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Total Domination Number of Regular Dendrimer Graph

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Abstract. In this paper total domination number is calculated for regular dendrimer graph. New equations are obtained for regular dendrimers by using geometric series properties.

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1. Introduction

Let $G = (V, E)$ be a simple connected graph whose vertex set V and the edge set E. For the open neighborhood of a vertex v in a graph G, the notation $N_G(v)$ is used as $N_G(v) = \{u|(u, v) \in E(G)\}$ and the closed neighborhood of v is used as $N_G[v] = N_G(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$.

A subset $D \subseteq V$ is a dominating set, if every vertex in G either is element of D or is adjacent to at least one vertex in D. The domination number of a graph G is denoted with $\gamma(G)$ and it is equal to the minimum cardinality of a dominating set in G. By a similar definition, a subset $D \subseteq V$ is a total dominating set if every vertex of G has a neighbor in D. The total domination number of a graph G is denoted with $\gamma_t(G)$ and it is equal to the minimum cardinality of a total dominating set in G. Fundamental notions of domination theory are outlined in the book [2, 3].

The number of papers about domination number of chemical graphs is limited. For example, number of dominating sets of cactus chains is determined in [5] and domination number of some classes of benzenoid chains is studied in [1, 4, 8, 10]. Moreover, domination number of regular dendrimer is studied in [9].

In this paper, we attain total domination number of regular dendrimers.

Dendrimers are highly branched trees [7]. A regular dendrimer $T_{k,d}$ is a tree with a central vertex v. Every non-pendant vertex of $T_{k,d}$ is of degree $d \geq 2$ and the radius is k, distance from v to each pendant vertex. Dendrimers $T_{2,4}$ and $T_{3,4}$ are demonstrated in Figure 1. Some properties of regular dendrimers are denoted in the following lemma [6].

Lemma 1.1. *If $T_{k,d}$ is a tree with central vertex v, then*

i) *The order of $T_{k,d}$ is* $1 + \dfrac{d[(d-1)^k - 1]}{d-2}$,

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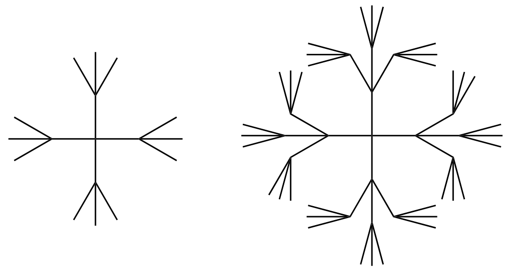

FIGURE 1. Dendrimers $T_{2,4}$ and $T_{3,4}$

ii) $T_{k,d}$ *has d branches,*

iii) *Each branch of* $T_{k,d}$ *has* $\dfrac{(d-1)^k - 1}{d-2}$ *vertices,*

iv) *Each branch of* $T_{k,d}$ *has* $(d-1)^{k-1}$ *pendant vertices,*

v) *Each branch of* $T_{k,d}$ *has* $\dfrac{(d-1)^{k-1} - 1}{d-2}$ *nonpendant vertices,*

vi) *The number of vertices on radius k is* $d(d-1)^{k-1}$.

2. Main Results

We remind domination number of paths and cycles in the next lemma.

Lemma 2.1 ([2]). *Let* P_n *path and* C_n *cycle with n vertices,*

$$\gamma(P_n) = \gamma(C_n) = \lceil \frac{n}{3} \rceil.$$

Observation 2.2. For every connected graph G, every support vertex is contained by every total dominating set.

Theorem 2.3. *If* $T_{k,d}$ *be a regular dendrimer, then total domination number of a regular dendrimer* $T_{k,d}$ *is*

$$\gamma_t(T_{k,d}) = \begin{cases} 2 + d^2(d-1)\dfrac{(d-1)^k - 1}{(d-1)^4 - 1}, & k \equiv 0 \ (mod\ 4) \\ 2 + d^2(d-1)^2\dfrac{(d-1)^{k-1} - 1}{(d-1)^4 - 1}, & k \equiv 1 \ (mod\ 4) \\ 1 + d + d^2(d-1)^3\dfrac{(d-1)^{k-2} - 1}{(d-1)^4 - 1}, & k \equiv 2 \ (mod\ 4) \\ d^2\dfrac{(d-1)^{k+1} - 1}{(d-1)^4 - 1}, & k \equiv 3 \ (mod\ 4) \end{cases}.$$

Proof. Let k is a multiple of four. In this case the dominating set of $T_{k,d}$ is consisted of central vertex v, a vertex from first radius and vertices on radius $k = 2, 3, 6, 7, \ldots, k-2, k-1$. Summation of all vertices is by Lemma 1.1 (vi),

$$\begin{aligned} \gamma_t(T_{k,d}) &= 1 + 1 + d(d-1) + d(d-1)^2 + d(d-1)^5 + d(d-1)^6 + \cdots + d(d-1)^{k-3} + d(d-1)^{k-2} \\ &= 2 + d(d-1) + d(d-1)^5 + \cdots + d(d-1)^{k-3} + d(d-1)^2 + d(d-1)^6 + \cdots + d(d-1)^{k-2} \\ &= 2 + d(d-1)[1 + (d-1)^4 + \cdots + (d-1)^{k-4}] + d(d-1)^2[1 + (d-1)^4 + \cdots + (d-1)^{k-4}] \\ &= 2 + (d(d-1) + d(d-1)^2)[1 + (d-1)^4 + \cdots + (d-1)^{k-4}]. \end{aligned}$$

