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ON $I-$DEFERRED STATISTICAL CONVERGENCE IN TOPOLOGICAL GROUPS

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Abstract. In this paper, the concepts of $I-$deferred statistical convergence of order α and $I-$deferred statistical convergence of order (α, β) in topological groups were defined. Also some inclusion relations between $I-$statistical convergence of order α, $I-$deferred statistical convergence of order α, $I-$statistical convergence of order (α, β) and $I-$deferred statistical convergence of order (α, β) in topological groups are given.

1. INTRODUCTION

The idea of statistical convergence was given by Zygmund [38] in the first edition of his monograph puplished in Warsaw in 1935. The consept of statistical convergence was introduced by Steinhaus [30] and Fast [13] and later reintroduced by Schoenberg [28] independently. Later on it was further investigated from the sequence space point of view and linked with summability theory by Çakallı ([2],[3],[4],[5],[6]), Çınar et al. [7], Et et al. ([9],[10],[11],[12],[24]), Fridy [14], Fridy and Orhan [15], Işık and Akbaş [17], Salat [22], Savaş [23], Sengul et. al. ([31],[32],[33],[34]), Srivastava and Et [29], Yıldız [37] and many others.

Let X be a non-empty set. Then a family of sets $I \subseteq 2^X$ (power sets of X) is said to be an *ideal* if I is additive *i.e.* A, $B \in I$ implies $A \cup B \in I$ and hereditary, *i.e.* $A \in I$, $B \subset A$ implies $B \in I$.

A non-empty family of sets $F \subseteq 2^X$ is said to be a *filter* of X if and only if (i) $\phi \notin F$, (ii) A, $B \in F$ implies $A \cap B \in F$ and (iii) $A \in F$, $A \subset B$ implies $B \in F$.

An ideal $I \subseteq 2^X$ is called *non-trivial* if $I \neq 2^X$.

A non-trivial ideal I is said to be *admissible* if $I \supset \{\{x\} : x \in X\}$.

If I is a non-trivial ideal in $X (X \neq \phi)$ then the family of sets $F(I) = \{M \subset X : (\exists A \in I)(M = X \setminus A)\}$ is a filter of X, called the *filter associated with I*.

Throughout the paper I will stand for a non-trivial admissible ideal of $\mathbb{N}$.

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The idea of $I-$convergence of real sequences was introduced by Kostyrko *et al.* [19] and also independently by Nuray and Ruckle [21] (who called it generalized statistical convergence) as a generalization of statistical convergence. Later on $I-$convergence was studied in ([20],[26],[27],[25],[35],[36]).

The order of statistical convergence of a sequence of numbers was given by Gadjiev and Orhan in [16] and after then statistical convergence of order α and strong $p-$Cesàro summability of order α studied by Çolak [8].

In 1932, R.P. Agnew [1] defined the deferred Cesaro mean $D_{p,q}$ of the sequence $x=(x_k)$ by

$$(D_{p,q}x)_n=\frac{1}{q(n)-p(n)}\sum_{p(n)+1}^{q(n)}x_k$$

where $(p(n))$ and $(q(n))$ are sequences of non-negative integers satisfying

$$p(n)<q(n) \text{ and } \lim_{n\to\infty}q(n)=+\infty. \tag{1.1}$$

Let K be a subset of $\mathbb{N}$, and denote the set $\{k:p(n)<k\leq q(n),k\in K\}$ by $K_{p,q}(n)$. Deferred density of K is defined by

$$\delta_{p,q}(K)=\lim_{n\to\infty}\frac{1}{q(n)-p(n)}|K_{p,q}(n)| \tag{1.2}$$

whenever the limit exists (finite or infinite). The vertical bars in (1.2) indicate the cardinality of the set $K_{p,q}(n)$.

