MINIMIZING MAKESPAN WITH FUZZY PROCESSING TIMES UNDER JOB DETERIORATION AND LEARNING EFFECT

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Abstract
This paper proposes a single machine/processor scheduling problem considering uncertain processing times under job deterioration and learning effect. In order to express uncertainty of parameters such as processing times, effects of deterioration and learning, fuzzy numbers are used. The objective function in this study is to minimize the makespan (maximum completion time) where the parameters of the problem are fully uncertain. In the literature, many single machine scheduling problems have been interested in deterministic model parameters such as processing times, due dates and release dates. This study introduces a way for decision makers to cope with real life ambiguity and imprecision in single machine scheduling problems with uncertain processing time under uncertain effects of job deterioration and learning. Due to complexity of the problem, most of constraints are non-linear. A numerical example is illustrated and a fuzzy mixed integer nonlinear programming model is proposed in this study.

BOZULMA VE ÖĞRENME ETKİLERİ ALTINDAKİ BULANIK İŞLEM SÜRELERİ İLE ÇİZELGE TAMAMLANMA SÜRESİNİN EN AZA İNDİRİLMESİ

Anahtar Kelimeler
Çizelgeleme, Bulanık, Bozulma, Öğrenme Etkisi, Karma Tam Sayılı Doğrusal Olmayan Programlama

Öz
Bu çalışma tek makine çizelgeleme problemlerinde öğrenme ve bozulma etkileri altındaki belirsiz işlem sürelerini incelemektedir. Öğrenme etkisi, bozulma etkisi ve işlem süresi gibi parametrelerdeki belirsizliği ifade edebilmek için bulanık sayılar kullanılmıştır. Çalışmaya konu olan ve belirsiz parametrelere sahip problemin amaç fonksiyonu çizelge tamamlanma süresinin en aza indirilmesidir. Literatürde birçok tek makine çizelgeleme problemi deterministik parametreler ile incelenmiştir. Bu çalışmada ise karar vericilerin öğrenme ve bozulma etkileri altındaki gerçek hayat tek makine çizelgeleme problemlerinin belirsizliği ile başa çıkabilmelerine olanak tanıyacak bir metot tanıtılmaktadır. Problemin karmaşıklığı nedeni ile birçok kısıt doğrusal değildir. Bulanık doğrusal olmayan karma tam sayılı bir matematiksel model problemin çözümü için önerilmiştir ve ayrıca bir sayısal örnek çalışma içerisinde verilmiştir.

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1. Introduction

In the literature, many researchers have examined single machine scheduling problems considering deterministic job deterioration and learning effects. In addition, many researchers have been interested in deterministic processing times and relase dates. In real life, processing time may not be defined as determinant due to the performance of a worker or the failure of a machine. This study investigates a

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single machine scheduling problem having fuzzy processing times under the effects of job deterioration and learning. The problem of single machine scheduling under job deterioration and learning effects with certain processing times to minimize makespan may be described by the notation of $1/LE, D/C_{Max}$ where LE implies learning effect and D implies deterioration effect. In this study, we assume that there is no preemption. The processing time of a job can be expressed as a fuzzy triangular number to present the degree of satisfaction of the decision maker.

Job deterioration indicates that it may be deteriorated while waiting in a job queue or being processed on the machine. Therefore, the processing time of the work under the influence of deterioration has an increased function depending on the start time. For example, any reduction in ingot temperature while waiting to enter the rolling machine requires reheating of the ingot before rolling, the time required to control a fire will be increased, if there is a delay in the fire-fighting efforts (Mazdeh et al., 2010). Many studies on the scheduling problem under the effect of deterioration assumed the job deterioration is deterministic. Arindam et al. (2007) used a fuzzy deterioration effect for a product with stock-based demand in a proposed inventory model. Mazdeh et al. (2010) examined the single machine scheduling problem with fuzzy deteriorating effects for fuzzy processing times. They developed a multipurpose fuzzy mathematical model to minimize total tardiness and total work-in-process costs. The expression of the learning effect, on the other hand, indicates that the consistent repetition of an activity or activities makes the worker or workers faster and this leads to a reduction in processing times.

In the literature, many studies have been conducted on single machine scheduling problems that are interesting in the deterioration of job and learning effects. Biskup (1999) was the pioneer who investigated learning effect on scheduling problems. Mosheiov (2001) followed Biskup's (1999) study of the effect of learning and proposed a parallel machine scheduling problem with learning effect. Other pioneering studies about learning effect in scheduling problems have been studied by Mosheiov and Sidney (2003), Bachman and Janiak (2004), and Kuo and Yang (2006a, 2006b).

