Results in Nonlinear Analysis 2 (2019) No. 4, 182–192
Available online at www.nonlinear-analysis.com

Results in Nonlinear Analysis
ISSN 2636-7556

Peer Reviewed Scientific Journal

Symmetry analysis and some new exact solutions of the Newell-Whitehead-Segel and Zeldovich equations

Abdullahi Yusuf[a], Behzad Ghanbari[b], Sania Qureshi[c], Mustafa Inc[d], Dumitru Baleanu[e],

[a]*Department of Mathematics, Federal University Dutse, Jigawa, Nigeria.*
[b]*Department of Engineering Science, Kermanshah University of Technology, Kermanshah, Iran.*
[c]*Department of Basic Sciences and Related Studies, Mehran University of Engineering and Technology, 76062 Jamshoro, Pakistani.*
[d]*Department of Mathematics, Firat University Turkey, Elazig, Turkey.*
[e]*Department of Mathematics, Cankaya University, Ankara, Turkey.*
Institute of Space Sciences, Magurele, Bucharest, Romania

Abstract

The present study offers an overview of Newel-Whitehead-Segel (NWS) and Zeldovich equations (ZEE) equations by Lie symmetry analysis and generalizes rational function methods of exponential function. Some novel complex and real-valued exact solutions for the equations under consideration are presented. Using a new conservation theorem, we construct conservation laws for the ZEE equation. The physical expression for some of the solutions is presented to shed more light on the mechanism of the solutions.

Keywords: Newell-Whitehead-Segel equation Zeldovich equation symmetries conservation laws
2010 MSC:

1. Introduction

Analysis of nonlinear partial differential equation (NLPDEs) has been very potent owing to their expansive range of applicability in various field of science, engineering, and dynamical systems. In the field of science, nonlinear physical phenomena is one of the most significant area of study and come into being in various scientific areas of study in engineering, such as, plasma physics, fluid mechanics, gas dynamics, elasticity, relativity, chemical reactions, ecology, optical fiber, solid state physics, biomechanics to mention few. All these equations are basically restrained by NLPDEs [1]-[6].

Email addresses: yusufabdullahi@fud.edu.ng (Abdullahi Yusuf), b.ghanbary@yahoo.com (Behzad Ghanbari), sania.qureshi@faculty.muet.edu.pk (Sania Qureshi), minc@firat.edu.tr (Mustafa Inc), dumitru@cankaya.edu.tr (Dumitru Baleanu)

Received October 18, 2019, Accepted: November 23, 2019, Online December 08, 2019.

It is imperative that numerous equations in engineering and science possess an empirical parameters. Therefore, establishing exact solutions gives liberty to researchers to design and run experiments, by constructing convenient or natural conditions, to evaluate such parameters. Therefore, analysis and obtaining exact travelling wave solutions is becoming more fascinating in nonlinear sciences. Moreover, obtaining exact solutions to NLPDEs gives us the liberty to present information on the characteristics of a complex physical phenomenon. Several analytical approaches have been used to establish travelling wave solutions for NLPDEs [7]-[13].

Furthermore, it is popularly known that conservation laws and symmetries exhibit numerous information on the real physical phenomena modelled by the differential equations. The symmetries are more vital in establishing solutions and conservation laws of differential equations. Conservation laws affirm the internal properties, integrability and proving existence and uniqueness of solutions. Motivated by the aforementioned importance for the conservation laws, symmetries and exact solutions, we construct and investigate the exact solutions, symmetries and conservation laws for some reaction diffusion equations.

2. Description of the method

Let us state the main steps of GERFM as follows [19]:

1. Let us take into account the NPDE in the form:

$$\mathcal{L}(\psi, \psi_x, \psi_t, \psi_{xx}, \ldots) = 0. \tag{1}$$

   Using the transformations $\psi = \psi(\xi)$ and $\xi = \sigma x - lt$, we reduce the nonlinear partial differential equation to the following ordinary differential equation:

$$\mathcal{L}(\psi, \psi', \psi'', \ldots) = 0, \tag{2}$$

   where the values of σ and l will be found later.

2. Consider Eq. (2) has the solution of the form

$$\psi(\xi) = A_0 + \sum_{k=1}^{M} A_k \Theta(\xi)^k + \sum_{k=1}^{M} B_k \Theta(\xi)^{-k}. \tag{3}$$

   where

$$\Theta(\xi) = \frac{p_1 e^{q_1 \xi} + p_2 e^{q_2 \xi}}{p_3 e^{q_3 \xi} + p_4 e^{q_4 \xi}}. \tag{4}$$

   The values of constants $p_i, q_i (1 \le i \le 4)$, A_0, A_k and $B_k (1 \le k \le M)$ are determined, in such a way that solution (3) always persuade Eq. (2). By considering the homogenous balance principle the value of M is determined.

