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Existence Results for Fractional Integrodifferential Systems with Interval Impulse via Sectorial Operator

Kandasamy Malar[a], Annamalai Anguraj[b],

[a]Department of Mathematics, Erode Arts and Science College, Erode-638009, Tamil Nadu, India.
[b]Department of Mathematics, PSG College of Arts and Science, Coimbatore-641 014, Tamil Nadu, India.

Abstract

This paper focuses on the existence results for nonlocal fractional integrodifferential equations with interval impulses and measure of noncompactness using Mönch fixed point theorem and sectorial operator. At the end of this paper, an example is given to illustrate the applications of the abstract results.

Keywords: Fractional Integrodifferential equations, Interval impulse, Fixed point theorem, Sectorial operators, Measure of noncompactness.
2010 MSC: 26A33, 34A37, 34K37, 34A08.

1. Introduction

In this paper, we study the existence results for the following nonlocal problem of non-instantaneous impulsive fractional integrodifferential equations

$$ {}_{s_i}D_t^{\alpha} u(t) = Au(t) + f(t, u(t), Gu(t)), \quad t \in (s_i, t_{i+1}], \quad i = 0, 1, ..., m \tag{1.1}$$

$$ u(t) = g_i(t, u(t)), \quad t \in (t_i, s_i], \quad i = 1, 2, ..., m \tag{1.2}$$

$$ u(0) = u_0 + k(u) \tag{1.3}$$

where $0 < \alpha < 1$ and A is a sectorial operator on a Banach space $(X, \|.\|)$. ${}_{s_i}D_t^{\alpha}$ is the Caputo fractional derivative of the order α and f is a given function $f : [0, a] \times X \times X \to X$,

$$ Gu(t) = \int_0^t H(t, s)u(s)ds, \ H \in C[D, R^+], \ D = \{(t, s) \in R^2 : 0 \le s \le t \le a\}, $$

Email addresses: malarganesaneac@gmail.com (Kandasamy Malar), angurajpsg@yahoo.com (Annamalai Anguraj)

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$0 = t_0 = s_0 < t_1 \leq s_1 \leq t_2 \leq t_m \leq .... \leq s_m \leq t_{m+1} = a$ are pre-fixed numbers, $k : X \to X$, $u_0 \in X$ and $g_i \in C\big((t_i, s_i] \times X; X\big)$ for all $i = 1, 2, ..., m$.

The main techniques relay on the impulsive fractional integro- differential equation, Mönch fixed point theorem via measure of noncompactness.

In the past few years, many researchers have focused their research on the study of fractional differential equations defined on bounded and unbounded intervals. Which have been receiving greater attention due to various theoritical results obtained. For a detailed study see the monogrphs of several authors [35, 3, 33, 31, 17, 10]. These reults are applied in different fields of science and engineering such as physics, biology and chemistry etc. Byszewski [18, 22, 30] initiated in many cases a non-local conditions $u(0) + g(u) = u_0$ in treating physical problems. So which has been studied extensively by several authors for different kinds of problems see [29, 14, 5, 11, 6] and the references therein.

Recently a great attention has been focused on the study of impulsive differential equations for which the impulses are not intantaneous and is also an important area of research. They appear in mathematical models of phenomenon in both physical and social sciences. Also such problems appear in mathematical models with suddenly changes of their state in population dynamics, pharmacology, optimal control etc. [21] . Impulsive problems for fractional equations are treated by topological methods in [20, 25].

Integrodifferential equations are important for investigating some problems raised from natural phenomena. They have been studied in many different aspects. In recent years, this theory has been applied to a large class of non-linear differential equations in Banach spaces. For further details, see [26, 32, 23, 24, 4, 19]. Shaochun.Ji, Gang Li [29] investigated a unified approach to nonlocal impulsive differential equations with measure of non-compactness. In [4], Ahmad et al. studies nonlocal problem of impulsive integrodifferential equations with measure of non-compactness by applying a new fixed point theorem. For some recent results, see [2, 13, 12]. Eduardo Hernandez and Donal O'Regan [16] established on a new class of abstract impulsive differential equations for which the impulses are not instantaneous. Jin Rong Wang et al. [34] also proved periodic BVP integer/fractional order nonlinear differential equations with non-instantaneous impulses.

In a recent paper [1], Agarwal.R et al. studies stability by Lyapunov like functions of non linear differential equations with non- instantaneous impulses.

Motivated by above works, we derive the existence results for the mild solutions of (1.1)-(1.3) combining impulsive conditions and nonlocal conditions. Our results are obtained by means of Mönch fixed point theorem and the technique of Hausdorff measure of noncompactness. This paper is organized as follows:

In section 2, we introduce some defections, notations and some preliminary notions. In section 3, we present out main results on existence results. Example is presented in section 4 illustrating the applicability of the improved conditions.

2. Preliminaries

Let $C([0, a]; X)$ denote the Banach space of all continuous functions from $[0, a]$ into X with the norm $\|u\|_C := \sup\{|u(t)| : t \in [0, a]\}$ for $u \in C([0, a]; X)$. Now we consider the space
$PC([0, a]; X) := \{u : [0, a] \to X : u \in C((t_k, t_{k+1}], X), k = 0, 1, ..., m$ and there exist $u(t_k^-)$ and $u(t_k^+)$, $k = 1, ...m$, with $u(t_k^-) = u(t_k)\}$ with the norm$\|u\|_{PC}$:=sup$\{|u(t)| : t \in [0, a]\}$. Set $PC^1([0, a]; X) := \{u \in PC([0, a]; X) : u' \in PC([0, a]; X)$ with $\|u\|_{PC^1} : \max\{\|u\|_{PC}, \|u'\|_{PC}\}$.
Clearly, $PC^1([0, a], X)$ endowed with the norm $\|.\|_{PC^1}$ is a Banach space.

