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Involute Curves in 4-Dimensional Galilean Space G4

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Abstract: In this paper, we define the (0,2)-involute of a given curve in 4-dimensional Galilean space, and for the curve with a generalized involute, the necessary and sufficient condition is obtained.

Keywords: Frenet formula, Galilean space, Involute curve. (Please, alphabetical order and at lease one keyword)

1 Introduction

Galilean geometry is one of the nine projective space geometries which was discussed by Cayley-Klein at the beginning of 20th century. After that, the curvature-related studies were maintained and the curve properties in Galilean space were studied in $[1, 2]$. The involute of a given curve in Euclidean space is a famous concept, whereas the idea of an involute string is due to C. Huygens, who is well known for his job in optics and who found involutes while attempting to construct a more accurate clock in 1968 $[3, 4]$. The theories of the Involute and Evolute Curves in Minkowski Space are extensively studied in $[5, 6, 7]$.

In classical differential geometry, an evolute of a curve is defined as the locus of the centers of curvatures of the curve, which is the envelope of the normal of the given curve. While an Involute of a specified curve is a curve in which all tangents of a specified curve are normal $[3, 8, 9, 10]$.

In $[11]$, the author created Frenet-Serret curve frame in the Galilean 4-space and acquired constant ratio curves in Galilean 4-space. Aydin and Ergüt constructed equiform differential geometry of curves and obtained the angle between the equiform Frenet vectors and their derivatives in G_4 $[12]$.

In $[13, 14]$, the authors studied some curves of Galilean geometry in both plane and space, they obtained the characterization of slant helices in 3- dimensional Galilean space G_3.

2 Preliminaries

The Galilean space can be described as a three dimentional complex projective space with absolute figures $\{m, l, p_1, p_2\}$ which consists of a real plane m, a real line $l \subset m$ and two complex conjugate points $p_1, p_2 \in l$.

The study of plane-parallel motion mechanics decreases the study of a 3-space geometry with $\{x, y, t\}$ coordinates by the motion formula $[2]$. This geometry can be described as geometry of Galilean 3-space. It is clarified in $[2]$ that four dimensional Galilean space, which studies all invariant features under object movements in space is even more complicated.

Moreover, it is indicated that this geometry can be more accurately defined as studying those four dimensional space characteristics with co-ordinates that are invariant under the general Galilean transformations as follows:

$$
\begin{aligned}
x^{'} &= (\cos\theta\cos\phi - \cos\gamma\sin\theta\sin\phi)\,x + (\sin\theta\cos\phi - \cos\gamma\cos\theta\sin\phi)\,y \\
&\quad + (\sin\gamma\sin\phi)\,z + (v\cos\beta_1)\,t + a \\
y^{'} &= -(\cos\theta\sin\phi + \cos\gamma\sin\theta\cos\phi)\,x + (-\sin\theta\sin\phi + \cos\gamma\cos\theta\cos\phi)\,y \\
&\quad + (\sin\gamma\cos\phi)\,z + (v\cos\beta_2)\,t + b \\
z^{'} &= (\sin\gamma\sin\theta)\,x - (\sin\gamma\cos\theta)\,y + (\cos\gamma)\,z + (v\cos\beta_3)\,t + c \\
t^{'} &= t + d
\end{aligned}
$$

with $\cos^2\beta_1 + \cos^2\beta_2 + \cos^2\beta_3 = 1$

The following chapter provides some basic characteristics of curves in Galilean 4-space for the uses of the conditions.

A curve $\alpha : I \to G_4$, $I \subset R$ can be given as

$$\alpha(t) = (x_1(t), x_2(t), x_3(t), x_4(t)),$$

where $x_i(t) \in C^4$ i=1,2,3,4 and $t \in I$. Let α be a curve in G_4, which is parameterized by arclength $t = s$, and its coordinate form can be written as

$$\alpha(s) = (s, x_2(s), x_3(s), x_4(s)).$$

In affine coordinates the Galilean inner product between two points $P_i = (x_{i1}, x_{i2}, x_{i3}, x_{i4})$, $i = 1, 2$, is defined by

$$\begin{aligned} g(P_1, P_2) &= |x_{21} - x_{11}|, \ if \ \ x_{21} \neq x_{11} \\ g(P_1, P_2) &= \sqrt{(x_{22} - x_{12})^2 + (x_{23} - x_{13})^2 + (x_{24} - x_{14})^2}, \ if \ \ x_{21} = x_{11} \end{aligned}$$

