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Controllability of Neutral Impulsive Stochastic Integrodifferential Systems with Unbounded Delay

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Abstract. This manuscript investigates the controllability of neutral impulsive stochastic integrodifferential systems with infinite delay in separable Hilbert space. The controllability results is obtained by using fixed-point technique and via resolvent operator.

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1. Introduction

The concept of controllability plays a major role in both finite and infinte dimensional spaces for systems represented by ordinary differential equations and partial differential equations. One of the basic qualitative behaviours of a dynamical system is the controllability. The problem of controllability is to show the existence of control function, which steers the solution of the system from its initial state to final state, where the initial and final states may vary over the entire space. Conceived by Kalman, the controllability concept has been studied extensively in the fields of finite and infinite-dimensional systems. If a system cannot be controlled completely then different types of controllability can be defined such as approximate, null, local null and local approximate null controllability. For more details the reader may refer to [3, 12, 13, 17, 18] and references therein.

Balachandran et al. [7] discussed the controllability of neutral functional integrodifferential systems in Banach spaces by using semigroup theory and the Nussbaum fixed point theorem. Recently, Balachandran et al. [5, 6], derived sufficient conditions for controllability of stochastic integrodifferential systems in finite dimensional spaces.

Recently, Park et al. [16] investigated the controllability of impulsive neutral integrodifferential systems with infinite delay in Banach spaces using Schauder-fixed point theorem. Very recently, [4, 10] established the existence, uniqueness and asymptotic behaviours of mild solutions to a class of impulsive neutral stochastic integrodifferential equations driven by a fractional Brownian motion with delays. Moreover, several upcoming researchers have keen interest to study the solvation of control problems in the field of stochastic systems. Through the survey of literature it reeveals

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that a very little work has been done for the fractional Brownian motion in stochastic control problems. Chen [9] concerned the approximate controllability for semilinear stochastic equations with fractional Brownian motion. Several researchers reported the use of fractional Brownian motion in stochastic integrodifferential equations (see refer to [1,2,14,15,17] and references therein). Moreover, the controllability of neutral impulsive stochastic integrodifferential systems with infinite delay is an untreated topic in the literature so far. Thus, we will make the first attempt to study such problem in this paper.

The goal of present research work is to focus the study of the controllability of neutral impulsive stochastic integrodifferential equations of the form:

$$\begin{aligned} d\left[x(t) - g(t, x_t, \int_0^t a_1(t,s,x_s))\right] &= \left[Ax(t) - g(t, x_t, \int_0^t a_1(t,s,x_s))ds\right]dt + f(t, x_t, \int_0^t a_2(t,s,x_s)ds)dt \\ &+ Bu(t)dt + \left[\int_0^t \gamma(t-s)\left[x(s) - g(s, x_s, \int_0^s a_1(s,r,x_r)dr)\right]ds\right]dt \\ &+ \sigma(t, x_t, \int_0^t a_3(t,s,x_s)ds)dw(t), \quad t \in I = [0,T], \ t \neq t_k, \qquad (1.1) \\ \Delta x\big|_{t-t_k} &= x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), \quad k = 1,...,m, \ m \in \mathbb{N}, \qquad (1.2) \\ x(t) &= \varphi(t) \in \mathcal{L}_2^0(\Omega, \mathscr{B}_h), \quad \text{for } a.e. \ t \in (-\infty, 0]. \qquad (1.3) \end{aligned}$$

Here, A is the infinitesimal generator of a strongly continuous semigroup $(T(t))_{t\geq 0}$ of bounded linear operators in a Hilbert space X; and the control function $u(\cdot)$ takes values in $\mathcal{L}^2([0,T], \mathrm{U})$, the Hilbert space of admissible control functions for a separable Hilbert space U; and B is a bounded linear operator from U into X. The history $x_t : (-\infty, 0] \to \mathrm{X}$, $x_t(\theta) = x(t+\theta)$, belongs to an abstract phase space $\mathscr{B}_h$ defined axiomatically, and $f, g : [0,T] \times \mathscr{B}_h \times \mathrm{X} \to \mathrm{X}$, $a_1, a_2, a_3 : \mathscr{D} \times \mathscr{B}_h \to \mathrm{X}$, $\sigma : [0,T] \times \mathscr{B}_h \times \mathrm{X} \to \mathcal{L}_2^0(\mathrm{Y}, \mathrm{X})$, are appropriate functions, where $\mathcal{L}_2^0(\mathrm{Y}, \mathrm{X})$ denotes the space of all Q-Hilbert-Schmit operators from Y into X and $\mathscr{D} = \{(s,t) \in I \times I : s < t\}$. Moreover, the fixed moments of time t_k satisfy $0 < t_1 < t_2 < ... < t_m < T$, $x(t_k^-)$ and $x(t_k^+)$ represent the left and right limits of $x(t)$ at time t_k respectively. $\Delta x(t_k)$ denotes the jump in the state x at time t_k with $I : \mathrm{X} \to \mathrm{X}$ determining the size of the jump.