In this paper, β denotes the Hausdorff measure of noncompactness on both X and $PC([0, a]; X)$. The following lemma describes some properties of the Hausdorff measure of noncompactness.

Lemma 1. *([7]) Let $\mathbb{Y}$ be a real Banach space and $B, C \subseteq \mathbb{Y}$ be bounded. Then*

1. $\beta_{\mathbb{Y}}(B) = 0 \Leftrightarrow \overline{B}$ is compact (B is relatively compact) *;*
2. $\beta_{\mathbb{Y}}(B) = \beta_{\mathbb{Y}}(\overline{B}) = \beta_{\mathbb{Y}}(convB)$*, where $\overline{B}$ and conv B mean the closure and convex hull of B respectively;*
3. $\beta_{\mathbb{Y}}(B) \leq \beta_{\mathbb{Y}}(C)$*, where $B \subseteq C$;*

4. $\beta_{\mathbb{Y}}(B+C) \leq \beta_{\mathbb{Y}}(B) + \beta_{\mathbb{Y}}(C)$, *where* $B + C = \{x + y : x \in B, y \in C\}$*;*
5. $\beta_{\mathbb{Y}}(B \cup C) \leq \max\{\beta_{\mathbb{Y}}(B), \beta_{\mathbb{Y}}(C)\}$*;*
6. $\beta_{\mathbb{Y}}(\lambda B) \leq |\lambda|\beta_{\mathbb{Y}}(B)$ *for any* $\lambda \in \mathbb{R}$*;*
7. *If the map* $Q : D(Q) \subseteq \mathbb{Y} \to \mathbb{Z}$ is Lipschitz continuous with constant k, then $\beta_{\mathbb{Z}}(QB) \leq k\beta_{\mathbb{Y}}(B)$ *for any bounded subset* $B \subseteq D(Q)$*, where* $\mathbb{Z}$ *be a Banach space;*
8. *If* $\{W_n\}_{n=1}^{+\infty}$ is decreasing sequence of bounded closed nonempty subsets of $\mathbb{Y}$ and $\lim_{n\to\infty} \beta_{\mathbb{Y}}(W_n) = 0$, then $\bigcap_{n=1}^{+\infty} W_n$ is nonempty and compact in $\mathbb{Y}$.

The map $Q : W \subseteq \mathbb{Y} \to \mathbb{Y}$ is said to be a $\beta_{\mathbb{Y}}$-contraction if there exists a constant $0 < k < 1$ such that $\beta_{\mathbb{Y}}(Q(B)) \leq k\beta_{\mathbb{Y}}(B)$ for any bounded closed subset $B \subseteq W$, where $\mathbb{Y}$ is a Banach space.

Lemma 2. *([7]). If* $W \subseteq PC([0, a]; X)$ *is bounded, then* $\beta(W(t)) \leq \beta_{PC}(W)$ *for all* $t \in [0, a]$*, where* $W(t) = \{u(t) : u \in W\} \subseteq X$*. Furthermore if* W *is equicontinuous on [0,a], then* $\beta(W(t))$ *is continuous on [0,a], and* $\beta_{PC}(W) = sup\{\beta(W(t)) : t \in [0, a]\}$*.*

Lemma 3. *([27]). If* $\{u_n\}_{n=1}^{\infty} \subset L^1(0, K; X)$ *is uniformly integrable, then* $\beta(\{u_n(t)\}_{n=1}^{\infty})$ *is measurable and*

$$\beta\Big(\Big\{\int_0^t u_n(s)ds\Big\}_{n=1}^{\infty}\Big) \leq 2\int_0^t \beta\{u_n(s)\}_{n=1}^{\infty} ds$$

Lemma 4. *([9]) . If* W *is bounded, then for each* $\varepsilon > 0$*, there is a sequence* $\{u_n\}_{n=1}^{\infty} \subseteq W$ *such that* $\beta(W) \leq 2\beta(\{u_n\}_{n=1}^{\infty}) + \varepsilon$*.*

Lemma 5. *([7]. Darbo-Sadovskii). If* $W \subseteq Y$ *is bounded closed and convex, the continuous map* $Q : W \to W$ *is an* $\beta-$ *contraction, then the map* Q *has at least one fixed point in* W*.*

The following fixed point theorem, a nonlinear alternative of Mönch type, plays a key role of the system (1.1)-(1.3).

Theorem 1. *([27]) Let* D *be a bounded, closed and convex subset of a Banach space such that* $0 \in D$*, and let* Γ *be a continuous mapping of* D *into itself. If the implication*
$Q = \overline{conv}\Gamma(Q)$ *or* $Q = \Gamma(Q) \cup \{0\} \Rightarrow \beta(Q) = 0$ *holds for every subset* Q *of* D*, then* Γ *has a fixed point.*

Now we introduce some notations about sectorial operators, solution operators, and analytic solution operators.

An operator A is said to be sectorial if there are constants $\mu \in R, \theta \in [\frac{\pi}{2}, \pi], M > 0$ such that the following two conditions are satisfied:

$$\begin{cases} (1)\rho(A) \subset \sum_{\theta,\mu} = \{\lambda \in C : \lambda \neq \mu, |arg(\lambda - \mu)| < \theta\}, \\ (2)\|R(\lambda, A)\|_{L(X)} \leq \frac{M}{|\lambda-\mu|}, \qquad \lambda \in \sum_{\theta,\mu} \end{cases}$$

Consider the following Cauchy problem for the Caputo fractional derivative evolution equation of order $\alpha(m - 1 < \alpha < m, m > 0$ is an integer):

$$D_*^{\alpha} u(t) = Au(t), \quad u(0) = x, \quad u^{(k)}(0) = 0, \quad k = 1, 2, ..., m - 1, \tag{2.1}$$

where A is a sectorial operator. The solution operators $S_\alpha(t)$ of (2.1) is defined by (see [8])

$$S_\alpha(t) = \frac{1}{2\pi i}\int_\gamma e^{\lambda t}\lambda^{\alpha-1}R(\lambda^\alpha, A)d\lambda,$$

where γ is a suitable path lying on $\sum_{\theta,\mu}$.

