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A PROPOSED RESPONSE SURFACE-BASED ROBUST DESIGN MODEL FOR QUALITY ENGINEERING PROBLEMS

Akın ÖZDEMİR[1,*]

[1] *Bayburt University, Department of Industrial Engineering, Bayburt Turkey*

ABSTRACT

The aim of robust design models is to reduce the variability reduction as small as possible. The process bias defined as a difference between the desired target value and the process mean is an important concern for quality engineering problems. In addition, the selection of different variability measures may also change optimal operating conditions for a response variable. Therefore, this paper is three-fold. One, another view of the dual response model is proposed with the three different variability measures in order to determine optimum robust design solutions for input variables while minimizing the process bias. Two, the linearization of constraints is performed using the sequential quadratic programming method as an effective optimization method. Three, a printing process from the literature is conducted to obtain the best optimal settings for input variables. Finally, the results of the proposed model show approximately % 16 more variance reduction than traditional models.

Keywords: Robust design, Dual response model, Response surface design, Variability measures, Sequential quadratic programming

KALİTE MÜHENDİSLİĞİ PROBLEMLERİ İÇİN ÖNERİLEN BİR YANIT YÜZEYİ TABANLI SAĞLAM TASARIM MODELİ

ÖZET

Sağlam tasarım modellerinin amacı değişkenliği mümkün olduğu kadar azaltmaktır. İstenen hedef değer ile işlem ortalaması arasındaki fark olarak tanımlanan işlem yanlılığı, kalite mühendisliği problemleri için önemli bir husustur. Ek olarak, farklı değişkenlik ölçümlerinin seçimi bir yanıt değişkeni için en uygun çalışma koşullarını da değiştirebilir. Bu nedenle, bu makalenin üç amaçlıdır. Birincisi, yanıt modelinin bir başka görünümü işlem yanlılığını en aza indirirken girdi değişkenleri için en iyi sağlam tasarım çözümlerini belirlemek amacıyla üç farklı değişkenlik ölçüsüyle önerilmiştir. İkincisi, kısıtlamaların doğrusallaştırılması, etkin bir optimizasyon yöntemi olarak sıralı ikinci dereceden programlama yöntemi kullanılarak gerçekleştirilir. Üçüncüsü, girdi değişkenleri için en uygun ayarları elde etmek için literatürden bir baskı işlemi süreci araştırılmıştır. Son olarak, önerilen modelin sonuçları geleneksel modellere göre yaklaşık % 16 daha fazla varyansın azaldığını gösterir.

Anahtar kelimeler: Sağlam tasarım, İkili tepki modeli, Yanıt yüzey tasarımı, Farklı değişkenlik ölçüleri, Sıralı ikinci dereceden programlama

1. INTRODUCTION

Robust design (RD), originally coined by Taguchi [1], has become one of the quality improvement methods in the quality engineering literature to find optimal design factor settings by minimizing the process variance as possible as and reaching the desired target value. Vining and Myers [2] offered a dual response-based RD model with a zero-bias assumption. Further, Del Castillo and Montgomery [3] improved the dual response model and they proposed more flexible and easier approach obtaining better solutions in the experimental region. The dual response approach could be improved to consider the process bias; therefore, Lin and Tu [4] offered a mean-squared error (MSE) criterion-based RD model. Cho et al. [5] developed the weighted MSE methods. Further, Copeland and Nelson [6] introduced an RD model with the desired distance for the bias. Kim and Lin [7] conducted another dual response modification using the fuzzy model approach to optimize input variables. In addition, Cho et al. [8] introduced the priority concept in the MSE approach. Tang and Xu [9] developed another approach with different weights for the process bias and variance. Similarly, Kim and Cho [10] developed a priority-based RD approach.

* Sorumlu yazar / Corresponding author, e-posta / e-mail: akinozdemir@bayburt.edu.tr
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In addition, Koksoy and Doganaksoy [11] suggested a joint optimization technique under no constraints. There also exist some research attempts in the multi-objective RD optimization problems. For example, Ding et al. [12] proposed weighted methods in multi-objective optimization problems. In addition to these studies, Romano et al. [13] modified the dual response model using the quality loss function. Further, Shin and Cho [14] offered another perspective on the dual response model using a bias-specified method and derived the Karush-Kuhn-Tucker conditions. Additionally, Koksoy [15] and Park et al. [16] separately proposed the MSE criterion for multi-response surface optimization problems. Robinson et al. [17] introduced another approach using generalized linear mixed models. In addition to low-order models, Shaibu and Cho [18] considered higher polynomial models for the MSE approach. Further, Costa [19] proposed a variant model with the estimated mean and standard deviation response functions. Along the same lines, Ozdemir and Cho [20] conducted one of the recent studies in the response surface-based robust design models using the standard deviation and variance estimators for an integer-constrained experimental design region. In addition, Chan and Ozdemir [21] proposed response surface-based robust design models with a skew-normal distribution while considering the truncated statistics. Further, Ozdemir and Cho [22] proposed a 0-1 mixed-integer nonlinear programming model for both qualitative and quantitative variables in the context of the response surface-based robust design model. Lu et al. [23] developed a robust design method to apply to the renewable energy system. Then, Chartterjee et al. [24] proposed a response surface modeling method to robust design problems while considering supersaturated designs. Ouyang et al. [25] developed an interval programming model in order to measure the level of quality while considering the decision maker’s preferences for continuous improvement. Ouyang et al. [25] also reviewed dual response approaches for process optimization. Finally, Ozdemir and Cho [26] developed the *D*-optimality-embedded response surface optimization models for nonlinearly constrained irregular experimental design region.

