Konuralp Journal of Mathematics, 8 (1) (2017) 97-105

Konuralp Journal of Mathematics

Journal Homepage: www.dergipark.gov.tr/konuralpjournalmath
e-ISSN: 2147-625X

Topological Properties of Networks Using M-Polynomial Approach

Sourav Mondal[1], Nilanjan De[2*] and Anita Pal[1]

[1] *Department of mathematics, NIT Durgapur, India.*
[2] *Department of Basic Sciences and Humanities (Mathematics), Calcutta Institute of Engineering and Management, Kolkata, India.*
[*] *Corresponding author*

Abstract

The M-polynomial is one of the algebraic polynomials, that is useful in theoretical chemistry. It plays significant role in computing the exact expressions of many degree based topological indices. In this report, the M-polynomial of the benzene ring embedded in P-type-surface in 2D network and the Tickysim SpiNNaker Model (TSM) sheet are derived. Using those M-polynomials, some degree based topological indices are derived. In addition, the results are interpreted graphically.

Keywords: *M-polynomial, Degree based topological index, benzene ring embedded in P-type-surface in 2D network, Tickysim SpiNNaker Model (TSM) sheet.*
2010 Mathematics Subject Classification: *Primary: 05C35; Secondary: 05C07, 05C40.*

1. Introduction

Throughout this article we consider simple connected graph. Let $V(G)$ and $E(G)$ be vertex set and edge set of a graph G, respectively. The total number of edges incident to $v \in V(G)$ is known as the degree of v and is denoted by d_v. Mathematical models play an important role in the analysis of important concepts of chemistry. Topological index plays key role in such modeling. Topological index is a mapping from the collection of all graphs to the set of real numbers that yields same value for isomorphic graphs. These numerical quantities corresponding to a graph are effective in correlating the structure with different physico-chemical properties, chemical reactivity, and biological activities. These indices are evaluated by formal definitions. Instead of calculating different topological indices of a specific category [1, 2], we can use a compact general method related to polynomial for calculating the same. For instance, in the domain of distance-based topological indices Wiener polynomial is a general polynomial whose derivatives at 1 yield Weiner and Hyper Weiner indices [2]. There are many such polynomials such as PI polynomial [3], Theta polynomial [4] etc.

In the area of degree-based topological indices, M-polynomial [1] perform similar role to compute closed expressions of many degree based topological indices [1, 5]. Thus computation of degree based topological indices reduce to evaluation of a single polynomial. Moreover, detailed analysis of this polynomial can yield new insights in the knowledge of degree based topological indices.

About a quarter of a century ÓKeeffe et al.[16] distributed a letter managing two 3D systems of benzene one of the structure (figure 3.1) called 6.82P (additionally polybenzene) and has a space gathering place Im3m, compared to the P-type surface. This actually inserts the hexagon-fix into the negative ebb and flow P surface. The P-type surface in the Euclidean space is coordinated with the Cartesian arrangements. The peruser can find out more about this intermittent surface in [17, 18]. This structure had to be combined as 3D carbon solids. This goal was to awaken researchers' enthusiasm for the atomic recognition in carbon nanoscience of such pleasant thoughts. The graph of the benzene ring embedded in P-type-surface network, shown in figure 3.1, contains $24mn$ nodes and $32mn-2m-2n$ edges.

Inter-chip architectures are considered as graphs in which nodes are supposed to be devices and the edges to be the topology used between devices. Hexagonal torus (12×12) is one of the network topologies utilized in this model. Each vertex is associated to six incident nodes in this topology. We also consider the finite TSM sheet which is obtained by hexagonal torus. For more discussion, see [19]. The graph of the Tickysim SpiNNaker Model (TSM) sheet, shown in figure 3.2, has mn nodes and $3mn-2m-2n+1$ edges.

A. Ahmed [20] discussed the topological properties of benzene ring embedded in P-type-surface network. In [21], topological properties of the Tickysim SpiNNaker Model are studied. Kwun et al [22] derived some degree based topological indices of V-Phenylenic Nanotubes and Nanotori using M-polynomial approach. Present authors [23] did the same work for line graph of subdivision graph of some composite graphs. For more works related to this topic, readers are referred to [24, 25, 26, 27]. The goal of this article is to compute some degree based topological indices of the benzene ring embedded in P-type-surface network and Tickysim SpiNNaker Model (TSM) sheet using M-polynomial.

Email addresses: souravmath94@gmail.com (Sourav Mondal), de.nilanjan@rediffmail.com (Nilanjan De), anita.buie@gmail.com (Anita Pal)

2. Basic definitions and literature review

Definition 2.1. *The M-polynomial of a graph G is defined as,*

$$M(G;x,y)=\sum_{i\leqslant j} m_{ij}(G)x^i y^j.$$

Where $m_{ij}(G)$ is the total count of edges $uv\in E(G)$ such that $\{d_u,d_v\}=\{i,j\}$. We use $M(G)$ for $M(G;x,y)$ in this article.