In the scheduling literature, the deterioration effect was firstly introduced by Gupta and Gupta (1988). In their study, they proposed heuristic algorithms for a non-linear processing time function. Browne and Yechiali (1990) set forth a stochastic single processor model under job deterioration to minimize the expected makespan value and they derived some optimal scheduling policies. Mosheiov (1991) showed that the optimal schedule can be obtained by using V-Shaped policies for scheduling deteriorating jobs when the objective is to minimize flow time and there are different rate of growth. Mosheiov (1994) considered a simple linear deterioration for processing times and showed some well-known single machine scheduling problems are polynomially solvable. Mosheiov (1995) introduced a new kind of deterioration effect. In his study, he considered that predetermined maintenance activities causes need for extra time to complete schedule and this increase was named as step-deterioration. Mosheiov (1996) handled a case of single machine scheduling problem when the objective is to minimize sum of weighted completion times with proportional weight for basic processing times under job-independent deterioration effect. In his research, he proved the optimal schedule can be obtained with Λ-shaped policy. The other pioneer studies about learning effect in single machine scheduling problems were investigated by Alidaee and Womer (1999), Bachman and Janiak (2000) and Hsu and Lin (2003) .

Wang and Cheng (2007) used position-dependent learning effect and time-dependent deterioration effect at same time to minimize makespan. They evaluated $1|p_{i,r} = (p_0 + a_i t)|C_{max}$ model of Browne and Yechiali (1990) and they revised that model as $1|p_{i,r} = (p_0 + a_i t)r^{\alpha}|C_{max}$ where $p_{i,r}$ is actual processing time, r^{α} is position based learning effect and $a_i t$ is time-dependent deterioration effect. Browne and Yechiali (1990) proved largest growing rate (LGR) dispatching rule assures the optimum schedule. However, under learning effect LGR dispatching rule is not optimum policy for the problem that was introduced by Wang and Cheng (2007). Wang (2007) investigated the single machine scheduling problem with the effects of learning and deterioration effects. Furthermore, its research proved that the shortest weighted processing time (WSPT) rule provides the most appropriate schedule to minimize the weighted sum of completion times, and also that the dispatching rule of the earliest due date (EDD) provides the most appropriate program to minimize the maximum lateness. . Cheng et al. (2008) presented polynomial-time optimal solutions to minimize the makespan and total completion time. They also noted that the problems of minimizing total weighted completion time and maximum lateness can be solved under some conditions. Toksari and Guner (2008) proposed a mixed integer nonlinear programming model to minimize earliness and tardiness costs on parallel machines under the effects of learning and deterioration with common due date for all jobs. In their following work (Toksarı and Guner, 2009), they showed that there is a V-shaped optimum policy for the problem of minimizing parallel machine earliness/tardiness costs with common due dates under position based learning effect and linear/non linear deterioration effect.

There has been a growing interest in scheduling problem in fuzzy numbers. To deal with the real life's ambiguity, most of researchers have begun to use fuzzy numbers to define processing times, due dates, release dates, completion times and et al. The first work on single machine scheduling with fuzzy numbers was proposed by Han et al. (1994). In this study, they worked to minimize maximum lateness on single machine by using fuzzy due date with a controllable speed. Ishii and Tada (1995) used fuzzy precedence relation instead of crisp binary relation to get relaxation and obtain desired satisfaction level of decision maker, and they examined two objectives these were minimizing maximum lateness and minimizing satisfaction level.

Liao and Liao (1998) proposed a model with due dates and processing times that are expressed with fuzzy numbers at the same time for minimizing the minimum grade of satisfaction of each job's fitting for its own due date. They also showed that their proposed model is solvable in polynomial time. Itoh and Ishii (1999) examined a model with fuzzy processing times and fuzzy due dates for minimizing the number of tardy jobs. Chanas and Kasperski (2001) investigated a single machine scheduling problem considering fuzzy processing times and fuzzy due dates for minimizing maximum fuzzy completion time and they proposed their model by using possibility and necessity functions. Lam and Cai (2002) proposed a single machine model with fuzzy due dates when jobs are released at different times to minimize maximum lateness. Wang et al. (2002) investigated single machine ready time scheduling problem considering fuzzy processing time and adapted chance constraint programming technique to the problem. Chanas and Kasperski (2003) proposed two similar single machine scheduling problems considering fuzzy processing times and fuzzy due dates. One of these problems was to minimize the maximum expected value of a fuzzy tardiness and the other one was to minimize the expected value of a maximum fuzzy tardiness. Furthermore, they analyzed their problems' computational complexity levels. Sung and Vlach (2003) considered the minimizing number of late jobs. In their study, processing times and due dates are expressed with fuzzy numbers. Muthusamy et al. (2003) proposed time delay constrains and precedence constraints are fuzzified to compare alternative schedules' makespans and degrees of satisfaction. Chanas and Kasperski (2004) investigated the problem of $1|| \sum w_i C_i$ with fuzzy processing time by using possibility and necessity functions. Itoh and Ishii (2005) proposed a single machine scheduling problem with fuzzy random due dates. They used exponential distribution to describe randomness of due dates. Harikrishnan and Ishii (2005) handled a resource allocation problem on single machine. Other some recent papers studied fuzziness in machine scheduling papers are conducted by Kasperski (2007), Duenas and Petrovic (2008), Cheng et al. (2010), Li et al. (2010), Moghaddam et al. (2010), Li et al. (2011), Ahmadizar and Hosseini (2011, 2013), Li et al. (2012) and Li et al. (2015).