The main objective of this research paper is three-fold. One, the existing RD approaches may not efficiently reach optimal operating conditions in a number of practical industrial applications, such as deterioration and shrinkage, due to less variance reduction and attaining more bias. For these reasons, we may desire to control the variability measure by a specified upper bound in a cuboidal or a spherical design region while we may still desire to minimize the process bias as small as possible. The selection of different variability measures may provide alternate solutions. Thus, another view of s robust design model with the three different variability measures is proposed. Two, some optimization techniques, such as the Lagrangian multipliers and the Karush-Kuhn-Tucker conditions, may not be appropriate or inefficient due to somehow misleading, more complexity, and more computational time to solve the proposed model. The feasibility conditions may not also hold true. Therefore, it is believed that the sequential quadratic programming (SQP) has been one of the most effective techniques for solving the proposed RD optimization problem. Three, we also experimentally investigate the results of optimal operating conditions for each variability measure.

This research article is organized as follows. First, material and method are discussed in Section 2. Then, a numerical example and comparisons of the results are then performed in Section 3. Finally, concluding remarks are drawn in Section 4.

2. MATERIAL AND METHOD

2.1. Material

Box and Draper [27] conducted an experimental study in order to investigate the effect of three input variables, namely x_1 (speed), x_2 (pressure), and x_3 (distance), on a printing process quality for package labels (*y*). This experiment is a three-level factorial design of the three input variables with three replicates at each experimental design point. It is analyzed the same data set from Box and Draper [27] in order to provide fair comparisons with the previously offered models.

Table 1 shows the abbreviations and notation used in this article.

Table 1. Abbreviations and notation

Abbreviations/ Notation	Meaning
y_{ui}	The u^{th} response variable of the i^{th} replicate where i= 1, 2, …, m
$\bar{y}_u$	Mean value of the u^{th} experimental run
x_i	The i^{th} input variable where i = 1, …, n
$\mathbf{x}$	A vector of input variables
$f(\mathbf{x})$	An objective function
$g_k(\mathbf{x})$	The k^{th} inequality constraint

$\hat{\mu}(\mathbf{x})$	The estimated mean response function
$\hat{\sigma}(\mathbf{x})$	The estimated standard deviation response function
$\hat{\sigma}^2(\mathbf{x})$	The estimated variance response function
$\ln\hat{\sigma}(\mathbf{x})$	The estimated logarithm response function of the standard deviation
σ_0	An upper bound for the process standard deviation
σ_0^2	An upper bound for the process variance
$\ln\ \sigma_0$	An upper bound for the logarithm of the standard deviation
τ_μ	Target value
$\lvert\hat{\mu}(\mathbf{x})-\tau_\mu\rvert$	An absolute value of the process bias function
s_u	The estimated standard deviation of the u^{th} run where i = 1, …, m
s_u^2	The estimated variance of the u^{th} run where i = 1, …, m
$\ln\ s_u$	The estimated logarithm of the standard deviation of the u^{th} run where i = 1, …, m
ρ^2	The radius of the experimental region
LB	Lower bound
UB	Upper bound

2.2. Proposed Method

The proposed method consists of five stages, which are the experimental phase, the regression model selection phase, the formulation phase, the optimization phase, and the verification phase. Each phase is described in what follows.

2.2.1. Experimental phase

The proposed model is aimed to work well in the second-order designs, such as the three-level factorial design. Table 2 shows an experimental design format for the proposed model estimating the process mean, standard deviation, variance, and the logarithm of the standard deviation.

Table 2. Experimental design format for data collection

Run (u)	**x**	*Replications*			$\bar{y}_u$	s_u	s_u^2	$\ln\ s_u$
1	Input variable settings	y_{11}	…	y_{1m}	$\bar{y}_1$	s_1	s_1^2	$\ln\ s_1$
2		y_{21}	…	y_{2m}	$\bar{y}_2$	s_2	s_2^2	$\ln\ s_2$
⋮		⋮	⋮	⋮	⋮	⋮	⋮	⋮
u		y_{u1}	…	y_{um}	$\bar{y}_u$	s_u	s_u^2	$\ln\ s_u$

2.2.2. Regression model selection phase

In general, approximation functions of the models are described as the first-order and second-order models in the literature. Firstly, if the response is modeled by just a linear function of input variables, then this function is denoted as the first-order model. The general formula of the first-order model is shown as follows:

$$\hat{y}=\varphi_0+\varphi_1x_1+\varphi_2x_2+\ldots+\varphi_nx_n+\varepsilon \tag{1}$$

where ε and φ_i represent the observed error and estimated regression coefficients for the response, respectively. Secondly, if there is curvature in the process, then quality engineers generally use second-order models. The general formula of the second-order model is given for the response by

$$\hat{y} = \varphi_0 + \sum_{i=1}^{n} \varphi_i x_i + \sum_{i=1}^{n} \varphi_{ii} x_i^2 + \sum_{i<j=2} \sum_{i=1}^{n} \varphi_{ij} x_i x_j + \varepsilon \tag{2}$$

