Content list available at JournalPark

Turkish Journal of Forecasting

Journal Homepage: tjforecast.xyz

A Fuzzy Modelling Approach to NSE Criterion on Robust Design

O. Köksoy[1], M. Zeybek[1,*]

[1]*Ege University, Faculty of Science, Department of Statistics, 35100 Izmir, Turkey*

ARTICLE INFO

Article history:

Received	13	November	2018
Revision	11	January	2019
Accepted	23	January	2019
Available online	31	August	2019

Keywords:
Fuzzy modelling
Response surface methodology
Robust design
Nash-Sutcliffe efficiency

ABSTRACT

Dual response methodology is a natural and effective tool for a reliable and robust operation process or product in modern quality engineering. Therefore, many of quality improvement techniques based on dual response methodology focus on being on target and reducing system variability. This study focuses on the model quality performance criterion and presents a fuzzy modelling approach based on the Nash-Sutcliffe efficiency (NSE) for a dual response problem. The proposed approach aims to determine a set of operating conditions that maximize the degree of satisfaction due to the Nash-Sutcliffe efficiency in a quality improvement context. The main advantage of the proposed approach is to allow the practitioners evaluating the model quality performance which acts as a measure of model efficiency. The procedure and the validity of the proposed approach are illustrated on a popular example, *i.e.*, the printing process study by comparing existing methods. Based on the obtained results, the proposed approach gives the smallest MSE and provides additional information including the model quality performance for the printing process study data.

1. Introduction

In the early 1980s, robust parameter design (RPD) based on experimental design and quality engineering concepts, is introduced by Japanese quality engineer Taguchi. RPD, along with Taguchi's philosophy, was popularized and received a great deal of attention by many quality improvement engineers and statistical communities. However, his experimental methodologies and analysis techniques have been criticized mostly in the statistical sense. Consequently, new techniques that aim to improve a product/process quality have been proposed. The response surface methodology (RSM), first developed by [1] has revisited around the early of 1990s and then got popularized. RSM defines a relationship between a quality characteristic of a system of interest and a set of independent factors by using stochastic models. Because of this feature, when high-cost complexity designs are not desirable, RSM provides a simplified relationship and more insights about the system.

Dual response surface (DRS) methodology, proposed by [2], is an effective procedure based on fitting separate response surfaces for the system's mean and variance. This technique optimizes one fitted response subject to a constraint on the value of the other fitted response. Utilizing nonlinear programming replacing the equality constraints by inequalities is suggested by [3]. Additionally, an approach based on minimizing the mean squared error criterion (MSE) is proposed by [4]. Their approach focuses on both the distance from the target and variability and involves determining the best operating condition which minimizes the MSE modelled by response surfaces. A slightly different version of the MSE criterion, based on considering how far the mean can be located from its target,

* Corresponding author.
E-mail addresses: onur.koksoy@ege.edu.tr (Onur Köksoy), *melis.zeybek@ege.edu.tr (Melis Zeybek)

is discussed by [5]. Further work has been conducted by [6]. They proposed an alternative formulation based on joint optimization of the mean and standard deviation responses under no constraints or minimally constrained. Following these articles, several procedures have been proposed for the DRS problems; see, for example [7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

A fuzzy optimization technique for the DRS problem is proposed by [17]. They introduce a fuzzy modelling which considers both the deviation of the mean from the target and the magnitude of standard deviation. Moreover, a fuzzy modelling approach based on upside-down normal loss function is studied by [18]. Their approach aims to identify a set of operating conditions to maximize the degree of satisfaction with respect to the expected loss fitted by mean and standard deviation response surfaces. In [19], a fuzzy mathematical model was developed by RSM technique and fuzzy logic to optimize drilling process optimization with multiple responses. Following these articles, several studies related to the decision making approaches have been proposed; as follows: [20, 21, 22, 23, 24].

