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On Design and Analysis of Damping Physiognomies of Reinforced Composite Loofah Sponge on Athlete's Shoe

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Abstract: In this paper, the vibration damping characteristic of reinforced composite loofah sponge as mid-sole, for running shoes was modelled and analysed. The reinforced composite loofah sponge was used as midsole for running shoes to actively and critically damp the vibration. The damped vibration prevents knee and ankle injuries during running exercise. The reinforced composite loofah sponge was prepared by washing the loofah sponge fibre with distilled water to make it soft and free of slippery. The dried mat and chopped loofah sponge fibres were treated with a molar solution of sodium hydroxide to increase its surface roughness for better mechanical interlock and reaction sites. Then, the matrix is applied as a binder and reinforcement to enhance its Young's modulus and loss factor. Dynamic impact vibration, compression, low cyclic fatigue (LCF) and high cyclic fatigue (HCF) tests were carried out. The results depict that the mid-sole is an excellent vibration damper with high durability.

Keywords: loofah sponge, vibration damping, composite material.

Nomenclatures

$\xi(t)$	strain rate	x	Compression
$\sigma(t)$	stress rate/impact force rate	$F(t)$	Impact force rate
η	loss factor	σ_D	Dashpot stress
μ	reduced mass/Design factor	ξ_S	Spring strain
λ	relaxation rate	K	Spring constant
t	time	C	Damping factor
σ_0	initial stress	τ	Relaxation time
A	sample surface area	$D(t)$	Energy dissipated rate per circle
m	mass of the athlete	$W(t)$	Vibration energy rate/Work done
x_0	Initial sample height		

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1. Introduction

Running is an anaerobic exercise that has health and economic benefits. It is a great and cheap way of improving cardiovascular health. It raises heart rate and increases the flow of oxygen-rich blood in the body, including the brain. In sub-Saharan Africa countries like Nigeria, the government encourages citizenry in running exercise by introducing marathon, which is practice yearly. The marathon usually attracts tourists into the state and thus, increases the economic prospect of the host state.

However, people complain of a lack of improved and relatively cheap facilities like shoes for smooth and injuries free running exercise. The statistical data available show that the frequency at which running athlete groans due to ankles and knees injuries during running exercise is on the rise. This is as a result of using shoes that cannot effectively damp vibration. The impact force during running causes strain, which is transmitted through the feet and ankles in a vertical sinusoidal form to the more significant part of the body [1]. The effects could be fatal as most times; the impact is greater than the attenuation ability of the muscles of the foot and leg, hence causes injuries [2][3]. Therefore, there is a need for improvement in the damping mechanism of running shoes to compliment the inbuilt intrinsic attenuation mechanism of the body.

The design of a well-structured and improved vibration damping mechanism in running shoes is necessary at this period. This is because the population of running athletes is increasing geometrically as people now take running as a career far beyond exercise. Since the overall idea of running exercise is geared toward a healthy nation, a healthy environment cannot be ruled out. However, knee and ankle injuries are frequent problems of runners [4]. This is caused by ineffective vibration-damping running shoes which people use during running exercise. The ineffectiveness of the shoes could be linked to the current available petroleum-based materials used for the design of midsole. These materials are known with severe adverse effects, including the enormous amount of carbon (II) oxide released into the environment resulting from the burning of the substances [4][5]. Also, due to their non-biodegradable nature, they cause an environmental nuisance.

Therefore, there is a need to introduce locally available material in the design of running shoe midsoles. The material(s) has to be biodegradable and environmental friendly with a high damping coefficient. However, ecological concern continues to prompt research into a total or partial replacement of synthetic reinforcement materials with vegetable fibres, which are biodegradable and non-toxic[4]. Loofah sponge fibres are relatively less expensive, low density, good mechanical strength, better noise and vibration reduction characteristics. It is readily available as the source is renewable [4][5][6]. Natural fibre materials such as loofah sponge could be used instead of chemical substances, in absorbing the vibration due to their high damping coefficient [7].

Hence, this study aims at improving the damping characteristics of running shoe midsole using reinforced composite loofah sponge fibres. It is also aimed at reducing the pressure on inbuilt intrinsic attenuation mechanism of the lower part of the human body and minimise frequent injuries that occur during running exercise.

