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Research Article

A New Asymptotic Series and Estimates Related to Euler Mascheroni Constant

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ABSTRACT. In this article, we give a new asymptotic series for a sequence (q_n) that converges to Euler-Mascheroni's constant with the convergence speed as n^{-4}. We present and prove a theorem about how to get the sequence (q_n). Using this asymptotic series, we establish the lower and upper bounds for the sequence (q_n).

Keywords: Euler-Mascheroni's constant, asymptotic series, inequalities.

2010 Mathematics Subject Classification: 26D15, 41A25, 34E05.

1. INTRODUCTION

One of the famous constants in mathematics is the Euler-Mascheroni's constant $\gamma = 0,57721566490153286...$. It is defined as the limit of the sequence:

$$\gamma_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} - \ln n$$

in honor of the Swiss mathematician Leonhard Euler (1707-1783) and the Italian mathematician Lorenzo Mascheroni (1750-1800), who studied the Euler-Mascheroni's constant γ. The sequence $(\gamma_n)_{n\geq 1}$ and the constant γ have many applications in several branches of mathematics as probability, analysis, special functions and number theory. The sequence $(\gamma_n)_{n\geq 1}$ converges very slowly to the constant γ, with the convergence speed as n^{-1}. In the beginning, Tims and Tyrell [18], and then Young [19] got the lower and upper bounds for the sequence $(\gamma_n)_{n\geq 1}$ as the following:

$$\frac{1}{2(n+1)} < \gamma_n - \gamma < \frac{1}{2n}$$

with the convergence speed as n^{-1}. Many authors [2, 3, 6, 7, 10, 12–17] interested in obtaining sequences that converge very fast to the limit γ. One of them is DeTemple [6], who introduced the sequence

$$R_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} - \ln\left(n + \frac{1}{2}\right)$$

that converges to the limit γ as n^{-2}. Then Mortici [12] has introduced the sequence

$$t_n = 1 + \frac{1}{2} + ... + \frac{1}{n-1} + \frac{1}{2n} - \frac{1}{2}\ln\left(n^2 - \frac{1}{6}\right) \tag{1.1}$$

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in order to obtain a faster convergence to the limit γ with the convergence speed as n^{-4} and the following limit:

$$\lim_{n\to\infty} n^4\left(t_n-\gamma\right)=\frac{11}{720}.$$

Then, Cristea [4] has showed in 2014, the following double inequality

$$\frac{11}{720n^4}-\frac{29}{9072n^6}<t_n-\gamma<\frac{11}{720n^4}$$

for all integers $n \geq 1$ and has got the following asymptotic series for the sequence (t_n) given in (1.1)

$$t_n=\gamma+\sum_{k=2}^{\infty}\frac{1}{2k}\left\{\frac{1}{6^k}-B_{2k}\right\}\frac{1}{n^{2k}}$$

or

$$t_n=\gamma+\frac{11}{720n^4}-\frac{29}{9072n^6}+\frac{221}{51\,840n^8}-\frac{6469}{855\,360n^{10}}+\cdots$$

Cristea and Mortici [5] have introduced the sequence

$$s_n=1+\frac{1}{2}+...+\frac{1}{n-2}+\frac{13}{12\left(n-1\right)}+\frac{5}{12n}-\ln n \tag{1.2}$$

that converges to the limit γ with the convergence speed as n^{-3} and have demonstrated the following double inequality

$$\frac{1}{12n^3}+\frac{11}{120n^4}<s_n-\gamma<\frac{1}{12n^3}+\frac{13}{120n^4}.$$

Then, X. Hu, D. Lu, X. Wang [9] have presented the following sequence:

$$r^3_{n,2}=1+\frac{1}{2}+...+\frac{1}{n}-\ln n-\frac{1}{2}\ln\left(1+\frac{1}{n-\frac{n}{3n+1}}\right)$$

that converges to the limit γ with the convergence speed as n^{-4}, with the following approximation:

$$\frac{1}{180\left(n+1\right)^4}<\gamma-r^3_{n,2}<\frac{1}{180n^4}.$$

The aim of the paper is to introduce a new sequence (q_n) that converges very fast to the limit γ and to establish the lower and upper bounds for this sequence. Motivated by Mortici [12] and Hu [9], we introduce new sequence