An operator A is said to belong to $\zeta^\alpha(X;M,\mu)$, or $\zeta^\alpha(M,\mu)$ if problem (2.1) has a solution operator $S_\alpha(t)$ satisfying $\|S_\alpha(t)\| \leq Me^{\mu t}, t \geq 0$. Denote $\zeta^\alpha(\mu) := \cup\{\zeta^\alpha(M,\mu) : M \geq 1\}$ and $\zeta^\alpha := \cup\{\zeta^\alpha(\mu) : \mu \geq 0\}$.
Definition 2.1([8]). A solution operator $S_\alpha(t)$ of (2.1) is called analytic if $S_\alpha(t)$ admits an analytic extension to a sector $\sum_{\theta_0} := \{\lambda \in C \setminus \{0\} : |arg\ \lambda| < \theta_0\}$ for some $\theta_0 \in (0, \frac{\pi}{2}]$. An anaytic soluton operator is said to be of analyticity type (θ_0, μ_0) for each $\theta < \theta_0$ and $\mu > \mu_0$ there is an $M = M(\theta,\mu)$ such that $\|S_\alpha(t)\| \leq M^{\mu Ret}, t \in \sum_\theta := \{t \in C \setminus \{0\} : |arg\ t| < \theta$. Denote $\mathcal{A}^\alpha(\theta_0,\mu_0) := \{\mathcal{A} \in \zeta^\alpha : \mathcal{A}$ generates analytic solution operators $S_{\alpha(t)}$ of type $(\theta_0,\mu_0)\}$.

Lemma 6. *([9, 28]). Let $\alpha \in (0,2)$. A linear closed densely defined operator A belongs to $\mathcal{A}^\alpha(\theta_0,\mu_0)$ iff $\lambda^\alpha \in \rho(A)$ for each $\lambda \in \sum_{\theta_0+\pi/2}$, and for any $\mu > \mu_0, \theta < \theta_0$, there is a constant $C = C(\theta,\mu)$ such that*

$$\|\lambda^{\alpha-1}R(\lambda^\alpha, A)\| \leq \frac{C}{|\lambda-\mu|}, \quad \text{for } \lambda \in \sum_{\theta+\pi/2}(\mu). \tag{2.2}$$

Consider the following cauchy problem

$$\begin{cases} (D_t^\alpha u) = Au(t) + f(t), & 0 < \alpha < 1, \\ u(0) = x_0 \in X, \end{cases} \tag{2.3}$$

where f is an abstract function defined on $[0,\infty)$ and with values in X, A is a sectorial operator.

Theorem 2. *([31]) If f satisfies the uniform Hölder condition with exponent $\beta \in (0,1]$ and A is a sectorial operator, then the unique solution of the Cauchy problem (2.3) is given by*

$$u(t) = S_\alpha(t)x_0 + \int_0^t T_\alpha(t-s)f(s)ds, \tag{2.4}$$

where

$$S_\alpha(t) = \frac{1}{2\pi i}\int_\Gamma e^{\lambda t}\lambda^{\alpha-1}R(\lambda^\alpha, A)d\lambda, \quad T_\alpha(t) := \frac{1}{2\pi i}\int_\Gamma e^{\lambda t}R(\lambda^\alpha, A)d\lambda$$

and Γ is a suitable path lying on $\sum_{\theta,\mu}$.

Theorem 3. *If $\alpha \in (0,1)$ and $A \in \mathcal{A}^\alpha(\theta_0,\mu_0)$, then for any $x \in X$ and $t > 0$, we have*

$$\|T_\alpha(t)\| \leq Ce^{\mu t}(1 + t^{\alpha-1}), \quad t > 0, \mu > \mu_0. \tag{2.5}$$

Definition 2.2. ([15]) The fractional integral of order α with the lower limit s_i for a function $f(t)$ is defined as

$$_{s_i}I_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_{s_i}^t (t-s)^{\alpha-1}f(s)ds \quad t > s_i,\ \alpha > 0$$

provided the right side is point-wise defined on$[\alpha,\infty)$, where $\Gamma(.)$ is the gamma function.
Definition 2.3. ([15]) Riemann- Liouville derivative of order order α with the lower limit s_i for a function $f(t)$ is defined as

$$^L_{s_i}(D_t^\alpha f)(t) = \frac{1}{\Gamma(n-\alpha)}(\frac{d}{dt})^n \times \int_{s_i}^t (t-s)^{n-\alpha-1}f(s)ds,\ t > s_i\ n-1 < \alpha < n.$$

Definition 2.4. ([15])The Caputo derivative of order α for the function $f(t)$ can be written as

$$^c_{s_i}D_t^\alpha f(t) =^L_{s_i} D_t^\alpha\Big(h(t) - \sum_{k=0}^{n-1}\frac{(t-s_i)^k}{k!}f^{(k)}(s_i)\Big),\ t > s_i, n-1 < \alpha < n.$$

Remark 2.1
(1) If $f(t) \in C^n[s_i, \infty)$, then

$$\begin{aligned} {}^{L}_{s_i}(D_t^{\alpha} f)(t) &= \frac{1}{\Gamma(n-\alpha)} \int_{s_i}^{t} (t-s)^{n-\alpha-1} f^{(n)}(s) ds \\ &=_{s_i} I_t^{n-\alpha} f^{(n)}(t),\ t > s_i\ n-1 < \alpha < n. \end{aligned}$$

(2) The Caputo derivative of a constant is equal to zero.
To study the existence results for the problem (1.1) − (1.3), we need the following lemma.