2. Materials and Sample

2.1. Fibre Extraction

The ripe and dried fruit of loofah sponge plant has a thick peel and a sponge gourd, which has a multidirectional array of fibres. The fibre comprises of a natural mat which is divided as an inner fibre core and an outer cylindrical core. The fibres were extracted manually by removing the peel [5][8]. Then, the cylindrical sponge gourd obtained was cut longitudinally to give a roughly rectangular mat.

2.2. Chemical Treatment

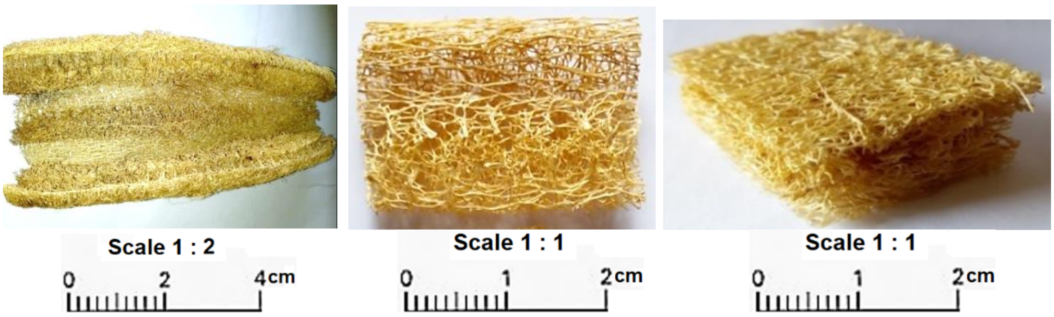
The rectangular loofah sponge fibres were first treated with distilled water. The loofah sponge fibre material was soaked, washed and rinsed with distilled water for an hour until it was soft and free of slippery. It was dried in an open space at room temperature. Secondly, the loofah sponge fibres were soaked in a molar solution of sodium hydroxide prepared by dissolving 40g sodium hydroxide pellets per cubic decimeters for two hours. Then, it was removed from the solution and allowed to dry in open space at room temperature.

2.3. Matrix

The thermoplastic resin DY250 was mixed with hardener TH7103 at the ratio 2:1 until curing occurred. The cured mixture was applied to the sample at a ratio of 10:1.

2.4. Preparation of Sample

The loofah sponge for the experiment is as shown in Fig. 1 (a). The loofah sponge fibres were cut to 65 mm by 50 mm by 2mm mats and two of such mats chopped as shown in Fig. 1 (b). The mat fibres were arranged in parallel layers with chopped fibres and matrix applied in between to give a rectangular box-like sample of 43 mm thickness. The sample was slightly compressed to ensure complete adhesion of the fibres and the matrix as shown in the Fig.1 (c) below.



(a) (b) (c)

FIGURE 1. (a) Untreated loofah sponge (b) Loofah sponge with matrix (c) Compressed loofah sponge with matrix

TABLE 1. Properties of reinforced composite loofah sponge

	Loofah Sponge	Epoxy Resin	Analytical Sample
Density (kg/m^3)	15	1.6x10^{-9}	13.50
Young modulus (GPa)	1.2x10^{-6}	3.42	9.0
Volume Fraction	0.1	0.9	1.0

3. Experimentation

3.1. Vibration Damping Test

3.1.1 Determination of loss factor

The loss factor of the sample was determined using a modal hammer Endevco 2302. The model hammer excites the sample with a constant impact force over a frequency range. The excitation responses to the selected frequencies were measured and recorded as shown in Table 2.

3.1.2 Determination of Young's modulus

The Young's modulus was evaluated using known resonance frequencies of the sample. The modulus was evaluated and recorded, as shown in Table 2.

TABLE 2. Vibration damping test

Model no	Loss factor (KJ/J)	Young modulus (GPa)
1	0.16	9.0
2	0.14	8.9
3	0.15	9.0
Average	0.15	9.0

3.2. Fatigue Test

The ASTM E606 machine was used that is calibrated to strain-controlled low and high cycle fatigue testing standard. The machine was loaded with the sample regulated to a temperature of 33^{o}C and made to deliver a dynamic impact force of 80kg weight. The relative humidity was regulated to 85%. The low, high strain rates and the relaxation time were measured and recorded. The machine was automated to repeat this for a certain number of circles, and the data are tabulated in Table 3.