$$q_n(a,b,c)=1+\frac{1}{2}+...+\frac{1}{n-2}+\frac{an+b}{n(n-1)}-\frac{1}{3}\ln\left(n^3+c\right), \tag{1.3}$$

where a,b,c are real parameters and for $a=\frac{3}{2}, b=-\frac{5}{12}, c=\frac{1}{4}$ the new sequence given by

$$q_n=q_n(\frac{3}{2},-\frac{5}{12},\frac{1}{4})=1+\frac{1}{2}+...+\frac{1}{n-2}+\frac{13}{12\left(n-1\right)}+\frac{5}{12n}-\frac{1}{3}\ln\left(n^3+\frac{1}{4}\right) \tag{1.4}$$

converges to the limit γ with the convergence speed as n^{-4}. We will show the following double inequality

$$\frac{11}{120n^4}+\frac{1}{12n^5}+\frac{181}{2016n^6}<q_n-\gamma<\frac{11}{120n^4}+\frac{1}{12n^5}+\frac{182}{2016n^6}$$

for all integers $n \geq 2$ in the left side inequality and for all integers $n \geq 225$ in the right side inequality. We will also construct the asymptotic series

$$q_n=\gamma+\frac{11}{120n^4}+\frac{1}{12n^5}+\frac{181}{2016n^6}+\frac{1}{12n^7}+\cdots$$

for the sequence (q_n) (1.4).

2. The results

We consider the sequence $(q_n(a,b,c))$ given by (1.3). To obtain the best real parameters a,b,c, for which the sequence $(q_n(a,b,c))$ converges to γ with the highest convergence speed, we prove the following theorem:

Theorem 2.1. *(i) If $a \neq \frac{3}{2}, b \neq -\frac{5}{12}$ and $c \neq \frac{1}{4}$ then the sequence $(q_n(a,b,c))_{n\geq 1}$ has the convergence speed as n^{-1}.*

(ii) If $a = \frac{3}{2}, b \neq -\frac{5}{12}$ and $c \neq \frac{1}{4}$ then the sequence $(q_n(a,b,c))_{n\geq 1}$ has the convergence speed as n^{-2}.
(iii) If $a = \frac{3}{2}, b = -\frac{5}{12}$ and $c \neq \frac{1}{4}$ then the sequence $(q_n(a,b,c))_{n\geq 1}$ has the convergence speed as n^{-3}.
(iv) If $a = \frac{3}{2}, b = -\frac{5}{12}$ and $c = \frac{1}{4}$ then the sequence $(q_n(a,b,c))_{n\geq 1}$ has the convergence speed as n^{-4}.

We will use the following:

Lemma 2.1. *If the sequence $(x_n)_{n\geq 1}$ converges to x and if there exists the limit*

$$\lim_{n\to\infty} n^k (x_n - x_{n+1}) = l \in \mathbb{R}$$

with $k > 1$, then there exists the limit

$$\lim_{n\to\infty} n^{k-1} (x_n - x) = \frac{l}{k-1}.$$

For the proof see [11]. This lemma is a form of Cesaro-Stolz's lemma. We utilize it in the construction of the asymptotics series and in order to estimate the convergence speed.

Proof. We compute the difference

$$q_n(a,b,c) - q_{n+1}(a,b,c) = \frac{an+b}{n(n-1)} - \frac{1}{n-1} - \frac{an+a+b}{n(n+1)}$$
$$-\frac{1}{3}\ln\left(n^3+c\right) + \frac{1}{3}\ln\left((n+1)^3+c\right).$$

Using a computer program as Maple, we get

$$q_n(a,b,c) - q_{n+1}(a,b,c) = \left(a-\frac{3}{2}\right)\frac{1}{n^2} + \left(a+2b-\frac{2}{3}\right)\frac{1}{n^3} + \left(a-c-\frac{5}{4}\right)\frac{1}{n^4}$$

$$+\left(a+2b+2c-\frac{4}{5}\right)\frac{1}{n^5} + O\left(\frac{1}{n^6}\right). \tag{2.5}$$

(i) If $a - \frac{3}{2} \neq 0$, then

$$\lim_{n\to\infty} n^2 \left(q_n(a,b,c) - q_{n+1}(a,b,c)\right) = \left(a-\frac{3}{2}\right) \neq 0$$

and Lemma 2.1 says that

$$\lim_{n\to\infty} n \left(q_n(a,b,c) - \gamma\right) = \left(a-\frac{3}{2}\right) \neq 0.$$