3. Putting Eq. (3) into Eq. (2) and collecting all terms, left-hand side of Eq. (2) give us an algebraic equation $P(Z_1, Z_2, Z_3, Z_4) = 0$ in terms of $Z_i = e^{q_i \xi}$ for $i = 1, \ldots, 4$. Setting each coefficient of P to zero, a system of nonlinear equations in terms of $p_i, q_i (1 \le i \le 4)$, and σ, l, A_0, A_k and $B_k (1 \le k \le M)$ is generated.

4. By solving the above system of equations using any symbolic computation software, the values of $p_i, q_i (1 \le i \le 4)$, A_0, A_k, and $B_k (1 \le k \le M)$ are determined, replacing these values in Eq. (3) provides us the soliton solutions of Eq.(1).

3. Exact solutions of the NWSE

In this section, we study the NWSE equation is given by [17]:

$$u_t + p u_{zz} + qu + ru^3 = 0. \tag{5}$$

Here, we assume that the formal solutions of Eq. (5) can be written as follows:

$$u = \mathcal{U}(\xi), \quad \xi = \mu\left(z - \kappa t\right), \tag{6}$$

where μ and κ are arbitrary constants to be determined.
Utilizing the wave transformation (6) in Eq. (5), the following nonlinear ODE is obtained as

$$-\mu\kappa\mathcal{U}' + p\mu^2\mathcal{U}'' + qU + rU^3 = 0, \tag{7}$$

Applying the balance principle on the terms of $\mathcal{U}''$ and $\mathcal{U}^3$ in Eq. (7) gives $3M = M + 2$, so $M = 1$.
Using $M = 2$ along with Eqs.(3) and (4), one gets:

$$\mathcal{U}(\xi) = A_0 + A_1\Theta(\xi) + B_1\Theta^{-1}(\xi). \tag{8}$$

Using a methodology similar to the one adopted in Subsection 2, we get some solutions of (5), as bellows:
Case 1: We attain $p = [0, -1, 1, 1]$ and $q = [0, 0, 1, 0]$, which gives

$$\Theta\left(\xi\right) = -\frac{1}{1 + e^{\xi}}. \tag{9}$$

We also obtain

$$\mu = \sqrt{\frac{q}{2p}}, \kappa = -3\sqrt{\frac{pq}{2}}, A_0 = 0, A_1 = \sqrt{\frac{-q}{r}}, B_1 = 0.$$

Putting values in Eqs.(8) and (9), one gets

$$\mathcal{U}\left(\xi\right) = \sqrt{\frac{-q}{r}}\frac{1}{\left(1 + \mathrm{e}^{\xi}\right)}.$$

In this case, Eq. (5) has the exact solutions:

$$u_1\left(z, t\right) = \sqrt{\frac{-q}{r}}\frac{1}{\left(1 + \mathrm{e}^{\sqrt{\frac{q}{2p}}\left(z + 3\sqrt{\frac{pq}{2}}t\right)}\right)} \tag{10}$$

Case 2: We attain $p = [0, 1, 1, 1]$ and $q = [0, 1, 0, 1]$, which gives

$$\Theta\left(\xi\right) = \frac{\mathrm{e}^{\xi}}{1 + \mathrm{e}^{\xi}}. \tag{11}$$

We also obtain

$$\mu = \sqrt{\frac{q}{2p}}, \kappa = 3\sqrt{\frac{pq}{2}}, A_0 = 0, A_1 = \sqrt{\frac{-q}{r}}, B_1 = 0.$$

Putting values in Eqs.(8) and (11), one gets

$$\mathcal{U}\left(\xi\right) = \sqrt{\frac{-q}{r}}\frac{\mathrm{e}^{\xi}}{\left(1 + \mathrm{e}^{\xi}\right)}.$$

In this case, Eq. (5) has the exact solutions:

$$u_2\left(z, t\right) = \sqrt{\frac{-q}{r}}\frac{\mathrm{e}^{\sqrt{\frac{q}{2p}}\left(z - 3\sqrt{\frac{pq}{2}}t\right)}}{\left(1 + \mathrm{e}^{\sqrt{\frac{q}{2p}}\left(z - 3\sqrt{\frac{pq}{2}}t\right)}\right)} \tag{12}$$

Case 3: We attain $p = [0, 1, 1, 1]$ and $q = [0, 1, 0, 1]$, which gives

$$\Theta(\xi) = \frac{e^{\xi}}{1 + e^{\xi}}. \tag{13}$$

We also obtain

$$\mu = \frac{1}{2}\sqrt{\frac{q}{2p}}, \kappa = 3\sqrt{\frac{pq}{2}}, A_0 = \frac{1}{2}\sqrt{\frac{-q}{r}}, A_1 = \frac{1}{2}\sqrt{\frac{-q}{r}}, B_1 = 0.$$

Putting values in Eqs.(8) and (13), one gets

$$\mathcal{U}(\xi) = \frac{1}{2}\sqrt{\frac{-q}{r}}\left(\frac{\cosh(\xi) + \sinh(\xi)}{\cosh(\xi)}\right).$$