A real valued sequence $x=(x_k)$ is said to be deferred statistical convergent to l, if

$$\lim_{n\to\infty}\frac{1}{q(n)-p(n)}|\{p(n)<k\leq q(n):|x_k-l|\geq\varepsilon\}|=0$$

for every $\varepsilon>0$. If $q(n)=n$, $p(n)=0$ then deferred statistical convergence coincides statistical convergence [18].

2. $I-$DEFERRED STATISTICAL CONVERGENCE OF ORDER α IN TOPOLOGICAL GROUPS

In this section, some inclusion relations between $I-$statistical convergence, $I-$statistical convergence of order α and $I-$deferred statistical convergence of order α in topological groups are given.

Definition 2.1. *Let $(p(n))$ and $(q(n))$ be two sequences of non-negative integers satisfying the conditions (1.1), X be an abelian topological Hausdorf group, $(x(k))$ be a sequence of real numbers and α be a positive real number such that $0<\alpha\leq 1$. The sequence $x=(x(k))$ is said to be $DS_{p,q}^{\alpha}(X,I)-$statistically convergent in topological groups to l (or $I-$deferred statistically convergent sequences of order α in topological groups to l) if there is a real number l for each neighbourhood U of 0 such that*

$$\left\{n\in\mathbb{N}:\frac{1}{(q(n)-p(n))^{\alpha}}|\{p(n)<k\leq q(n):x(k)-l\notin U\}|\geq\delta\right\}\in I.$$

In this case we write $DS_{p,q}^{\alpha}(I)-\lim x(k)=l$ or $x(k)\to l\left(DS_{p,q}^{\alpha}(I)\right)$. The set of all $DS_{p,q}^{\alpha}(X,I)-$ statistically convergent sequences in topological groups will be denoted by $DS_{p,q}^{\alpha}(X,I)$. If $\alpha=1$, then $I-$deferred statistical convergence *of order α coincides then $I-$deferred statistical convergence in topological groups ($DS_{p,q}(X,I)-$ convergence) and if $q(n)=n$, $p(n)=0$ then $I-$deferred statistical convergence of order α coincides $I-$statistical convergence of order α in topological groups ($S^{\alpha}(X,I)-$convergence). If $q(n)=n$, $p(n)=0$ and $\alpha=1$, then $I-$deferred statistical convergence of order α coincides $I-$statistical convergence in topological groups ($S(X,I)-$convergence).*

Theorem 2.1. *Let $(p(n))$ and $(q(n))$ be two sequences of non-negative integers satisfying the conditions (1.1) and α,β be positive real numbers such that $0<\alpha\leq\beta\leq 1$ then $DS^{\alpha}_{p,q}(X,I)\subseteq DS^{\beta}_{p,q}(X,I)$ and the inclusion is strict.*

Proof. Omitted. □

Theorem 2.1 yields the following corollary.

Corollary 2.2. *If a sequence is $DS^{\alpha}_{p,q}(X,I)-$statistically convergent of order α to l, then it is $DS_{p,q}(X,I)-$statistically convergent to l.*

Theorem 2.3. *Let $(p(n))$ and $(q(n))$ be two sequences of non-negative integers satisfying the conditions (1.1) and α be a positive real number such that $0<\alpha\leq 1$. If $\liminf_n \frac{q(n)}{p(n)}>1$, then $S^{\alpha}(X,I)\subset DS^{\alpha}_{p,q}(X,I)$.*

Proof. Suppose that $\liminf_n \frac{q(n)}{p(n)}>1$; then there exists an $a>0$ such that $\frac{q(n)}{p(n)}\geq 1+a$ for sufficiently large n, which implies that

$$\frac{q(n)-p(n)}{q(n)}\geq\frac{a}{1+a}\Longrightarrow\left(\frac{q(n)-p(n)}{q(n)}\right)^{\alpha}\geq\left(\frac{a}{1+a}\right)^{\alpha}\Longrightarrow\frac{1}{q(n)^{\alpha}}\geq\frac{a^{\alpha}}{(1+a)^{\alpha}}\frac{1}{(q(n)-p(n))^{\alpha}}.$$