Response surface design (RSD) is a critical approach of experimental designs for developing new processes, optimizing the operating conditions, and enhancing the design. Many researchers and practitioners have combined robust design philosophies with the RSD to formulate the response as a function of input variables. In a number of situations, the response function depends on input variables. Furthermore, the exact response function is challenging or is unknown Thus, the RSD is applied to find the fitted response functions. The estimated response functions are found as follows:

$$\hat{\mu}(\mathbf{x}) = \mathbf{X}\hat{a} \text{ where } \hat{a} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\bar{\mathbf{y}},\ \mathbf{X} = \begin{pmatrix} 1 & x_{11} & \cdots & x_{1i} & x_{11}^2 & \cdots & x_{1i}^2 & x_{11}x_{12} & \cdots & x_{1n-1}x_{1n} \\ 1 & x_{21} & \cdots & x_{2i} & x_{21}^2 & \cdots & x_{2i}^2 & x_{21}x_{22} & \cdots & x_{2n-1}x_{2n} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{n1} & \cdots & x_{ni} & x_{n1}^2 & \cdots & x_{ni}^2 & x_{n1}x_{n2} & \cdots & x_{nn-1}x_{nn} \end{pmatrix}, \tag{3}$$

and $\bar{\mathbf{y}} = [\bar{y}_1,\ \bar{y}_2,\ ...,\ \bar{y}_n]'$

$$\hat{\sigma}(\mathbf{x}) = \mathbf{X}\hat{b} \text{ where } \hat{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{s},\ \mathbf{s} = [s_1,\ s_2,\ ...,\ s_n]' \tag{4}$$

$$\hat{\sigma}^2(\mathbf{x}) = \mathbf{X}\hat{c} \text{ where } \hat{c} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{s}^2,\ \mathbf{s}^2 = [s_1^2,\ s_1^2,\ ...,\ s_n^2]' \tag{5}$$

$$\ln\hat{\sigma}(\mathbf{x}) = \mathbf{X}\hat{d} \text{ where } \hat{d} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\ln\mathbf{s},\ \ln\mathbf{s} = [\ln s_1,\ \ln s_2,\ ...,\ \ln s_n]' \tag{6}$$

2.2.3. Formulation phase

In this section, a nonlinear programming model is formulated to obtain optimum operating conditions of the mean, standard deviation, and variance of the response in terms of input variables. It will be proposed the model with different variability measures. It is also known that the selection of the different variability measures may change optimal operating conditions in the RSDs [28]. Even the smallest variation is desired, it may be also considered to minimize the process bias and seek an optimal solution in an experimental region. Thus, the absolute value of the bias function is minimized when the estimated response function of the variation is controlled by a specified upper bound. The objective of the proposed model will minimize the process bias function; therefore, the process mean will be closer to the desired target value achieving more variability reduction for the process. The first constraint in Table 3 is that the estimated process standard deviation should be less than or equal to an upper bound for the process standard deviation. We can use the process variance as a variability measure. Therefore, the second constraint in Table 3 is that the estimated process variance should be less than or equal to an upper bound for the process variance. In some cases, the estimated logarithm of the standard deviation may give better solutions. Thus, the third constraint in Table 3 is that the estimated logarithm of the standard deviation should be less than or equal to an upper bound for the logarithm of the standard deviation. In addition, the boundary constraints are given in Table 3 for a cuboidal or a spherical design region. It is also noted that the boundary requirements should be satisfied in order to obtain optimal solutions for input variables. Table 3 shows the proposed robust design model with different variability measures.

Table 3. Proposed model with the different variability measures

Minimize $f(\mathbf{x}) =$	$\left\lvert \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\bar{\mathbf{y}} - \tau_\mu \right\rvert \Rightarrow \left\lvert \varphi_0 + \sum_{i=1}^{n} \varphi_i x_i + \sum_{i=1}^{n} \varphi_{ii} x_i^2 + \sum_{i<j=2} \sum_{i=1}^{n} \varphi_{ij} x_i x_j - \tau_\mu \right\rvert$
Satisfy to	Constraint 1
	$\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{s} \le \sigma_0 \Rightarrow b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} b_{ii} x_i^2 + \sum_{i<j=2} \sum_{i=1}^{n} b_{ij} x_i x_j \le \sigma_0$
	or Constraint 2
	$\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{s}^2 \le \sigma_0^2 \Rightarrow c_0 + \sum_{i=1}^{n} c_i x_i + \sum_{i=1}^{n} c_{ii} x_i^2 + \sum_{i<j=2} \sum_{i=1}^{n} c_{ij} x_i x_j \le \sigma_0^2$
	or Constraint 3

Given	$(\mathbf{X}^{'}\mathbf{X})^{-1}\mathbf{X}^{'}\ln\mathbf{s} \le \ln\sigma_0 \Rightarrow d_0 + \sum_{i=1}^{n} d_i x_i + \sum_{i=1}^{n} d_{ii} x_i^2 + \sum_{i<j=2} \sum_{i=1}^{n} d_{ij} x_i x_j \le \ln\sigma_0$
	μ_τ and the variability measure (σ_0 or σ_0^2 or $\ln\sigma_o$)
	Design regions $\begin{cases} \text{A cuboidal design region} \Rightarrow \mathbf{x} \in [LB,\ UB] \\ \text{A spherical design region} \Rightarrow \sum_{i=1}^{n} x_i^2 \le \rho^2 \text{ where } i = 1,\ 2,\ ...,\ n \end{cases}$
Find	Input variable settings $\mathbf{x}^*$ and $f(\mathbf{x})$

2.2.4. Optimization phase with the sequential quadratic programming procedure and verification phase for the results

The SQP procedure is an effective method for nonlinearly constrained models. This approach solves a sequence of quadratic sub-problems with the linearization of constraints iteratively. The SQP is an appropriate method for small and large scale optimization problems. It is believed that it is well-suited to solving robust design problems with nonlinearly constrained optimization schemes. In addition, a detailed discussion of the SQP can be found in Ruszczyński [29] and Fishback [30].