2. Fuzzy Model

In this study, we declare that we comply with scientific and ethical principles. All used materials and resources are cited in the reference section. In this section, we represent our proposed model with fuzzy processing times under the fuzzy job deterioration effect and the fuzzy learning effect. The learning effect can be presented with fuzzy numbers, because each repetition for same kind of jobs may lead different effects for machine or worker due to job's characteristics, work place's conditions, different method improvement effects or learning capacity of the worker. A decision maker wants workers learn more than previous work to minimize maximum completion time because learning effect has a reducing impact on next actual processing time. The decision maker should observe the work place's conditions and learning capacity of the workers and he should design the learning effect in form of fuzzy numbers i.e., decision maker can decide use triangular fuzzy numbers (see Fig. 1) to express different rates that $\tilde{B} = (B^L, B^C, B^R)$ where $\tilde{B}$ denotes the fuzzy position-based learning effect and $-1 < \tilde{B} < 0$. The expression of B^L denotes the left side number of position-based learning effect and it has the biggest effect to decrease actual processing time and the expression of B^R has the least effect to minimize actual processing time (see Fig. 1).

As shown in Fig. 2, the decision maker can express triangular fuzzy learning effect $\tilde{B}$ with a membership function on the interval of $[B^L, B^R]$. The membership function of learning effect is expressed in below Eq. (1).

$$\mu_{\tilde{B}}^{(x)} = \begin{cases} \dfrac{B^R - x}{B^R - B^C}, & if\ B^R \geq x > B^C \\ \dfrac{x - B^L}{B^C - B^L}, & if\ B^C \geq x \geq B^L \\ 0, & otherwise \end{cases} \tag{1}$$

Furthermore, job deterioration effect $\tilde{a}$ can be presented with triangular fuzzy numbers because of work place conditions or machine breakdowns. The job deterioration has an increasing effect on actual processing time and it can be defined as a time-dependent function (see Fig. 3). In our study, we will use a time-dependent job deterioration effect expressed with the triangular fuzzy number. Job deterioration $\tilde{a}$ is between $[a^L, a^R]$ and $0 < \tilde{a} < 1$.

As shown in Fig. 4, the decision maker can express triangular fuzzy job deterioration effect $\tilde{a}$ with a membership function on the interval of $[a^L, a^R]$. The membership function of job deterioration effect is expressed in below Eq. (2). Furthermore, each side of job deterioration effect can be observed in Fig. 4.

$$\mu_{\tilde{a}}^{(x)} = \begin{cases} \dfrac{a^R - x}{a^R - a^C}, & if\ a^R \geq x > a^C \\ \dfrac{x - a^L}{a^C - a^L}, & if\ a^C \geq x \geq a^L \\ 0, & otherwise \end{cases} \tag{2}$$

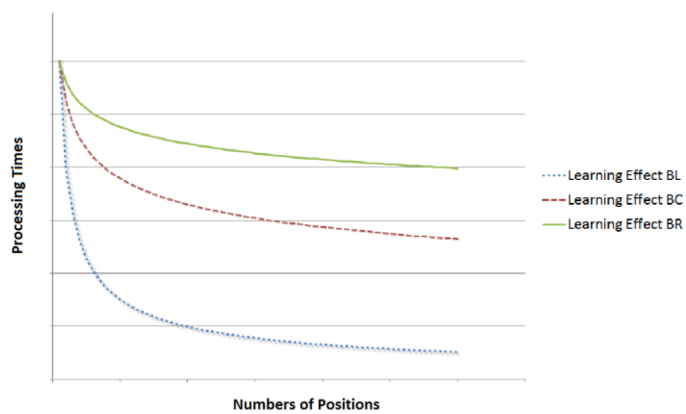

Figure 1. Actual Position-Based Processing Times for Each Side of Triangular Learning Effect