For the vectors $p = (p_1, p_2, p_3, p_4)$, $q = (q_1, q_2, q_3, q_4)$ and $r = (r_1, r_2, r_3, r_4)$, Galilean cross product in G_4 is defined as follows:

$$p \wedge q \wedge r = \begin{vmatrix} 0 & e_2 & e_3 & e_4 \\ p_1 & p_2 & p_3 & p_4 \\ q_1 & q_2 & q_3 & q_4 \\ r_1 & r_2 & r_3 & r_4 \end{vmatrix}$$

where e_i are the standard basis vectors.

The notation $< x, y >_G$ we use in this paper denotes the inner product of the vectors x, y in Galilean space.

Let $\alpha(s) = (s, x_2(s), x_3(s), x_4(s))$ be a curve parameterized by arclength s in G_4, the Frenet formulas can written as

$$\begin{bmatrix} T' \\ N' \\ B_1' \\ B_2' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & 0 & 0 \\ 0 & 0 & k_2 & 0 \\ 0 & -k_2 & 0 & k_3 \\ 0 & 0 & -k_3 & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B_1 \\ B_2 \end{bmatrix} \tag{2.1}$$

where T, N, B_1, B_2 are mutually orthogonal vector fields which the following equations hold

$$\begin{aligned} &< T, T >_G = < N, N >_G = < B_1, B_1 >_G = < B_2, B_2 >_G = 1 \\ &< T, N >_G = < T, B_1 >_G = < T, B_2 >_G = < N, B_1 >_G = < N, B_2 >_G = < B_1, B_2 >_G = 0. \end{aligned}$$

We use some terms in this journal. The plane spanned by { T, B_1 } is called (0,2)-tangent plane at any point of the curve ϕ. The plane spanning { N, B_2 } is called the (1,3)-normal plane of ϕ.

Let $\phi : I \to G_4$ and $\phi^* : I \to G_4$, $I \subset R$ be two regular parameterized curves in Galilean 4-space G_4. Let $s^* = f(s)$ be an arc-length parameter of ϕ^*. $\forall\ s \in I$, if the $(0, 2)$-tangent plane at $\phi(s)$ of ϕ overlaps with the $(1, 3)$-normal plane of ϕ^*at $\phi^*(s)$, then ϕ^* is said to be $(0, 2)$-involute curve of ϕ in G_4 while ϕ is called $(1, 3)$-evolute curve of ϕ^* in G_4.

3 The (0,2)-involute curve in a Galilean 4-space G_4

In this chapter, we investigate the existence and representation of the (0,2)-involute curve in Galilean 4-space.

Let $\phi : I \subset R \to G_4$ be a regular parameterized curve, and k_1, k_2 and k_3 to be its curvatures $k_i \neq 0$, and let $\phi^* : I \subset R \to G_4$ be a $(0, 2)$-involute curve of ϕ. Donate $\{T^*, N^*, B_1^*, B_2^*\}$ to be the Frenet Frame along ϕ^* and k_1^*, k_2^* and k_3^* to be the curvatures of ϕ^*. Then

$$\begin{aligned} span\{T, B_1\} &= span\{N^*, B_2^*\} \\ span\{N, B_2\} &= span\{T^*, B_1^*\} \end{aligned} \tag{3.1}$$

and

$$\left\langle T^*, T \right\rangle = 0.$$

Moreover, α^* can be expressed as

$$\phi^*(s) = \phi(s) + a(s)T(s) + b(s)B_1(s) \tag{3.2}$$

where $a, b \in C^\infty$ functions on I.