If $S^{\alpha}(I)-\lim_{k\to\infty}x(k)=l$, then for each neighbourhood U of 0 and for sufficiently large n, we have

$$\begin{aligned}\frac{1}{q(n)^{\alpha}}\left|\{k\leq q(n):x(k)-l\notin U\}\right| &\geq \frac{1}{q(n)^{\alpha}}\left|\{p(n)<k\leq q(n):x(k)-l\notin U\}\right|\\ &\geq \frac{a^{\alpha}}{(1+a)^{\alpha}}\frac{1}{(q(n)-p(n))^{\alpha}}\left|\{p(n)<k\leq q(n):x(k)-l\notin U\}\right|.\end{aligned}$$

Therefore, we can write

$$\begin{aligned}&\left\{n\in\mathbb{N}:\frac{1}{(q(n)-p(n))^{\alpha}}\left|\{p(n)<k\leq q(n):x(k)-l\notin U\}\right|\geq\delta\right\}\\ \subseteq &\left\{n\in\mathbb{N}:\frac{1}{q(n)^{\alpha}}\left|\{k\leq q(n):x(k)-l\notin U\}\right|\geq\delta\frac{a^{\alpha}}{(1+a)^{\alpha}}\right\}\in I.\end{aligned}$$

This implies that $S^{\alpha}(X,I)\subset DS^{\alpha}_{p,q}(X,I)$. □

Theorem 2.4. *Let $(p(n))$ and $(q(n))$ be two sequences of non-negative integers satisfying the conditions (1.1) and α be a positive real number such that $0<\alpha\leq 1$. If $\liminf_n \frac{(q(n)-p(n))^{\alpha}}{n}>0$ and $q(n)<n$, then $S(X,I)\subset DS^{\alpha}_{p,q}(X,I)$.*

Proof. For each neighbourhood U of 0, we have

$$\{k\leq n:x(k)-l\notin U\}\supset\{p(n)<k\leq q(n):x(k)-l\notin U\}.$$

Therefore,

$$\begin{aligned}\frac{1}{n}\left|\left\{k\leq n:x\left(k\right)-l\notin U\right\}\right| &\geq \frac{1}{n}\left|\left\{p\left(n\right)<k\leq q\left(n\right):x\left(k\right)-l\notin U\right\}\right| \\ &= \frac{\left(q\left(n\right)-p\left(n\right)\right)^{\alpha}}{n}\frac{1}{\left(q\left(n\right)-p\left(n\right)\right)^{\alpha}}\left|\left\{p\left(n\right)<k\leq q\left(n\right):x\left(k\right)-l\notin U\right\}\right|.\end{aligned}$$

Hence, we can write

$$\begin{aligned}&\left\{n\in\mathbb{N}:\frac{1}{\left(q\left(n\right)-p\left(n\right)\right)^{\alpha}}\left|\left\{p\left(n\right)<k\leq q\left(n\right):x\left(k\right)-l\notin U\right\}\right|\geq\delta\right\} \\ \subseteq\ &\left\{n\in\mathbb{N}:\frac{1}{n}\left|\left\{k\leq n:x\left(k\right)-l\notin U\right\}\right|\geq\delta\frac{\left(q\left(n\right)-p\left(n\right)\right)^{\alpha}}{n}\right\}\in I.\end{aligned}$$

Consequently, $S\left(X,I\right)\subset DS_{p,q}^{\alpha}\left(X,I\right).$ □

Theorem 2.5. *Let $\left(p\left(n\right)\right),\left(q\left(n\right)\right),\left(p'\left(n\right)\right),\left(q'\left(n\right)\right)$ be four sequences of non-negative integers such that $p\left(n\right)<q\left(n\right)$, $p'\left(n\right)<q'\left(n\right)$ and $q\left(n\right)-p\left(n\right)\leq q'\left(n\right)-p'\left(n\right)$ for all $n\in\mathbb{N}$, let U be any neighbourhood of 0 and let α and β be such that $0<\alpha\leq\beta\leq1$.*