2. Preliminaries

Let X, Y be real separable Hilbert spaces and $\mathcal{L}(\mathrm{Y}, \mathrm{X})$ be the space of bounded linear operators mapping Y into X. Let $(\Omega, \mathfrak{I}, \mathbb{P})$ be a complete probability space with an increasing right continuous family $\{\mathfrak{I}_t\}_{t\geq 0}$ of complete sub σ algebra of $\mathfrak{I}$. Let $\{w(t) : t \geq 0\}$ denote a Y-valued Wiener process defined on the probability space $(\Omega, \mathfrak{I}, \mathbb{P})$ with covariance operator Q, that is

$$\mathbf{E} < w(t), x >_{\mathrm{Y}} < w(s), y >_{\mathrm{Y}} \;=\; (t \wedge s) < Qx, y >_{\mathrm{Y}}, \quad \text{for all } \ x, y \in \mathrm{Y},$$

whare Q is a positive, self-adjoint, trace class operator on Y. We assume that there exists a complete orthonormal system $\{e_i\}_{i\geq 1}$ in Y, a bounded sequence of non-negative real numbers λ_i such that $Qe_i = \lambda_i e_i$, $i = 1, 2, ...$, and a sequence $\{\beta_i\}_{i\geq 1}$ of independent Brownian motions such that

$$\langle w(t), e\rangle \;=\; \sum_{n=1}^{\infty} \sqrt{\lambda_i}\,\langle e_i, e\rangle\,\beta_i(t), \quad e \in \mathrm{Y},$$

and $\mathfrak{I}_t = \mathfrak{I}_t^w$, where $\mathfrak{I}_t^w$ is the sigma algebra generated by $\{w(s) : 0 \leq s \leq t\}$. Let $\mathcal{L}_2^0 = \mathcal{L}_2(\mathrm{Y}_0, \mathrm{X})$ denote the space of all Hilbert-Schmidt operators from Y_0 into X. It turns out to be a separable Hilbert space equipped with the norm

$$\|\zeta\|^2_{\mathcal{L}_2^0} \;=\; \mathrm{tr}\left((\zeta Q^{\frac{1}{2}})(\zeta Q^{\frac{1}{2}})^*\right)$$

for any $\zeta \in \mathcal{L}_2^0$. Clearly for any bounded operators $\zeta \in \mathcal{L}(\mathrm{Y}, \mathrm{X})$ this norm reduces to

$$\|\zeta\|^2_{\mathcal{L}_2^0} \;=\; \mathrm{tr}\,(\zeta Q \zeta^*)\,.$$

We assume that the phase space $\mathscr{B}_h$ is a linear space of functions mapping $(-\infty, 0]$ into X, endowed with a norm $\|\cdot\|_{\mathscr{B}_h}$. First, we present the abstract phase space $\mathscr{B}_h$. Assume that $h : (-\infty, 0] \to [0, +\infty)$ is a continuous function with

$$l = \int_{-\infty}^{0} h(s)ds < +\infty.$$

We define the abstract phase space $\mathscr{B}_h$ by $\mathscr{B}_h = \Big\{\zeta : (-\infty, 0] \to \mathrm{X}$ for any $\tau > 0$, $(\mathbf{E}\,\|\zeta\|^2)^{1/2}$ is bounded and measurable function $[\tau, 0]$ and $\int_{-\infty}^{0} h(t) \sup_{t\le\tau\le 0}(\mathbf{E}\,\|\zeta(s)\|^2)^{1/2}dt < +\infty\Big\}$. If this space with the norm

$$\|\zeta\|_{\mathscr{B}_h} \quad = \quad \int_{-\infty}^{0} h(t) \sup_{t\le s\le 0}(\mathbf{E}\,\|\zeta\|^2)^{1/2}dt,$$

then it is clear that $(\mathscr{B}_h, \|\cdot\|_{\mathscr{B}_h})$ is a Banach space.

We now consider the space $\mathscr{B}_{\mathscr{D}I}$ [$\mathscr{D}$ and I stand for delay and impulse, respectively] given by $\mathscr{B}_{\mathscr{D}I} = \Big\{x : (-\infty, T] \to \mathrm{X} : x|I_k \in \mathscr{C}(I_k, \mathrm{X})$ and $x(t_k^+), x(t_k^-)$ exist with $x(t_k^+) - x(t_k^-), k - 1, 2, ..., m$ $x_0 - \varphi \in \mathscr{B}_h$ and $\sup_{0\le t\le T}\mathbf{E}(\|x(t)\|^2) < \infty\Big\}$, where $x|I_k$ is the restriction of x to the interval $I_k = (t_k, t_{k+1}]$, $k = 1, 2, ..., m$. Then the function $\|\cdot\|_{\mathscr{B}_h}$ to be a semi-norm in $\mathscr{B}_{\mathscr{D}I}$, it is defined by

$$\|x\|_{\mathscr{B}_{\mathscr{D}I}} \quad = \quad \|x_0\|_{\mathscr{B}_h} + \sup_{0<t<T}(\mathbf{E}(\|x(t)\|^2))^{1/2}.$$

The following lemma is a common property of phase spaces.

