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Exact Solutions of the Two Dimensional KdV-Burger Equation by Generalized Kudryashov Method

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ABSTRACT: In this article, the generalized Kudryashov method which provides the exact solutions is examined. It is possible to obtain new exact solutions of the nonlinear differential equations with this method. By implementing this developed method to the two-dimensional KdV-Burger equation, new exact solutions of this equation are found. This new exact solutions are the solutions that are not in the literature. In addition, two and three-dimensional graphics of these new exact solutions have been drawn and their physical behavior has been demonstrated.

Keywords: Generalized Kudryashov method, nonlinear partial differential equations, two dimensional KdV-Burger equation, exact solution

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INTRODUCTION

Problems arising in many areas are modeled and solved with mathematical arguments. With this modeling, solutions of many problems are tried to be understood by using differential equations. Therefore, obtaining the solutions of differential equations is very important. More importantly, since the nonlinear differential equations will set light to the solution of the different problems, the studies on these equations have become more intense. As a result of these intensive studies, many methods have been improved to analyze the partial differential equations. The obtained solutions by these methods have contributed to the understanding and interpretation of many physical events. Wave concept is used in order to make these kinds of equations understandable without a specific solution. These nonlinear natural phenomena are seen in the fields of biology, plasma physics, chemical kinematics, solid state physics, geochemistry, chemical physics, optical fiber and engineering. Since the application of the waves is quite large, a lots of active and vigorous methods have been introduced by different scientists to enable them to get exact solutions of the partial differential equations. Therefore, different approximation methods have been proposed and developed. Examples of these exact solution methods are exp function method (He and Wu, 2006; Ravi et al., 2017), tanh function method (Malfliet and Hereman, 1996; Malfliet, 2004), (G'/G)-expansion method (Akbar et al., 2013; Shakeel and Mohyud-Din, 2015), trial equation method (Liu, 2006; Liu, 2010; Gurefe et al., 2011; Gurefe, 2012), improved (G'/G)-expansion method (Zhang et al., 2010; Guo and Zhou, 2010), extended trial equation method (Pandir et al., 2012; Pandir et al., 2013; Gurefe et al., 2013), multiple extended trial equation method (Pandir et al., 2013), Jakobi elliptical function method (Fu, 2001; Shen and Pan, 2003), Kudryashov method (Kudryashov, 2012; Ryabov et al., 2011; Lee and Sakthivel, 2013), modified Kudryashov method (Pandir, 2014; Tandogan et al., 2013), a new version of the generalized F-expansion method (Pandir and Turhan, 2014), a new type of the generalized F-expansion method (Pandir, 2017).

Nikolay A. Kudryashov (Kudryashov, 2012) proposed an effective method to obtain the exact solutions of the nonlinear differential equations. Later, with the development of this proposed method, different versions were brought into the literature by many researchers. Recently, the proposed method was further developed and introduced into the literature as the generalized Kudryashov method by Pandır et al. (Pandir et al., 2016; Demiray et al., 2015; Demiray et al., 2015; Pandir et al., 2015).

In this study the generalized Kudryashov method is implemented to the two dimensional KdV-Burger equation. By improving the algorithms required for the generalized Kudryashov equation method and writing the codes according to the improved algorithm, different exact solutions which is not found in the literature was obtained. In the next section, the generalized Kudryashov method is explained in detail.

MATERIALS AND METHODS

In this section, it is purposed to get new exact solutions of the partial differential equations with the generalized Kudryashov method developed based on the Kudryashov method. The outlines of the generalized Kudryashov method are explicted in detail. Let's consider the general form of the partial differential equation with detached variables $x, y, z, ..., t$ as

$$\Lambda\left(u, u_x, u_y, u_z ..., u_t, ..., u_{xx}, u_{xy}, u_{xz}, ..., u_{xt}, ...\right) = 0 \tag{1}$$

The travelling wave transformation is used for the Eq.(1) as follows

$$u(x,y,z,\dots,t)=u(\eta), \quad \eta=h_1x+h_2y+h_3z+...+h_mt \tag{2}$$

where $h_j\neq 0(j=1,2,3,...,m)$. Replacing Eq. (2) into Eq. (1) abates a ordinary differential equation

$$\Upsilon(u,u',u'',u''',...)=0 \tag{3}$$

Let's consider the exact solutions of Eq. (3) as follows

$$u(\eta)=\frac{\sum_{i=0}^{N}a_iQ^i(\eta)}{\sum_{j=0}^{M}b_jQ^j(\eta)}=\frac{A[Q(\eta)]}{B[Q(\eta)]} \tag{4}$$

where $Q(\eta)=\frac{1}{1\pm e^{\eta}}$, $a_N\neq 0$, $b_M\neq 0$. We ensure that the function Q is solution of equation