(i) If

$$\lim_{n\to\infty}\inf\frac{\left(q\left(n\right)-p\left(n\right)\right)^{\alpha}}{\left(q'\left(n\right)-p'\left(n\right)\right)^{\beta}}>0 \tag{2.1}$$

then $DS_{p',q'}^{\beta}(X,I)\subseteq DS_{p,q}^{\alpha}(X,I)$,

(ii) If

$$\lim_{n\to\infty}\frac{q'\left(n\right)-p'\left(n\right)}{\left(q\left(n\right)-p\left(n\right)\right)^{\beta}}=1 \tag{2.2}$$

then $DS_{p,q}^{\alpha}(X,I)\subseteq DS_{p',q'}^{\beta}(X,I)$.

Proof. (i) Let (2.1) be satisfied. For given $\varepsilon>0$ and each neighbourhood U,W of 0 such that $W\subset U$, we have

$$\left\{p'\left(n\right)<k\leq q'\left(n\right):x\left(k\right)-l\notin W\right\}\supseteq\left\{p\left(n\right)<k\leq q\left(n\right):x\left(k\right)-l\notin U\right\},$$

and so

$$\begin{aligned}&\frac{1}{\left(q'\left(n\right)-p'\left(n\right)\right)^{\beta}}\left|\left\{p'\left(n\right)<k\leq q'\left(n\right):x\left(k\right)-l\notin W\right\}\right| \\ \geq\ &\frac{\left(q\left(n\right)-p\left(n\right)\right)^{\alpha}}{\left(q'\left(n\right)-p'\left(n\right)\right)^{\beta}}\frac{1}{\left(q\left(n\right)-p\left(n\right)\right)^{\alpha}}\left|\left\{p\left(n\right)<k\leq q\left(n\right):x\left(k\right)-l\notin U\right\}\right|\end{aligned}$$

for all $n\in\mathbb{N}$, where $p\left(n\right)<q\left(n\right)$, $p'\left(n\right)<q'\left(n\right)$ and $q\left(n\right)-p\left(n\right)\leq q'\left(n\right)-p'\left(n\right).$

Then we can write

$$\begin{aligned}&\left\{n\in\mathbb{N}:\frac{1}{\left(q\left(n\right)-p\left(n\right)\right)^{\alpha}}\left|\left\{p\left(n\right)<k\leq q\left(n\right):x\left(k\right)-l\notin U\right\}\right|\geq\delta\right\} \\ \subseteq\ &\left\{n\in\mathbb{N}:\frac{1}{\left(q'\left(n\right)-p'\left(n\right)\right)^{\beta}}\left|\left\{p'\left(n\right)<k\leq q'\left(n\right):x\left(k\right)-l\notin W\right\}\right|\geq\delta\frac{\left(q\left(n\right)-p\left(n\right)\right)^{\alpha}}{\left(q'\left(n\right)-p'\left(n\right)\right)^{\beta}}\right\}\in I.\end{aligned}$$

This completes the proof.

(ii) Omitted. □

Corollary 2.6. *Let $(p(n)), (q(n)), (p'(n)), (q'(n))$ be four sequences of non-negative integers such that $p(n) < q(n)$, $p'(n) < q'(n)$ and $q(n) - p(n) \leq q'(n) - p'(n)$ for all $n \in \mathbb{N}$ and $0 < \alpha \leq 1$.*

If (2.1) *holds then,*

(i) $DS^{\alpha}_{p',q'}(X,I) \subseteq DS^{\alpha}_{p,q}(X,I)$,

(ii) $DS_{p',q'}(X,I) \subseteq DS^{\alpha}_{p,q}(X,I)$,

(iii) $DS_{p',q'}(X,I) \subseteq DS_{p,q}(X,I)$.