Lemma 7. *A function u is given by*

$$u(t) = \begin{cases} S_\alpha(t)[u_0 - k(u)] + \int_0^t T_\alpha(t-\theta)h(\theta)d\theta, & t \in [0, t_1], \\ G_i(t), \quad t \in (t_i.s_i], \quad i = 1, 2, ..., m, \\ S_\alpha(t-s_i)G_i(s_i) + \int_{s_i}^t T_\alpha(t-\theta)h(\theta)d\theta, & t \in (s_i, t_{i+1}], \\ & i = 0, 1, ..., m \end{cases} \tag{2.6}$$

is a solution of the following problem

$$\begin{cases} {}_{s_i}D_t^{\alpha} u(t) = h(t), \quad t \in (s_i, t_{i+1}], \quad i = 0, 1, ..., m \\ u(t) = G_i(t), \quad t \in (t_i.s_i], \quad i = 1, 2, ..., m, \\ u(0) = u_0 + k(u) \end{cases} \tag{2.7}$$

Proof: Assume that u satisfies (2.7). If $t \in [0, t_1]$. Integrating the first equation of (2.7) from zero to t, we have

$$u(t) = S_\alpha(t)[u_0 - k(u)] + \int_0^t T_\alpha(t-\theta)h(\theta)d\theta \tag{2.8}$$

On the other hand, if $t \in (s_i, t_{i+1}]$, $i = 0, 1, ..., m$ and one can apply the impulsive condition of (2.7) to derive.

$$u(t) = S_\alpha(t-s_i)G_i(s_i) + \int_{s_i}^t T_\alpha(t-\theta)h(\theta)d\theta. \tag{2.9}$$

So, we have

$$u(0) = S_\alpha(t-s_m)G_m(s_m) + \int_{s_m}^t T_\alpha(t-\theta)h(\theta)d\theta. \tag{2.10}$$

It is clear that(2.8),(2.9) and (2.10) implies that (2.6).

Definition 2.5 A function $u : [0, a] \to X$ is called a mild solution of system(1.1) − (1.3) if $u \in PC([0, a], X)$ satisfies the following equation

$$u(t) = \begin{cases} S_\alpha(t)[u_0 - k(u)] + \int_0^t T_\alpha(t-\theta)f(\theta, u(\theta), Gu(\theta))d\theta, & t \in [0, t_1], \\ g_i(t, u(t)), \quad t \in (t_i.s_i], \quad i = 1, 2, ..., m, \\ S_\alpha(t-s_i)g_i(s_i, u(s_i)) + \int_{s_i}^t T_\alpha(t-\theta)f(\theta, u(\theta), Gu(\theta))d\theta, \\ \qquad t \in (s_i, t_{i+1}],\ i = 0, 1, ..., m \end{cases} \tag{2.11}$$

3. Main Results

In this section, we give the existence of mild solutions for the impulsive system (1.1) − (1.3). If $A \in \mathcal{A}^{\alpha}(\theta_0.\mu_0)$, then $\|S_\alpha(t)\| \le Me^{\mu t}$ and $\|T_\alpha(t)\| \le Ce^{\mu t}(1 + t^{\alpha-1})$
Let

$$\widetilde{M_S} := \sup_{0\le t\le a} \|S_\alpha(t)\|_{\mathcal{L}(X)}, \quad \widetilde{M_T} := \sup_{0\le t\le a} Ce^{\mu t}(1 + t^{1-\alpha}) \tag{3.1}$$

where $\mathcal{L}(X)$ is the Banach space of bounded linear operators from X into X equipped with its natural topology.
So we have $\|S_\alpha(t)\|_{\mathcal{L}(X)} \le \widetilde{M_S}, \quad \|T_{\alpha(t)}\|_{\mathcal{L}(X)} \le t^{\alpha-1}\widetilde{M_T}$.
For some real constants r and w, we define

$$W = \{u \in PC(X), \|u(t)\| \le r, \|Gu(t)\| \le w \quad \forall t \in [0, a]\}. \tag{3.2}$$

Now we introduce the following hypotheses:
(H_1) (i) The functions g_i are continuous and there are positive constants L_{g_i} such that $\|g_i(t, x) - g_i(t, y)\| \le L_{g_i}[\|x - y\|]$ for all $x, y \in X,\ t \in (t_i, s_i]$ and each $i = 0, 1, ..., m$.
(ii) There is a function $\varphi_i(t)$, i= 1,2,...m such that $\|g_i(t, x)\| \le \varphi_i(t)$ for each $t \in (t_i, s_i]$ and all $u \in X$. Setting $M_i = \sup_{t\in(t_i,si]} \varphi_i(t) < \infty, i = 1, 2, ..., m$.
(H_2)For $x \in X$, the function $f(., ., x)$ is strongly measurable on $[0, a]$ and $f(t, ., .) \in C(X, X)$ for $t \in [0, a]$, there are $m_f \in L^1([0, a]; R^+)$ and a non-decreasing function $W_f \in C([0.\infty); R^+)$ such that
$\|f(t, x, y)\| \le m_f(t)W_f(\|x\| + \|y\|)$, for all $(t, x, y) \in [0, a] \times X \times X$.
$(H_3)k : X \to X$ is continuous and there exists positive constants c and d such that $\|k(u) - k(v)\| \le c\|u - v\|$ and $\|k(u)\| \le c\|u\| + d$, for all $u \in PC(X)$.
(H_4) There is an integrable function $\eta : [0, a] \to [0, +\infty)$ such that
$\beta(f(t, D_1, D_2)) \le \eta(t)\big[\sup_{-\infty<\theta\le 0}\beta(D_1(\theta)) + \beta(D_2)\big]$ for a.e $t \in [0, a]$ and any bounded subsets $D_1, D_2 \subset X$ and β is the Hausdorff measure of noncompactness with $\zeta^* = \sup_{t\in[0,a]} \int_0^t \eta(s)ds < \infty$
(H_5) For each bounded subset $D \subset X$ we have $\beta(g_i(t, D)) \le M_i\beta(D)$,
$i = 1, 2, ..., m$.