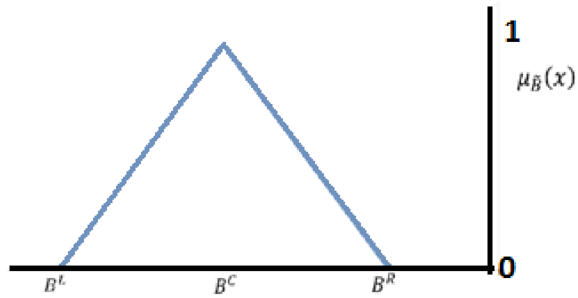

Figure 2. Learning Effect Presented with Triangular Fuzzy Numbers

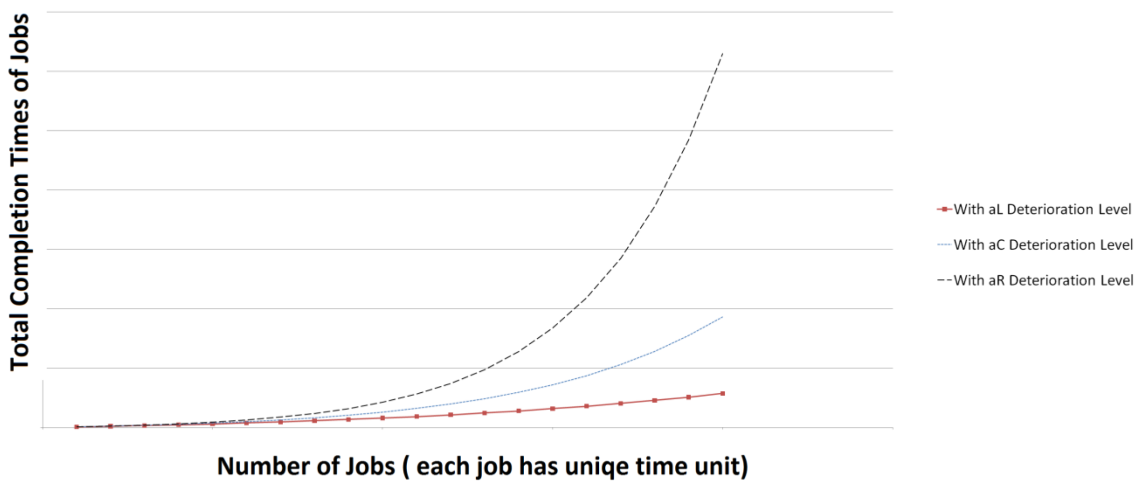

Figure 3. Total Completion Times of Jobs for Each Side of Triangular Job Deterioration Effect

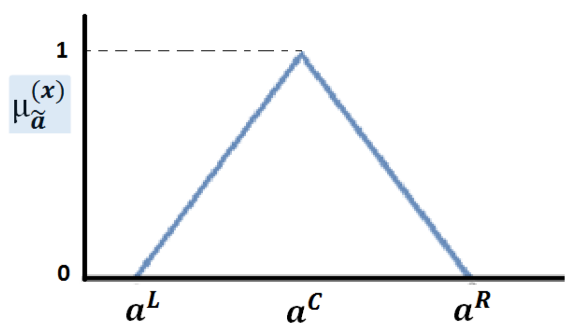

Figure 4. Job Deterioration Effect Presented with Triangular Fuzzy Numbers

The membership function of processing time expressed with fuzzy numbers is presented in Fig. 5 and Eq. (3).

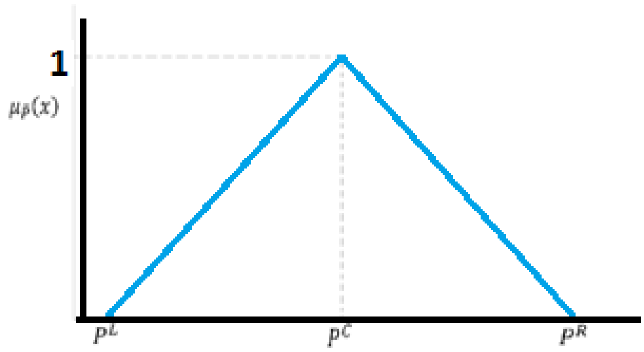

Figure 5. Fuzzy Processing Time

$$\mu_{\tilde{P}}^{(x)} = \begin{cases} \dfrac{P^R - x}{P^R - P^C}, & if\ P^R \geq x > P^C \\ \dfrac{x - P^L}{P^C - P^L}, & if\ P^C \geq x \geq P^L \\ 0, & otherwise \end{cases} \tag{3}$$

2.1 Fuzzy Nonlinear Programming Model

Indexes

π : *index of all possible sequences (permutations) for N positions* $\pi = 1, \ldots, \Pi$

r : *index of position numbers in the sequence* $r = 1, \ldots, N$

Parameters

$\tilde{P}_{r(\pi)}$: *Basic processing time in form of triangular fuzzy numbers for the job in r^{th} position in the schedule* π

$\tilde{B}$: *Fuzzy learning effect*

$\tilde{a}$: *Fuzzy deterioration effect*

A : *The time when machine can start to process jobs*

Decision Variable

$\tilde{C}_{[r](\pi)}$: *Fuzzy competition time of the job in r^{th} position in schedule* π

$\tilde{P}_{[r](\pi)}$: *Actual fuzzy processing time of the job in r^{th} position in schedule* π

$\tilde{C}_{\max(\pi)}$: *Maximum competition time of the schedule (Makespan) in schedule* π

$\mu_{\tilde{C}_{max(\pi)}}^{(C_{max})}$: *Membership Function value of Makespan value in the schedule* π

<u>*Model*</u>

$$Maximize \min_{\pi} \left\{ \mu_{\tilde{C}_{max(\pi)}}^{(C_{max})} \right\} \tag{4}$$

Subject to

$$\tilde{C}_{[r](\pi)} \cong \tilde{C}_{[r-1](\pi)} + \tilde{P}_{[r](\pi)} \ \forall\, r, \pi \tag{5}$$

$$\tilde{P}_{[r](\pi)} \cong \left(\tilde{P}_{r(\pi)} + \tilde{a} * \tilde{C}_{[r-1](\pi)} \right) r^{\tilde{B}} \quad \forall\, r, \pi \tag{6}$$

$$\tilde{C}_{[r-1](\pi)} \cong A \quad \forall \pi \tag{7}$$

$$\tilde{C}_{max(\pi)} \gtrapprox \tilde{C}_{[r](\pi)} \quad \forall\, r, \pi \tag{8}$$

$$\tilde{C}_{[r](\pi)}, \tilde{P}_{[r](\pi)}, \tilde{C}_{max(\pi)} \gtrapprox 0 \ \forall\, r, \pi \tag{9}$$