If (2.2) *holds then,*

(i) $DS^{\alpha}_{p,q}(X,I) \subseteq DS^{\alpha}_{p',q'}(X,I)$,

(ii) $DS^{\alpha}_{p,q}(X,I) \subseteq DS_{p',q'}(X,I)$,

(iii) $DS_{p,q}(X,I) \subseteq DS_{p',q'}(X,I)$.

3. $I-$DEFERRED STATISTICAL CONVERGENCE OF ORDER(α, β) IN TOPOLOGICAL GROUPS

In this section, the results which were given in the previous section are generalized. Some inclusion relations between $I-$statistical convergence of order (α, β) and $I-$deferred statistical convergence of order (α, β) in topological groups are given.

Definition 3.1. *Let $(p(n))$ and $(q(n))$ be two sequences of non-negative integers satisfying the conditions* (1.1)*, X be an abelian topological Hausdorff group, $(x(k))$ be a sequence of real numbers and α, β be positive real numbers such that $0 < \alpha \leq \beta \leq 1$. The sequence $x = (x(k))$ is said to be $I-$deferred statistical convergent of order (α, β) in topological groups to l (or $DS^{\alpha,\beta}_{p,q}(X,I)-$statistically convergent to l), if there is a real number l, for each neighbourhood U of* 0 *such that*

$$\left\{ n \in \mathbb{N} : \frac{1}{(q(n) - p(n))^{\alpha}} \left| \{ p(n) < k \leq q(n) : x(k) - l \notin U \} \right|^{\beta} \geq \delta \right\} \in I.$$

In this case we write $DS^{\alpha,\beta}_{p,q}(I) - \lim x(k) = l$ or $x(k) \to l \left(DS^{\alpha,\beta}_{p,q}(I) \right)$. The set of all $DS^{\alpha,\beta}_{p,q}(X,I)-$statistically convergent sequences in topological groups will be denoted by $DS^{\alpha,\beta}_{p,q}(X,I)$. If $q(n) = n$, $p(n) = 0$ and $\alpha = \beta = 1$, then $I-$deferred statistical convergence of order (α, β) coincides $I-$statistical convergence in topological groups ($S(X,I)-$convergence).

Theorem 3.1. *Let $(p(n))$ and $(q(n))$ be two sequences of non-negative integers satisfying the conditions* (1.1) *and $\alpha_1, \alpha_2, \beta_1$ and β_2 be positive real numbers such that $0 < \alpha_1 \leq \alpha_2 \leq \beta_1 \leq \beta_2 \leq 1$, then $DS^{\alpha_1,\beta_2}_{p,q}(X,I) \subseteq DS^{\alpha_2,\beta_1}_{p,q}(X,I)$ and the inclusion is strict.*

Proof. Omitted. □

Theorem 3.2. *Let $(p(n))$ and $(q(n))$ be two sequences of non-negative integers satisfying the conditions* (1.1) *and α, β be two positive real numbers such that $0 < \alpha \leq \beta \leq 1$. If $\liminf_n \frac{q(n)}{p(n)} > 1$, then $S^{\alpha,\beta}(X,I) \subset DS^{\alpha,\beta}_{p,q}(X,I)$.*

Proof. The proof is similar to that of Theorem 2.3. □

Theorem 3.3. *Let $(p(n)), (q(n)), (p'(n))$ and $(q'(n))$ be four sequences of non-negative integers such that $p(n) < q(n), p'(n) < q'(n)$ and $q(n) - p(n) \leq q'(n) - p'(n)$ for all $n \in \mathbb{N}$, let U be any neighbourhood of 0 and let $\alpha_1, \alpha_2, \beta_1$ and β_2 be such that $0 < \alpha_1 \leq \alpha_2 \leq \beta_1 \leq \beta_2 \leq 1$.*