$$Q'_\eta=Q^2-Q. \tag{5}$$

Handling the solution function (4), the corresponding derivatives in the differential equation (3) is calculated as

$$u'(\eta)=\frac{A'Q'B-AB'Q'}{B^2}=(Q^2-Q)\frac{A'B-AB'}{B^2} \tag{6}$$

$$u''(\eta)=\frac{Q^2-Q}{B^2}\left[(2Q-1)(A'B-AB')+\frac{Q^2-Q}{B}\left(B(A''B-AB'')-2B'A'B+2AB'^2\right)\right] \tag{7}$$

$$\begin{aligned}u'''(\eta)&=(Q^2-Q)^3\left[\frac{(A'''B-AB'''-3A''B'-3B''A')B+6B(AB''+B'A')}{B^3}\right]\\&+3(Q^2-Q)^2(2Q-1)\left[\frac{B(A''B-AB'')-2B'A'B+2AB'^2}{B^3}\right]\\&+(Q^2-Q)(6Q^2-6Q+1)\left[\frac{A'B-AB'}{B^2}\right]\\&\vdots\end{aligned} \tag{8}$$

When the obtained derivatives in (6-8) expressions are examined, a polynomial expression of a rational Q function is obtained

$$u(\eta)=\frac{a_0+a_1Q+a_2Q^2+\cdots+a_NQ^N}{b_0+b_1Q+b_2Q^2+\cdots+b_MQ^M} \tag{9}$$

as stated in the solution function (4). In order to determine the numbers N and M in Eq. (4), balancing is applied between the non-linear high order term and the term with the highest order derivative in Eq. (3). Thus, the N and M numbers necessary for the solution function are determined depending on each other. If the solution function is rewritten according to the determined numbers and derivatives are calculated accordingly, a zero polynomial is obtained related to Q function. An algebraic equation system is obtained by equating the coefficients to zero in the zero polynomial.

When this obtained algebraic equation system is solved with the help of Mathematica 10 package program, the $a_i\left(i=0,\cdots,N\right)$, $b_j\left(j=0,\cdots,M\right)$ ve $h_j\ \left(j=1,2,3,...,m\right)$ coefficients are obtained. These obtained coefficients are replaced in the solution function (4). Thus, when inverse transforation is applied to the obtained $u\left(\eta\right)$ solution functions new exact wave solutions of Eq. (1) are attained.

RESULTS AND DISCUSSION

In this chapter, we search the exact solutions of the two dimensional KdV-Burger equation by use of the generalized Kudryashov method. The nonlinear two dimensional KdV-Burger equation is described as follows

$$\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+p\frac{\partial^3 u}{\partial x^3}-q\frac{\partial^2 u}{\partial x^2}\right)_x+r\frac{\partial^2 u}{\partial y^2}=0 \tag{10}$$

where p,q and r are high-handed constants (Seadawy, 2013). The Korteweg-de Vries-Burgers (KdVB) equation is one of the most proverbial partial differential equations. It is a general equation that indicative especially propagation of waves on a viscous fluid, fluids containing gas bubbles and an elastic tube filled with turbulence. In the literature, various studies have been carried out using a kind of methods to attain the exact solutions for the KdV-Burgers equation. Also, a exhaustive description of the travelling wave solutions to the KdV-Burgers equation can be existed in the review article by Jeffrey and Kakutani (Jeffrey and Kakutani, 1972).

First let's assume travelling wave transformation to implement the generalized Kudryashov method to Eq. (10)

$$u\left(x,y,t\right)=u\left(\tau\right),\ \tau=\alpha x+\beta y+ct \tag{11}$$

where α,β and c are high-handed constants. When travelling wave transformation is applied to Eq. (10) and the integral is taken twice according to τ (integration constant taken as zero), then Eq.(10) is degraded to a nonlinear second order differential equation

$$\left(\alpha c+r\beta^2\right)u\left(\tau\right)+\frac{\alpha^2}{2}u^2\left(\tau\right)-q\alpha^3u'\left(\tau\right)+\alpha^4pu''\left(\tau\right)=0. \tag{12}$$

Before determining the proposed solution function for the obtained Eq. (12), it is necessary to apply the balancing procedure to determine the N and M numbers included in the solution function. Let's apply the balancing procedure among the highest order derivative u'' term and the highest order nonlinear u^2 terms in Eq. (12). The terms required for the balancing procedure are briefly determined from the necessary correlations as follows

$$u^2=Q\left(\tau\right)^{2N-2M}+\ldots \qquad u''\left(\tau\right)\cong Q\left(\tau\right)^{N-M+2}+\ldots \tag{13}$$