Lemma 2.1. *Suppose $x \in \mathscr{B}_{\mathscr{D}I}$, then for all $t \in [0, T]$, $x_t \in \mathscr{B}_h$ and*

$$l(\mathbf{E}(\|x(t)\|^2))^{\frac{1}{2}} \quad \le \quad l \sup_{0\le s\le t}(\mathbf{E}\,\|x(s)\|^2)^{\frac{1}{2}} + \|x_0\|_{\mathscr{B}_h},$$

where $l = \int_{-\infty}^{0} h(s)ds < \infty$.

2.1. **Partial integrodifferential equations in Banach spaces.** In the present section, we recall some defintions and properties needed in the sequel. In what follows, X will denote a Banach space, A and $\gamma(t)$ are closed linear operators on X. Y represents the Banach space $\mathscr{D}(A)$, the domain of operator A, equipped with the graph norm

$$|y|_{\mathrm{Y}} := |Ay| + |y| \quad \text{for} \quad y \in \mathrm{Y}.$$

The notation $\mathscr{C}([0, +\infty); \mathrm{Y})$ stands for the space of all continuous functions from $[0, +\infty)$ into Y. We consider the following Cauchy problem

$$\begin{cases} v'(t) = Av(t) + \displaystyle\int_0^t \gamma(t-s)v(s)ds \;\; \text{for} \;\; t \ge 0, \\ v(0) = v_0 \in \mathrm{X}. \end{cases} \tag{2.1}$$

Definition 2.2 ([11]). A resolvent operator for equation (2.1) is a bounded linear operator valued function $R(t) \in \mathcal{L}(\mathrm{X})$ for $t \ge 0$, satisfying the following properties:
(i) $R(0) = I$ and $\|R(t)\| \le Me^{\lambda t}$ for some constants M and β.
(ii) For each $x \in \mathrm{X}$, $R(t)x$ is strongly continuous for $t \ge 0$.
(iii) For $x \in \mathrm{Y}$, $R(\cdot)x \in \mathscr{C}^1([0, +\infty); \mathrm{X}) \bigcap \mathscr{C}([0, +\infty); \mathrm{Y})$ and

$$R^{'}(t)x \quad = \quad AR(t)x + \int_0^t \gamma(t-s)R(s)xds = R(t)Ax + \int_0^t R(t-s)B(s)xds \;\; \text{for } t \ge 0.$$

For additional details on resolvent operators, we refer the reader to [11]. In what follows we suppose the following assumptions:
(H1) A is the infintesimal generator of a C_0-semigroup $(R(t))_{t\ge 0}$ on X.
(H2) For all $t \ge 0$, $\gamma(t)$ is a continuous linear operator from $(\mathrm{Y}, |\cdot|_{\mathrm{Y}})$ into $(\mathrm{X}, |\cdot|_{\mathrm{X}})$. Moreover, there exists an integrable function $\mathscr{C} : [0, +\infty) \to \mathbb{R}^+$ such that for any $y \in \mathrm{Y}$, $y \to \gamma(t)y$ belongs to $W^{1,1}([0, +\infty); \mathrm{X})$ and

$$\left|\frac{d}{dt}\gamma(t)(t)y\right|_{\mathrm{X}} \quad \le \quad \mathscr{C}(t)\,|y|_{\mathrm{Y}} \quad \text{for } y \in \mathrm{Y} \;\text{ and } t \ge 0.$$

Theorem 2.3. *Assume that hypotheses* **(H1)** *and* **(H2)** *hold. Then equation* (2.1) *admits a resolvent operator* $(R(t))_{t\ge 0}$.

Theorem 2.4. *Assume that hypotheses* **(H1)** *and* **(H2)** *hold. Let $R(t)$ be a compact operator for $t > 0$. Then, the corresponding resolvent operator $R(t)$ of equation* (2.1) *is continuous for $t > 0$ in the operator norm, for all $t_0 > 0$, it holds that* $\lim_{h\to 0}\|R(t_0 + h) - R(t_0)\| = 0$.

In the sequel, we recall some results on existence of solutions for the following integrodifferential equation

$$\begin{cases} v'(t) = Av(t) + \int_0^t \gamma(t-s)v(s)ds + q(t) \ \text{ for } t \geq 0, \\ v(0) = v_0 \in \mathrm{X}. \end{cases} \tag{2.2}$$

where $q : [0, +\infty[\to \mathrm{X}$ is a continuous function.

Definition 2.5. A continuous function $v : [0, +\infty) \to \mathrm{X}$ is said to be a strict solution of equation (2.2) if
(i) $v \in \mathscr{C}^1([0, +\infty); \mathrm{X}) \bigcap \mathscr{C}([0, +\infty); \mathrm{Y})$,
(ii) v satisfies equation (2.2) for $t \geq 0$.

Remark 2.6. From this definition we deduce that $v(t) \in \mathscr{D}(A)$, and the function $\gamma(t-s)v(s)$ is integrable, for all $t > 0$ and $s \in [0, +\infty)$.