(i) If

$$\lim_{n\to\infty} \inf \frac{(q(n) - p(n))^{\alpha_1}}{(q'(n) - p'(n))^{\alpha_2}} > 0 \tag{3.1}$$

then $DS^{\alpha_2,\beta_2}_{p',q'}(X, I) \subseteq DS^{\alpha_1,\beta_1}_{p,q}(X, I)$,

(ii) If

$$\lim_{n\to\infty} \frac{q'(n) - p'(n)}{(q(n) - p(n))^{\alpha_2}} = 1 \tag{3.2}$$

then $DS^{\alpha_1,\beta_2}_{p,q}(X, I) \subseteq DS^{\alpha_2,\beta_1}_{p',q'}(X, I)$.

Proof. (i) Let $\lim_{n\to\infty} \inf \frac{(q(n)-p(n))^{\alpha_1}}{(q'(n)-p'(n))^{\alpha_2}} > 0$. For given $\varepsilon > 0$ and each neighbourhood U, W of 0 such that $W \subset U$, we have

$$\begin{aligned} &\frac{1}{(q'(n) - p'(n))^{\alpha_2}} \left|\{p'(n) < k \leq q'(n) : x(k) - l \notin W\}\right|^{\beta_2} \\ \geq\ &\frac{(q(n) - p(n))^{\alpha_1}}{(q'(n) - p'(n))^{\alpha_2}} \frac{1}{(q(n) - p(n))^{\alpha_1}} \left|\{p(n) < k \leq q(n) : x(k) - l \notin U\}\right|^{\beta_1} \end{aligned}$$

for all $n \in \mathbb{N}$.

Therefore, we can write

$$\begin{aligned} &\left\{n \in \mathbb{N} : \frac{1}{(q(n) - p(n))^{\alpha_1}} \left|\{p(n) < k \leq q(n) : x(k) - l \notin U\}\right|^{\beta_1} \geq \delta\right\} \\ \subseteq\ &\left\{n \in \mathbb{N} : \frac{1}{(q'(n) - p'(n))^{\alpha_2}} \left|\{p'(n) < k \leq q'(n) : x(k) - l \notin W\}\right|^{\beta_2} \geq \delta \frac{(q(n) - p(n))^{\alpha_1}}{(q'(n) - p'(n))^{\alpha_2}}\right\} \in I. \end{aligned}$$

This completes the proof.

(ii) Omitted. □

Corollary 3.4. *Let $(p(n)), (q(n)), (p'(n))$ and $(q'(n))$ be four sequences of non-negative integers such that $p(n) < q(n)$, $p'(n) < q'(n)$ and $q(n) - p(n) \leq q'(n) - p'(n)$ for all $n \in \mathbb{N}$ and $0 < \alpha_1 \leq \alpha_2 \leq \beta_1 \leq \beta_2 \leq 1$.*

If (3.1) holds then,

(i) $DS^{\alpha_2}_{p',q'}(X, I) \subseteq DS^{\alpha_1}_{p,q}(X, I)$ for $\beta_1 = \beta_2 = 1$,

(ii) $DS_{p',q'}(X, I) \subseteq DS^{\alpha_1}_{p,q}(X, I)$ for $\alpha_2 = \beta_1 = \beta_2 = 1$,

(iii) $DS_{p',q'}(X, I) \subseteq DS_{p,q}(X, I)$ for $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1$.

If (3.2) holds then,

(i) $DS^{\alpha_1}_{p,q}(X, I) \subseteq DS^{\alpha_2}_{p',q'}(X, I)$ for $\beta_1 = \beta_2 = 1$,

(ii) $DS^{\alpha_1}_{p,q}(X, I) \subseteq DS_{p',q'}(X, I)$ for $\alpha_2 = \beta_1 = \beta_2 = 1$,

(iii) $DS_{p,q}(X, I) \subseteq DS_{p',q'}(X, I)$ for $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1$.

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