In this case, Eq. (5) has the exact solutions:

$$u_3(z,t) = \frac{1}{2}\sqrt{\frac{-q}{r}}\left(\frac{\cosh\left(\frac{1}{2}\sqrt{\frac{q}{2p}}\left(z - 3\sqrt{\frac{pq}{2}}t\right)\right) + \sinh\left(\frac{1}{2}\sqrt{\frac{q}{2p}}\left(z - 3\sqrt{\frac{pq}{2}}t\right)\right)}{\cosh\left(\frac{1}{2}\sqrt{\frac{q}{2p}}\left(z - 3\sqrt{\frac{pq}{2}}t\right)\right)}\right). \tag{14}$$

Case 4: We attain $p = [1, 1, -1, 1]$ and $q = [1, -1, 1, -1]$, which gives

$$\Theta(\xi) = -\frac{\cosh(\xi)}{\sinh(\xi)}. \tag{15}$$

We also obtain

$$\mu = \frac{1}{4}\sqrt{\frac{q}{2p}}, \kappa = -3\sqrt{\frac{pq}{2}}, A_0 = \frac{1}{2}\sqrt{\frac{-q}{r}}, A_1 = \frac{1}{4}\sqrt{\frac{-q}{r}}, B_1 = \frac{1}{4}\sqrt{\frac{-q}{r}}.$$

Putting values in Eqs.(8) and (15), one gets

$$\mathcal{U}(\xi) = -\frac{1}{4}\sqrt{\frac{-q}{r}}\frac{(\coth(\xi) - 1)^2}{\coth(\xi)}.$$

In this case, Eq. (5) has the exact solutions:

$$u_4(z,t) = -\frac{1}{4}\sqrt{\frac{-q}{r}}\frac{\left(\coth\left(\frac{1}{4}\sqrt{\frac{q}{2p}}\left(z + 3\sqrt{\frac{pq}{2}}t\right)\right) - 1\right)^2}{\coth\left(\frac{1}{4}\sqrt{\frac{q}{2p}}\left(z + 3\sqrt{\frac{pq}{2}}t\right)\right)}. \tag{16}$$

Case 5: We attain $p = [-1, 3, 1, -1]$ and $q = [1, -1, 1, -1]$, which gives

$$\Theta(\xi) = \frac{\cosh(\xi) - 2\sinh(\xi)}{\sinh(\xi)}. \tag{17}$$

We also obtain

$$\mu = \frac{1}{2}\sqrt{\frac{q}{2p}}, \kappa = -3\sqrt{\frac{pq}{2}}, A_0 = \frac{1}{2}\sqrt{\frac{-q}{r}}, A_1 = \frac{1}{2}\sqrt{\frac{-q}{r}}, B_1 = 0.$$

Putting values in Eqs.(8) and (17), one gets

$$\mathcal{U}(\xi) = \frac{1}{2}\sqrt{\frac{-q}{r}}\left(\frac{\cosh(\xi) - \sinh(\xi)}{\sinh(\xi)}\right).$$

In this case, Eq. (5) has the exact solutions:

$$u_5(z,t) = \frac{1}{2}\sqrt{\frac{-q}{r}}\left(\frac{\cosh\left(\frac{1}{2}\sqrt{\frac{q}{2p}}\left(z + 3\sqrt{\frac{pq}{2}}t\right)\right) - \sinh\left(\frac{1}{2}\sqrt{\frac{q}{2p}}\left(z + 3\sqrt{\frac{pq}{2}}t\right)\right)}{\sinh\left(\frac{1}{2}\sqrt{\frac{q}{2p}}\left(z + 3\sqrt{\frac{pq}{2}}t\right)\right)}\right). \tag{18}$$

Case 6: We attain $p = [1, 1, 1, -1]$ and $q = [1, -1, 1, -1]$, which gives

$$\Theta(\xi) = \Theta(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)}. \tag{19}$$

We also obtain

$$\mu = \frac{1}{4}\sqrt{\frac{q}{2p}}, \kappa = 3\sqrt{\frac{pq}{2}}, A_0 = \frac{1}{2}\sqrt{\frac{-q}{r}}, A_1 = \frac{1}{4}\sqrt{\frac{-q}{r}}, B_1 = \frac{1}{4}\sqrt{\frac{-q}{r}}.$$

Putting values in Eqs.(8) and (19), one gets

$$\mathcal{U}(\xi) = \frac{1}{2}\sqrt{\frac{-q}{r}}\frac{\left(2\cosh^2(\xi) + 2\cosh(\xi)\sinh(\xi) - 1\right)}{\sinh(\xi)\cosh(\xi)}.$$

In this case, Eq. (5) has the exact solutions:

$$u_6(z,t) = \frac{1}{2}\sqrt{\frac{-q}{r}}\frac{\left(2\cosh^2\left(\frac{1}{4}\sqrt{\frac{q}{2p}}\left(z - 3\sqrt{\frac{pq}{2}}t\right)\right)\right) + \mathbf{A}}{\sinh\left(\frac{1}{4}\sqrt{\frac{q}{2p}}\left(z - 3\sqrt{\frac{pq}{2}}t\right)\right)\cosh\left(\frac{1}{4}\sqrt{\frac{q}{2p}}\left(z - 3\sqrt{\frac{pq}{2}}t\right)\right)}, \tag{20}$$

where $\mathbf{A} = 2\cosh\left(\frac{1}{4}\sqrt{\frac{q}{2p}}\left(z - 3\sqrt{\frac{pq}{2}}t\right)\right)\sinh\left(\frac{1}{4}\sqrt{\frac{q}{2p}}\left(z - 3\sqrt{\frac{pq}{2}}t\right)\right) - 1$.