We get that the sequence $(q_n(a,b,c))_{n\geq 1}$ has the convergence speed as n^{-1}.
(ii) If $a = \frac{3}{2}, b \neq -\frac{5}{12}$ and $c \neq \frac{1}{4}$ then the relation (2.5) is written as

$$q_n(a,b,c) - q_{n+1}(a,b,c) = \left(2b + \frac{5}{6}\right)\frac{1}{n^3} + \left(\frac{1}{4} - c\right)\frac{1}{n^4}$$

$$+ \left(\frac{7}{10} + 2b + 2c\right)\frac{1}{n^5} + O\left(\frac{1}{n^6}\right). \tag{2.6}$$

If $b \neq -\frac{5}{12}$, then from the relation (2.6), we get

$$\lim_{n\to\infty} n^3 \left(q_n(a,b,c) - q_{n+1}(a,b,c)\right) = \left(2b + \frac{5}{6}\right) \neq 0$$

and Lemma 2.1 says that

$$\lim_{n\to\infty} n^2 \left(q_n(a,b,c) - \gamma\right) = \frac{1}{2}\left(2b + \frac{5}{6}\right) \neq 0.$$

We obtain that the sequence $\left(q_n(\frac{3}{2},b,c)\right)_{n\geq 1}$ has the convergence speed as n^{-2}.
(iii) If $a = \frac{3}{2}, b = -\frac{5}{12}$ and $c \neq \frac{1}{4}$ then the relation (2.5) is written as

$$q_n(a,b,c) - q_{n+1}(a,b,c) = \left(\frac{1}{4} - c\right)\frac{1}{n^4} + \left(-\frac{2}{15} + 2c\right)\frac{1}{n^5} + O\left(\frac{1}{n^6}\right). \tag{2.7}$$

Then from the relation (2.7), we get

$$\lim_{n\to\infty} n^4 \left(q_n(a,b,c) - q_{n+1}(a,b,c)\right) = \left(\frac{1}{4} - c\right) \neq 0$$

and Lemma 2.1 says that

$$\lim_{n\to\infty} n^3 \left(q_n(a,b,c) - \gamma\right) = \frac{1}{3}\left(\frac{1}{4} - c\right) \neq 0.$$

We get that the sequence $\left(q_n(\frac{3}{2},-\frac{5}{12},c)\right)_{n\geq 1}$ has the convergence speed as n^{-3}.
(iv) If $a = \frac{3}{2}, b = -\frac{5}{12}$,and $c = \frac{1}{4}$ then the relation (2.5) is written as

$$q_n(a,b,c) - q_{n+1}(a,b,c) = \frac{11}{30n^5} + O\left(\frac{1}{n^6}\right) \tag{2.8}$$

and Lemma 2.1 says that

$$\lim_{n\to\infty} n^4 \left(q_n(a,b,c) - \gamma\right) = \frac{11}{120}.$$

We get that the sequence $\left(q_n(\frac{3}{2},-\frac{5}{12},\frac{1}{4})\right)_{n\geq 1}$ has the convergence speed as n^{-4}. □

We notice that (2.8) gives us the approximation

$$q_n - \gamma \approx \frac{11}{120n^4} \quad \text{as } n \to \infty.$$

We give the following theorem related to the estimates of (q_n) given in (1.4):

Theorem 2.2. *We have the following double inequality for all integers $n \geq 2$ in the left side inequality and for all integers $n \geq 225$ in the right side inequality:*

$$\frac{11}{120n^4} + \frac{1}{12n^5} + \frac{181}{2016n^6} < q_n - \gamma < \frac{11}{120n^4} + \frac{1}{12n^5} + \frac{182}{2016n^6}.$$