Theorem 2.7. *Assume that* **(H1)**-**(H2)** *hold. If v is a strict solution of equation* (2.2)*, then the following variation of constants formula holds*

$$v(t) \;=\; R(t)v_0 + \int_0^t R(t-s)q(s)ds \quad \text{for} \;\; t \geq 0.$$

Definition 2.8. An X-valued process $\{x(t) : t \in (-\infty, T]\}$ is a mild solution of (1.1)-(1.3) if
1. $x(t)$ is measurable for each $t > 0$, $x(t) = \varphi(t)$ on $(\infty, 0]$,

$$\Delta x\Big|_{t-t_k} = I_k(x(t_k^-)), \;\; k = 1, 2, ...m$$

the restriction of $x(\cdot)$ to $[0, T] = \{t_1, t_2, ...t_m\}$ is continuous.
2. For every $0 \leq s \leq t$, the process x satisfies the following integral equation

$$\begin{aligned} x(t) \;=\;& R(t)\left[\varphi(0) - g(0, \varphi, 0)\right] + g(t, x_t, \int_0^t a_1(t, s, x_s)ds) + \int_0^t R(t-s)Bu(s)ds \\ &+ \int_0^t R(t-s)f(s, x_s, \int_0^s a_2(s, r, x_r)dr)ds + \int_0^t R(t-s)\sigma(s, x_s, \int_0^s a_3(s, r, x_r)dr)dw(s) \\ &+ \sum_{0<t_k<t} R(t-t_k)I_k(x(t_k^-)), \;\; \mathbb{P}-a.s. \end{aligned} \tag{2.3}$$

3. Controllability Result

Definition 3.1. System (1.1)-(1.3) is said to be controllable on the interval $(-\infty, T]$ if for every initial stochastic process φ defined on $(-\infty, T]$, there exists a stochastic control $u \in \mathcal{L}^2([0, T]; \mathrm{U})$ such that the mild solution $x(\cdot)$ of (1.1)-(1.3) satisfies $x(T) = x_1$.

In order to establish the controllability of (1.1)-(1.3), we impose the following hypotheses:

- **(H3)** There exist constants $M \geq 1$ such that $\|R(t)\|^2 \leq M$.
- **(H4)** The mapping $g : I \times \mathscr{D} \times \mathscr{B}_h \to \mathrm{X}$ satisfies the following conditions
  (i) The function $a_1 : \mathscr{D} \times \mathscr{B}_h \to \mathrm{X}$ satisfies the following condition. There exists a constant $k_1 > 0$, for $x_1, x_2 \in \mathscr{B}_h$ such that

$$\mathbf{E}\left\|\int_0^t [a_1(t, s, x_1) - a_1(t, s, x_2)]ds\right\|^2 \;\;\leq\;\; k_1 \|x_1 - x_2\|^2_{\mathscr{B}_h}, \quad (t, s) \in \mathscr{D},$$

  and

$$\bar{k_1} = \sup_{(t,s)\subset\mathscr{D}} \left\|\int_0^t a_1(t, s, 0)ds\right\|^2.$$

(ii) g is a continuous function and there exists constants $k_2 > 0$ such that for $x_1, x_2 \in \mathscr{B}_h$, $y_1, y_2 \in$ X and satisfies for all $t \in [0, T]$

$$\mathbf{E}\,\|g(t, x_1, y_1) - g(t, x_2, y_2)\|^2 \quad \leq \quad k_2\Big[\,\|x_1 - x_2\|^2_{\mathscr{B}_h} + \mathbf{E}\,\|y_1 - y_2\|^2\,\Big],$$

$$\lim_{t \to s} \mathbf{E}\,\|g(t, x_1, y_1) - g(t, x_2, y_2)\|^2 \quad = \quad 0.$$

and

$$\bar{k_2} \quad = \quad \sup_{t \subset [0,T]} \|g(t, 0, 0)\|^2 .$$

(H5) The mapping $f : I \times \mathscr{B}_h \times \mathrm{X} \to \mathrm{X}$ satisfies the following Lipschitz conditions

(i) There exist positive constants $k_3, \bar{k_3}$ for $t \in [0, T]$, $x_1, x_2 \in \mathscr{B}_h$, $y_1, y_2 \in$ X such that

$$\mathbf{E}\,\|f(t, x_1, y_1) - f(t, x_2, y_2)\|^2 \quad \leq \quad k_3\left[\|x_1 - x_2\|^2_{\mathscr{B}_h} + \mathbf{E}\,\|y_1 - y_2\|^2\right],$$

and

$$\bar{k_3} \quad = \quad \sup_{t \in [0,T]} \|f(t, 0, 0)\|^2 .$$

(ii) The function $a_2 : \mathscr{D} \times \mathscr{B}_h \to$ X satisfies the following condition. There exists a constant $k_4 > 0$, for $x_1, x_2 \in \mathscr{B}_h$ such that

$$\mathbf{E}\left\|\int_0^t [a_2(t, s, x_1) - a_2(t, s, x_2)]ds\right\|^2 \quad \leq \quad k_4\,\|x_1 - x_2\|^2_{\mathscr{B}_h}, \quad (t, s) \in \mathscr{D},$$

and

$$\bar{k_4} = \sup_{(t,s) \subset \mathscr{D}} \left\|\int_0^t a_2(t, s, 0)ds\right\|^2 .$$

(H6) The mapping $\sigma : I \times \mathscr{B}_h \times \mathrm{X} \to \mathcal{L}(\mathrm{Y}, \mathrm{X})$ satisfies the following Lipschitz conditions