MSE and its general normalization version, NSE, are the two most widely used criteria for the model efficiency. In fact, NSE can be interpreted as a classic skill score and acts a measure of model performance. Therefore, especially in hydrological modelling and image quality assessment, NSE criterion is an effective alternative to MSE since it is dimensionless, being scaled onto the interval $(-\infty, 1]$. The general formulation of the NSE criterion can be given by the following equation,

$$NSE = 1 - \frac{MSE}{\sigma^2} \tag{1}$$

where σ^2 is the variance of the data. As evident to Equation (1), NSE and MSE are closely similar with each other. However, the difference in the optimization phase is that MSE is subject to minimization and NSE is subject to maximization. In the study of [25], a new optimization approach based on the NSE is configured on optimizing the fitted NSE response surface for the “target is best” case. The general applications of the NSE criterion can be found in the papers of [26, 27, 28].

In this article, we introduce a fuzzy modelling approach to optimize the NSE criterion for a given system. The proposed approach aims to identify a set of process parameter conditions to maximize the degree of satisfaction with respect to NSE fitted by mean and standard deviation response surfaces. The remainder of this manuscript is divided into three sections. Section 2 provides the proposed optimization technique. All findings are illustrated in an example in Section 3, before the manuscript finally ends with a conclusion.

2. The Proposed Fuzzy Modelling Approach based on NSE Criterion

In this study, we take into account the case where the response is affected by k control variables, xs. Therefore, the process mean and standard deviation can be modelled by the second order response surfaces as follows,

$$\hat{\mu}(x) = \hat{\alpha}_0 + \sum_{j=1}^{k} \hat{\alpha}_j x_j + \sum_{j=1}^{k} \hat{\alpha}_{jj} {x_j}^2 + \sum_{j<t}^{k} \sum \hat{\alpha}_{jt} x_j x_t \tag{2}$$

and

$$\hat{\sigma}(x) = \hat{\beta}_0 + \sum_{j=1}^{k} \hat{\beta}_j x_j + \sum_{j=1}^{k} \hat{\beta}_{jj} {x_j}^2 + \sum_{j<t}^{k} \sum \hat{\beta}_{jt} x_j x_t \tag{3}$$

Hence, the MSE criterion is modelled by the following formulation in [4],

$$MSE = (\hat{\mu}(x) - \tau)^2 + \hat{\sigma}^2(x) \tag{4}$$

where τ is the target value. Consequently, taking into account these equations, the NSE criterion given by Equation (1) can be modelled as follows,

$$NSE = 1 - \frac{(\hat{\mu}(x) - \tau)^2 + \hat{\sigma}^2(x)}{\sigma^2} \tag{5}$$

where $\hat{\mu}(x)$ and $\hat{\sigma}^2(x)$ are given in Equations (2) and (3), σ^2 is the desired system requirement about the system variability. This proposed NSE modelled by response surface is used for the proposed optimization technique. Note that, when $\hat{\mu}(x) = \tau$ and $\hat{\sigma}^2(x) = 0$, then the NSE takes its maximum value of 1. As an important situation, when the mean response hits the target, $\hat{\mu}(x) = \tau$, and the variance response reaches the process requirement about the system variability, *i.e.*, $\hat{\sigma}^2(x) = \sigma^2$, then the $NSE = 0$. On the other hand, while the mean response moves away from the target, the bias and the estimated variance response tends to increase, so, the value of the NSE ranges to $-\infty$. However, it should be pointed out that, this is not a desired situation due to the aim of the quality improvement studies, and is worked out by adding some constraints about the system requirements for the optimization phase, such as $MSE \leq 2\sigma^2$. This constraint should hold the optimal operating conditions in the desired region and will adjust the mean and variance responses depending on the requirement of the system variance. Therefore, the minimum value for the proposed NSE response turns out to be the value of -1 for the proposed problem.