The objective function (4) maximizes minimum membership value for each possible schedule. To find decision maker's satisfaction degree for any makespan value that is on the interval of $\left[min_{\pi}\{\tilde{C}_{\max(\pi)}\},\ max_{\pi}\{\tilde{C}_{\max(\pi)}\}\right]$, we need to determine the membership functions of $\tilde{C}_{\max(\pi)}$. After converting our fuzzy nonlinear programming model (FNPM) to fuzzy mixed integer nonlinear programming model (FMINPM), the membership function of $\tilde{C}_{\max(\pi)}$ will be obtained by solving problem to find minimum and maximum $\tilde{C}_{\max(\pi)}$ values. Constrain (5) implies that fuzzy completion time at r^{th} position in schedule π essentially equals to the previous job's completion time and actual processing time at r^{th} position under job deterioration and learning effects. Constrain (6) implies that the actual fuzzy processing time at r^{th} position is essentially equal to the sum of previous job's completion time and basic fuzzy processing time under time dependent deterioration and position based learning effects. Constrain (7) shows at the $(r-1)^{th}$ position, fuzzy completion time essentially equals to machine ready time A. Constraint (8) assures that the maximum completion time (makespan) of the schedule is essentially greater than or equal to each completion time. In short makespan value must be equal to last job's completion time in a single machine scheduling problem. Constraint (9) shows that all actual processing times, completion times of positions in the schedule are essentially equal or greater than zero and they are essentially non-negative values.

The concept of fuzzy decision making was firstly introduced by Belman and Zadeh (1970). They introduced a new concept of decision making and this concept lets decision maker build objectives and constraints with non-rigid expressions. Tanaka et al. (1984) used this new concept to introduce fuzzy linear programming with fuzzy numbers. Zimmermann (1983) put forth a frame for fuzzy mathematical programming. After these pioneers' works, there have been lots of theoretical and technical works in the literature. Fuzzy nonlinear programming has been interested in some researchers. Sakawa and Yano (1989) proposed a method for multi-objective nonlinear programming problems with fuzzy numbers. Furthermore, they introduced M-α-Pareto optimality. M-α-Pareto optimality was also studied by Osman and El-Banna (1993), Kassem and Ammar (1995) and Kanaya (2010). Nasseri (2008) introduced the general concept of fuzzy non-linear programming in his research. Toksarı and Arık (2017) proposed a credibility based chance constrained solution approach for single machine scheduling problems where the processing times are uncertain and there is uncertain learning consideration. Arık and Toksarı (2018a) considered a parallel machine scheduling problem that has multiple objective functions and fuzzy processing times under the effects of fuzzy learning and deterioration. Furthermore, they adopted different fuzzy mathematical modelling techniques to their problem and they proposed a local search algorithm for the fuzzy parallel machine scheduling problems. They compared their local search method with other fuzzy mathematical programming model. Arık and Toksarı (2018b) proposed credibility based programming model for fuzzy earliness/tardiness scheduling problem on single machine. Arık (2019a) used credibility based chance constrained programming technique in project scheduling problems where tasks durations are expressed with fuzzy numbers. Toksarı and Arık (2018) proposed a genetic algorithm for flow shop scheduling problems with fuzzy common due date and the effects of fuzzy learning and fuzzy deteriorations. Arık and Toksarı (2019) investigated different types of learning and deterioration effects with fuzzy number for fuzzy parallel machine scheduling problems. Arık (2019b) investigated single machine earliness/tardiness problem considering the decision maker's tolerances for earliness and tardiness durations in case of a restrictive common due date. Arık (2019b) relaxed common due date in a single machine scheduling problem with lower and upper bounds and these bounds were used for illustrating the decision maker's tolerances or satisfaction levels or dissatisfaction levels by using fuzzy sets. Furthermore, Arık (2019b) used dissatisfaction levels of completing jobs in order to introduce a new objective criterion that minimizes the products of earliness and tardiness durations with dissatisfaction levels.

2.2 Fuzzy Mixed Integer Non-Linear Programming Model

Indexes

$i : index\ of\ jobs \quad i = 1, \dots, N$

$r : index\ of\ position\ numbers\ in\ the\ sequence \quad r = 1, \dots, N$

Parameters

P_i^L : $Left\ side\ value\ of\ job\ i^{th}\ processing\ time$

P_i^C : $Center\ value\ of\ job\ i^{th}\ processing\ time$

P_i^R : $Right\ side\ value\ of\ job\ i^{th}\ processing\ time$

B^R : $Right\ side\ value\ of\ Learning\ effect$

B^C : $Center\ value\ of\ Learning\ effect$

B^L : $Left\ side\ value\ of\ Learning\ effect$

a^R : $Right\ side\ value\ of\ Deterioration\ effect$

a^C : $Center\ value\ of\ Deterioration\ effect$

a^L : $Left\ Side\ value\ of\ Deterioration\ effect$

A : $The\ time\ when\ machine\ can\ process\ jobs$

$\overline{Cmax} =\ The\ greatest\ total\ completition\ time\ of\ all\ possible\ schedules$