(i) There exist positive constants $k_5, \bar{k_5}$ for $t \in [0, T]$, $x_1, x_2 \in \mathscr{B}_h$, $y_1, y_2 \in$ X such that

$$\mathbf{E}\,\|\sigma(t, x_1, y_1) - \sigma(t, x_2, y_2)\|^2 \quad \leq \quad k_5\left[\|x_1 - x_2\|^2_{\mathscr{B}_h} + \mathbf{E}\,\|y_1 - y_2\|^2\right],$$

and

$$\bar{k_5} \quad = \quad \sup_{t \in [0,T]} \|f(t, 0, 0)\|^2 .$$

(ii) The function $a_3 : \mathscr{D} \times \mathscr{B}_h \to$ X satisfies the following condition. There exists a constant $k_6 > 0$, for $x_1, x_2 \in \mathscr{B}_h$ such that

$$\mathbf{E}\left\|\int_0^t [a_3(t, s, x_1) - a_3(t, s, x_2)]ds\right\|^2 \quad \leq \quad k_6\,\|x_1 - x_2\|^2_{\mathscr{B}_h}, \quad (t, s) \in \mathscr{D},$$

and

$$\bar{k_6} = \sup_{(t,s) \subset \mathscr{D}} \left\|\int_0^t a_3(t, s, 0)ds\right\|^2 .$$

(H7) The impulses functions I_k for $k = 1, 2, ..., m$, satisfies the following condition. There exists positive constants M_k, $\widetilde{M_k}$ such that

$$\|I_k(x) - I_k(y)\|^2 \quad \leq \quad M_k\,\|x - y\|^2\, and\, \|I_k(x)\|^2 \leq \widetilde{M_k} \;\; forall \;\; x, y \in \mathscr{B}_h.$$

(H8) The linear operator W from U into X defined by

$$Wu \quad = \quad \int_0^T R(T - s)Bu(s)ds$$

has an inverse operator W^{-1} that takes values in $\mathcal{L}^2([0, T], \mathrm{U})$ ker W, where $kerW = \left\{x \in \mathcal{L}^2([0, T], \mathrm{U}) : Wx = 0\right\}$.

(H9) There exists a constant $\lambda > 0$ such that

$$\lambda = 10l^2\left(1+4MM_bM_WT^2\right)\left[k_2(1+2k_1)+MT^2k_3(1+k_4)+Mc_pk_5(1+k_6)+mM\sum_{k=1}^{m}M_k\right] < 1.$$

The main result of this paper is given in the next theorem.

Theorem 3.2. *Suppose that* **(H1)**-**(H9)** *hold. Then, the system* (1.1)-(1.3) *is controllable on* $(-\infty, T]$ *provide that*

$$6l^2\left(1+7MM_bM_WT^2\right)\left[8[k_2(1+2k_1)]+8MT^2[k_3(1+2k_4)]+8Mc_p[k_5(1+2k_6)]\right] < 1. \tag{3.1}$$

Proof. Using **(H8)** for an arbitrary function $x(\cdot)$, define the control

$$\begin{aligned} u_x(t) &= W^{-1}\bigg[x_1 - R(T)\left[\varphi(0)-g(0,x_0,0)\right] - g(T,x_T,\int_0^T a_1(T,s,x_s)ds)) \\ &+ \int_0^T R(T-s)f(s,x_s,\int_0^s a_2(s,r,x_r)dr)ds + \int_0^t R(t-s)\sigma(s,x_s,\int_0^s a_3(s,r,x_r)dr)dw(s) \\ &+ \sum_{0<t_k<t} R(T-t_k)I_k(x(t_k^-))\bigg](t). \end{aligned}$$

Now, put the control $u(\cdot)$ into the stochastic control system (2.4) and obtain a nonlinear operator Γ on $\mathscr{B}_{\mathscr{D}I}$ given by

$$\Gamma(x)(t) = \begin{cases} \varphi(t), \quad \text{for } t \in (-\infty,0], \\ R(t)\left[\varphi(0)-g(0,\varphi,0)\right] + g(t,x_t,\int_0^t a_1(t,s,x_s)ds) + \int_0^t R(t-s)Bu_x(s)ds \\ + \int_0^t R(t-s)f(s,x_s,\int_0^s a_2(s,r,x_r)dr)ds + \int_0^t R(t-s)\sigma(s,x_s,\int_0^s a_3(s,r,x_r)dr)dw(s) \\ + \sum_{0<t_k<t} R(t-t_k)I_k(x(t_k^-)), \quad \text{if} \quad t\in[0,T]. \end{cases}$$

Then it is clear that to prove the existence of mild solutions to equations (1.1)-(1.3) is equivalent to find a fixed point for the operator. Clearly, $\Gamma x(T) = x_1$, which means that the control u steers the system grom the initial state φ to x_1 in time T, provided we can obtain a fixed point of the operator Γ which implies that the system in contrillable.