Theorem 4. *Assume the hypotheses (H_1)- (H_3) are satisfied then the nonlocal impulsive problem (1.1)−(1.3) has at least one integral solution on $[0, a]$ provided that*

$$\widetilde{M_S}\,[c + L_{g_i}] < 1. \tag{3.3}$$

Proof. Let $\Gamma : PC(X) \to PC(X)$ be defined by

$\Gamma u(0) = u_0 + k(u), \Gamma u(t) = g_i(t, u(t))$ for $t \in (t_i, s_i]$ and

$$\Gamma u(t) = \begin{cases} S_\alpha(t)[u_0 - k(u)] + \int_0^t T_\alpha(t-\theta)f(\theta, u(\theta), Gu(\theta))d\theta, & t \in [0, t_1], \\ g_i(t, u(t)), \quad t \in (t_i.s_i], \quad i = 1, 2, ..., m, \\ S_\alpha(t - s_i)g_i(s_i, u(s_i) + \int_{s_i}^t T_\alpha(t-\theta)f(\theta, u(\theta), Gu(\theta))d\theta, \\ \qquad\qquad t \in (s_i, t_{i+1}],\ i = 0, 1, ..., m \end{cases} \tag{3.4}$$

Here Γ is well defined on $PC(X)$.

It is easy to see that the fixed point of Γ is the integral solution of the nonlocal impulsive problem (1.1) − (1.3). Subsequently we will prove that Γ has a fixed point by using Lemma 1.

First, we prove that the mapping Γ is continuous on $PC([0, a]; X)$. Let $\{u_n\}_{n=1}^{\infty}$ be a sequence in $PC([0, a]; X)$ with $\lim_{n\to\infty} u_n = u$ in $PC([0, a]; X)$. It follows that $f(s, u_n(s), Gu_n(s)) \to f(s, u(s), Gu(s))$

as $n \to \infty$.

Case 1. Now, for every $t \in [0, t_1]$, we have

$$\begin{aligned}\|\Gamma u_n(t) - \Gamma u(t)\| \le \widetilde{M_S}\|k(u_n) - k(u)\| \\ + \widetilde{M_T}\int_0^t (t-\theta)^{\alpha-1}\big\|f(\theta, u_n(\theta), Gu_n(\theta)) - f(\theta, u(\theta), Gu(\theta))\big\|d\theta \\ \le \widetilde{M_S}\|k(u_n) - k(u)\| + \frac{\epsilon T^\alpha \widetilde{M_T}}{\alpha}, \quad \epsilon > 0,\, \epsilon \to 0(n \to \infty).\end{aligned}$$

Case 2. For every $t \in (t_i, s_i]$, i=1,2,...,m, we get
$$\|\Gamma u_n(t) - \Gamma u(t)\| \le L_{g_i}\|u_n - u\|$$
Case 3. For each $t \in (s_i, t_{i+1}]$, i=1,2,...,m, we get

$$\|\Gamma u_n(t) - \Gamma u(t)\| \le \widetilde{M_S} L g_i \|u_n - u\| + \frac{\epsilon T^\alpha \widetilde{M_T}}{\alpha} \quad \epsilon > 0,\, \epsilon \to 0(n \to \infty)$$

for every $t \in (s_i, t_{i+1}]$.

Thus
$$\lim_{n\to\infty} \|\Gamma u_n - \Gamma u\| = 0$$

Which implies that the mapping Γ is continuous on $PC([0, a]; X)$.

Secondly, we claim that $\Gamma W \subseteq W$. It suffices to prove that for any $r > 0$ there exists $\gamma > 0$ such that $\|\Gamma u\| \le \gamma$ for each $u \in W \subseteq PC([0, a]; X)$, we have

Case 1. Now, for every $t \in [0, t_1]$, we get

$$\begin{aligned}\|\Gamma u(t)\| &\le \|S_\alpha(t)\|\|u_0 - k(u)\| + \int_0^t \|T_\alpha(t-\theta)\|\|f(\theta, u(\theta), Gu(\theta)\|d\theta \\ &\le \widetilde{M_S}[\|u_0\| + (c\|u\| + d)] + \frac{T^\alpha \widetilde{M_T}}{\alpha} m_f(\theta) W_f(r + w) \\ &\le \widetilde{M_S}[\|u_0\| + (cr + d)] + \frac{T^\alpha \widetilde{M_T}}{\alpha} m_f(\theta) W_f(r + w)\end{aligned}$$

Case 2. For every $t \in (t_i, s_i]$, i=1,2,...,m, we have

$$\begin{aligned}\|\Gamma u(t)\| &\le \|g_i(s_i, u(s_i))\| \\ &\le M_i\end{aligned}$$

Case 3. For each $t \in (s_i, t_{i+1}]$, i=1,2,...,m, we get

$$\begin{aligned}\|\Gamma u(t)\| &\le \|S_\alpha(t - s_i)\|\|g_i(s_i, u(s_i))\| + \int_{s_i}^t \|T_\alpha(t-\theta)\|\|f(\theta, u(\theta), Gu(\theta)\|d\theta \\ &\le \widetilde{M_S} M_i + \frac{T^\alpha \widetilde{M_T}}{\alpha} m_f(\theta) W_f(r + w)\end{aligned}$$

From the above, we obtain for $t \in [0, a]$,

$$\begin{aligned}\|\Gamma u(t)\| &\le \widetilde{M_S}[\|u_0\| + (cr + d) + M_i] + \frac{T^\alpha \widetilde{M_T}}{\alpha} m_f(\theta) W_f(r + w) \\ &\le \gamma\end{aligned}$$

which implies that $\Gamma : W \to W$ is a bounded operator.