The general form of the proposed model is denoted as follows:

$$\begin{aligned} &\text{Minimize } f(\mathbf{x}) = \left|\mathbf{X}(\mathbf{X}^{'}\mathbf{X})^{-1}\mathbf{X}^{'}\bar{\mathbf{y}} - \tau_\mu\right| \\ &\text{subject to } g_1(\mathbf{x}) \le 0 \\ &\qquad\qquad \mathbf{x} \in X \end{aligned} \tag{7}$$

where $X \subseteq R = \{\mathbf{x} \in R^n \mid LB \le x_i \le UB\}$ and $g_1(\mathbf{x}) = \mathbf{X}(\mathbf{X}^{'}\mathbf{X})^{-1}\mathbf{X}^{'}\mathbf{s} - \sigma_0$ or $g_1(\mathbf{x})=\mathbf{X}(\mathbf{X}^{'}\mathbf{X})^{-1}\mathbf{X}^{'}\mathbf{s}^2 - \sigma_0^2$ or $g_1(\mathbf{x}) = (\mathbf{X}^{'}\mathbf{X})^{-1}\mathbf{X}^{'}\ln\mathbf{s} - \ln\sigma_0$. Further, $f: R^n \to R$ and $g_1(\mathbf{x}): R^n \to R$ are twice continuously differentiable. We then formulate the Lagrangian function $L: R^n \times R^m \to R$ as follows:

$$L(\mathbf{x}, \boldsymbol{\lambda}) = f(\mathbf{x}) + \langle \boldsymbol{\lambda},\ g(\mathbf{x}) \rangle \tag{8}$$

In addition, the necessary conditions of the proposed model are given as follows:

$$\nabla_x L(\mathbf{x}, \boldsymbol{\lambda}) = 0,\ g(\mathbf{x}) \le 0,\ \boldsymbol{\lambda} \ge 0 \text{ and } \langle \boldsymbol{\lambda},\ g(\mathbf{x}) \rangle = 0 \tag{9}$$

Next, the functions $\nabla_x L(\mathbf{x}, \boldsymbol{\lambda})$ and $g(\mathbf{x})$ are linearized at a given point ($\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}}$) by

$$\begin{aligned} &\nabla_x L(\bar{\mathbf{x}}, \boldsymbol{\lambda}) + \nabla_{xx}^2 L(\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})(\mathbf{x} - \bar{\mathbf{x}}) = 0 \\ &g(\bar{\mathbf{x}}) + g^T(\bar{\mathbf{x}})(\mathbf{x} - \bar{\mathbf{x}}) \le 0 \end{aligned} \tag{10}$$

It is also given the quadratic programming problem as follows:

$$\begin{aligned} &\text{Minimize } \langle \nabla f(\mathbf{x}),\ \mathbf{x} - \bar{\mathbf{x}} \rangle + \tfrac{1}{2}\langle \mathbf{x} - \bar{\mathbf{x}},\ [\nabla_{xx}^2 L(\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})](\mathbf{x} - \bar{\mathbf{x}}) \rangle \\ &\text{subject to } g(\bar{\mathbf{x}}) + g^T(\bar{\mathbf{x}})(\mathbf{x} - \bar{\mathbf{x}}) \le 0 \end{aligned}$$

(11)

Given the approximation of the solution $\mathbf{x}^k$ and multiplier $\boldsymbol{\lambda}^k$ at iteration *k*, the sequential quadratic programming model is solved as follows:

$$\begin{aligned} &\text{Minimize } \langle \nabla f(\mathbf{x}),\ \Delta\mathbf{x} \rangle + \tfrac{1}{2}\langle \Delta\mathbf{x},\ [\mathbf{H}_k]\Delta\mathbf{x} \rangle \\ &\text{subject to } g(\mathbf{x}^k) + g^T(\mathbf{x}^k)\Delta\mathbf{x} \le 0 \end{aligned} \tag{12}$$

where $\Delta\mathbf{x} = \mathbf{x}^{k+1} - \mathbf{x}^k$ and $\mathbf{H}_k = \nabla_{xx}^2 L(\mathbf{x}^k, \boldsymbol{\lambda}^k)$. This algorithm is terminated when $\Delta\mathbf{x}$ is smaller than the sufficiently chosen tolerance. Further, $\mathbf{x}^k = \hat{\mathbf{x}}$, $\boldsymbol{\lambda}^k = \hat{\boldsymbol{\lambda}}$, the point $\Delta\mathbf{x} = 0$, and $\boldsymbol{\lambda} = \hat{\boldsymbol{\lambda}}$ satisfy second-order sufficient conditions of optimality in Equation (12) where $\hat{\mathbf{x}}$ is the global minimum with the multipliers $\hat{\boldsymbol{\lambda}}$ due to the bounded convex set. These second-order sufficient conditions are the same as in Equation (7). The second-order sufficient conditions of optimality are true at the solution ($\Delta\mathbf{x}$, $\boldsymbol{\lambda}$) by the continuity of $\mathbf{H}_k$. The solution provided that ($\mathbf{x}^k$, $\boldsymbol{\lambda}^k$) are in ($\hat{\mathbf{x}}$, $\hat{\boldsymbol{\lambda}}$) based on the chosen tolerance. Thus, an optimal solution ($\mathbf{x}^*$) is a global minimum in Equation (7).

2.2.5. Verification phase

The verification phase is the last phase of this study. Verification is an important phase that is used together for investigating that a process meets requirements and boundaries. In this paper, we compare to the proposed robust design model with the existing approaches in the current literature to provide fair comparisons in the last phase. It is also known that the variance reduction is an important issue for quality engineering problems. The last phase verifies that the proposed model is able to achieve more variance reduction than the traditional counterparts for a numerical example.