Proof. We consider the following sequences

$$a_n = (q_n - \gamma) - \left(\frac{11}{120n^4} + \frac{1}{12n^5} + \frac{181}{2016n^6}\right)$$

and

$$b_n = (q_n - \gamma) - \left(\frac{11}{120n^4} + \frac{1}{12n^5} + \frac{182}{2016n^6}\right)$$

that converges to zero. To prove that $a_n > 0$ and $b_n < 0$, it suffices to show that $(a_n)_{n\geq 1}$ is strictly decreasing and $(b_n)_{n\geq 1}$ is strictly increasing. Let $f_1(n) = a_{n+1} - a_n$ and $f_2(n) = b_{n+1} - b_n$, where

$$\begin{aligned} f_1(x) &= \frac{8}{12x} + \frac{5}{12(x+1)} - \frac{1}{12(x-1)} + \frac{1}{3}\ln\left(x^3 + \frac{1}{4}\right) - \frac{1}{3}\ln\left((x+1)^3 + \frac{1}{4}\right) \\ &\quad - \left(\frac{11}{120(x+1)^4} - \frac{11}{120x^4}\right) - \left(\frac{1}{12(x+1)^5} - \frac{1}{12x^5}\right) - \left(\frac{181}{2016(x+1)^6} - \frac{181}{2016x^6}\right) \end{aligned}$$

and

$$\begin{aligned} f_2(x) &= \frac{8}{12x} + \frac{5}{12(x+1)} - \frac{1}{12(x-1)} + \frac{1}{3}\ln\left(x^3 + \frac{1}{4}\right) - \frac{1}{3}\ln\left((x+1)^3 + \frac{1}{4}\right) \\ &\quad - \left(\frac{11}{120(x+1)^4} - \frac{11}{120x^4}\right) - \left(\frac{1}{12(x+1)^5} - \frac{1}{12x^5}\right) - \left(\frac{182}{2016(x+1)^6} - \frac{182}{2016x^6}\right). \end{aligned}$$

We get

$$f_1'(x) = \frac{P(x-2)}{1680(x+1)^7(x-1)^2(4x^3+1)^1(12x+12x^2+4x^3+5)^1 x^5} > 0 \tag{2.9}$$

for all real numbers $x \geq 2$ and

$$f_2'(x) = -\frac{Q(x-225)}{120(x+1)^7(x-1)^2(12x+12x^2+4x^3+5)^1(4x^3+1)^1 x^7} < 0 \tag{2.10}$$

for all real numbers $x \geq 225$, where

$$\begin{aligned} P(x) &= 8615781393 + 48322358\,535x + 124\,451770884x^2 + 195088765300x^3 \\ &\quad + 207843366162x^4 + 159018283386x^5 + 89932803430x^6 + 38082594545x^7 \\ &\quad + 12078804629x^8 + 2834912752x^9 + 478671564x^{10} + 55071128x^{11} \\ &\quad + 3869824x^{12} + 125440x^{13} \end{aligned}$$

and

$$
\begin{aligned}
Q(x) \quad = \quad & 22876\,348962124636919596278035200 \\
& +1561258918341618251050908\,15353280x \\
& +89646892057928206975675131563 75x^{2} \\
& +23829891358302962648588882 5003x^{3} \\
& +387400193922908539529966 0913x^{4} \\
& +42953509800254866165809975x^{5} \\
& +3429542980886586835373 31x^{6} \\
& +202851374032512781609 3x^{7} \\
& +89992142959018019 73x^{8} \\
& +29943893833882652x^{9} \\
& +73805584698144x^{10} \\
& +130981721712x^{11} \\
& +158491784x^{12} \\
& +117200x^{13} \\
& +40x^{14}
\end{aligned}
$$

are two polynomials with positive integers coefficients for all real numbers $x \geq 2$ and respectively for all real numbers $x \geq 225$. Then, from (2.9), we have f_1 is strictly increasing on $[2, \infty)$ and from (2.10), we have f_2 is strictly decreasing on $[225, \infty)$. It follows that from $f_1(\infty) = f_2(\infty) = 0$, we have $f_1 < 0$ on $[2, \infty)$ and $f_2 > 0$ on $[225, \infty)$. Thus, $(a_n)_{n \geq 2}$ is strictly decreasing and $(b_n)_{n \geq 225}$ is strictly increasing. This concludes the proof. □

We can get the asymptotic series of the sequence (q_n) , using the sequence (h_n)

$$h_n = 1 + \frac{1}{2} + ... + \frac{1}{n-2} + \frac{1}{n-1} + \frac{1}{n}$$

harmonic sum in terms of digamma function ψ

$$h_n = \gamma + \frac{1}{n} + \psi(n),$$

with the digamma function defined by

$$\psi(x) = \frac{d}{dx}\left(\ln \Gamma(x)\right) = \frac{\Gamma'(x)}{\Gamma(x)}.$$