Now according to Lemma 1, it remains to prove that Q is a β- contraction in W. By using condition (H_1) and (H_3), we get
Case 1. For every $t \in [0, t_1]$, we have

$$\Gamma u(t) = S_\alpha(t)[u_0 - k(u)] + \int_0^t T_\alpha(t-\theta) f(\theta, u(\theta), Gu(\theta)) d\theta$$

$$(\Gamma u)(t) = (\Gamma_1 u)(t) + (\Gamma_2 u)(t) \quad \text{with}$$

$$(\Gamma_1 u)(t) = S_\alpha(t)[u_0 - k(u)]$$
$$(\Gamma_2 u)(t) = \int_0^t T_\alpha(t-\theta) f(\theta, u(\theta), Gu(\theta)) d\theta$$

$$\begin{aligned}\|(\Gamma_1 u)(t) - (\Gamma_1 v)(t)\| &= \sup_{t\in[0,t_1]} \|S_\alpha(t)[k(u) - k(v)]\| \\ &\leq \widetilde{M_S}\, c \|u - v\|_{PC}\end{aligned}$$

Thus, by Lemma 1, we obtain that $\beta(\Gamma_1 W) \leq \widetilde{M_S} c \beta(W)$.

Now we prove that $\Gamma_2 : W \to PC([0, a]; X)$ is a compact operator by using Arzela-Ascoli's theorem. From this, we conclude that Γ_2 is compact. Thus $\beta(\Gamma_2 W) = 0$.
Case 2. For every $t \in (t_i, s_i]$, i=1,2,...,m, we get

$$\begin{aligned}\|(\Gamma u)(t) - (\Gamma v)(t)\| &\leq \|g_i(s_i, u(s_i)) - g_i(s_i, v(s_i))\| \\ &\leq L_{g_i} \|u - v\|\end{aligned}$$

Thus, by Lemma 1, we obtain that $\beta(\Gamma W) \leq L_{g_i}\ \beta(W)$.
Case 3. For each $t \in (s_i, t_{i+1}]$, $i = 1, 2, ..., m$, we get

$$(\Gamma u)(t) = S_\alpha(t - s_i) g_i(s_i, u(s_i)) + \int_{s_i}^t T_\alpha(t-\theta) f(\theta, u(\theta), Gu(\theta)) d\theta$$

$$(\Gamma u)(t) = (\Gamma_1 u)(t) + (\Gamma_2 u)(t) \quad \text{with}$$

$$(\Gamma_1 u)(t) = S_\alpha(t - s_i) g_i(s_i, u(s_i))$$
$$(\Gamma_2 u)(t) = \int_{s_i}^t T_\alpha(t-\theta) f(\theta, u(\theta), Gu(\theta)) d\theta$$

$$\begin{aligned}\|(\Gamma_1 u)(t) - (\Gamma_1 v)(t)\| &\leq \|S_\alpha(t - s_i)\| \|g_i(s_i, u(s_i)) - g_i(s_i, v(s_i))\| \\ &\leq \widetilde{M_S}\, L_{g_i} \|u - v\|_{PC}\end{aligned}$$

Thus, by Lemma 1, we obtain that $\beta(\Gamma_1 W) \leq \widetilde{M_S}\, L_{g_i}\ \beta(W)$.

Now we prove that $\Gamma_2 : W \to PC([0, a]; X)$ is a compact operator by using Arzela-Ascoli's theorem. From this, we conclude that Γ_2 is compact. Thus $\beta(\Gamma_2 W) = 0$.

From the above, we obtain for $t \in [0, a]$,

$$\beta(\Gamma W) \leq \widetilde{M_S}\, [c + L_{g_i}]\ \ \beta(W)$$

Since the condition (3.3), $\widetilde{M_S}\, [c + L_{g_i}] < 1$, the mapping Γ is an $\beta-$ contraction in W. By Darbo-Sadovskii's fixed point theorem, the operator Γ has a fixed point in W, which is the integral solution of the nonlocal impulsive problem$(1.1) - (1.3)$.

□

Theorem 5. *Asssume the hypotheses* (H_1) - (H_5) *are full filled, then the system* $(1.1)-(1.3)$ *has at least one mild solution provided that*

$$Z^* = M_i + 2\frac{T^\alpha \widetilde{M_T}}{\alpha}(1+k)\zeta^* < 1. \tag{3.5}$$

Proof. Define operator $\Gamma : PC(X) \to PC(X)$ by

$$\Gamma u(t) = \begin{cases} S_\alpha(t)[u_0 - k(u)] + \int_0^t T_\alpha(t-\theta)f(\theta, u(\theta), Gu(\theta))d\theta, \quad t \in [0, t_1], \\ g_i(t, u(t)), \quad t \in (t_i.s_i], \quad i = 1, 2, ..., m, \\ S_\alpha(t-s_i)g_i(s_i, u(s_i)) + \int_{s_i}^t T_\alpha(t-\theta)f(\theta, u(\theta), Gu(\theta))d\theta, \\ \qquad\qquad t \in (s_i, t_{i+1}], \ i = 0, 1, ..., m \end{cases} \tag{3.6}$$

Here Γ is well defined on $PC([0,a], X)$ and shows that the operator Γ satisfied the hypotheses of Theorem 1. The proof consists of following steps.
Step 1. Γ is continuous

Let $\{u_n\}$ be a sequence in $PC(X)$ such that $u_n \to u$ in $PC(X)$. Then $f(s, u_n(s), Gu_n(s)) \to f(s, u(s), Gu(s))$ as $n \to \infty$.
Case 1. Now, for every $t \in [0, t_1]$, we have

$$\|\Gamma u_n(t) - \Gamma u(t)\| \le \widetilde{M_S}\|k(u_n) - k(u)\| + \frac{\epsilon T^\alpha \widetilde{M_T}}{\alpha}, \quad \epsilon > 0, \ \epsilon \to 0 (n \to \infty).$$