By differentiating (3.2) with respect to s and using (2.1)

$$\phi^{*'}(s) = \phi'(s) + a'(s)T(s) + a(s)T'(s) + b'(s)B_1 + b(s)B_1'(s) \tag{3.3}$$

$$f'T^* = (1 + a')\,T + (ak_1 - bk_2)\,N + b'B_1 + bk_3B_2.$$

So by taking dot product on both-sides of (3.3) with T and B_1

$$\begin{aligned}\left\langle f^{'}T^{^{*}},T\right\rangle &= \left\langle\left(1+a'\right)T+\left(ak_1-bk_2\right)N+b^{'}B_1+bk_3B_2,T\right\rangle\\ 0 &= 1+a'\\ a' &= -1\end{aligned}$$

integrate both sides of the above equation

$$\begin{aligned}\int\frac{da}{ds}ds &= -\int ds\\ a &= a_0-s, \quad (a_0 \text{ is a constant})\end{aligned}$$

and

$$\begin{aligned}\left\langle f^{'}T^{^{*}},B_1\right\rangle &= \left\langle\left(1+a'\right)T+\left(ak_1-bk_2\right)N+b^{'}B_1+bk_3B_2,B_1\right\rangle\\ 0 &= b^{'},\end{aligned}$$

which implies that b is a constant, thus (3.3) turns to

$$f^{'}T^{^{*}}=\left(ak_1-bk_2\right)N+bk_3B_2, \tag{3.4}$$

let

$$\delta=\frac{\left(ak_1-bk_2\right)}{f'} \quad \text{and} \quad \gamma=\frac{bk_3}{f^{'}}, \tag{3.5}$$

therefore

$$T^{^{*}}=\delta N+\gamma B_2, \tag{3.6}$$

$$\delta^2+\gamma^2=1.$$

Case 1

$b\neq 0$, in this case $\gamma=\frac{bk_3}{f'}\neq 0$. Denote $\frac{\delta}{\gamma}=t_1$, then $\delta=\gamma t_1$ and

$$f^{'}=\frac{bk_3}{\gamma}=b\gamma^{-1}k_3 \tag{3.7}$$

From (3.5) and (3.7)

$$\delta=\frac{\left(ak_1-bk_2\right)}{f'}$$

$$bt_1k_3=ak_1-bk_2. \tag{3.8}$$

From (3.6)

$$\delta^2+\gamma^2=1$$

$$\gamma^2=\frac{1}{t_1^2+1}. \tag{3.9}$$

Differentiate (3.6) with respect to s and using (2.1)

$$T^{^{*'}}=\delta^{'}N+\delta N^{'}+\gamma^{'}B_2+\gamma B_2^{'}$$

$$f^{'}k_1^{^{*}}N^{^{*}}=\delta^{'}N+\left(\delta k_2-\gamma k_3\right)B_1+\gamma^{'}B_2 \tag{3.10}$$

So by taking dot product on both-sides of (3.10) with N and B_2

$$\begin{aligned}\left\langle f^{'}k_1^{^{*}}N^{^{*}},N\right\rangle &= \left\langle\delta^{'}N+\left(\delta k_2-\gamma k_3\right)B_1+\gamma^{'}B_2,N\right\rangle\\ 0 &= \delta^{'}\\ \left\langle f^{'}k_1^{^{*}}N^{^{*}},B_2\right\rangle &= \left\langle\delta^{'}N+\left(\delta k_2-\gamma k_3\right)B_1+\gamma^{'}B_2,B_2\right\rangle\\ 0 &= \gamma^{'}\end{aligned}$$

which implies that γ and δ are constants, thus (3.10) turns to

$$f^{'}k_1^{*}N^{*} = (\delta k_2 - \gamma k_3)\, B_1. \tag{3.11}$$

We suppose that

$$f^{'}k_1^{*} = \delta k_2 - \gamma k_3,$$
$$N^{*} = B_1. \tag{3.12}$$

Differentiate (3.12) with respect to s

$$N^{*'} = B_1^{'}$$
$$f^{'}k_2^{*}B_1^{*} = -k_2 N + k_3 B_2. \tag{3.13}$$

Let

$$c = \frac{-k_2}{f' k_2^{*}}, \quad e = \frac{k_3}{f' k_2^{*}}, \tag{3.14}$$

then (3.13) turns into

$$B_1^{*} = cN + eB_2,$$
$$c^2 + e^2 = 1.$$

Let $\frac{c}{e} = t_2$, then $c = et_2$, from (3.14)

$$\begin{aligned} c &= \frac{-k_2}{f' k_2^{*}} \\ et_2 &= \frac{-k_2}{f' k_2^{*}} \\ \frac{k_3}{f' k_2^{*}} t_2 &= \frac{-k_2}{f' k_2^{*}} \end{aligned}$$