In the phase of fuzzy modelling of the NSE criterion, first, one should determine the degree of satisfaction of the decision maker. In this purpose, taking into account the scaled range of the NSE, the satisfaction level with respect to the NSE criterion can be modelled by a monotonically decreasing function. According to [17] and [18], such function is referred as a membership function, which reflects the decision maker's belief, as in fuzzy set theory. A nonlinear membership function offers potential benefits in terms of realism and is chosen with the varying perception of the decision maker (see, [17]). In the study [17], the standardized forms of the mean and variance membership functions are used. In this study, according to Equations (1) and (4), the NSE criterion is also a standardized function of the mean and variance responses.

In the light of these beneficial comments; we suggest the following membership function to the NSE response surface,

$$m(NSE) = \begin{cases} \frac{e^d - e^{d|NSE|}}{e^d - 1} & , \quad if\ d \neq 0 \\ 1 - |NSE| & , \quad if\ d = 0 \end{cases} \tag{6}$$

where d is an exponential constant. When $d < 0, d = 0, d > 0$, $m(NSE)$ can be convex, linear and concave shapes, respectively. Note that, when the value of NSE is its maximum or minimum, then $m(NSE) = 0$. $m(NSE)$ achieves its maximum value, *i.e.*,1, when the mean response hits its target, and the variance response reaches the process requirement about the system variability, *i.e.*, $NSE = 0$. As a result, the function $m(NSE)$ given in Equation (6) provides a reasonable and flexible representation of a human perception.

Finally, the proposed optimization problem can be stated as,

$$\text{Maximize} \quad \delta$$
$$\text{Subject to} \quad m(NSE) \geq \delta \ \text{ and } \ x \in \Omega$$

where Ω defines a feasible region of x. This optimization problem aims to identify x^* which would maximize the minimum degree of satisfaction, δ, with respect to the expected loss within a feasible region.

3. Example: The Printing Process Study

This printing process study example is borrowed from [29], which is revisited by [2,4,17,18]. This experiment was conducted to find the optimum combination of the effects of speed (x_1), pressure (x_2) and distance (x_3) factors on the quality of a printing process (y). A 3^3 factorial design with three replicates was used to fit the response and the design of the experiment is given in Table 1.

For the illustrated purposes, the fitted NSE response surface and the exponential membership function is obtained under the assumptions that the specification bound is (490, 510), the target value for this problem is 500, and the desired standard deviation is smaller than 60.

The fitted response surfaces from [2] are as follows,

$$\hat{\mu}(x) = 327.6 + 177.0x_1 + 109.4x_2 + 131.5x_3 + 32.0{x_1}^2 - 22.4{x_2}^2 - 29.1{x_3}^2 + 66.0x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3 \tag{7}$$

$$\hat{\sigma}(x) = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2{x_1}^2 - 1.3{x_2}^2 + 16.8{x_3}^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3 \tag{8}$$

and the fitted response surface function of the NSE criterion is,

$$NSE = 1 - \frac{(\hat{\mu}(x) - 500)^2 + \hat{\sigma}^2(x)}{60^2} \tag{9}$$

where $\hat{\mu}(x)$ and $\hat{\sigma}(x)$ are given in Equations (7) and (8).