Accordingly, the balance term is obtained as $N=M+2$ from the equivalence of the $u''\approx u^2$ terms. If we prefer $M=1$ and $N=3$, then the general form of the new solution function of the Eq. (10) is determined as

$$u\left(\tau\right)=\frac{a_0+a_1Q+a_2Q^2+a_3Q^3}{b_0+b_1Q}. \tag{14}$$

The related derivatives in Eq. (12) is calculated as

$$u'(\tau)=\left(Q^2-Q\right)\frac{\left(a_1+2a_2Q+3a_3Q^2\right)\left(b_0+b_1Q\right)-\left(a_0+a_1Q+a_2Q^2+a_3Q^3\right)b_1}{\left(b_0+b_1Q\right)^2}$$

$$u''(\tau)=\frac{Q^2-Q}{\left(b_0+b_1Q\right)^2}\left[\begin{array}{l}\left(2Q-1\right)\left(a_1+2a_2Q+3a_3Q^2\right)\left(b_0+b_1Q\right)-\left(a_0+a_1Q+a_2Q^2+a_3Q^3\right)b_1\\ +\dfrac{Q^2-Q}{b_0+b_1Q}\left(\begin{array}{l}\left(b_0+b_1Q\right)^2\left(2a_2+6a_3Q\right)\\ -2b_1\left(a_1+2a_2Q+3a_3Q^2\right)\left(b_0+b_1Q\right)\\ +2b_1^2\left(a_0+a_1Q+a_2Q^2+a_3Q^3\right)\end{array}\right)\end{array}\right]. \tag{15}$$

When the correlations is substituted in Eq. (12), there is a polynomial equation linked to the $Q(\tau)$ function. If this polynomial is considered a zero polynomial, a system of algebraic equation is obtained. When this system is solved with the help of Mathematica 10 package program according to the related algorithms, a_0,a_1,a_2,a_3,b_0,b_1 and α,β,c coefficients are obtained. By replacing these coefficients in the solution function (14), exact solutions of the Eq. (10) are attained as follows.

Case 1:

$$a_0=-\frac{12q^2b_0}{25p},\ a_1=a_3+\frac{24q^2b_0}{25p},\ a_2=-2a_3-\frac{12q^2b_0}{25p},\ a_3=a_3,\ b_0=b_0,$$

$$b_1=-\frac{25pa_3}{12q^2},\ \alpha=\frac{q}{5p},\ c=\frac{6q^3}{125p^2}-\frac{5pr\beta^2}{q}. \tag{16}$$

When we subrogate Eq. (16) into Eq. (14), we find respectively dark soliton solutions of the Eq. (10) as follows

$$u_{1,1}(x,y,t)=D_1\left[\tanh^2(\tau_1)+2\tanh(\tau_1)+1\right] \tag{17}$$

$$u_{1,2}(x,y,t)=D_1\left[\coth^2(\tau_1)+2\coth(\tau_1)+1\right] \tag{18}$$

where $D_1=\dfrac{-3q^2}{25p}$, $\tau_1=\dfrac{q}{10p}x+\dfrac{\beta}{2}y+\left(\dfrac{3q^3}{125p^2}-\dfrac{5pr\beta^2}{2q}\right)t$. Here, an equation relation is used for the $Q(\tau)=\dfrac{1}{1+e^{\tau}}=\dfrac{1}{2}\left[1-\tanh\left(\dfrac{\tau}{2}\right)\right]$. Thus, we show that the exact solution for the nonlinear differential equation can be presented in the terms of hyperbolic functions.

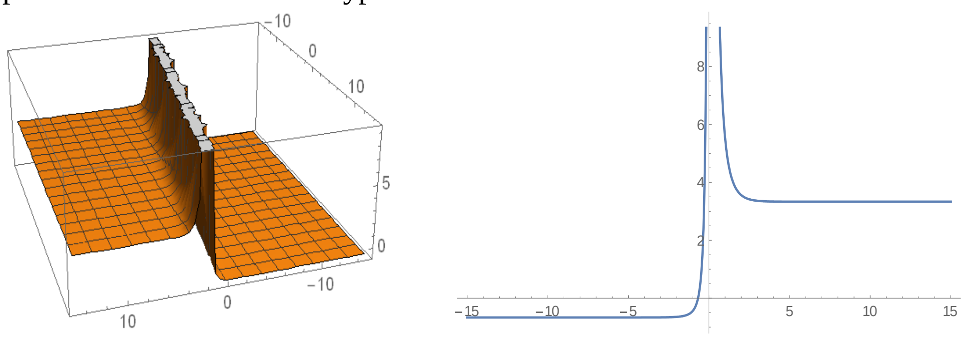
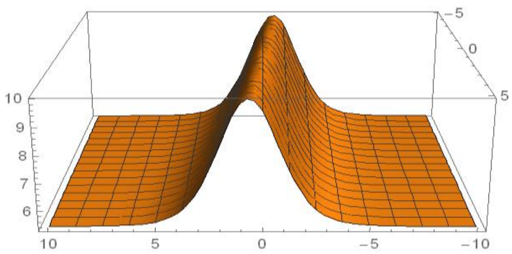


Figure 1. Three and two dimensional graphical for $\beta=1,\ r=2,\ q=5,\ p=i$ of the solution (17).