4. Exact solutions of the ZE

In this section, we study the NWSE equation is given by [17]:

$$u_t + pu_{zz} + qu^2 + ru^3 = 0. \tag{21}$$

Here, we assume that the formal solutions of Eq. (5) can be written as follows:

$$u = \mathcal{U}(\xi), \quad \xi = \mu(z - \kappa t), \tag{22}$$

where μ and κ are arbitrary constants to be determined.
Utilizing the wave transformation (22) in Eq. (21), the following nonlinear ODE is obtained

$$-\mu\kappa\mathcal{U}' + p\mu^2\mathcal{U}'' + qU^2 + rU^3 = 0, \tag{23}$$

Applying the balance principle on the terms of $\mathcal{U}''$ and $\mathcal{U}^3$ in Eq. (23) gives $3M = M + 2$, so $M = 1$.
Using $M = 1$ along with Eqs.(3) and (4), one gets:

$$\mathcal{U}(\xi) = A_0 + A_1\Theta(\xi) + B_1\Theta^{-1}(\xi). \tag{24}$$

Using a methodology similar to the one adopted in Subsection 2, we get some solutions of (5), as bellows:
Case 1: We attain $p = [1, 2, 1, 1]$ and $q = [0, 1, 0, 1]$, which gives

$$\Theta(\xi) = \frac{1 + 2\,\mathrm{e}^{\xi}}{1 + \mathrm{e}^{\xi}}. \tag{25}$$

We also obtain

$$\mu = \frac{q}{\sqrt{2pr}}, \kappa = -q\sqrt{\frac{p}{2r}}, A_0 = -\frac{2q}{r}, A_1 = \frac{q}{r}, B_1 = 0.$$

Putting values in Eqs.(24) and (25), one gets

$$\mathcal{U}(\xi) = -\frac{q}{r(1 + \mathrm{e}^{\xi})}.$$

In this case, Eq. (21) has the exact solutions:

$$u_1(z,t) = -\frac{q}{r\left(1+\mathrm{e}^{\frac{q}{\sqrt{2pr}}\left(z+q\sqrt{\frac{p}{2r}}t\right)}\right)}. \tag{26}$$

Case 2: We attain $p = [-1, 0, 1, 1]$ and $q = [0, 0, 0, 1]$, which gives

$$\Theta(\xi) = -\frac{1}{1+\mathrm{e}^{\xi}}. \tag{27}$$

We also obtain

$$\mu = \frac{q}{\sqrt{2pr}}, \kappa = q\sqrt{\frac{p}{2r}}, A_0 = -\frac{q}{r}, A_1 = -\frac{q}{r}, B_1 = 0.$$

Putting values in Eqs.(24) and (11), one gets

$$\mathcal{U}(\xi) = -\frac{q\mathrm{e}^{\xi}}{r(1+\mathrm{e}^{\xi})}.$$

In this case, Eq. (21) has the exact solutions:

$$u_2(z,t) = -\frac{q\mathrm{e}^{\frac{q}{\sqrt{2pr}}\left(z-q\sqrt{\frac{p}{2r}}t\right)}}{r\left(1+\mathrm{e}^{\frac{q}{\sqrt{2pr}}\left(z-q\sqrt{\frac{p}{2r}}t\right)}\right)}. \tag{28}$$

Case 3: We attain $p = [-1, 3, 1, -1]$ and $q = [1, -1, 1, -1]$, which gives

$$\Theta(\xi) = \frac{\cosh(\xi) - 2\sinh(\xi)}{\sinh(\xi)}. \tag{29}$$

We also obtain

$$\mu = \frac{q}{2\sqrt{2pr}}, \kappa = -q\sqrt{\frac{p}{2r}}, A_0 = \frac{2q}{r}, A_1 = \frac{q}{2r}, B_1 = 0.$$

Putting values in Eqs.(24) and (29), one gets

$$\mathcal{U}(\xi) = \frac{q}{2r}\left(\frac{\cosh(\xi) - \sinh(\xi)}{\sinh(\xi)}\right).$$