Table 1. The printing process study data

u	x_1	x_2	x_3	y_1	y_2	y_3	$\bar{y}$	s
1	-1	-1	-1	34	10	28	24	12.5
2	0	-1	-1	115	116	130	120.3	8.4
3	1	-1	-1	192	186	263	213.7	42.8
4	-1	0	-1	82	88	88	86	3.5
5	0	0	-1	44	178	188	136.7	80.4
6	1	0	-1	322	350	350	340.7	16.2
7	-1	1	-1	141	110	86	112.3	27.6
8	0	1	-1	259	251	259	256.3	4.6
9	1	1	-1	290	280	245	271.7	23.6
10	-1	-1	0	81	81	81	81	0.0
11	0	-1	0	90	122	93	101.7	17.7
12	1	-1	0	319	376	376	357	32.9
13	-1	0	0	180	180	154	171.3	15
14	0	0	0	372	372	372	372	0.0
15	1	0	0	541	568	396	501.7	92.5
16	-1	1	0	288	192	312	264	63.5
17	0	1	0	432	336	513	427	88.6
18	1	1	0	713	725	754	730.7	21.1
19	-1	-1	1	364	99	199	220.7	133.8
20	0	-1	1	232	221	266	239.7	23.5
21	1	-1	1	408	415	443	422	18.5
22	-1	0	1	182	233	182	199	29.4
23	0	0	1	507	515	434	485.3	44.6
24	1	0	1	846	535	640	673.7	158.2
25	-1	1	1	236	126	168	176.7	55.5
26	0	1	1	660	440	403	501	138.9
27	1	1	1	878	991	1161	1010	142.5

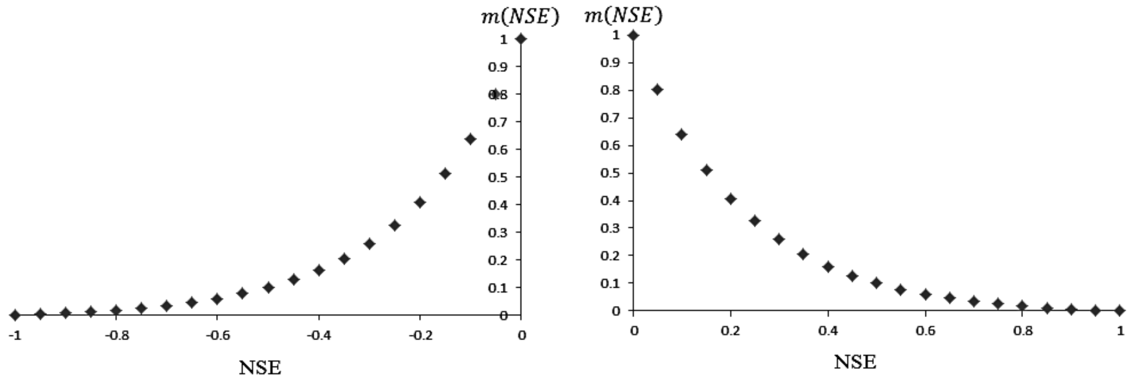
From the studies [17,18], a convex-shaped exponential membership function with $d = -4.39$ with respect to the Equation (9) is chosen. Thus, the membership function takes the following form,

$$m(NSE) = \frac{e^{-4.39} - e^{-4.39|NSE|}}{e^{-4.39} - 1} \quad (10)$$

The complete optimization formulation of this problem is conducted as follows,

$$\begin{aligned} &\text{Maximize} \quad \delta \\ &\text{Subject to} \quad \frac{e^{-4.39} - e^{-4.39|NSE|}}{e^{-4.39} - 1} \geq \delta \\ &-1 \leq x \leq 1 \\ &0 \leq \hat{\sigma}(x) \leq 60 \\ &(\hat{\mu}(x) - 500)^2 + \hat{\sigma}^2(x) \leq 7200 \end{aligned}$$

The optimal operating condition obtained from the proposed approach turns out to be $x^* = (1.00, 0.084, -0.254)$ where $\hat{\mu}(x) = 495.84$ and $\hat{\sigma}(x) = 44.60$ and the resulting $\delta = 0.987$. When $d = -4.39$, the membership function is highly convex and indicates the high stringency. As a result, the proposed approach performs better for both estimated mean and standard deviation. Figure 1 illustrates the relationship between membership function in Equation (9) and the fitted response of NSE criterion in Equation (8) for various values.



(a) **(b)**

Figure 1. The relationship between $m(NSE)$ and NSE: a) the value of NSE ranges 0 to -1; b) the value of NSE ranges 0 to 1

The results of the proposed approach are summarized in Table 2 and compared with those of [4], [17], and [18].