TABLE 3. Fatigue test

Mode No	No of circles	Relaxation time (sec)	Low level (cm)	High level (cm)	Strain Amplitude
1	102,000	14.28	1.40	1.52	0.0618
2	127,000	14.59	1.40	1.50	0.0536
3	178,000	14.93	1.40	1.49	0.0466
4	298,000	15.30	1.40	1.48	0.0417
5	700,000	15.70	1.38	1.45	0.0375
6	1,276,000	16.13	1.38	1.44	0.0347

3.3. Compression Test

The compression machine was loaded with the sample and 80kg compression force applied for about 2.5 hours. The low level was measured and recorded. The load was released and the high level was recorded at an interval of 2 seconds until the sample attained its original height. The compression analysis test curve is shown in Fig. 2.

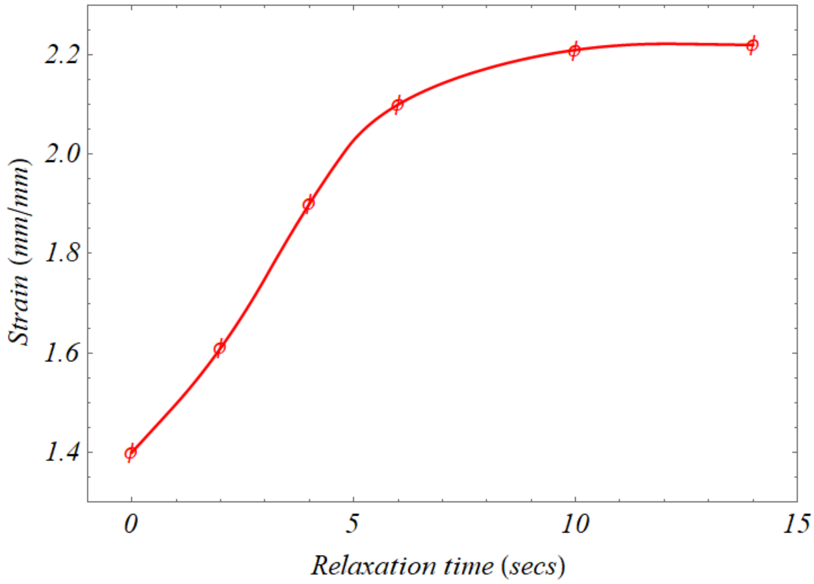


FIGURE 2. Compression analysis test curve

4. Governing Equation

The reinforced loofah sponge fibre exhibits properties of viscous and elastic materials. Hence, it is classified as a viscoelastic material. A material that consists of long flexible fibre like particles is in nature viscoelastic [9]. The viscoelastic nature of the material is due to its molecular structure. The shape of the particles and intermolecular bond enables the material to undergo compression and relaxation upon cyclic load. And, the particles slide easily along each other due to their flexibility which constitutes flow. In this project, the mechanical properties of the viscoelastic material, reinforced loofah sponge is mathematically modelled using the Kelvin-Voigt model [10]. This is ideal for this work as it is quite realistic, predicts strain to tend to σ/E as time continues to infinity and also best for organic materials. This consists of viscous dashpot in parallel with spring, as shown in Fig. 3.

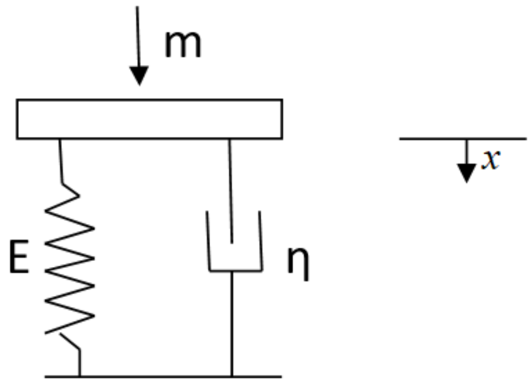


FIGURE 3. Kelvin-Voigt model

Therefore, when the runner is set to move at time, *t*, the impact force, F(*t*) exerted on the midsole of surface area, A is given as:

$$F(t) = A\sigma(\mathrm{t}) \tag{1}$$

The compression, x and strain rate, $\xi(t)$ recorded by midsole of the original length, x_0 is stated as

$$x = x_0\xi(t) \tag{2}$$

By differentiation, we have

$$\frac{dx}{dt} = x_0 \frac{d\xi(t)}{dt} \tag{3}$$

From Fig. 3, it is evident that the strain rates on the spring and dashpot are equal at time, *t*. Hence,