3. RESULTS OF THE NUMERICAL EXAMPLE AND DISCUSSIONS

The experiment and data are given in Table 4. A second-order model is used in this paper and it is also assumed that the quadratic models were adequate regardless of the significance levels of the model fitting functions. The estimated regression functions using the JMP [31] software are found as follows:

The estimated mean response function using Equation (3):

$$\hat{\mu}(\mathbf{x}) = 327.6 + 177.0x_1 + 109.4x_2 + 131.5x_3 + 32.0x_1^2 - 22.4x_2^2 - 29.1_3^2 + 66.0x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3 \tag{13}$$

The estimated standard deviation response function using Equation (4):

$$\hat{\sigma}(\mathbf{x}) = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2x_1^2 - 1.3x_2^2 + 16.8x_3^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3 \tag{14}$$

The estimated variance response function using Equation (5):

$$\hat{\sigma}^2(\mathbf{x}) = 2348.8 + 1742.3x_1 + 1893.7x_2 + 4401.6x_3 + 684.1x_1^2 - 456.5x_2^2 + 3027.7x_3^2 + 2352.1x_1x_2 + 1840.3x_1x_3 + 2049.7x_2x_3 \tag{15}$$

The estimated logarithm response function of the standard deviation using Equation (6):

$$\ln\hat{\sigma}(\mathbf{x}) = 3.5 + 0.25x_1 + 0.27x_2 + 0.68x_3 + 0.08x_1^2 - 0.02x_2^2 - 0.09x_3^2 - 0.002x_1x_2 - 0.16x_1x_3 + 0.28x_2x_3 \tag{16}$$

The proposed robust design model for the printing process experiment is given to the different variability measures in Table 5.

The optimal solutions of each variability measure are found using the MAPLE [32] software package. In addition, Table 6 shows optimal solutions for the printing process. Note that the necessary and second-order sufficient conditions are satisfied.

The results of the proposed model with the three variability measures show that the logarithmic fitted function approximates an optimal operating condition more effective than both the variance and standard deviation fitted functions.

Notice that process conditions are stable ($\overline{CV} \approx 0.1$). Further, response surface plots of the model with the three different variability measures are shown in Figures 1-3. Note that Figures 1-3 provide the graphical view of each optimal solution with the three different variability measures.

Table 4. The printing process experiment and collected data by Box and Draper [27]

u	x_1	x_2	x_3	y_{u1}	y_{u2}	y_{u3}	$\bar{y}_u$	s_u	s_u^2	$\ln s_u$
1	-1	-1	-1	34	10	28	24	12.49	156.00	2.52
2	0	-1	-1	115	116	130	120.3	8.39	70.33	2.13
3	1	-1	-1	192	186	263	213.7	42.83	1834.33	3.76
4	-1	0	-1	82	88	88	86	3.46	12.00	1.24
5	0	0	-1	44	178	188	136.7	80.41	6465.33	4.39
6	1	0	-1	322	350	350	340.7	16.17	261.33	2.78
7	-1	1	-1	141	110	86	112.3	27.57	760.33	3.32
8	0	1	-1	259	251	259	256.3	4.62	21.33	1.53
9	1	1	-1	290	280	245	271.7	23.63	558.33	3.16
10	-1	-1	0	81	81	81	81	0	0.00	N/A
11	0	-1	0	90	122	93	101.7	17.67	312.33	2.87
12	1	-1	0	319	376	376	357	32.91	1083.00	3.49
13	-1	0	0	180	180	154	171.3	15.01	225.33	2.71
14	0	0	0	372	372	372	372	0	0.00	N/A
15	1	0	0	541	568	396	501.7	92.5	8556.33	4.53
16	-1	1	0	288	192	312	264	63.5	4032.00	4.15
17	0	1	0	432	336	513	427	88.61	7851.00	4.48
18	1	1	0	713	725	754	730.7	21.08	444.33	3.05
19	-1	-1	1	364	99	199	220.7	133.82	17908.33	4.90
20	0	-1	1	232	221	266	239.7	23.46	550.33	3.16
21	1	-1	1	408	415	443	422	18.52	343.00	2.92
22	-1	0	1	182	233	182	199	29.44	867.00	3.38
23	0	0	1	507	515	434	485.3	44.64	1992.33	3.80
24	1	0	1	846	535	640	673.7	158.21	25030.33	5.06
25	-1	1	1	236	126	168	176.7	55.51	3081.33	4.02
26	0	1	1	660	440	403	501	138.94	19303.00	4.93
27	1	1	1	878	991	1161	1010	142.45	20293.00	4.96

Table 5. Proposed robust design model for the printing process experiment

Minimize $f(\mathbf{x})$ =	$\lvert 327.6+177.0x_1+109.4x_2+131.5x_3+32.0x_1^2-22.4x_2^2-29.1x_3^2$ $+66.0x_1x_2+75.5x_1x_3+43.6x_2x_3-500\rvert$
Subject to	Constraint 1 $\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{s}\le\sigma_0 \Rightarrow 34.9+11.5x_1+15.3x_2+29.2x_3+4.2x_1^2-1.3x_2^2$ $+16.8x_3^2+7.7x_1x_2+5.1x_1x_3+14.1x_2x_3\le\sigma_0$ or Constraint 2 $\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{s}^2\le\sigma_0^2 \Rightarrow 2348.8+1742.3x_1+1893.7x_2+4401.6x_3+684.1x_1^2$ $-456.5x_2^2+3027.7x_3^2+2352.1x_1x_2+1840.3x_1x_3+2049.7x_2x_3\le\sigma_0^2$ or Constraint 3