Let $y : (-\infty, T] \to \mathrm{X}$ be the function defined by

$$y(t) = \begin{cases} \varphi(t), \quad \text{if } t\in(-\infty,0], \\ R(t)\varphi(0), \quad \text{if } t\in[0,T]. \end{cases}$$

then, $y_0 = \varphi$. For each function $z \in \mathscr{B}_{\mathscr{D}I}$, set

$$x(t) = z(t) + y(t).$$

It is obvious that x satisfies the stochastic control system (2.4) if and only if z satisfies $z_0 = 0$ and

$$\begin{aligned} z(t) &= g(t,z_t+y_t,\int_0^t a_1(t,s,z_s+y_s)ds) - R(t)g(0,\varphi,o) + \int_0^t R(t-s)B_{z+y}(s)ds \\ &+ \int_0^t R(t-s)f(s,z_s+y_s,\int_0^s a_2(s,r,z_r+y_r)dr)ds \\ &+ \int_0^t R(t-s)\sigma(s,x_s,\int_0^s a_3(s,r,x_r)dr)dw(s) \\ &+ \sum_{0<t_k<t} R(t-t_k)I_k[z(t_k^-)-y(t_k^-)], \quad \text{if } t\in[0,T], \end{aligned}$$

where

$$\begin{aligned} u_{z+y}(t) &= W^{-1}\Big[x_1 - R(T)\,[\varphi(0) - g(0, z_0 + y_0, 0)] - g(T, z_T + y_T, \int_0^T a_1(T, s, z_s + y_s)ds) \\ &- \int_0^T R(T-s) f(s, z_s + y_s, \int_0^s a_2(s, r, z_r + y_r)dr)ds - \int_0^t R(t-s)\sigma(s, x_s, \int_0^s a_3(s, r, x_r)dr)dw(s) \\ &- \sum_{0<t_k<T} R(T-t_k) I_k[z(t_k^-) + y(t_k^-)]\Big](t). \end{aligned}$$

Set

$$\mathscr{B}^0_{\mathscr{D}I} = \{z \in \mathscr{B}_{\mathscr{D}I} : z_0 = 0\},$$

for any $z \in \mathscr{B}^0_{\mathscr{D}I}$, we have

$$\|z\|_{\mathscr{B}^0_{\mathscr{D}I}} = \|z_0\|_{\mathscr{B}_h} + \sup_{t\in[0,T]} (\mathbf{E}\,\|z(t)\|^2)^{\frac{1}{2}} = \sup_{t\in[0,T]} (\mathbf{E}\,\|z(t)\|^2)^{\frac{1}{2}}.$$

Then, $(\mathscr{B}^0_{\mathscr{D}I}, \|\cdot\|_{\mathscr{B}^0_{\mathscr{D}I}})$ is a Banach space. Define the operator $\Theta : \mathscr{B}^0_{\mathscr{D}I} \to \mathscr{B}^0_{\mathscr{D}I}$ by

$$(\Theta z)(t) = \begin{cases} 0 \;\text{ if }\; t \in (-\infty, 0], \\ g(t, z_t + y_t, \int_0^t a_1(t, s, z_s + y_s)ds) - R(t)g(0, \varphi, o) + \int_0^t R(t-s)B_{z+y}(s)ds \\ + \int_0^t R(t-s) f(s, z_s + y_s, \int_0^s a_2(s, r, z_r + y_r)dr)ds \\ + \int_0^t R(t-s)\sigma(s, x_s, \int_0^s a_3(s, r, x_r)dr)dw(s) \\ + \sum_{0<t_k<t} R(t-t_k) I_k[z(t_k^-) - y(t_k^-)], \quad \text{if }\; t \in [0, T]. \end{cases}$$

Set

$$\mathscr{B}_k = \Big\{z \in \mathscr{B}^0_{\mathscr{D}I} : \|z\|^2_{\mathscr{B}^0_{\mathscr{D}I}} \le k\Big\}, \quad \text{for some }\; k \ge 0,$$

then $\mathscr{B}_k \subseteq \mathscr{B}^0_{\mathscr{D}I}$ is a bounded closed convex set, and for $z \in \mathscr{B}_k$, we have

$$\begin{aligned} \|z_t + y_t\|_{\mathscr{B}_{\mathscr{D}I}} &\le 2\left(\|z_t\|^2_{\mathscr{B}_{\mathscr{D}\mathscr{I}}} + \|y_t\|^2_{\mathscr{B}_{\mathscr{D}\mathscr{I}}}\right) \\ &\le 4\left(l^2 \sup_{0\le s\le t} \mathbf{E}\,\|z(s)\|^2 + \|z_0\|^2_{\mathscr{B}_h} + l^2 \sup_{0\le s\le t} \mathbf{E}\,\|y(s)\|^2 + \|y_0\|^2_{\mathscr{B}_h}\right) \\ &\le 4l^2\left(k + M\mathbf{E}\,\|\varphi(0)\|^2\right) + 4\,\|y\|^2_{\mathscr{B}_h} \\ &:= r^*. \end{aligned}$$