$\underline{Cmax} = The\ smallest\ total\ completition\ time\ of\ all\ possible\ schedules$

Decision Variable

$CL_{[r]}$: $Actual\ competition\ time\ of\ the\ job\ in\ r^{th}\ position, calculated\ by\ using$

$left\ \ side\ values\ of\ processingtime, job\ deterioration\ \ and\ learning\ effects$

$CC_{[r]}: Actual\ competition\ time\ of\ the\ job\ in\ r^{th}\ position, calculated\ by\ using\ center\ values\ of$

$processingtime, job\ deterioration\ \ and\ learning\ effects$

$CR_{[r]}: Actual\ competition\ time\ of\ the\ job\ in\ r^{th}\ position, calculated\ by\ using$

$right\ \ side\ values\ of\ processingtime, job\ deterioration\ \ and\ learning\ effects$

$CL^{\lambda}_{[r]}: Left\ side\ value\ of\ Actual\ competition\ time\ func. of\ the\ job\ in\ r^{th}\ position\ at\ \lambda\ level$

$CR^{\lambda}_{[r]}: Right\ side\ value\ of\ actual\ competition\ time\ func. of\ the\ job\ in\ r^{th}\ position\ at\ \lambda\ level$

$C^{\lambda}_{[r]}: Deffuzicated\ actual\ competition\ time\ of\ the\ job\ in\ r^{th}\ position\ at\ \lambda\ level$

$PL^{\lambda}_{[r]}: Left\ side\ value\ of\ actual\ processing\ time\ function\ of\ the\ job\ in\ r^{th}\ position\ at\ \lambda\ level$

$PL^{\lambda}_{[r]}: Right\ side\ value\ of\ actual\ processing\ time\ function\ of\ the\ job\ in\ r^{th}\ position\ at\ \lambda\ level$

$PL_{[r]}: Actual\ processing\ time\ \ of\ the\ job\ in\ r^{th}\ position, calculated\ by\ using\ left\ \ side\ values\ of$

$processingtime, job\ deterioration\ \ and\ learning\ effects$

$PC_{[r]}: Actual\ processing\ time\ \ of\ the\ job\ in\ r^{th}\ position, calculated\ by\ using\ center\ values\ of$

$processingtime, job\ deterioration\ \ and\ learning\ effects$

$PR_{[r]}: Actual\ processing\ time\ of\ the\ job\ in\ r^{th}\ position, calculated\ by\ using\ right\ \ side\ values\ of$

$processingtime, job\ deterioration\ \ and\ learning\ effects$

$X_{i,[r]}: \begin{cases} 1, & if\ job\ i\ is\ in\ r^{th}\ position\ of\ the\ sequence \\ 0, & otherwise \end{cases}$

$C_{max}: Maximum\ competition\ time\ of\ the\ schedule\ (Makespan)$

$E(C_{max}): Expected\ maximum\ competition\ time\ of\ the\ schedule\ (Makespan)$

$\lambda: Decision\ maker's\ satisfaction\ level$

$\lambda_B: Membership\ value\ of\ job\ deterioration\ effect$

$\lambda_a: Membership\ value\ of\ learning\ deterioration\ effect$

$\lambda_{P_i}: Membership\ value\ of\ i^{th}\ job's\ \ processing\ time$

$\lambda_{P_r}: Membership\ value\ of\ r^{th}\ job's\ actual\ processing\ time$

$\lambda_{C_r}: Membership\ value\ of\ completion\ time\ ar\ r^{th}\ position$

<u>*Model*</u>

$$Maximize\ \lambda \tag{10}$$

Subject to

$$CL_{[r]} = CL_{[r-1]} + PL_{[r]} \quad \forall r \tag{11}$$

$$CC_{[r]} = CC_{[r-1]} + PC_{[r]} \quad \forall r \tag{12}$$

$$CR_{[r]} = CR_{[r-1]} + PR_{[r]} \quad \forall r \tag{13}$$

$$CL_{[r]}^{\lambda} = CL_{[r-1]}^{\lambda} + PL_{[r]}^{\lambda} \quad \forall r \tag{14}$$

$$CR_{[r]}^{\lambda} = CR_{[r-1]}^{\lambda} + PR_{[r]}^{\lambda} \quad \forall r \tag{15}$$

$$PL_{[r]} = \left(\sum_{i=1}^{N} X_{i,[r]} * P_i^L + a^L * CL_{[r-1]}\right) r^{B^L} \quad \forall r \tag{16}$$

$$PC_{[r]} = \left(\sum_{i=1}^{N} X_{i,[r]} * P_i^C + a^C * CC_{[r-1]}\right) r^{B^C} \quad \forall r \tag{17}$$

$$PR_{[r]} = \left(\sum_{i=1}^{N} X_{i,[r]} * P_i^R + a^R * CL_{[r-1]}\right) r^{B^R} \quad \forall r \tag{18}$$