$$\xi_s(t) = \xi_D(t) \tag{4}$$

giving the total stress rate, $\sigma(t)$ as

$$\sigma(t) = \sigma_S(t) + \sigma_D(t) \tag{5}$$

The governing constitutive equation for the model is

$$\sigma(t) = E\xi(\mathrm{t}) + \eta\frac{d\xi(\mathrm{t})}{dt} \tag{6}$$

where E and η are Young's modulus of the spring and loss factor of the dashpot respectively. Using the integrating factor method, the solution of Eq. (6) was obtained as

$$\xi(t) = \frac{\sigma_0}{E}(1 - e^{-\frac{E}{\eta}t}) \tag{7}$$

where the rate of relaxation, $\lambda = E/\eta$ and relaxation time, $\tau = \eta/E$, making Eq. (7) to become

$$\xi(t) = \frac{\sigma_0}{E}(1 - e^{-\lambda t}) \tag{8}$$

By applying Newton's third law of motion and balance of forces, the inertia of mass becomes

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t) \tag{9}$$

where k and c are stiffness of the spring and damping factor of the dashpot respectively. By substituting Eqs. (1) and (3) into Eq.(9), we have

$$x_0 m\frac{d^2\xi(t)}{dt^2} + \eta x_0\frac{d\xi(t)}{dt} + x_0 E\xi(t) = A\sigma(\mathrm{t}) \tag{10}$$

The design factor, μ is a control factor that is crucial in this design due to its pivotal role in the damping ability of the midsole. It is a function of the area, the thickness of the midsole and the weight of the running athlete. Research shows that the design factor, μ is proportional to the weight, m of the runner and inversely to the impact area, A of the midsole of the shoe[7][11].

Mathematically put,

$$\mu = \frac{m\,x_0}{A} \tag{11}$$

where x_0 is the initial sample height Eq.(11) in standard form with the substitution for A from the design factor Eq. (11) and equation (6) gives a second-order ordinary differential equation as the governing equation:

$$\frac{d^2\xi(t)}{dt^2} + \frac{\eta}{m}\frac{d\xi(t)}{dt} + \frac{E}{m}\xi(t) = \frac{1}{\mu}\left(E\xi(\mathrm{t}) + \eta\frac{d\xi(\mathrm{t})}{dt}\right) \tag{12}$$

5. Damping Energy Equation

The rate at which the system converts vibration energy to heat defined the efficiency of the vibration damping midsole. Monica *et al.*[12] and Neilson [13]gave the energy dissipation per cycle, as

$$\eta = \frac{D(t)}{2\pi W(t)} \tag{13}$$

where $W(t) = mx\, d^2x \big/ dt^2$. Differentiating Eq. (3) with the result and Eq. (11) substituted into Eq. (13) gives

$$D(t) = 2\eta\pi\mu A \frac{d^2\xi(t)}{dt^2} \tag{14}$$

Substituting Eq. (14) into Eq. (12) gives

$$\begin{gathered} \frac{D(\mathrm{t})}{2\eta\pi\mu A} + \frac{\eta}{m}\frac{d\xi(t)}{dt} + \frac{E}{m}\xi(t) = \frac{1}{\mu}\left(E\xi(t) + \eta\frac{d\xi(t)}{dt} \right) \\ \frac{d\xi(t)}{dt} + \frac{E}{m}\xi(t) = \frac{D(t)}{2\pi\eta^2 A\left(1 - \frac{\mu}{m}\right)} \end{gathered} \tag{15}$$

Using the integrating factor method, the solution of Eq. (15) is obtained as

$$\xi(t) = \frac{\eta}{E} pD(t) + \mathrm{C}e^{-\frac{E}{\eta}t} \tag{16}$$

At $t = 0$, $\xi(\mathrm{t}) = 0$ and $D(\mathrm{t}) = 0$

Therefore, $C = 0$ and the energy dissipation rate equation of the system is

$$D(t) = 2\pi\eta EA(1 - \frac{\mu}{m})\xi(t) \tag{17}$$

The energy dissipation rate of the system is exothermal as evidenced by the negative sign from Eq. (17). This shows that the design converts a large fraction of the vibration energy to heat energy, which radiates into the surrounding.