In this case, Eq. (5) has the exact solutions:

$$u_3(z,t) = \frac{q}{2r}\left(\frac{\cosh\left(\frac{q}{2\sqrt{2pr}}\left(z+q\sqrt{\frac{p}{2r}}t\right)\right) - \sinh\left(\frac{q}{2\sqrt{2pr}}\left(z+q\sqrt{\frac{p}{2r}}t\right)\right)}{\sinh\left(\frac{q}{2\sqrt{2pr}}\left(z+q\sqrt{\frac{p}{2r}}t\right)\right)}\right). \tag{30}$$

Case 4: We attain $p = [-1, 3, 1, -1]$ and $q = [1, -1, 1, -1]$, which gives

$$\Theta(\xi) = \frac{\cosh(\xi) + \sinh(\xi)}{\sinh(\xi)}. \tag{31}$$

We also obtain

$$\mu = \frac{q}{\sqrt{2pr}}, \kappa = q\sqrt{\frac{p}{2r}}, A_0 = -\frac{2q}{r}, A_1 = \frac{q}{r}, B_1 = 0.$$

Putting values in Eqs.(24) and (31), one gets

$$\mathcal{U}(\xi) = -\frac{q}{2r}\left(\frac{\cosh(\xi) + \sinh(\xi)}{\sinh(\xi)}\right)..$$

In this case, Eq. (21) has the exact solutions:

$$u_4(z,t) = -\frac{q}{2r}\left(\frac{\cosh\left(\frac{q}{\sqrt{2pr}}\left(z - q\sqrt{\frac{p}{2r}}t\right)\right) + \sinh\left(\frac{q}{\sqrt{2pr}}\left(z - q\sqrt{\frac{p}{2r}}t\right)\right)}{\sinh\left(\frac{q}{\sqrt{2pr}}\left(z - q\sqrt{\frac{p}{2r}}t\right)\right)}\right). \tag{32}$$

Case 5: We attain $p = [i, -i, 1, 1]$ and $q = [i, -i, i, -i]$, which gives

$$\Theta(\xi) = \Theta(\xi) = -\frac{\sinh(\xi)}{\cosh(\xi)}. \tag{33}$$

We also obtain

$$\mu = \frac{q}{4\sqrt{2pr}}, \kappa = -q\sqrt{\frac{p}{2r}}, A_0 = -\frac{2q}{r}, A_1 = -\frac{iq}{r}, B_1 = \frac{iq}{r}.$$

Putting values in Eqs.(24) and (33), one gets

$$\mathcal{U}(\xi) = -\frac{q}{2r}\left(\frac{2i\cos^2(\xi) + 2\sin(\xi)\cos(\xi) - i}{\cos(\xi)\sin(\xi)}\right).$$

In this case, Eq. (21) has the exact solutions:

$$u_5(z,t) = -\frac{q}{2r}\left(\frac{2i\cos^2\left(\frac{q}{4\sqrt{2pr}}\left(z + q\sqrt{\frac{p}{2r}}t\right)\right) + \mathbf{B}}{\cos\left(\frac{q}{4\sqrt{2pr}}\left(z + q\sqrt{\frac{p}{2r}}t\right)\right)\sin\left(\frac{q}{4\sqrt{2pr}}\left(z + q\sqrt{\frac{p}{2r}}t\right)\right)}\right), \tag{34}$$

where $\mathbf{B} = 2\sin\left(\frac{q}{4\sqrt{2pr}}\left(z + q\sqrt{\frac{p}{2r}}t\right)\right)\cos\left(\frac{q}{4\sqrt{2pr}}\left(z + q\sqrt{\frac{p}{2r}}t\right)\right) - i$

5. Symmetry analysis

This portion will be dedicated to the investigation of symmetry analysis, and conservation laws for (21). The Lie point symmetries of (21) are generated by a vector field of the form

$$\mathcal{X} = \xi^1(z,t,u)\frac{\partial}{\partial z} + \xi^2(z,t,u)\frac{\partial}{\partial t} + \eta(z,t,u)\frac{\partial}{\partial u}. \tag{35}$$

It can be shown by a well established procedure that the (21) admits an infinitesimals given by

Case 1 $p \neq 0$, $r \neq 0$ and $q = 0$

$$\begin{aligned} \xi^1(z,t,u) &= c_1 + tc_2, \\ \xi^2(z,t,u) &= \frac{zc_2}{2} + c_3, \\ \eta(z,t,u) &= -\frac{uc_3}{2}, \end{aligned} \tag{36}$$

(21) admits the algebra of Lie point symmetries given by

$$\begin{aligned} \mathcal{X}_1 &= \partial_t, \\ \mathcal{X}_2 &= \partial_z, \\ \mathcal{X}_3 &= 2t\partial_t + z\partial_z - u\partial_u. \end{aligned} \tag{37}$$

Case 2 $p \neq 0$, $r \neq 0$ and $q \neq 0$

$$\begin{aligned} \xi^1(z,t,u) &= c_1, \\ \xi^2(z,t,u) &= c_2, \\ \eta(z,t,u) &= 0, \end{aligned} \tag{38}$$

admits the algebra of Lie point symmetries given by

$$\begin{aligned}\mathcal{X}_1 &= \partial_t,\\ \mathcal{X}_2 &= \partial_z.\end{aligned} \tag{39}$$