$$PL_{[r]}^{\lambda} = \left(\sum_{i=1}^{N} X_{i,[r]} * \left(\lambda_{P_i} * (P_i^C - P_i^L) + P_i^L\right) + (a^R - \lambda_{a_r}(a^R - a^L)) * CL_{[r-1]}\right) r^{(B^L + \lambda_B(B^C - B^L))} \quad \forall r \tag{19}$$

$$PR_{[r]}^{\lambda} = \left(\sum_{i=1}^{N} X_{i,[r]} * \left(P_i^R - \lambda_{P_i} * (P_i^R - P_i^C)\right) + (a^R - \lambda_{a_r}(a^R - a^L)) * CR_{[r-1]}\right) r^{(B^R - \lambda_B(B^R - B^C))} \quad \forall r \tag{20}$$

$$CL_{[0]}^{\lambda} = A \tag{21}$$

$$CR_{[0]}^{\lambda} = A \tag{22}$$

$$CL_{[0]} = A \tag{23}$$

$$CC_{[0]} = A \tag{24}$$

$$CR_{[0]} = A \tag{25}$$

$$C_{[r]}^{\lambda} = \frac{(CL_{[r]}^{\lambda} + CR_{[r]}^{\lambda})}{2} \quad \forall r \tag{26}$$

$$\sum_{i=1}^{N} X_{i,[r]} = 1 \quad \forall r \tag{27}$$

$$\sum_{r=1}^{N} X_{i,[r]} = 1 \quad \forall\, i \tag{28}$$

$$Cmax = C_{[N]}^{\lambda} \tag{29}$$

$$E(Cmax) = (CL_{[N]} + 2 * CC_{[N]} + CR_{[N]})\frac{1}{4} \tag{30}$$

$$\lambda \leq \frac{\overline{Cmax} - Cmax}{\overline{Cmax} - \underline{Cmax}} \tag{31}$$

$$\lambda_{C_r} \leq \lambda_{p_r} \qquad \forall\, r \tag{32}$$

$$\lambda_{p_r} \leq \lambda_{C_{r-1}} \qquad \forall\, r \tag{33}$$

$$\lambda_{P_r} \leq \lambda_{B_r} \qquad \forall\, r \tag{34}$$

$$\lambda_{P_r} \leq \lambda_{a_r} \qquad \forall\, r \tag{35}$$

$$\lambda_{P_r} \leq \sum_{r=1}^{N} X_{i,[r]} * \lambda_{P_i} \ \forall\, r \tag{36}$$

$$\lambda \leq \lambda_{C_r} \qquad \forall\, r \tag{37}$$

$$0 \leq \lambda \leq 1 \tag{38}$$

$$0 \leq \lambda_{C_r} \leq 1 \qquad \forall\, r \tag{39}$$

$$0 \leq \lambda_{P_r} \leq 1 \qquad \forall\, r \tag{40}$$

$$0 \leq \lambda_{P_i} \leq 1 \qquad \forall\, i \tag{41}$$

$$0 \leq \lambda_{B_r} \leq 1 \qquad \forall\, r \tag{42}$$

$$0 \leq \lambda_{a_r} \leq 1 \qquad \forall\, r \tag{43}$$

$$\lambda_{C_0} = 1 \tag{44}$$

$$X_{i,[r]} \,\epsilon\, \{0,1\} \quad \forall\, i, r \tag{45}$$

$$C_{[r]}, CL_{[r]}, CC_{[r]}, CR_{[r]}, PL_{[r]}, PC_{[r]}, PR_{[r]}, C_{max}, E(C_{max}), CL_{[r]}^{\lambda}, CC_{[r]}^{\lambda} \geq 0 \ \forall\, r \tag{46}$$

Objective function (10) maximizes the decision maker's satisfaction level to minimize maximum completion time under fuzzy job deterioration and learning effects with fuzzy processing time. Since processing times, job deterioration and learning effects are presented with triangular fuzzy numbers, they have piecewise functions those have left (optimistic), center (most likely) and right (pessimistic) values dependent on λ =1 and λ =0. Therefore, the completion time and the actual processing time for each r^{th} position are expressed with triangular fuzzy numbers. Thus, the actual processing time and the completion time have optimistic values at λ =0, most likely values at λ =1 and pessimistic values λ =0. Constraints (11-13) show calculations of optimistic, most likely and pessimistic values, respectively. Constraints (14-15) show calculations of optimistic and pessimistic values with any confidence level λ each r^{th} position, respectively. Constraints (14-15) use each parameter's own confidence level such as $\lambda_{P_i}, \lambda_{a_r}$ and λ_{B_r}. Constraints (11-15) imply that each completion time at r^{th} position in the sequence is dependent on the completion time of the job in the previous position and the actual processing time of the current job that is already under deterioration and learning effects. Constraints (16-18) show calculations of optimistic, most likely and pessimistic actual processing times at r^{th} position with λ =0 and λ =1, respectively. Constraints (19-20) show calculations of optimistic and pessimistic actual processing times at r^{th} position with any confidence level λ, respectively. Constraints (16-20) imply that the actual processing time at r^{th} position is dependent on previous job's completion time and basic processing time. Constraints (21-25) show that at the 0^{th} position, the completion time equals to machine ready time A. Constraint (26) is min-max deffuzzication method for completion times at r^{th} position with any confidence level λ. Constraint (27) ensures that each job must be assigned to one position in the schedule. Constraint (28) ensures that each position must be used for only one job. Constraint (29) is to obtain the deffuzzied maximum completion time (makespan) of the schedule. The makespan value is the last position's completion time for this problem. Constraint (30) is to obtain the expected makespan value of the schedule. Constraint (31) forces the mathematical model to select the schedule having minimum makepan value with the highest satisfaction level. Constraints (32-36) are to maximize the minimum value of confidence level λ_{C_r} for completion times at r^{th} position. Constraint (37) is to maximize the minimum value of decision maker's satisfaction level for all positions in the sequence. Constraints (38-43) imply that all confidence levels must be on an interval of [0,1]. Constraint (44) shows that the confidence level of completion time at the 0^{th} position is equal to 1 because completion time at the 0^{th} position is a crisp value. Constraint (45) shows that $X_{i,[r]}$ decision variables are binary. Finally, constraint (46) shows that all actual processing times, completion times of positions in the schedule are equal or greater than zero and they are non-negative values.