$$k_3 = \frac{-k_2}{t_2}, \tag{3.16}$$

from (3.15)

$$c^2 + e^2 = 1$$
$$e^2 = \frac{1}{t_2^2 + 1}, \tag{3.17}$$

from (3.8) and (3.16)

$$\begin{aligned} bt_1 k_3 &= ak_1 - bk_2 \\ bt_1 \left(\frac{-k_2}{t_2}\right) &= ak_1 - bk_2 \\ \tau &= \frac{k_2}{k_1} = \frac{at_2}{b\,(t_2 - t_1)} \end{aligned}$$

$$\tau = \frac{\frac{a}{b} t_2}{(t_2 - t_1)}. \tag{3.18}$$

From (3.16)

$$k_2 = -k_3 t_2, \tag{3.19}$$

substitute (3.19) in (3.8)

$$bt_1 k_3 = ak_1 - bk_2$$

$$\frac{k_3}{k_1} = \frac{a}{(bt_1 - bt_2)} = -\frac{1}{t_2}\tau. \tag{3.20}$$

Let $\frac{\gamma}{e} = t_3$, then $\gamma = et_3$, from (3.14)

$$e = \frac{k_3}{f' k_2^*}$$

$$f' k_2^* = \frac{k_3 t_3}{\gamma} = e^{-1} k_3 \tag{3.21}$$

but

$$\begin{aligned} t_3 &= \frac{\gamma}{e} \\ t_3^2 &= \frac{\gamma^2}{e^2}, \end{aligned}$$

substitute (3.9) and (3.17) in the above equation

$$t_3^2 = \frac{\gamma^2}{e^2}$$

$$t_3^2 = \frac{1 + t_2^2}{1 + t_1^2}. \tag{3.22}$$

Differentiate (3.15) with respect to s and using (2.1)

$$B_1^{*\prime} = c' N + c N' + e' B_2 + e B_2'$$

$$f' k_3^* B_2^* = f' k_2^* N^* + c' N + (c k_2 - e k_3) B_1 + e' B_2. \tag{3.23}$$

So by taking inner product on both-sides of (3.23) with N and B_2

$$\begin{aligned} \left\langle f' k_3^* B_2^*, N \right\rangle &= \left\langle f' k_2^* N^* + c' N + (c k_2 - e k_3) B_1 + e' B_2, N \right\rangle \\ 0 &= c' \\ \left\langle f' k_3^* B_2^*, B_2 \right\rangle &= \left\langle f' k_2^* N^* + c' N + (c k_2 - e k_3) B_1 + e' B_2, B_2 \right\rangle \\ 0 &= e', \end{aligned}$$

which implies that c and e are constants, thus (3.23) turns to

$$f' k_3^* B_2^* = f' k_2^* N^* + (c k_2 - e k_3) B_1, \tag{3.24}$$

substitute (3.12) and (3.21) in (3.24)

$$f' k_3^* B_2^* = e^{-1} k_3 B_1 + (c k_2 - e k_3) B_1,$$

$$f' k_3^* B_2^* = c (t_2 k_3 + k_2) B_1, \tag{3.25}$$

we may choose that

$$B_2^* = c B_1 \tag{3.26}$$

$$f' k_3^* = (t_2 k_3 + k_2).$$

Summarising the above discussion, we obtain the following

Theorem 1. *Let $\phi : I \subset R \to G_4$ be a regular parameterized curve and k_1, k_2 and k_3 are its curvatures $k_i \neq 0$. If ϕ has the $(0,2)$-involute mate curve $\phi^*(s) = \phi(s) + (a_{0-}s)T(s) + b(s)B_1(s)$ with $b \neq 0$, then k_1, k_2 and k_3 satisfy*

$$\frac{k_2}{k_1} = \tau, \quad \frac{k_3}{k_1} = -\frac{1}{t_2}\tau \;\; and \;\; \tau = \frac{(a_0 - s)\, t_2}{b\,(t_2 - t_1)},$$

where a_0, b and t_2 are constants, moreover, the three curvatures of $\phi^*(s)$ are given by

$$k_1^* = -\frac{(t_1 t_2 + 1)}{b\left(1 + t_2^2\right)}, \quad k_2^* = \frac{t_3}{b} \;\; and \;\; k_3^* = 0,$$

its frenet frame can be written as

$$\begin{aligned} T^{*} &= et_3 \left(t_1 N + B_2\right), \\ N^{*} &= B_1 \\ B_1^{*} &= e\left(t_2 N + B_2\right) \\ B_2^{*} &= et_2 B_1. \end{aligned}$$