Considering the system (21), the formal Lagrangian can be given by:

$$L = \upsilon(z,t)\left(qu(z,t)^2 + ru(z,t)^3 + pu_{zz} + u_t\right), \tag{40}$$

where υ is a new-dependent variables called the nonlocal variables. The adjoint system can be obtained using

$$\mathcal{F}^* = \frac{\delta\mathcal{L}}{\delta u} = 0, \tag{41}$$

where

$$\frac{\delta\mathcal{L}}{\delta u} = \frac{\partial\mathcal{L}}{\partial u} - D_t\frac{\partial\mathcal{L}}{\partial u_t} - D_z\frac{\partial\mathcal{L}}{\partial u_z} + (D_z)^2\frac{\partial\mathcal{L}}{\partial u_{zz}}. \tag{42}$$

On the basis of Lagrangian reported in (40), one can get the adjoint system as

$$\mathcal{F}^* = \upsilon(z,t)\left(3ru(z,t)^2 + q\right) + p\upsilon_{zz} - \upsilon_t = 0. \tag{43}$$

5.1. Conservation laws

In this portion, we establish the Cls of the (21) [30-33]. We recall the following theorem from :

Theorem 5.1. *The system* (21) *with symmetry reported in* (35) *satisfy the conservation equation*

$$D_i\left(C^i\right)|_{(21)=0} = 0, \tag{44}$$

where

$$\begin{aligned} C^i = \xi_i\mathcal{L} + W^{\bar\alpha}\left[\frac{\partial\mathcal{L}}{\partial u_i^{\bar\alpha}} - D_j\left(\frac{\partial\mathcal{L}}{\partial u_{ij}^{\bar\alpha}}\right) + D_jD_k\left(\frac{\partial\mathcal{L}}{\partial u_{ijk}^{\bar\alpha}}\right) - ...\right] + \\ D_j\left(W^{\bar\alpha}\right)\left[\frac{\partial\mathcal{L}}{\partial u_{ij}^{\bar\alpha}} - D_k\left(\frac{\partial\mathcal{L}}{\partial u_{ijk}^{\bar\alpha}}\right) + ...\right] + D_jD_k\left(W^{\bar\alpha}\right)\left[\frac{\partial\mathcal{L}}{\partial u_{ijk}^{\bar\alpha}} + ...\right], \end{aligned} \tag{45}$$

and $W^{\bar\alpha} = \eta_{\bar\alpha} - \xi_j u_j^{\bar\alpha}$. The expression C^i represent the conserved vectors.

We compute the Cls for the (21) using the Lie point symmetries Equations (21).

Case 1

i. The symmetry $\mathcal{X}_2 = \partial_t$ admits the conserved vectors:

$$\begin{aligned} C_1^t &= p\left(u_t\upsilon_z - u_{zt}\upsilon(z,t)\right),\\ C_1^z &= \upsilon(z,t)\left(qu(z,t) + ru(z,t)^3 + pu_{zz}\right). \end{aligned} \tag{46}$$

ii. The symmetry $\mathcal{X}_1 = \partial_z$ admits the conserved vectors:

$$\begin{aligned} C_2^t &= u_t\upsilon(z,t) + qu(z,t)\upsilon(z,t) + ru(z,t)^3\upsilon(z,t) + pu_z\upsilon_z,\\ C_2^z &= u_z(-\upsilon(z,t)). \end{aligned} \tag{47}$$

iii. The symmetry $\mathcal{X}_3 = 2t\partial_t + z\partial_z - u\partial_u$ admits the conserved vectors:

$$\begin{aligned} C_3^t &= u^2(z,t)\left(qz\upsilon(z,t)+p\upsilon_z\right)+\left(zu_t-2p\left(tu_{zt}+u_z\right)\right)\upsilon(z,t)+rzu(z,t)^3\upsilon(z,t)+\mathbf{E}, \\ C_3^t &= \upsilon(z,t)\left((2qt-1)u^2(z,t)+2rtu(z,t)^3+2ptu_{zz}-zu_z\right), \end{aligned} \tag{48}$$

where $\mathbf{E} = p\upsilon_z\left(2tu_t+zu_z\right)$.