3. Numerical Example

Let us have twenty jobs. Processing times of those jobs are expressed with fuzzy numbers. The job deterioration and learning effects are also expressed with triangular fuzzy numbers. The learning effect $\tilde{B}$ is between [-0.2, -0.1] and it's optimistic, most likely and pessimistic values are respectively -0.2, -0.15 and -0.1. The job deterioration effect $\tilde{a}$ is between [0.1, 0.2] and it's optimistic, most likely and pessimistic values are respectively 0.1, 0.15 and 0.2. Each processing time has membership function defined in Equation 4. Membership functions of learning effect and job deterioration effect introduced in Equation 2 and 3, respectively. The early time that the machine can start (A) is equal to 5 time units. Processing times and their expected values are given in Table 1.

Table 1
Data for the Numerical Example

Job No: $i = (1 \dots 20)$	Processing Time $\{P^L, P^C, P^R\}$	Expected Value of Processing Time $EV(\tilde{P})$
1	{9,12,16}	12,25
2	{8,17,18}	15
3	{16,17,21}	17,75
4	{6,9,13}	9,25
5	{8,13,17}	12,75
6	{12,15,16}	14,5
7	{14,16,19}	16,25
8	{10,12,22}	14
9	{7,8,13}	9
10	{11,12,20}	13,75
11	{12,13,14}	13
12	{11,14,15}	13,5
13	{13,15,18}	15,25
14	{18,19,21}	19,25
15	{5,6,9}	6,5
16	{7,8,12}	8,75
17	{12,13,15}	13,25
18	{16,17,18}	17
19	{17,18,20}	18,25
20	{10,11,15}	11,75

In order to determine $\overline{Cmax}$ and $\underline{Cmax}$, the problem was solved two time by maximizing the makespan $\overline{Cmax}$ and minimizing the makespan $\underline{Cmax}$. The results are $\overline{Cmax}$ = 1091.121, $E(Cmax)$ = 893.968 and λ=0 when the problem is solved for maximizing the makespan. The results are $\underline{Cmax}$ = 499.415, $E(Cmax)$ = 660.247 and λ=0 when the problem is solved for minimizing the makespan. By using $\overline{Cmax}$ and $\underline{Cmax}$ values, if the problem is resolved, the obtained results are $Cmax$ = 507.616, $E(Cmax)$ = 660.247 and λ=0.986 with a proposed schedule $S = \{15, 16, 9, 4, 20, 8, 10, 1, 5, 17, 11, 12, 6, 13, 7, 2, 18, 3, 19, 14\}$. The satisfaction level of λ=0.986 is so close to 1. Therefore we can say that the obtained schedule S satisfies the decision-maker with the degree of %98.6. When we examine the job order in the schedule S, the schedule does not fit any of well-known dispatching rules such as SPT that is acceptable for deterministic single machine scheduling problems under deterministic learning and job deterioration effects. Thus, we can deduce that new polynomial solution approaches should be examined for the proposed fuzzy single machine problem with consideration of fuzzy learning and job deterioration effects.

4. Conclusion

This paper investigated a single machine scheduling problem under fully fuzzy environment. The processing times of the problem assumed as fuzzy numbers and there is fuzzy learning and job deterioration consideration for the investigated problem where the objective is to minimize fuzzy makespan to assure highest satisfaction level of the decision maker. A fuzzy model o the problem is proposed and also a mixed integer nonlinear model for that model is also introduced to minimize the fuzzy makespan considering satisfaction level of the decision maker. A small example is illustrated. For future researches, polynomial time solution approaches can be examined for the proposed problem. Furthermore, the problem environment can be extended to multi-machine scheduling problem such as job shop, flow shop or open shop.

Contributions of Authors

Dr. Oğuzhan Ahmet Arık contributed to the mathematical model and experimental study in the paper. Prof. Dr. Mehmet Duran Toksarı designed the investigated problem and he contributed to the editing and writing of the paper.

Confilict of Interest

The authors declared that there is no confilict of interest.

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