6. Numerical Application

For this work, the design factor is 1000 Kg/m for 80 kg weight runner and initial sample height of 0.043 m calculated using Eq. (11). The averages Loss factor and Young's modulus are 0.15KJ/J and 9 Gpa respectively.

Applying Eq. (12) with initial values of $\xi(0) = 1, \frac{d\xi(0)}{dt} = 0$, Eq. (15) becomes

$$\frac{d^2\xi(t)}{dt^2}+\frac{150}{80}\frac{d\xi(t)}{dt}+\frac{9\times10^9}{80}\xi(t)=\frac{1}{1000}\left(9\times10^9\xi(\text{t})+150\frac{d\xi(\text{t})}{dt}\right)$$
$$0=\frac{d^2\xi(t)}{dt^2}+1.725\frac{d\xi(t)}{dt}+1.035\times10^8\xi(t) \quad (18)$$

giving the characteristic equation as;

$$\lambda^2+1.725\lambda+1.035\times10^8=0 \quad (19)$$

with $\lambda=-0.8625\pm10173i$ as the solution. Using the general equation:

$$\xi(t)=c_1e^{\lambda_0 t}\sin[\omega\text{t}]+c_2e^{\lambda_0 t}\cos[\omega\text{t}] \quad (20)$$

and taking $\lambda_\circ = -0.8625$, and $\omega=10173$, the solution becomes

$$\xi(t)=c_1e^{-0.8625t}\sin[10173\text{t}]+c_2e^{-0.8625t}\cos[10173\text{t}] \quad (21)$$

Which reduces to Eq. (22) after invoking the initial conditions

$$\xi(t)=e^{-0.8625t}\{8.48\times10^{-5}\sin[10173\text{t}]+\cos[10173\text{t}]\} \quad (22)$$

The optimal thickness for the design was determined to be 0.043m. The system equation was resolved using the results in Fig. 4, case 3. The optimal thickness was arrived at after analysing the system with the following 0.023m, 0.033m, 0.043m, 0.053m and 0.066m and the results as shown in Fig. 4, cases 1, 2, 3, 4, 5, respectively.

7. Discussion and Analysis

7.1 Sample Height

The height of the designed midsole is critical in the vibration damping characteristic of the system [12]. It influences the contact point of the shoe on the ground and the impact force [14][15]. The impact force controls the dynamic of vibration within the system. It varies proportionately with the distance between the contact point of the midsole and the ground. The data within the premises of this research, as shown in Fig. 4, case 1 to 5 indicate that the damping characteristics of the midsoles approach steady-state with increasing thickness of the midsole. The system attains asymptotic stability at the least possible time, as shown in Fig. 4, case 3 when the thickness is 0.043 m. In injurious damping vibration, which is the focal point of this research, the time to bring the system to asymptotic stability is paramount. Hence, the weight of the running athlete, 80kg and thickness of the midsole, 0.043 m are set as a standard for this design. This is explicitly explained in Table 4. The damping efficiency of the system is a function of the design factor of the midsole. The design factor depends on the thickness of the midsole as evidence in Eq. (11).

The thickness of the midsole also influences the speed of the athlete during running. The potential energy of an athlete using a high midsole is high. This is converted to high kinetic energy and momentum during running exercise. Therefore, the potential energy of the system is proportional to the midsole thickness. The thickness also provides a sort of relief and effective damping mechanism during running exercise.

7.2 Damping Behaviour

The midsole was designed to effectively convert vibration energy to heat energy and as well reduces the strain rate of change with time. The midsole also reserves marginal energy for continual retrogressive vibration during low or high cyclic load. The marginal energy makes it possible for superimposition to occur. The superimposition creates resonance, which provides comfort during running.

The energy dissipation rate per circle increases proportionately with time as evidence in Eq. (17). The vibration energy is converted to heat energy which radiates to the surrounding, thereby reducing the strain amplitude to a steady-state. Hence, asymptotic stability is attained in the shortest possible time, as shown in Fig. 4, case 3. The knees and ankles injuries that usually occur during running exercise are due to poor damping mechanism shoes. Such shoes could not damp the rise and fall of strain amplitude resulting from vibration for a prolonged time. In the case of this design, such a situation is critically aborted, and the system approaches the asymptotically steady state in the shortest possible time.

The damping characteristic of the midsole is described as underdamped. This is a function of the control factors such as loss factor, Young’s modulus and design factor as evidence in Eq. (12) and Fig. 4. The design was made to ensure a balance between the loss factor, Young’s modulus and the design factor for comfort and high damping efficiency during running.