Case 2

i. The symmetry $\mathcal{X}_2 = \partial_t$ admits the conserved vectors:

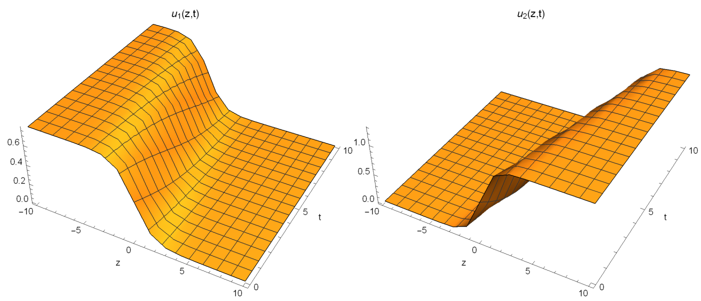
$$\begin{aligned} C_1^t &= p\left(u_t\upsilon_z - u_{zt}\upsilon(z,t)\right), \\ C_1^z &= \upsilon(z,t)\left(qu^2(z,t)+ru(z,t)^3+pu_{zz}\right). \end{aligned} \tag{49}$$

ii. The symmetry $\mathcal{X}_1 = \partial_z$ admits the conserved vectors:

$$\begin{aligned} C_2^t &= u_t\upsilon(z,t)+qu^2(z,t)\upsilon(z,t)+ru(z,t)^3\upsilon(z,t)+pu_z\upsilon_z, \\ C_2^z &= u_z(-\upsilon(z,t)). \end{aligned} \tag{50}$$

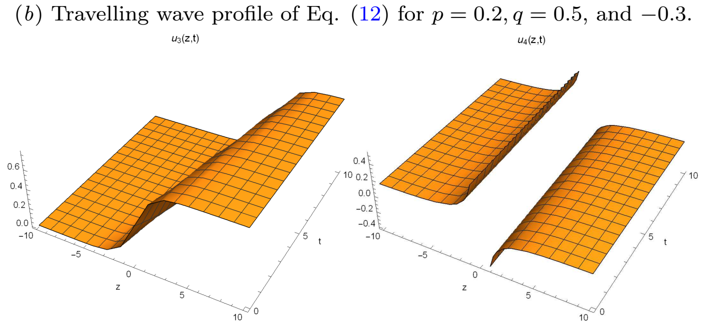
6. The graphical representation of solutions

In this part, we have plotted all acquired hyperbolic and trigonometric functions as solutions of the Newell-Whitehead-Segel and Zeldovich equations in Figures below. In each figure, some proper values for the parameters have been considered. As we can see, in all Figures the initial wave with travels at a constant speed along the space axis z. Indeed they preserve their heights and shapes during travel.



(a) Travelling wave profile of Eq. (10) for $p = 0.1, q = 0.3$, and -0.5.

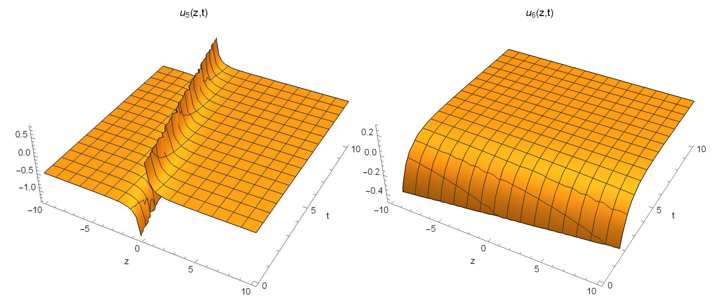
(b) Travelling wave profile of Eq. (12) for $p = 0.2, q = 0.5$, and -0.3.



(c) Travelling wave profile of Eq. (14) for $p = -0.1, q = -0.3$, and 0.5.

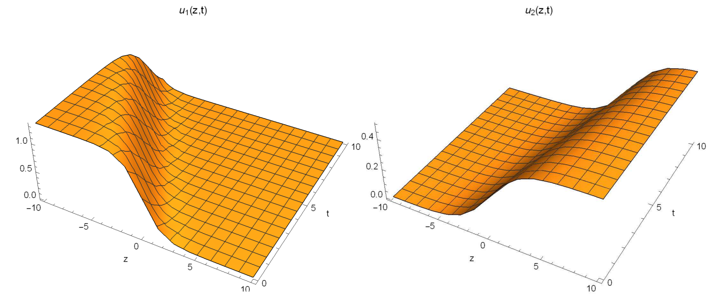
(d) Travelling wave profile of Eq. (16) for $p = -0.03, q = -0.01$, and 0.5.

Figure 1: Physical features for (10), (12), (14), and (16) for some values.



(a) Travelling wave profile of Eq. (18) for $p = 0.1, q = 0.4$, and -1.7.

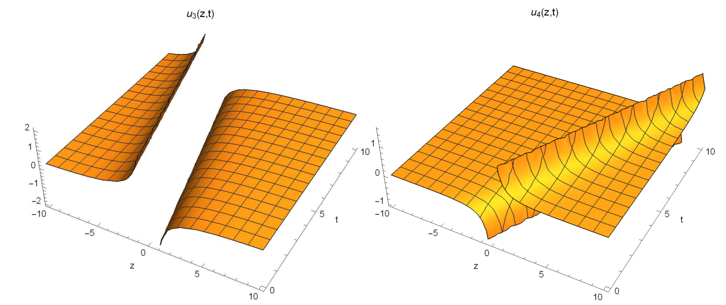
(b) Travelling wave profile of Eq. (20) for $p = -10, q = -5$, and $r = 6$.



(c) Travelling wave profile of Eq. (39) for $p = -1, q = 2$, and $r = -1.5$.