The second term of the equation is a geometric series such that $r = (d-1)^4$ and then,

$$\begin{aligned} \gamma_t(T_{k,d}) &= 2 + d^2(d-1)\frac{r^{\frac{k-4}{4}+1} - 1}{r - 1} \\ &= 2 + d^2(d-1)\frac{(d-1)^k - 1}{(d-1)^4 - 1}. \end{aligned}$$

Now let $k \equiv 1$ $(mod\ 4)$. For this case, the dominating set of $T_{k,d}$ is consisted of central vertex v, a vertex from first radius and vertices on radius $k = 3, 4, 7, 8, \ldots, k-2, k-1$. Summation of all vertices is,

$$\begin{aligned}
\gamma_t(T_{k,d}) &= 1 + 1 + d(d-1)^2 + d(d-1)^3 + d(d-1)^6 + d(d-1)^7 + \cdots + d(d-1)^{k-3} + d(d-1)^{k-2} \\
&= 2 + d(d-1)^2 + d(d-1)^6 + \cdots + d(d-1)^{k-3} + d(d-1)^3 + d(d-1)^7 + \cdots + d(d-1)^{k-2} \\
&= 2 + d(d-1)^2[1 + (d-1)^4 + \cdots + (d-1)^{k-5}] + d(d-1)^3[1 + (d-1)^4 + \cdots + (d-1)^{k-5}] \\
&= 2 + (d(d-1)^2 + d(d-1)^3)[1 + (d-1)^4 + \cdots + (d-1)^{k-5}].
\end{aligned}$$

The second term of the equation is a geometric series such that $r = (d-1)^4$ and then,

$$\begin{aligned}
\gamma_t(T_{k,d}) &= 2 + d^2(d-1)^2 \frac{r^{\frac{k-5}{4}+1} - 1}{r-1} \\
&= 2 + d^2(d-1)^2 \frac{(d-1)^{k-1} - 1}{(d-1)^4 - 1}.
\end{aligned}$$

Now we assume that $k \equiv 2$ $(mod\ 4)$. For this case, the dominating set of $T_{k,d}$ is consisted of central vertex v, a vertex from first radius and vertices on radius $k = 1, 4, 5, 8, 9, \ldots, k-2, k-1$. Summation of all vertices is,

$$\begin{aligned}
\gamma_t(T_{k,d}) &= 1 + d + d(d-1)^3 + d(d-1)^4 + d(d-1)^7 + d(d-1)^8 + \cdots + d(d-1)^{k-3} + d(d-1)^{k-2} \\
&= 1 + d + d(d-1)^3 + d(d-1)^7 + \cdots + d(d-1)^{k-3} + d(d-1)^4 + d(d-1)^8 + \cdots + d(d-1)^{k-2} \\
&= 1 + d + +d(d-1)^3[1 + (d-1)^4 + \cdots + (d-1)^{k-6}] + d(d-1)^4[1 + (d-1)^4 + \cdots + (d-1)^{k-6}] \\
&= 1 + d + (d(d-1)^3 + d(d-1)^4)[1 + (d-1)^4 + \cdots + (d-1)^{k-6}].
\end{aligned}$$

The second term of the equation is a geometric series such that $r = (d-1)^4$ and then,

$$\begin{aligned}
\gamma_t(T_{k,d}) &= 1 + d + d^2(d-1)^3 \frac{r^{\frac{k-6}{4}+1} - 1}{r-1} \\
&= 1 + d + d^2(d-1)^3 \frac{(d-1)^{k-2} - 1}{(d-1)^4 - 1}.
\end{aligned}$$

Finally, we assume that $k \equiv 3$ $(mod\ 4)$. For this case, the dominating set of $T_{k,d}$ is consisted of vertices on radius $k = 1, 2, 5, 6, 9, 10, \ldots, k-2, k-1$. Thus,

$$\begin{aligned}
\gamma_t(T_{k,d}) &= d + d(d-1) + d(d-1)^4 + d(d-1)^5 + d(d-1)^8 + d(d-1)^9 + \cdots + d(d-1)^{k-3} + d(d-1)^{k-2} \\
&= d + d(d-1)^4 + d(d-1)^8 + \cdots + d(d-1)^{k-3} + d(d-1) + d(d-1)^5 + d(d-1)^9 + \cdots + d(d-1)^{k-2} \\
&= d[1 + (d-1)^4 + \cdots + (d-1)^{k-3}] + d(d-1)[1 + (d-1)^4 + \cdots + (d-1)^{k-3}] \\
&= (d(d-1) + d)[1 + (d-1)^4 + \cdots + (d-1)^{k-3}].
\end{aligned}$$

We have $r = (d-1)^4$ and from this,

$$\begin{aligned}
\gamma_t(T_{k,d}) &= d^2 \frac{r^{\frac{k-3}{4}+1} - 1}{r-1} \\
&= d^2 \frac{(d-1)^{k+1} - 1}{(d-1)^4 - 1}.
\end{aligned}$$

□

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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