Case 2
$b = 0$, in this case (3.2) turns to

$$\phi^*(s) = \phi(s) + (a_0 - s)T(s). \tag{3.27}$$

Differentiate (3.27) with respect to s and using (2.1)

$$f^{'} T^{*} = (a_0 - s)k_1 N, \tag{3.28}$$

we suppose that

$$f^{'} = (s - a_0)k_1$$

$$T^{*} = -N. \tag{3.29}$$

Differentiate (3.29) with respect to s and using (2.1)

$$\begin{aligned} T^{*'} &= -N^{'} \\ f^{'} k_1^{*} N^{*} &= -k_2 B_1. \end{aligned}$$

Let

$$N^{*} = eB_1 \tag{3.30}$$

$$e = \frac{-k_2}{f' k_1^{*}}.$$

By differentiating (3.30) with respect to s we get

$$f^{'} k_2^{*} B_1^{*} = -ek_2 N + e^{'} B_1 + ek_3 B_2, \tag{3.31}$$

so by taking dot product on both-sides of (3.31) with B_1

$$\begin{aligned} \left\langle f^{'} k_2^{*} B_1^{*}, B_1 \right\rangle &= \left\langle -ek_2 N + e^{'} B_1 + ek_3 B_2, B_1 \right\rangle \\ 0 &= e^{'} \end{aligned}$$

which implies that e is a constant, therefore (3.31) turns to

$$f^{'} k_2^{*} B_1^{*} = -ek_2 N + ek_3 B_2, \tag{3.32}$$

let

$$p = \frac{ek_3}{f' k_2^{*}}, \quad q = \frac{-ek_2}{f' k_2^{*}} \tag{3.33}$$

$$\begin{aligned} B_1^{*} &= pB_2 + qN \\ p^2 + q^2 &= 1 \end{aligned}$$

from (3.33) we get

$$pk_2 + qk_3 = 0. \tag{3.34}$$

From (3.34) we get

$$\frac{k_3}{k_1} = -\frac{p}{q}\tau. \tag{3.35}$$

Let $\frac{e}{p} = t_1$, $e = pt_1$, from (3.33)

$$\begin{aligned} p &= \frac{ek_3}{f'k_2^*} \\ \frac{e}{t_1} &= \frac{ek_3}{f'k_2^*} \end{aligned}$$

$$f' k_2^* = t_1 k_3, \tag{3.36}$$

By differentiating (3.33) we get

$$f' k_3^* B_2^* = f' k_2^* N^* + q' N + (qk_2 - pk_3) B_1 + p' B_2, \tag{3.37}$$

so by taking dot product on both-sides of (3.37) with N and B_2

$$\begin{aligned} \left\langle f' k_3^* B_2^*, N \right\rangle &= \left\langle f' k_2^* N^* + q' N + (qk_2 - pk_3) B_1 + p' B_2, N \right\rangle \\ 0 &= q' \\ \left\langle f' k_3^* B_2^*, B_2 \right\rangle &= \left\langle f' k_2^* N^* + q' N + (qk_2 - pk_3) B_1 + p' B_2, B_2 \right\rangle \\ 0 &= p' \end{aligned}$$

which implies that p and q are constants, thus (3.37) turns to

$$f' k_3^* B_2^* = f' k_2^* N^* + (qk_2 - pk_3) B_1, \tag{3.38}$$

by substituting (3.33) and (3.36) in (3.38) we have

$$f' k_3^* B_2^* = k_1 \left\{ p\frac{k_3}{k_1} \left(t_1^2 - 1 \right) + q\frac{k_2}{k_1} \right\} B_1 = \frac{k_1 \tau}{q} \left(1 - e^2 \right) B_1. \tag{3.39}$$

We suppose that

$$f' k_3^* = k_1 \tau \left(e^2 - 1 \right) \tag{3.40}$$

$$B_2^* = -q^{-1} B_1$$

Summarising the above discussion, we obtain the following.