(d) Travelling wave profile of Eq. (28) for $p = -0.5, q = 1$, and $r = -2$.

Figure 2: Physical features for (18), (20), (39), and (28) for some values.



(a) Travelling wave profile of Eq. (30) for $p = -16, q = 0.55$, and $r = -10$.

(b) Travelling wave profile of Eq. (32) for $p = -1.1, q = 2.5$, and $r = -4.2$.

Figure 3: Physical features for (30), (32) for some values.

7. Conclusion

In this study, Lie symmetry approach and the generalized exponential rational function method is applied to reach conservation laws and some complex and real valued exact solutions for the NWS and ZEE equations. The NWS equation exhibits the relation between a continuous finite bandwidth of modes and a post critical Rayleigh-Benard convection by the space-time tardily varying amplitudes while ZEE equation explains the evolution of a grove population. The exact solutions are in form of periodic wave, trigonometric, algebraic and hyperbolic solutions. Using a new conservation theorem, we constructed conservation laws for the ZEE equation. The dynamic behavior of some acquired solutions have been included to find out the actual mechanism of the solutions.

References

[1] M. J. Ablowitz and P. A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform (Cambridge University Press, Cambridge, 1990).

[2] F. Tchier, A. I. Aliyu, A. Yusuf, and M. Inc. Dynamics of solitons to the ill-posed Boussinesq equation. Eur. Phys. J. Plus **132**, 136 (2017).

[3] F. Tchier, A. Yusuf, A. I. Aliyu, and M. Inc. Soliton solutions and conservation laws for lossy nonlinear transmission line equation. Superlattices Microstruct **107**, 320 (2017).

[4] W. X. Ma. A soliton hierarchy associated with so(3,R). Appl. Math. Comput. **220**, 117 (2013).

[5] R. Dodd, J. Eilbeck, J. Gibbon, and H. Morris, Solitons and Nonlinear Wave Equations (Academic Press, 1988).

[6] N. Zabusky, A Synergetic Approach to Problems of Nonlinear Dispersive Wave Propagation and Interaction (Academic Press, 1967).

[7] J. H. He. Application of homotopy perturbation method to nonlinear wave equations, Chaos, Solitons and Fractals **26(3)**, (2005) 695-700.

[8] J. H. He, Variational principles for some nonlinear partial differential equations with variable coefficients, Chaos, Solitons and Fractals, **19**, (2004) 4.

[9] G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Academic Publishers, Boston, 1994.

[10] K. Khan, M. A. Akbar, Exact and solitary wave solutions for the Tzitzeica-Dodd-Bullough and the modified KdV-Zakharov-Kuznetsov equations using the modified simple equation method, Ain Shams Engr. J. **4(4)** (2013) 903-909.

[11] K. Khan, M. A. Akbar. Traveling wave solutions of the (2+1)-dimensional Zoomeron equation and the Burgers equations via the MSE method and the Exp-function method. Ain Shams Engr. J. **5(1)** (2014) 247-256.

[12] H. I. Abdel-Gawad, M. Tantawy, Mustafa Inc and Abdullahi Yusuf. On multi-fusion solitons induced by inelastic collision for quasi-periodic propagation with nonlinear refractive index and stability analysis. Modern Physics Letters B Vol. 32, No. 29 (2018) 1850353.

[13] H. O. Roshid, N. Rahman, M.A. Akbar. Traveling waves solutions of nonlinear Klein Gordon equation by extended (G/G)-expasion method. Annals of Pure and Appl. Math.**3**, (2013) 10-16.

[14] U. Khan, R. Ellahi, R. Khan, S. T. Mohyud-Din, Extracting new solitary wave solutions of BennyâĂŞLuke equation and Phi-4 equation of fractional order by using (G'/G)-expansion method, Opt Quant Electron (2017) 49:362.

[15] Behzad Ghanbari, Abdullahi Yusuf and Mustafa Inc. Dark optical solitons and modulation instability analysis of nonlinear Schrodinger equation with higher order dispersion and cubic-quintic. J. Coupled Syst. Multiscale Dyn. 6, 217-227 (2018) nonlinearity

[16] Abdullahi Yusuf, Mustafa Inc, and Mustafa Bayram. Stability Analysis and Conservation Laws via Multiplier Approach for the Perturbed Kaup-Newell. J. Adv. Phys. 7, 451-0453 (2018)

[17] B.H. Gilding, R. Kersner, Traveling Waves in Nonlinear Diffusion-convection-reaction, University of Twente, Memorandum, 1585 (2001).

[18] A. Korkmaz, Complex wave solutions to mathematical biology models I:Newell-Whitehead-Segel and Zeldovich equations, journal of computational and nonlinear dynamics, 13(8), 081004.

[19] B. Ghanbari, M. Inc, A new generalized exponential rational function method to find exact special solutions for the resonance nonlinear Schrödinger equation, Eur. Phys. J. Plus (2018) 133: 142.