Table 2. Comparison of optimal settings for the printing process study

	Optimal setting	$\hat{\mu}(x)$	$\hat{\sigma}(x)$	**MSE**	$m(NSE)$	**NSE**
[4]	(1.00, 0.07, -0.25)	494.44	44.43	2005.14	Unknown	Unknown
[17]	(1.00, 0.086, -0.254)	496.08	44.63	2007.07	Unknown	Unknown
[18]	(1.00, 0.076, -0.252)	494.84	44.48	2005.10	Unknown	Unknown
The proposed approach	(0.987, 0.092, -0.250)	494.99	44.60	1984.99	0.158	0.44

The solution of the proposed approach is adapted to the relaxing zero-bias assumption of [4]. The obtained results from the proposed modelling are much closer to [4], [17] and [18] in terms of the estimated mean and standard deviation. However, when the obtained values of MSEs are compared, the proposed approach has the smallest MSE. Furthermore, the proposed method provides additional information such as $NSE = 0.44$. In fact, since the NSE acts as a model performance criterion, it provides the quality of the estimated mean and variance with respect to the target and variance requirement of the printing process problem. In the statistical sense, the estimated mean and variance ensure the requirements for the printing process study with a desired quality. However, it should be pointed out that, the results from different approaches cannot be compared in a straightforward manner since the methods differ in terms of their optimization criteria.

4. Conclusion

The optimization algorithms based quality technologies have received considerable attention in recent years. In particular, researchers have sought to understand the extent of the quality of the obtained best-operating conditions from different quality improvement approaches. This manuscript is presented as an effective methodology based on a fuzzy modelling approach to NSE criterion, which quantifies the quality performance of the model and acts as a measure of model efficiency. The NSE criterion a widely used criterion for the model efficiency, thus adapting this fuzzy modelled version in the field of quality improvement not only provides a wide range of engineering information about the system but also offers a comprehensive solution to quality engineering. The proposed approach aims to identify a set of operating conditions to maximize the degree of satisfaction with respect to the NSE criterion fitted by mean and standard deviation response surfaces. Additionally, the proposed approach is examined with a well-known design of experiment, printing process data. It was shown that the proposed approach can model the decision maker's preference for the following responses: *i*. the estimated mean, *ii*. the estimated variance, and *iii*. the estimated NSE criterion. The proposed method achieves a better balance between bias and variability. Besides these advantages, this approach gives additional information such as a measure of model performance and this allows the estimation of which operating conditions offer the perfect quality.

References

[1] G.E.P. Box and K.B. Wilson, On the Experimental Attainment of Optimum Conditions, J. Roy. Statist. Soc. Ser. B Metho. 13 (1951) 1-45.

[2] G.G. Vining and R.H. Myers, Combining Taguchi and Response Surface Philosophies: A Dual Response Approach, J. Qual. Technol. 22 (1990) 38-45.

[3] E. Del Castillo and D.C. Montgomery, A Nonlinear Programming Solution to the Dual Response Problem, J. Qual. Technol. 25 (1993) 199-204.

[4] D.K.J. Lin and W. Tu, Dual Response Surface Optimization, J. Qual. Technol. 27 (1995) 34-39.

[5] K.A. Copeland and P.R. Nelson, Dual Response Optimization via Direct Function Minimization, J. Qual. Technol. 28 (1996) 331-336.

[6] O. Köksoy and N. Doganaksoy, Joint Optimization of Mean and Standard Deviation in Response Surface Experimentation, J. Qual. Technol. 35 (2003) 239-252.

[7] A.C. Shoemaker, K.L, Tsui, and C.F.J. Wu, Economical Experimentation Methods for Robust Parameter Design, Technometrics. 33 (1991) 415-427.

[8] J.M. Lucas, How to Achieve a Robust Process Using Response Surface Methodology, J. Qual. Technol. 26 (1994). 248-260.