7.3 Fatigue Strength

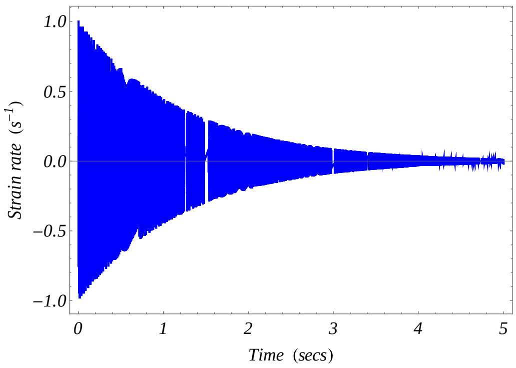
The durability and effectiveness of the midsole is a function of the materials selected for the design. The experimental data obtained, as shown in Fig. 5 show that the strength of the design varies with impact force rate and strain rate. The fatigue life of the damper is within 0 to 1,200,000 of circles, beyond which fatigue would set in. That is, the strain amplitude rate tends to be asymptotical stable within this range. This is equivalent to 1,720 km distance. Once an athlete covers this distance in using shoes with the designed midsole for exercises, the midsole is due to be replaced.

This design is suitable for long and short distance running exercises and also for low and high-speed running exercises. The fatigue test results and Fig. 5 show that the strain amplitude rate decreases with increasing impact rate. This indicates that the strength of the midsole decreases with increasing load per time.

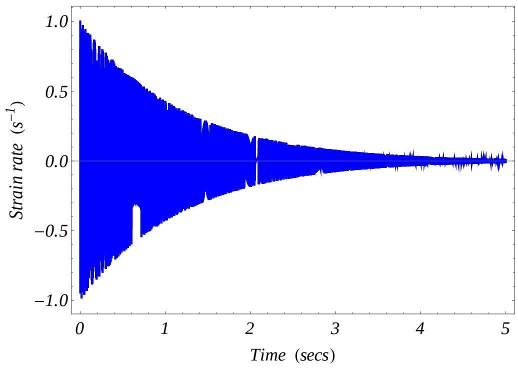
However, the design is suitable for training and running exercise at low or high speed. It is also an idea midsole for tournaments for effective impact damping to avoid injuries and provide comfort while running.

7.4 Elastic Behaviour

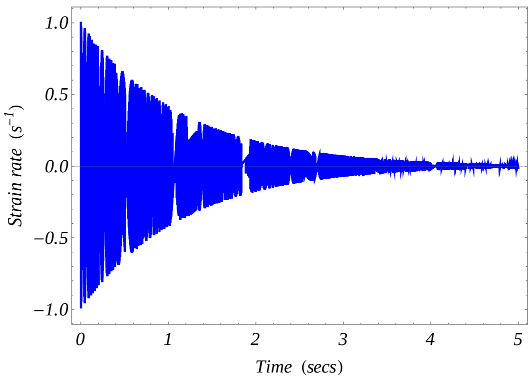
The midsole exhibits elastic property; this is as illustrated in Fig. 2. It relaxes quickly on a prolong compression. The elasticity of the midsole is a function of the materials selected for the design. The loofah sponge fibres and the thermoplastic matrix influence the viscoelastic properties of the midsole. The elastic characteristic of the thermoplastic matrix is thermal-induced. The heat generated by and within the midsole while on cyclic impact load is enough to cause structural and bond delocalisation. This is not high enough to create flow. Therefore, the mild structural adjustment paved the way for compression and relaxation of the material while on cyclic impact load. The fibre has spring nature, which is enhanced by the matrix that also provides reinforcement. Hence, the reinforced composite loofah sponge, midsole exhibits excellent elasticity characteristics.