Case 2. Now, for every $t \in (t_i, s_i]$,$i = 1, 2, ..., m$, we get
$\|\Gamma u_n(t) - \Gamma u(t)\| \le L_{g_i}\|u_n - u\|$.
Case 3. Now, for each $t \in (s_i, t_{i+1}]$, $i = 1, 2, ..., m$, we get

$$\|\Gamma u_n(t) - \Gamma u(t)\| \le \widetilde{M_S} L g_i \|u_n - u\| + \frac{\epsilon T^\alpha \widetilde{M_T}}{\alpha}, \quad \epsilon > 0, \ \epsilon \to 0(n \to \infty)$$

for every $t \in (s_i, t_{i+1}]$
Thus

$$\lim_{n\to\infty} \|\Gamma u_n - \Gamma u\| = 0.$$

Step 2. $\Gamma(W)$ is equicontinuous where W is defined by (3.2)
Case 1. For all $\tau_1, \tau_2 \in [0, t_1]$ and for each $\Gamma \in W(u)$ with $\tau_1 < \tau_2$, by $\|S_\alpha(t)\| \le Me^{\mu t}$, we have

$$\begin{aligned} \|(\Gamma u)(\tau_2) - (\Gamma u)(\tau_1)\| &\le M[\|u_0\| + (cr + d)]|e^{\mu\tau_2} - e^{\mu\tau_1}| \\ &\quad + m_f(\theta)W_f(r+w)\widetilde{M_T}\Big(\int_0^{\tau_2}(\tau_2-\theta)^{\alpha-1}d\theta - \int_0^{\tau_1}(\tau_2-\theta)^{\alpha-1}d\theta\Big) \\ &\le M[\|u_0\| + (cr+d)]|e^{\mu\tau_2} - e^{\mu\tau_1}| + \frac{m_f(\theta)W_f(r+w)\widetilde{M_T}(\tau_2^\alpha - \tau_1^\alpha)}{\alpha} \end{aligned}$$

Case 2. For every $\tau_1, \tau_2 \in (t_i, s_i]$, i=1,2,...,m, we have

$$\|(\Gamma u)(\tau_2) - (\Gamma u)(\tau_1)\| \le L_{g_i}\|u(\tau_2) - u(\tau_1)\|$$

Case 3. For each $\tau_1, \tau_2 \in (s_i, t_{i+1}]$, i=1,2,...,m, we get

$$\|(\Gamma u)(\tau_2) - (\Gamma u)(\tau_1)\| \le ML_{g_i}|e^{\mu\tau_2} - e^{\mu\tau_1}| + \frac{m_f(\theta)W_f(r+w)\widetilde{M_T}(\tau_2^\alpha - \tau_1^\alpha)}{\alpha}$$

From the above, we obtain for $t \in [0,a]$,

$$\lim_{\tau_1 \to \tau_2} \|(\Gamma u)(\tau_2) - (\Gamma u)(\tau_1)\| = 0$$

So, $\Gamma(W)$ is equicontinuous.

Step 3. Suppose that $Q \subseteq W$ is countable and $Q \subseteq \overline{conv}\big(\{0\} \cup \Gamma(Q)\big)$. We show that $\beta(Q) = 0$ where β is the Hausdorff measure of noncompactness. Without loss of generality, we may assume that $Q = \{u_n\}_{n=1}^{\infty}$ and we can easily verify that Q is bounded and equicontinuous.

Now we need to show that $\Gamma Q(t)$ is relatively compact in X for each $t \in [0,a]$.

Case 1. For each $t \in [0,t_1]$, we get

$$\begin{aligned}
\beta\Big(\{\Gamma u_n\}_{n=1}^{\infty}\Big) &\leq \beta\Big(\Big\{\int_0^t T_\alpha(t-\theta) f(\theta, u(\theta), Gu(\theta)) d\theta\Big\}_{n=1}^{\infty}\Big) \\
&\leq 2\frac{T^\alpha \widetilde{M_T}}{\alpha} \int_0^t \beta\Big(f(\theta, \{u_n(\theta)\}_{n=1}^{\infty}, G\{u_n(\theta)\}_{n=1}^{\infty}\Big) d\theta \\
&\leq 2\frac{T^\alpha \widetilde{M_T}}{\alpha} \int_0^t \eta(\theta)\Big(\beta\big(\{u_n(\theta)\}_{n=1}^{\infty}\big) + \beta\big(G\{u_n(\theta)\}_{n=1}^{\infty}\big)\Big) d\theta \\
&\leq 2\frac{T^\alpha \widetilde{M_T}}{\alpha} \int_0^t \eta(\theta)\Big(\big(\beta(Q(\theta))\big) + k\big(\beta(Q(\theta))\big)\Big) d\theta \\
&\leq 2\frac{T^\alpha \widetilde{M_T}}{\alpha}(1+k) \int_0^t \eta(s) ds \quad \big(\beta(Q(s))\big)
\end{aligned}$$

That is

$$\beta_{PC}\big(\Gamma Q\big) \leq 2\frac{T^\alpha \widetilde{M_T}}{\alpha}(1+k)\zeta^* \quad \big(\beta_{PC}(Q(s))\big)$$

Case 2. For each $t \in (t_i, s_i]$, i=1,2,...,m, we have

$$\begin{aligned}
\beta\Big(\{\Gamma u_n\}_{n=1}^{\infty}\Big) &\leq \beta(g_i(s_i, u(s_i))) \\
&\leq M_i\Big(\beta\{u_n(s_i)\}_{n=1}^{\infty}\Big)
\end{aligned}$$