[9] S.K.S. Fan, A generalized global optimization algorithm for dual response systems, J Qual Technol. 32 (2000). 444-456.

[10] O. Köksoy, Multiresponse Robust Design: Mean Square Error (MSE criterion), Appl. Math. Comput. 175 (2006) 1716-1729.

[11] H. Bayrak, B. Özkaya, M.A. Tekindal, Birinci derece faktoriyel denemelerde verimlilik için optimum noktalarin belirlenmesi: bir uygulama. Türkiye Klinikleri Biyoistatistik Dergisi, 2 (2010) 18-26.

[12] O. Köksoy and S.S. Fan, An upside-down normal loss function-based method for quality improvement, Eng Optimiz. 44 (2012) 935-945.

[13] M. A. Tekindal, H. Bayrak, B. Ozkaya, Y. Yavuz, Second-order response surface method: factorial experiments an alternative method in the field of agronomy. Turkish Journal of Field Crops, 19 (2014) 35-45.

[14] M. Zeybek and O. Köksoy, Optimization of correlated multi-response quality engineering by the upside-down normal loss function, Eng Optimiz. 48(2016) 1419-1431.

[15] M. Zeybek and O. Köksoy, The effects of gamma noise on quality improvement. Commun Stat Simulat (2018). DOI:10.1080/03610918.2018.1506030

[16] M. Zeybek M, Process capability: A new criterion for loss function–based quality improvement, Süleyman Demirel University Journal of Natural and Applied Sciences 22 (2018) 470-477.

[17] K. Kim and D.K.J. Lin, Dual Response Surface Optimization: A Fuzzy Modeling Approach, J. Qual. Technol. 30 (1998) 1-10.

[18] M. Zeybek and O. Köksoy, A Fuzzy Modelling Approach to Robust Design via Loss Functions, Turkish Journal of Forecasting. 01 (2017) 40-45.

[19] A. I. Boyaci, T. Hatipoglu, and E. Balci, Drilling process optimization by using fuzzy-based multi-response surface methodology, Advances in Production Engineering & Management 12 (2017) 163.

[20] H.J. Zimmermann, Fuzzy Sets, Decision Making, and Expert Systems, Kluwer Academic Publishers, Boston, M.A. (1987).

[21] H. Moskowitz and K. Kim, On Assessing the H Value in Fuzzy Linear Regression, Fuzzy Sets Syst. 58 (1993) 303-327.

[22] C.W. Kirkwood, Strategic Decision Making, Duxbury Press, Belmont, C.A. (1996).

[23] O. Turksen, Analysis of response surface model parameters with bayesian approach and fuzzy approach, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 24 (2016) 109-122.

[24] K. Youssef, Ç. Huseyin, E. Engin, Adaptive neuro-fuzzy inference system (ANFIS) and response surface methodology (RSM) prediction of biodiesel dynamic viscosity at 313 K, Procedia Computer Science 120 (2017) 521-528.

[25] M. Zeybek, Nash-Sutcliffe efficiency approach for quality improvement, Journal of Applied Mathematics and Computation, 2 (2018) 496-503.

[26] A.H. Murphy, Skill Scores Based on the Mean Square Error and Their Relationships to the Correlation Coefficient, Mon Weather Rev. 116 (1988) 2417-2424.

[27] S. Weglarzyk, The Interdependence and Applicability of Some Statistical Quality Measures for Hydrological Models, J Hydrol. 206 (1998) 98-103.

[28] H.V. Gupta, H. Kling, K.K. Yilmaz and G.F. Martinez, Decomposition of the Mean Squared Error and NSE Performance Criteria: Implications for Improving Hydrological Modelling, J Hydrol. 377 (2009) 80-91.

[29] G.E.P. Box and N.R. Draper, Empirical Model Building and Response Surfaces, John&Sons, New York, N.Y. (1987).