Given	$(\mathbf{X}^{'}\mathbf{X})^{-1}\mathbf{X}^{'}\ln\mathbf{s} \le \ln\sigma_0 \Rightarrow 3.5 + 0.25x_1 + 0.27x_2 + 0.68x_3 + 0.08x_1^2$ $-0.02x_2^2 - 0.09x_3^2 - 0.002x_1x_2 - 0.16x_1x_3 + 0.28x_2x_3 \le \ln\sigma_0$
	$\mu_\tau = 500,\ \sigma_0 = 45,\ \sigma_0^2 = 2025,$ and $\ln\sigma_o$=3.807
	$\mathbf{x} \in [LB,\ UB]$ (a cuboidal region) $\Rightarrow -1 \le x_i \le 1$ and $\forall x_i \in R\ (i = 1,2,3)$
Find	Factor settings $\mathbf{x}^* = (x_1^*,\ x_2^*,\ x_3^*)^T$ and an objective function value of the model

Table 6. Optimal solutions of each variability measure for the printing process

Estimator type	$\mathbf{x}^*$	$\hat{\mu}(\mathbf{x})$	$\hat{\sigma}(\mathbf{x})$	$\lvert\hat{\mu}(\mathbf{x}) - \tau_\mu\rvert$
$\hat{\sigma}(\mathbf{x})$	(1.000, 0.112, -0.259)	499.082	44.975	0.918
$\hat{\sigma}^2(\mathbf{x})$	(1.000, -0.474, -0.131)	423.492	44.962	76.053
$\ln\hat{\sigma}(\mathbf{x})$	(0.523, 0.558, -0.011)	500.103	44.288	0.103