That is

$$\beta_{PC}(\Gamma Q) \leq M_i\Big(\beta_{PC}(Q(s))\Big)$$

Case 3. Now, for each $t \in (s_i, t_{i+1}]$, i=1,2,...,m, we get

$$\begin{aligned}
\beta\Big(\{\Gamma u_n\}_{n=1}^{\infty}\Big) &\leq \beta\Big(\Big\{\int_{s_i}^t T_\alpha(t-\theta) f(\theta, u(\theta), Gu(\theta)) d\theta\Big\}_{n=1}^{\infty}\Big) \\
&\leq 2\frac{T^\alpha \widetilde{M_T}}{\alpha} \int_{s_i}^t \beta\Big(f(\theta, \{u_n(\theta)\}_{n=1}^{\infty}, G\{u_n(\theta)\}_{n=1}^{\infty}\Big) d\theta \\
&\leq 2\frac{T^\alpha \widetilde{M_T}}{\alpha} \int_{s_i}^t \eta(\theta)\Big(\beta\big(\{u_n(\theta)\}_{n=1}^{\infty}\big) + \beta\big(G\{u_n(\theta)\}_{n=1}^{\infty}\big)\Big) d\theta \\
&\leq 2\frac{T^\alpha \widetilde{M_T}}{\alpha} \int_{s_i}^t \eta(\theta)\Big(\big(\beta(Q(\theta))\big) + k\big(\beta(Q(\theta))\big)\Big) d\theta \\
&\leq 2\frac{T^\alpha \widetilde{M_T}}{\alpha}(1+k) \int_{s_i}^t \eta(s) ds \quad \Big(\beta(Q(s))\Big)
\end{aligned}$$

That is

$$\beta_{PC}\big(\Gamma Q\big) \leq 2\frac{T^{\alpha}\widetilde{M_T}}{\alpha}(1+k)\zeta^* \ \ \Big(\beta_{PC}(Q(s))\Big)$$

From the above, we obtain for $t \in [0,a]$,

$$\beta_{PC}\big(\Gamma Q\big) \leq \Big(M_i + 2\frac{T^{\alpha}\widetilde{M_T}}{\alpha}(1+k)\zeta^*\Big)\beta_{PC}(Q)$$

That is, $\beta_{PC}\big(\Gamma Q\big) \leq Z^*\beta(Q)$, where Z^* is defined by condition (3.5).

Thus from Mönch condition,we get

$\beta_{PC}(Q) \leq \beta_{PC}\Big(\overline{conv}\{0\} \cup \Gamma(Q)\Big) = \beta_{PC}(\Gamma(Q)) \leq Z^*\beta_{PC}(Q)$ which implies that $\beta_{PC}(Q) = 0$.

Hence using Theorem 1, Γ has a fixed point u in W. Which is a mild solution of $(1.1) - (1.3)$. This completes the proof. □

Example 1. *Consider a nonlocal problem of fractional impulsive integrodifferential equations given by*

$$\frac{\partial^{\alpha} u(t,w)}{\partial t} = \frac{\partial^2 u(t,w)}{\partial w^2} - \mu u(t,w) + \int_0^t h(t,s,u(s,w))ds + F(t,u(t,w)), \qquad (3.7)$$
$$(t,w) \in \cup_{i=1}^m [s_i, t_{i+1}] \times [0,\pi],$$

$$u(t,0) = u(t,\pi) = 0, \quad t \in [0,a], \qquad (3.8)$$

$$u(0,w) + \sum_{j=1}^m c_j u(t_j,w) = u_0(w), \ \ w \in [0,\pi], \qquad (3.9)$$

$$u(t,w) = G_i(t,u(t,w)), \quad w \in [0,\pi], \ \ t \in (t_i,s_i], \qquad (3.10)$$

with $0 = t_0 = s_0 < t_1 \leq s_1 \leq t_2 < ..... < t_m \leq s_m \leq t_{m+1} = a$ *are fixed real numbers, and* $u_0 \in X$, $F \in C([0,b] \times R; R)$ *and* $G_i \in C((t_i,s_i] \times R; R)$ *for all* $i = 1,...,m$.

Let $X = L^2([0,\pi],R)$ *and the operator* A *defined on* X *by* $Ax = x'' - vx$, $(v > 0)$ *with the domain* $D(A) = \{x \in X : x'' \in X, x(0) = x(\pi) = 0\}$. *It is well known that* $\Delta x = x''$ *is the infinitesimal generator of a non-compact semigroup* $\{T(t)\}_{t\geq 0}$ *on* X *and* $\beta(T(t)D) \leq \beta(D)$, *where* β *is the Hausdorff measure of non compactness. Hence* A *is sectorial of type* $\mu = -v < 0$.

To represent the problem $(3.7) - (3.10)$ *in the abstract form* $(1.1) - (1.3)$, *we assume that*
(i) $f : [0,a] \times X \to X$ *defined by* $f(t,x)(w) = \int_0^t h(t,s,x(w))ds + F(t,x(w))$, *for* $t \in [0,a], w \in [0,\pi]$.

(ii) $k : PC([0,a];X) \to X$ *is continuous function defined by* $k(u)(w) = u_0(w) - \sum_{i=1}^m c_j u(t_j)(w)$, $t \in [0,b]$, $w \in [0,\pi]$, *where* $u(t)(w) = u(t,w), t \geq 0$, $w \in [0,\pi]$.

(iii) $g_i : (t_i,s_i] \times X \to X$ *defined by* $g_i(t,x)(w) = G_i(t,x(w))$.

Now, we say that $u \in PC(X)$ *is a mild solution of* $(3.7)-(3.10)$ *if u(.) is a mild solution of the associated abstract problem* $(1.1) - (1.3)$.

Conclusion 1. *We studied a new class of fractional integrodifferential systems with interval impulses in Banach spaces. More precisely, by using Sectorial operator, Darbo-sadovskii and Mönch fixed point theorems combined with the Hausdorff measure of noncompactness techniques, we investigated the existence of mild solutions of the impulsive fractional integrodifferential equations. Finally, an application is provided to illustrate the applicability of the new results.*

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