Theorem 2. *Let $\phi : I \subset R \to G_4$ be a regular parameterized curve and k_1, k_2 and k_3 are its curvatures $k_i \neq 0$. If ϕ has the $(0, 2)$-involute mate curve $\phi^*(s) = \phi(s) + (a_{0-}s)T(s)$, then k_2 and k_3 satisfy*

$$pk_2 + qk_3 = 0, \tag{3.41}$$

$$\frac{k_2}{k_1} = \tau, \quad \frac{k_3}{k_1} = -\frac{p}{q}\tau,$$

where a_0, p and q are given constants, moreover, the three curvatures of $\alpha^*(s)$ are given by

$$k_1^* = -\frac{k_2}{e\,(s - a_0)\,k_1}, \quad k_2^* = \frac{-pt_1\tau}{q\,(s - a_0)} \text{ and } k_3^* = \frac{\tau \left(e^2 - 1 \right)}{(s - a_0)},$$

its frenet frame can be written as

$$\begin{aligned} T^* &= -N \\ N^{*\prime} &= pt_1 B_1 \\ B_1^* &= qN + pB_2 \\ B_2^* &= -q^{-1} B_1. \end{aligned}$$

Remark 1. *From theorems 1 and 2 we can see that the above two cases are quite different with each other.*

4 References

[1] A. O. Ogrenmis, M. Ergut, M., Bektas, *On the helices in the Galilean space G3*. Iranian J. of sci. Tec, Trans. A., 31(2)(2007), 177-181.
[2] M. Bektas, M. Ergut, A. O. Ogrenmis, *Special curves of 4D in Galilean space* . arXiv:1111.0419v1 [math.DG] 2 Nov 2011.
[3] M. Hanif and Z. H. Hou, *Generalized involute and evolute curve-couple In Euclidean space*, Int. J. Open Problems Compt. Math., 11(2)(2018).
[4] E. Özyılmaz, S. Yılmaz, *Involute-Evolute curve couples in Euclidean 4-space*, Int. J. Open Problems Compt. Math., 2 (2)(2009).
[5] B. Bukcu, M. K. Karacan, *On the Involute and Evolute Curves of the Spacelike Curve with a Spacelike Binormal in Minkowski 3-Space*, Int. J. Contemp. Math. Sciences, (2)(5)(2007), 221– 232.
[6] M. Hanif , Z. H. Hou, K. S. Nisar, *On special kinds of involute and evolute curves in 4-dimensional Minkowski space*. Symmetry, (10)(317)(2018).
[7] U. Ozturk, E. B. K. Ozturk, K. Ilarslan, *On the involute-evolute of pseudonull curves in Minkowski 3-space*. J. Appl. Math., Volume 2013, Article ID 651495.
[8] M.Y. Yılmaz, M. Bektas, M. Ergut, *The pair of involute-evolute curves in Firsler manifold* F^3, Pure and Appl. Maths J. of Inst. of Kyrgyzstan National Academy, No. 39, Bişkek, (2008), 156-163.
[9] IM. Yaglom, *A Simple Non-Euclidean Geometry and Its Physical Basis*, Springer-Verlag, New York, 1979.
[10] M. Akyigit, A. Z. Azak, S. Ersoy, *Involute-Evolute Curves in Galilean Space G3*.Scientia Magna, 6(2010), 75-80.
[11] H. Öztekin, T. Körpinar, G. Altay, *Constant ratio curves in the four dimensional Galilean space*, South Asian J. of Maths Vol. 7 (2)(2017),148-154.
[12] M. E. Aydin, M. Ergut, *The equiform differential geometry of curves in 4-dimensional Galilean space G4*, Stud. Univ. Babeş-Bolyai Math., (58)(3)(2013), 393-400.
[13] H. B. Oztekin, S. Tatlıpınar, *On Some Curves in Galilean Plane and 3-Dimensional Galilean Space*. J. Dyn. Sys. Geom. Theo., 10(2012),189-196.
[14] S. Büyükkütük, I. Kisi, V. N. Mishra, G. Oztürk, *Some Characterizations of Curves in Galilean 3-space G3*, Facta Uni.(NIS), Ser. Math. Inform.(31)(2)(2016), 503-512.