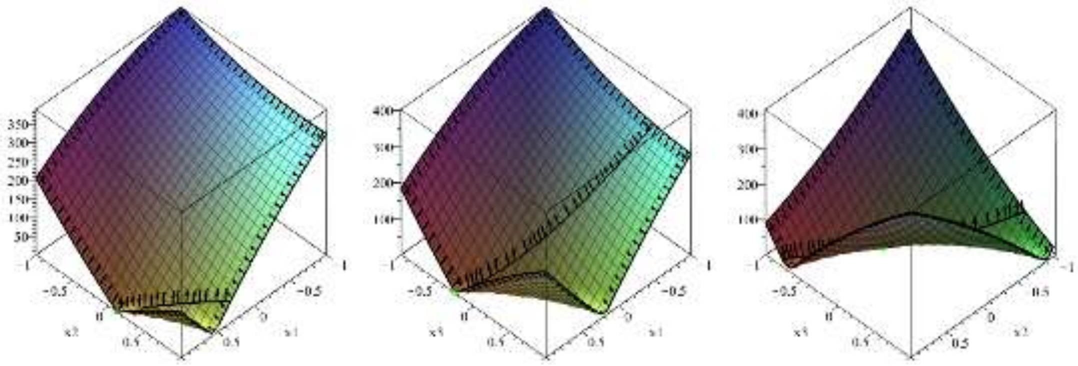

Figure 1. Surface plots of the proposed model with $\hat{\sigma}(\mathbf{x})$

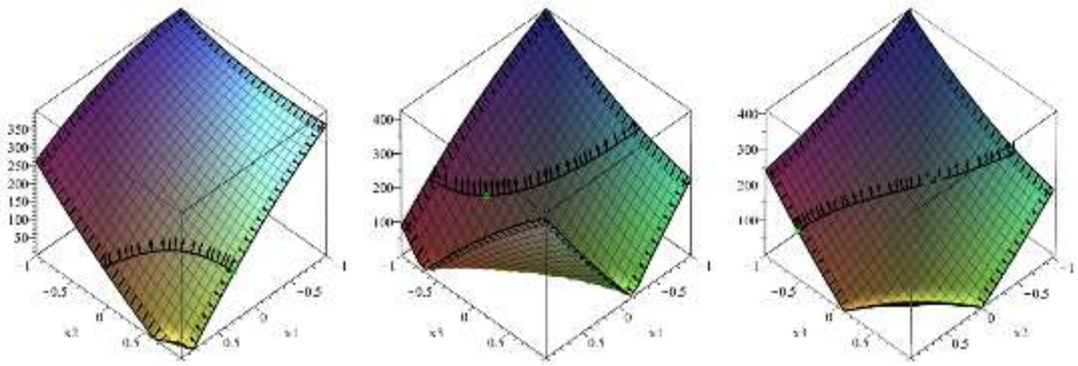

Figure 2. Surface plots of the proposed model with $\hat{\sigma}^2(\mathbf{x})$

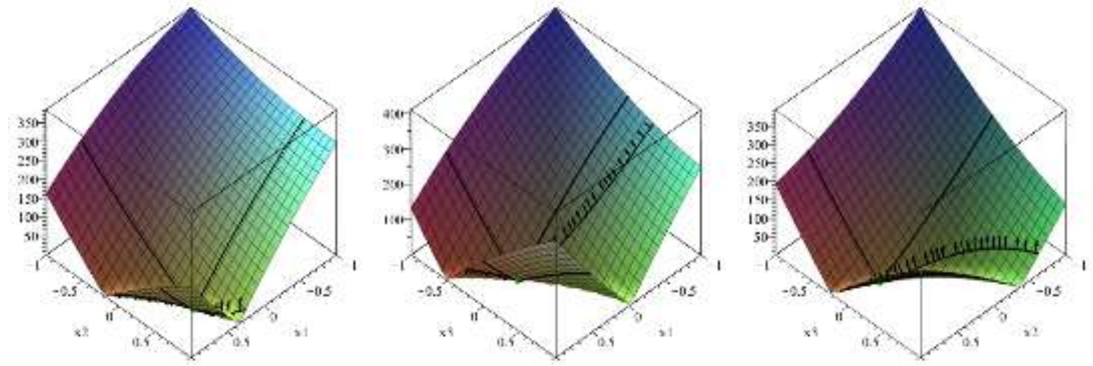

Figure 3. Surface plots of the proposed model with $\ln\hat{\sigma}(\mathbf{x})$

In Figures 1-3, y-axes represent the estimated bias values for the proposed model with $\hat{\sigma}(\mathbf{x})$, $\hat{\sigma}^2(\mathbf{x})$, and $\ln\hat{\sigma}(\mathbf{x})$, respectively. In Figure 1, the points (1.000, 0.112), (1.000, -0.259), and (0.112, -0.259) are the minimum estimated bias value. In Figure 2, the points (1.000, -0.474), (1.000, -0.131), and (-0.474, -0.131) are the minimum estimated bias value. In Figure 3, the points (0.523, 0.558), (0.523, -0.011), and (0.558, -0.011) are the minimum estimated bias value. Other points will increase the estimated bias values for the proposed model with $\hat{\sigma}(\mathbf{x})$, $\hat{\sigma}^2(\mathbf{x})$, and $\ln\hat{\sigma}(\mathbf{x})$.

For the verification phase, a number of models from the literature are used to compare with the proposed model with the three variability measures in Table 7.

Table 7. Comparing solutions to the models for the printing process

Proposed by	$\mathbf{x}^*$	$\hat{\mu}(\mathbf{x})$	$\hat{\sigma}(\mathbf{x})$	$\lvert\hat{\mu}(\mathbf{x})-\tau_{\mu}\rvert$
Vining and Myers [2]	(0.614, 0.228, 0.100)	500.00	51.766	0.000
Del Castillo and Montgomery [3]	(1.000, 0.1184, -0.259)	500.00	45.097	0.000
Lin and Tu [4]	(1.000, 0.070, -0.250)	494.44	44.429	5.560
Copeland and Nelson [6]	(0.9809, 0.0427, -0.1898)	499.00	45.200	1.000
Kim and Lin [7]	(1.000, 0.0860, -0.254)	496.08	44.628	3.920
Costa [19]	(1.000, 0.2049, -0.3180)	500.00	45.132	0.000
The proposed model using $\hat{\sigma}(\mathbf{x})$	(1.000, 0.112, -0.259)	499.082	44.975	0.918
The proposed model using $\hat{\sigma}^2(\mathbf{x})$	(1.000, -0.474, -0.131)	423.492	44.962	76.508
The proposed model using $\ln\hat{\sigma}(\mathbf{x})$	(0.523, 0.558, -0.011)	500.103	44.288	0.103

From Table 7, the Vining and Myers [2], Del Castillo and Montgomery [3], and Costa [9] models allow no process bias. The estimated process standard deviation values are 51.766, 45.097, and 45.132 for the Vining and Myers [2], Del Castillo and Montgomery [3], and Costa [9] models, respectively. The estimated process standard deviation values are greater than the proposed model using $\hat{\sigma}(\mathbf{x})$, $\hat{\sigma}^2(\mathbf{x})$, and $\ln\hat{\sigma}(\mathbf{x})$. Therefore, the results of these models are improved using the proposed model with $\hat{\sigma}(\mathbf{x})$, $\hat{\sigma}^2(\mathbf{x})$, and $\ln\hat{\sigma}(\mathbf{x})$. The estimated process standard deviation value of the Copeland and Nelson [6] model is also greater than the proposed model with $\hat{\sigma}(\mathbf{x})$, $\hat{\sigma}^2(\mathbf{x})$, and $\ln\hat{\sigma}(\mathbf{x})$. On the other hand, the Lin and Tu [4] model, the Kim and Lin model [7], and the proposed model with the three variability measures achieved more variance reduction than the Vining and Myers [2], Del Castillo and Montgomery [3], and Costa [9] models with resulted in zero bias values. It is also noted that the Lin and Tu [4] model is able to achieve variance reduction while attaining great process bias. Further, the proposed model with the logarithmic fitted function provides the smallest estimated standard deviation response function, 44.288, while attaining a small process bias value, which is 0.103. Finally, it is concluded that the proposed robust design model identifies the smallest estimation of the variability for quality engineering problems.

4. CONCLUDING REMARKS

One of the significant quality associated concerns is to find optimum operating conditions for input variables. In addition, the process variation is also another significant concern and it is desired to restrict by a specified upper bound when the process bias is minimized as small as possible. In this paper, the proposed model with the three variability measures is offered to optimize input variables. We then discuss the SQP technique to obtain optimal operating conditions efficiently. It is also verified the proposed model in the numerical example section. It is concluded that the proposed robust design model plays a critical role based on the variability measure and an upper bound for the variation in order to improve quality on a continuous basis. Finally, the proposed model with the three variability measures is compared with the existing RD models from the literature.

In this paper, there are two limitations. The first limitation is that this paper focuses on just controllable input variables. The second limitation is that a single quality characteristic is considered for the proposed methodology. For future studies, further research work could be involving both controllable and noise input variables at the same time, and modeling in multiple quality characteristics for the methodology.