See, e.g., [1, p. 258, Rel. 6.3.2]. We have the following asymptotic expansion for the digamma function ψ that

$$\psi(x) = \ln x - \frac{1}{2x} - \sum_{k=1}^{\infty} \frac{B_{2k}}{2kx^{2k}},$$

where B_j is the jth Bernoulli numbers given by

$$\frac{1}{e^t - 1} + \frac{1}{2} - \frac{1}{t} = \sum_{j=1}^{\infty} (-1)^{j-1} \frac{t^{2j}}{(2j)!} B_j.$$

We will demonstrate the following theorem related to the asymptotic expansion of q_n :

Theorem 2.3. *We get the following asymptotic expansion of* (q_n) *as* $n \to \infty$:

$$q_n = \gamma + \frac{1}{12n(n-1)} - \sum_{k=1}^{\infty} \frac{1}{k} \left\{ \frac{(-1)^{k-1}}{3 \cdot 4^k n^{3k}} + \frac{B_{2k}}{2n^{2k}} \right\}.$$

Proof. We get

$$\begin{aligned} q_n &= h_n - \frac{1}{n} + \frac{1}{12(n-1)} + \frac{5}{12n} - \frac{1}{3}\ln\left(n^3 + \frac{1}{4}\right) \\ &= \gamma + \psi(n) + \frac{1}{12(n-1)} + \frac{5}{12n} - \frac{1}{3}\ln\left(n^3 + \frac{1}{4}\right) \\ &= \gamma + \psi(n) - \ln n + \frac{1}{12(n-1)} + \frac{5}{12n} - \frac{1}{3}\ln\left(1 + \frac{1}{4n^3}\right) \\ &= \gamma + \frac{1}{12(n-1)} - \frac{1}{2n} + \frac{5}{12n} - \sum_{k=1}^{\infty} \frac{B_{2k}}{2kn^{2k}} - \frac{1}{3}\ln\left(1 + \frac{1}{4n^3}\right) \\ &= \gamma + \frac{1}{12n(n-1)} - \sum_{k=1}^{\infty} \frac{1}{k} \left\{ \frac{(-1)^{k-1}}{3 \cdot 4^k n^{3k}} + \frac{B_{2k}}{2n^{2k}} \right\}. \end{aligned}$$

Using the binomial theorem given in [8], we get

$$\frac{1}{12n(n-1)} = \frac{1}{12n^2\left(1 - \frac{1}{n}\right)} = \frac{1}{12n^2} + \frac{1}{12n^3} + \frac{1}{12n^4} + \frac{1}{12n^5} + \cdots$$

We get an explicite form as

$$q_n = \gamma + \frac{11}{120n^4} + \frac{1}{12n^5} + \frac{181}{2016n^6} + \frac{1}{12n^7} + \cdots \tag{2.11}$$

We notice that the three terms of the asymptotic series (2.11) were used for the estimate of q_n. We give the table with the above sequences:

n	$\lvert t_n - \gamma \rvert$	$\lvert s_n - \gamma \rvert$	$\lvert r_{n,2}^3 - \gamma \rvert$	$\lvert q_n - \gamma \rvert$
250	1.30935×10^{-17}	4.26667×10^{-12}	2.25298×10^{-14}	2.03175×10^{-18}
500	2.04586×10^{-19}	2.66667×10^{-13}	7.07570×10^{-16}	3.1746×10^{-20}
1000	3.19665×10^{-21}	1.66667×10^{-14}	2.21668×10^{-17}	4.96032×10^{-22}
10000	3.19665×10^{-27}	1.66667×10^{-18}	2.22167×10^{-22}	4.96032×10^{-28}
50000	2.04586×10^{-31}	2.66667×10^{-21}	7.11076×10^{-26}	3.1746×10^{-32}

Using the values from the above table, we conclude the superiority of the sequence $(q_n)_{n\geq 225}$ over Mortici's sequence $(t_n)_{n\geq 225}$, Lu's sequence $\left(r_{n,2}^3\right)_{n\geq 225}$, Cristea and Mortici's sequence $(s_n)_{n\geq 225}$. □

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