1} + c_{n-1}) \in even(\Phi(C^{\alpha_1,\alpha_2})).$$

On the other hand,

$r' = (r_1, r_3, ..., r_{n-1}) = (a_1 + b_1.u + c_1.u^2, a_3 + b_3.u + c_3.u^2, ..., a_{n-1} + b_{n-1}.u + c_{n-1}.u^2) \in even(C^{\alpha_1,\alpha_2})$. Then

$$\Phi(r') = (c_1, c_3, ..., c_{n-1}, a_1 + c_1, a_3 + c_3, ..., a_{n-1} + c_{n-1}, b_1 + c_1, b_3 + c_3, ..., b_{n-1} + c_{n-1}, a_1 + b_1 + c_1, a_3 + b_3 + c_3, ..., a_{n-1} + b_{n-1} + c_{n-1}) \in \Phi(even(C^{\alpha_1,\alpha_2})).$$

This Preposition can be written for the odd codes.

Example 3.3 : Write the matrix $M^{0,1}$ to define the code $C^{0,1}$.

$M^{0,1}=\begin{bmatrix}1 & 1\\ 0 & u^2\end{bmatrix}_{2x2}$. Then the elements of the code $C^{0,1}$ are of the form $c=(c_1,c_2).M^{0,1}$,where $c_1\in R_2$, $c_2\in IF_2$.

$$C^{0,1}=\{\ 0\ 0,0\ u^2\ ,11,11+u^2\ ,u\,u\ ,u\,u+u^2\ ,u^2\ u^2\ ,\ u^2\ 0,1+u1+u\ ,1+u1+u+u^2\ ,1+u^2\,1+u^2,\ 1+u^2\,1,u+u^2\ u+u^2\ ,u+u^2\ u\ ,1+u+u^2\,1+u+u^2\ ,\ 1+u+u^2\,1+u\}\subseteq R_2^2$$

It is seen that $d_L(C^{0,1})=4$ and $\left|C^{0,1}\right|=16$ and then this is a $(2,16,4)\_$ code.Therefore

$$\Phi(C^{0,1})=\{00000000,01010101,00110011,\ 01100110,11111111,10101010,\ 11001100,10011001,00001111,\ 01011010,00111100,01101001,\ 11110000,10100101,11000011,\ 10010110\}\subseteq IF_2^8$$

is a $(8,16,4)\_$ Hadamard code. Let $A_4=\begin{bmatrix}1 & 1 & 1 & 1\\ -1 & 1 & -1 & 1\\ -1 & -1 & 1 & 1\\ -1 & -1 & -1 & 1\end{bmatrix}_{4x4}$ be a normalized Hadamard matrix.

Writing 0 instead of 1 and 1 instead of -1 , the vectors 0000 ,1010,1100 and 0110 are obtained. Adding the complements of these vectors to these vectors the new vectors 0000 ,1010,1100,0110 ,1111,0101,0011 and 1001 are obtained. Then using the method given above, the new codewords

$00000000,01010101,00110011,01100110,11111111,10101010,11001100,10011001,$
$00001111,01011010,00111100,01101001,11110000,10100101,11000011,10010110$

are obtained. The code formed by these codewords is $\Phi(C^{0,2})$ which is a $(8,16,4)\_$ Hadamard code. Moreover $(C^{0,1})^{\perp}=\left\{00,u\,u,u^2\,u^2\ ,u+u^2\,u+u^2\right\}\subseteq IF_2^2$ and

$$\Phi((C^{0,1})^{\perp})=\{00000000\ ,\ 00001111\ ,\ 11111111\ ,\ 11110000\}\subseteq IF_2^8 .$$

$C^{0,1}$ is a cyclic code such that the equation $\tau(C^{0,1}) = C^{0,1}$ is provided. Similarly $\Phi(C^{0,1})$ is quasi-cyclic code of index 4 such that the equation $\sigma^{\otimes 4}(\Phi(C^{0,1})) = \Phi(C^{0,1})$ is satisfied. For the code $C^{0,1}$; $even(C^{0,1}) = \{0,1,u,u^2,1+u,1+u^2,u+u^2,1+u+u^2\} = C^{0,0} = R_2$ is obtained.

$\Phi(even(C^{0,1})) = \left\{ \begin{matrix} 0000, 0011, 1111, 1100, \\ 0101, 0110, 1010, 1001 \end{matrix} \right\}$. Hence $even(\Phi(C^{0,2})) = \Phi(even(C^{0,2}))$ is obtained.

4. Conclusions

In this paper the codes over the a finite chain $IF_2 + uIF_2 + u^2 IF_2$ where $u^3 = 0$ defining by matrices given in section 2 are Classified. The codes over the ring defining by matrices given lexicographically feature are described. These codes over the ring are obtained images of codes in the Galois field. Moreover even codes, odd codes and recurrent formsa are obtained.

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