REFERENCES

[1] G. Taguchi, Introduction to Quality Engineering. New York: UNIPUB/Kraus International, 1986.
[2] G. G. Vining and R. H. Myers, “Combining Taguchi and response surface philosophies: a dual response approach,” Journal of Quality Technology, vol. 22, no. 1, pp. 38-45, 1990.
[3] E. Del Castillo and D. C. Montgomery, “A nonlinear programming solution to the dual response problem,” Journal of Quality Technology, vol. 25, no. 3, pp. 199-204, 1993.
[4] D. K. J. Lin and W. TU, “Dual response surface optimization,” Journal of Quality Technology, vol. 27, no. 1, pp. 34-39, 1995.
[5] B. R. Cho, M. D. Philips, and K. C. Kapur, “Quality improvement by RSM modelling for robust design,” Proceedings of the Fifth Industrial Engineering Research Conference, 1996, pp. 650-655.
[6] K. A. F. Copeland and P. R. Nelson, “Dual response optimization via direct function minimization,” Journal of Quality Technology, vol. 28, no. 3, pp. 331-336, 1996.
[7] K. J. Kim and D. K. J. Lin, “Dual response surface optimization: a fuzzy modelling approach,” Journal of Quality Technology, vol. 30, no. 1, pp. 1-10, 1998.
[8] B. R. Cho, Y. J. Kim, D. L. Kimbler, and M. D. Phillips, “An integrated joint optimization procedure for robust and tolerance design,” International Journal of Production Research, vol. 38, no. 10, pp. 2309-2325, 2000.
[9] L. C. Tang and K. Xu, “A unified approach for dual response surface optimization,” Journal of Quality Technology, vol. 34, no. 4, pp. 437-447, 2002.
[10] Y. J. Kim and B. R. Cho, “Development of priority-based robust design,” Quality Engineering, vol. 14, no. 3, pp. 355-363, 2002.
[11] O. Köksoy and N. Doganaksoy, “Joint optimization of mean and standard deviation using response surface methods,” Journal of Quality Technology, vol. 35, no. 3, pp. 239-252, 2003.
[12] R. Ding, D. K. J. Lin, and D. Wei, “Dual-response surface optimization: a weighted MSE approach,” Quality Engineering, vol. 16, no. 3, pp. 377-385, 2004.
[13] D. Romano, M. Varetto, and G. Vicario, “Multiresponse robust design: a general framework based on combined array,” Journal of Quality Technology, vol. 36, no. 1, pp. 27-37, 2004.
[14] S. Shin and B. R. Cho, “Bias-specified robust design optimization and its analytical solutions,” Computers & Industrial Engineering, vol. 48, no. 1, pp. 129-140, 2005.
[15] O. Köksoy, “Multiresponse robust design: mean square error (MSE) criterion,” Applied Mathematics and Computation, vol. 175, no. 2, pp. 1716-1729, 2006.
[16] H. Park, S. H., Park, H. B. Kong, and I. Lee, “Weighted sum mse minimization under per-BS power constraint for network MIMO systems,” Communications Letters, IEEE, vol. 16, no. 3, pp. 360-363, 2012.
[17] T. J. Robinson, S. S. Wulff, D. C. Montgomery, and A. I. Khuri, “Robust parameter design using generalized linear mixed models,” Journal of Quality Technology, vol. 38, no. 1, pp. 65-75, 2006.
[18] A. B. Shaibu and B. R. Cho, “Another view of dual response surface modeling and optimization in robust parameter design,” The International Journal of Advanced Manufacturing Technology, vol. 41, no. 7-8, pp. 631-641, 2009.
[19] N. R. P. Costa, “Simultaneous optimization of mean and standard deviation,” Quality Engineering, vol. 22, no. 3, pp. 140-149, 2010.
[20] A. Ozdemir and B. R. Cho, “A nonlinear integer programming approach to solving the robust parameter design optimization problem,” Quality and Reliability Engineering International, vol. 32, no. 8, pp. 2859-2870, 2016.

[21] H. L. Chan and A. Ozdemır, "The development of a customer-centred, response surface methodology-based robust parameter design optimisation model under a moderately skewed production process," International Journal of Experimental Design and Process Optimisation, vol. 5, no. 4, pp. 255-284, 2017.

[22] A. Ozdemir and B. R. Cho, "Response surface-based robust parameter design optimization with both qualitative and quantitative variables," Engineering Optimization, vol. 49, no. 10, pp. 1796-1812, 2017.

[23] Y. Lu, S. Wang, C. Yan, and Z. Huang, "Robust optimal design of renewable energy system in nearly/net zero energy buildings under uncertainties," Applied Energy, vol. 187, pp. 62-71, 2017.

[24] K. Chatterjee, K. Drosou, S. D. Georgiou, and C. Koukouvinos, "Response modelling approach to robust parameter design methodology using supersaturated designs," Journal of Quality Technology, vol. 50, no. 1, pp. 66-75, 2018.

[25] L. Ouyang, Y. Ma, J. Wang, Y. Tu, and J. H. Byun, "An interval programming model for continuous improvement in micro-manufacturing," Engineering Optimization, vol. 50, no. 3, pp. 400-414, 2018.

[26] A. Ozdemir and B. R. Cho, "Response surface optimization for a nonlinearly constrained irregular experimental design space," Engineering Optimization, vol. 51, no. 12, pp. 2030-2048, 2019.

[27] G. E. P. Box and N. R. Draper, Empirical Model Building and Response Surfaces, New York: Wiley, 1987.

[28] P. Goethals, L. Aragon, and B. R. Cho, "Experimental investigations of estimated response surface functions with different variability measures," International Journal of Experimental Design and Process Optimisation, vol. 1, no. 2-3, pp. 123-163, 2009.

[29] A. P, Ruszczyński, Nonlinear Optimization, New Jersey: Princeton University Press, 2006.

[30] P. E. Fishback, Linear and Nonlinear Programming with Maple: An Interactive, Applications-Based Approach, Florida: Chapman and Hall/CRC, 2009.

[31] SAS Institute, Using JMP 11, Cary: SAS Institute, 2013.

[32] Maple Version 17, Waterloo Maple Software, Waterloo: University of Waterloo, 2013.