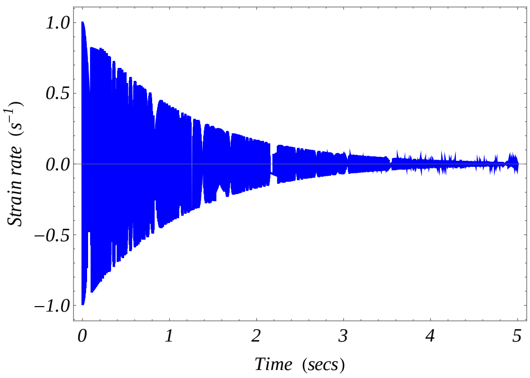
Case 1: Vibration Damping Impulse Response at $x_0 = 2.3$ cm



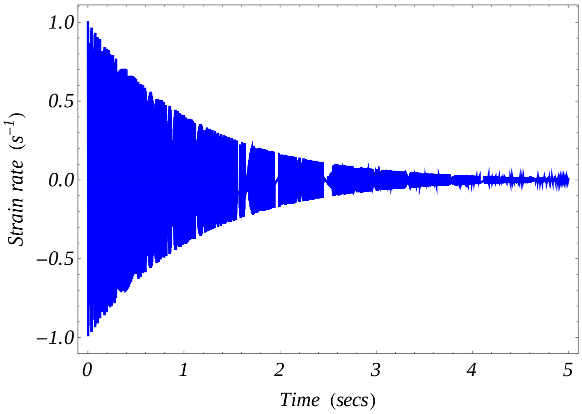
Case 2: Vibration Damping Impulse Response at $x_0 = 3.3$ cm



Case 3: Vibration Damping Impulse Response at $x_0 = 4.3$ cm



Case 4: Vibration Damping Impulse Response at $x_0 = 5.3$ cm



Case 5: Vibration Damping Impulse Response at $x_0 = 6.6$ cm

FIGURE 4. Effects of midsole thickness on vibration damping.

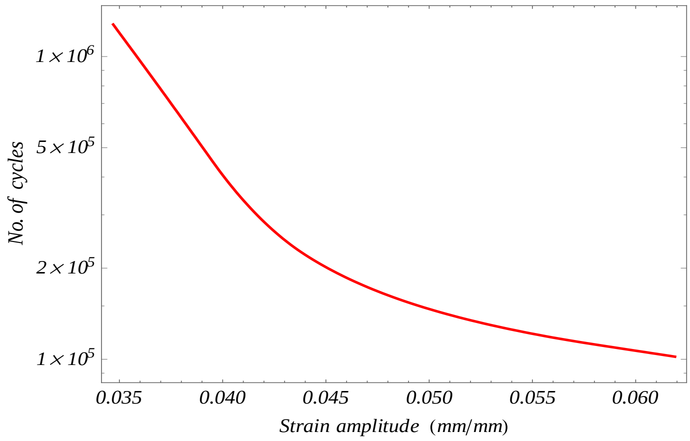


FIGURE 5. Fatigue analysis

TABLE 4. Midsole Design Parameters

Weight, m (kg)	Design Factor, μ (kg/m)	Midsole Thickness, x_0 (m)	Midsole Surface Area, $A(m^2)$
$m \leq 27$	300	0.014	
$28 \leq m \leq 35$	400	0.017	
$36 \leq m \leq 43$	500	0.021	
$44 \leq m \leq 51$	600	0.026	$A = \frac{mx_0}{\mu}$
$52 \leq m \leq 59$	700	0.030	
$60 \leq m \leq 67$	800	0.034	
$68 \leq m \leq 75$	900	0.039	
$76 \leq m \leq 83$	1000	0.043	
$84 \leq m \leq 91$	1100	0.047	$A = \frac{mx_0}{\mu}$
$92 \leq m \leq 95$	1200	0.051	

The design factors were round off to the nearest hundred, and the average thickness for a range of weight of runners was evaluated as in Table 4. The midsole surface area could be calculated using the formula. This is because the midsole surface area is different within the weight ranges.

8. Conclusion

The experimental data and analysis show that reinforced composite loofah sponge is an ideal mechanically improved material for the design and effective vibration damping of impact on the ground while running. This study dealt with the overall factors toward ensuring safety and comfort during running exercise.

The data within the premises of this study showed that the designed midsole could last eighteen (18) months for a tournament only, six (6) to nine (9) months for training and day to day running exercises including the tournament.

The design is the ideal material for running shoe midsoles. This is because of its ability to effectively damp vibration due to impact while running. It is locally available, relatively cheap and environmentally friendly. The adverse effects are mild on the environment as it is biodegradable.

Therefore, the ability of this design to effectively damp vibration results in minimal effects on the knees and ankles. Its durability with retentive damping characteristic over time and its environmental compatibility placed the design above the elastomers, which are also used for making running shoe midsoles.

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