Next,

$$\begin{aligned} \mathbf{E}\left\|u_{z+y}\right\|^2 &\le 7M_W\Big[\|x_1\|^2 + M\mathbf{E}\,\|\varphi(0)\|^2 + 2M[k_2\,\|y\|^2_{\mathscr{B}_h} + \bar{k}_2] + 2[k_2(1+2k_1)r^* + 2k_2\bar{k}_1 + \bar{k}_2] \\ &+ 2MT^2[k_3(1+2k_4)r^* + 2k_3\bar{k}_4 + \bar{k}_3] + 2Mc_p[k_5(1+2k_6)r^* + 2k_5\bar{k}_6 + \bar{k}_5] \\ &+ mM\sum_{k=1}^m \widetilde{M}_k\Big] := \mathcal{G} \end{aligned}$$

and

$$\begin{aligned} \mathbf{E}\left\|u_{z+y} - u_{v+y}\right\|^2 &\le 4M_W\Big[k_2(1+2k_1) + MT^2k_3(1+2k_4) + Mc_pk_5(1+2k_6) \\ &+ mM\sum_{k=1}^m M_k\Big]\mathbf{E}\,\|z_t - v_t\|^2_{\mathscr{B}_h}. \end{aligned} \tag{3.2}$$

It is clear that the operator Γ has a fixed point if and only if Θ has one, so it turns to prove that Θ has a fixed point. Since all functions involved in the operator are continuous therefore Θ is continuous. The proof will be given in following steps.

Step 1: We claim that there exists a positive number k, such that $\Theta(x) \in \mathscr{B}_k$ whenever $x \in \mathscr{B}_k$. If it is not true, then for each positive number k, there is a function $z^k(\cdot) \in \mathscr{B}_k$, but $\Theta(z^k) \notin \mathscr{B}_k$, that is $\mathbf{E}\left\|\Theta(z^k)(t)\right\|^2 > k$ for some $t \in [0,T]$. However, on the other hand, we have

$$\begin{aligned} k &< \mathbf{E}\left\|\Theta(z^k)(t)\right\|^2 \\ &\le 6\Big[2M(k_2\|y\|^2_{\mathscr{B}_h} + \bar{k}_2) + 2[k_2(1+2k_1)r^* + 2k_2\bar{k}_1 + \bar{k}_2] + 2MT^2[k_3(1+2k_4)r^* + 2k_3\bar{k}_4 + \bar{k}_3] \\ &+ 2Mc_p[k_5(1+2k_6)r^* + 2k_5\bar{k}_6 + \bar{k}_5] + mM\sum_{k=1}^{m}\widetilde{M}_k + MM_bT^2\mathcal{G}\Big] \\ &\le 6\left(1+7MM_bM_WT^2\right)\Big[2M(k_2\|y\|^2_{\mathscr{B}_h} + \bar{k}_2) + 2[k_2(1+2k_1)r^* + 2k_2\bar{k}_1 + \bar{k}_2] + 2MT^2[k_3(1+2k_4)r^* \\ &+ 2k_3\bar{k}_4 + \bar{k}_3] + 2Mc_p[k_5(1+2k_6)r^* + 2k_5\bar{k}_6 + \bar{k}_5] + mM\sum_{k=1}^{m}\widetilde{M}_k + MM_bT^2\mathcal{G}\Big] \\ &+ 7MM_bM_WT^2\left(\|x_1\|^2 + M\mathbf{E}\|\varphi(0)\|^2\right) \\ &\le \widetilde{\mathcal{G}} + 6\left(1+7MM_bM_WT^2\right)\Big[2[k_2(1+2k_1)]r^* + 2MT^2[k_3(1+2k_4)r^*] + 2Mc_p[k_5(1+2k_6)r^*\Big], \end{aligned}$$

where

$$\begin{aligned} \widetilde{\mathcal{G}} &= 6\left(1+7MM_bM_WT^2\right)\Big[2M(k_2\|y\|^2_{\mathscr{B}_h} + \bar{k}_2) + 2[2k_2\bar{k}_1 + \bar{k}_2] + 2MT^2[2k_3\bar{k}_4 + \bar{k}_3] \\ &+ 2Mc_p[2k_5\bar{k}_6 + \bar{k}_5] + mM\sum_{k=1}^{m}\widetilde{M}_k + 7MM_bM_WT^2\left(\|x_1\|^2 + M\mathbf{E}\|\varphi(0)\|^2\right)\Big] \end{aligned}$$

is independent of k. Dividing both sides by k and taking the limit as $k \to \infty$, we get

$$6l^2\left(1+7MM_bM_WT^2\right)\Big[8[k_2(1+2k_1)] + 8MT^2[k_3(1+2k_4)] + 8Mc_p[k_5(1+2k_6)]\Big] \quad \ge \quad 1.$$

This contradicts (3.1). Hence for some positive k,

$$(\Theta)(\mathscr{B}_k) \quad \subseteq \quad \mathscr{B}_k.$$

Step 2: Θ is a contraction. Let $t \in [0,T]$ and $z^1, z^2 \in \mathscr{B}^0_{\mathscr{D}I}$, we have