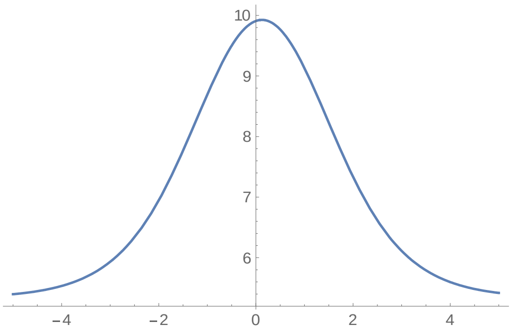


Figure 2. Three and two dimensional graphical for $\beta=1,\ r=2,\ q=5,\ p=i$ of the solution (18).

Case 2:

$$a_0=0,\ a_1=\frac{24q^2b_0}{25p},\ a_2=-2a_3-\frac{12q^2b_0}{25p},\ a_3=a_3,\ b_0=b_0,$$

$$b_1=-\frac{25pa_3}{12q^2},\ \alpha=\frac{q}{5p},\ c=-\frac{6q^3}{125p^2}-\frac{5pr\beta^2}{q} \tag{19}$$

When we replace Eq. (19) into Eq. (14), we attain respectively dark soliton solutions of the Eq. (10) as follows

$$u_{2,1}(x,y,t)=D_1\left[\tanh^2(\tau_2)+2\tanh(\tau_2)-3\right] \tag{20}$$

$$u_{2,2}(x,y,t)=D_1\left[\coth^2(\tau_2)+2\coth(\tau_2)-3\right] \tag{21}$$

where $\tau_2=\frac{q}{10p}x+\frac{\beta}{2}y-\left(\frac{3q^3}{125p^2}+\frac{5pr\beta^2}{2q}\right)t.$

Case 3:

$$a_0=\frac{12q^2b_0}{25p},\ a_1=-a_3,\ a_2=-\frac{12q^2b_0}{25p},\ a_3=a_3,\ b_0=b_0,$$

$$b_1=-\frac{25pa_3}{12q^2},\ \alpha=-\frac{q}{5p},\ c=\frac{6q^3}{125p^2}+\frac{5pr\beta^2}{q} \tag{22}$$

When we put Eq. (22) into Eq. (14), we gain respectively exact soliton solutions of the Eq. (10) as follows

$$u_{3,1}(x,y,t)=D_1\left[\operatorname{sech}^2(\tau_3)\right] \tag{23}$$

$$u_{3,2}(x,y,t)=D_1\left[\operatorname{csch}^2(\tau_3)\right] \tag{24}$$

where $\tau_3=-\frac{q}{10p}x+\frac{\beta}{2}y+\left(\frac{3q^3}{125p^2}+\frac{5pr\beta^2}{2q}\right)t.$

Case 4:

$$a_0=a_1=0,\ a_2=-\frac{12q^2b_0}{25p},\ a_3=a_3,\ b_0=b_0,\ b_1=-\frac{25pa_3}{12q^2},\ \alpha=-\frac{q}{5p},\ c=-\frac{6q^3}{125p^2}+\frac{5pr\beta^2}{q} \tag{25}$$

When we replace Eq. (25) into Eq. (14), we find respectively exact soliton solutions of the Eq. (10) as follows

$$u_{4,1}(x,y,t)=D_1\left[\tanh^2(\tau_4)-2\tanh(\tau_4)+1\right] \quad (26)$$

$$u_{4,2}(x,y,t)=D_1\left[\coth^2(\tau_4)-2\coth(\tau_4)+1\right] \quad (27)$$

where $\tau_4=-\frac{q}{10p}x+\frac{\beta}{2}y+\left(-\frac{3q^3}{125p^2}+\frac{5pr\beta^2}{2q}\right)t.$

When all obtained solutions from two dimensional KdV-Burger equation are examined, the obtained exact solutions of Eq. (20) and Eq. (21) are similar to the exact solutions of Seadawy (Seadawy, 2013) (12) and (10), respectively. Other exact solutions are solutions that are not found in the literature. It has been checked that the new exact solutions obtained here provide the two-dimensional KdV-Burger equation. In addition, two and three dimensional graphics of the obtained solution functions are shown in Figure 1-2.

CONCLUSION

In this paper, the generalized Kudryashov method has been utilized to find a new exact solutions of the two dimensional KdV-Burger equation. This method makes it possible to obtain the dark soliton solutions, travelling wave solution, soliton solution. We consider that generalized Kudryashov method can be implement to other partial differential equations.

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