$$\begin{aligned} &\mathbf{E}\left\|\Theta z^1(t) - \Theta z^2(t)\right\|^2 \\ &\le 5\mathbf{E}\left\|\int_0^t R(t-s)B[u_{z^1+y}(s) - u_{z^2+y}(s)]ds\right\|^2 \\ &+ 5\mathbf{E}\left\|\sum_{0<t_k<t} R(T-t_k)[I_k(z^1(t_k^-) + y(t_k^-)) - I_k(z^2(t_k^-) + y(t_k^-))]\right\|^2 \\ &+ 5\mathbf{E}\left\|g(t, z_t^1 + y_t, \int_0^t a_1(t,s,z_s^1 + y_s)ds) - g(t, z_t^2 + y_t, \int_0^t a_1(t,s,z_s^2 + y_s)ds)\right\|^2 \\ &+ 5\mathbf{E}\left\|\int_0^t R(t-s)[f(s, z_s^1 + y_s, \int_0^s a_2(s,r,z_r^1 + y_r)dr) - f(s, z_s^2 + y_s, \int_0^s a_2(s,r,z_r^2 + y_r)dr)]ds\right\|^2 \\ &+ 5\mathbf{E}\left\|\int_0^t R(t-s)[\sigma(s, z_s^1 + y_s, \int_0^s a_3(s,r,z_r^1 + y_r)dr) - \sigma(s, z_s^2 + y_s, \int_0^s a_3(s,r,z_r^2 + y_r)dr)]dw(s)\right\|^2. \end{aligned}$$

On the other hand from **(H1)**-**(H9)** combined with (3.2), we obtain

$$\begin{aligned}
\mathbf{E}\left\|\Theta z^1(t)-\Theta z^2(t)\right\|^2 &\le 5\left(1+4MM_bM_WT^2\right)\Big[k_2(1+2k_1)+MT^2k_3(1+k_4)+Mc_pk_5(1+k_6)\\
&+ mM\sum_{k=1}^{m}M_k\Big]\mathbf{E}\left\|z_t^1-z_t^2\right\|^2_{\mathscr{B}_h}\\
&\le 10\left(1+4MM_bM_WT^2\right)\Big[k_2(1+2k_1)+MT^2k_3(1+k_4)+Mc_pk_5(1+k_6)\\
&+ mM\sum_{k=1}^{m}M_k\Big]\times\left\{l^2\sup_{0\le s\le t}\mathbf{E}\left\|z^1(s)-z^2(s)\right\|^2+\left\|z_0^1-z_0^2\right\|^2_{\mathscr{B}_h}\right\}\\
&\le \lambda\sup_{0\le s\le T}\mathbf{E}\left\|z^1(s)-z^2(s)\right\|^2 \qquad \text{since } (z_0^1=z_0^2=0).
\end{aligned}$$

Taking supremum over t,

$$\left\|\Theta z^1-\Theta z^2\right\|_{\mathscr{B}_{\mathscr{D}I}} \le \lambda\left\|z^1-z^2\right\|_{\mathscr{B}_{\mathscr{D}I}},$$

where

$$\lambda = 10l^2\left(1+4MM_bM_WT^2\right)\Big[k_2(1+2k_1)+MT^2k_3(1+k_4)+Mc_pk_5(1+k_6)+mM\sum_{k=1}^{m}M_k\Big].$$

By condition **(H9)**, we have $\lambda<1$, hence Θ is a contraction mapping on $\mathscr{B}^0_{\mathscr{D}I}$ and therefore has a unique fixed point, which is a mild solution of equation (1.1)-(1.3) on $(-\infty,T]$. Clearly, $(\Theta x)(T)=x_1$ which implies that the system (1.1)-(1.3) is controllable on $(-\infty,T]$. This complete the proof. □

Remark 3.3. When the impulses disappear, that is $M_k=\widetilde{M_k}=0$, $k=1,2,...,m$ then the system (1.1)-(1.3) reduces to the following neutral stochastic integrodifferential equation:

$$\begin{aligned}
d\Big[x(t)-g(t,x_t,\int_0^t a_1(t,s,x_s))\Big] &= \Big[Ax(t)-g(t,x_t,\int_0^t a_1(t,s,x_s))ds\Big]dt+f(t,x_t,\int_0^t a_2(t,s,x_s)ds)dt\\
&+ Bu(t)dt+\Big[\int_0^t\gamma(t-s)\Big[x(s)-g(s,x_s,\int_0^s a_1(s,r,x_r)dr)\Big]ds\Big]dt\\
&+ \sigma(t,x_t,\int_0^t a_3(t,s,x_s)ds)dw(t),\quad t\in I=[0,T],\ t\ne t_k, \qquad (3.3)\\
x(t) &= \varphi(t)\in\mathcal{L}_2^0(\Omega,\mathscr{B}_h),\quad \text{for } a.e.\ t\in(-\infty,0]. \qquad (3.4)
\end{aligned}$$

where the operator A,g,f,σ,a_1,a_2 and a_3 are defined as same as before. Here $\mathscr{C}=\{x:(-\infty,T]\to \mathrm{X}: x(t) \text{ is continuous}\}$, Banach space of all stochastic processes $x(t)$ from $(-\infty,T]$ into $\mathscr{X}$, equipped with the supremum norm

$$\|\phi\|^2_{\mathscr{C}} = \sup_{s\in(-\infty,T]}\mathbf{E}\|\phi(s)\|^2,\quad \text{for } \phi\in\mathscr{C}.$$

By using the same technique in Theorem 3.2, we can easily deduce the following corollary.

Corollary 3.4. *Suppose that* **(H1)**-**(H9)** *hold. Then, the system* (3.3)-(3.4) *is controllable on* $(-\infty,T]$ *provide that the condition* (3.1) *is satisfied.*

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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