Any degree based topological indices for a graph G can be expressed as,

$$I(G)=\sum_{uv\in E(G)} f(d_u,d_v),$$

where $f=f(x,y)$ is a function appropriately selected for possible chemical applications [6]. The above result can also be written as

$$I(G)=\sum_{i\leqslant j} m_{ij}(G)f(i,j).$$

Gutman and Trinajstić introduced Zagreb indices [7].
The first Zagreb index is defined as,

$$M_1(G)=\sum_{v\in V(G)}(d_v)^2$$

The second Zagreb index is defined as,

$$M_2(G)=\sum_{uv\in E(G)} d_u d_v.$$

The forgotten topological index is defined as,

$$F(G)=\sum_{uv\in E(G)}[d_u^2+d_v^2].$$

The second modified Zagreb index is defined as,

$${}^mM_2(G)=\sum_{uv\in E(G)}\frac{1}{d_u d_v}$$

Bollobas and Erdos [8] and Amic et al. [9] presented the idea of the generalized Randić index and discussed widely in both chemistry and mathematics [10]. For more discussion, readers are referred [11, 12].
The general Randić index is defined as,

$$R_\alpha(G)=\sum_{uv\in E(G)}(d_u d_v)^\alpha$$

The inverse Randić index is defined as,

$$RR_\alpha(G)=\sum_{uv\in E(G)}\frac{1}{(d_u d_v)^\alpha}$$

The Symmetric division index of a connected graph G, is defined as,

$$SDD(G)=\sum_{uv\in E(G)}\{\frac{min(d_u,d_v)}{max(d_u,d_v)}+\frac{max(d_u,d_v)}{min(d_u,d_v)}\}.$$

The Harmonic index [13] is defined as,

$$H(G)=\sum_{uv\in E(G)}\frac{2}{d_u+d_v}.$$

The inverse sum indeg index [14] is given by

$$I(G)=\sum_{uv\in E(G)}\frac{d_u d_v}{d_u+d_v}.$$

The augmented Zagreb index proposed by Furtula et al. [15] is defined as,

$$A(G)=\sum_{uv\in E(G)}\{\frac{d_u d_v}{d_u+d_v-2}\}^3.$$

The relations of some degree-based topological indices with the M-polynomial are shown in the table 1.

Table 1: Derivation of some degree based topological indices

Topological Index	f(x,y)	Derivation from $M(G)$
M_1	$x+y$	$(D_x+D_y)(M(G))\|_{x=y=1}$
M_2	xy	$(D_xD_y)(M(G))\|_{x=y=1}$
F	x^2+y^2	$(D_x^2+D_y^2)(M(G))\|_{x=y=1}$
$^mM_2$	$\frac{1}{xy}$	$(S_xS_y)(M(G))\|_{x=y=1}$
R_α	$(xy)^\alpha$	$(D_x^\alpha D_y^\alpha)(M(G))\|_{x=y=1}$
RR_α	$\frac{1}{(xy)^\alpha}$	$(S_x^\alpha S_y^\alpha)(M(G))\|_{x=y=1}$
SDD	$\frac{x^2+y^2}{xy}$	$(D_xS_y+S_xD_y)(M(G))\|_{x=y=1}$
H	$\frac{2}{x+y}$	$2S_xJ(M(G))\|_{x=1}$
I	$\frac{xy}{x+y}$	$S_xJD_xD_y(M(G))\|_{x=1}$
A	$\frac{xy}{(x+y-2)^3}$	$S_x^3Q_{-2}JD_x^3D_y^3(M(G))\|_{x=1}$

Where,

$$D_x(f(x,y)) = x\frac{\partial(f(x,y))}{\partial x}, D_y(f(x,y)) = y\frac{\partial(f(x,y))}{\partial y},$$

$$S_x(f(x,y)) = \int_0^x \frac{(f(t,y))}{t}dt, S_y(f(x,y)) = \int_0^y \frac{(f(x,t))}{t}dt$$

$$J(f(x,y)) = f(x,x), Q_\alpha(f(x,y)) = x^\alpha f(x,y).$$

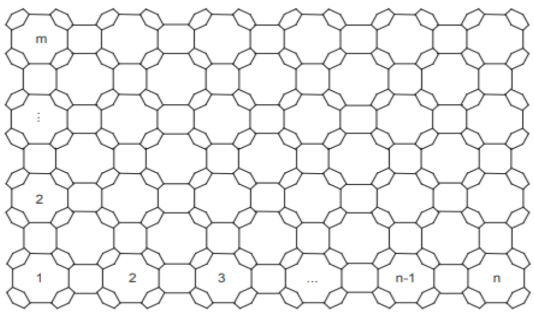


Figure 3.1: Benzene ring embedded in p-type surface in 2D network

.

3. Main Results

In this part, we give our main computational results and divide the section in two subsections.

3.1. Computational aspects of Benzene ring embedded in p-type surface in 2D network.

We compute the M-polynomial of the benzene ring embedded in p-type surface network in the following theorem.

Theorem 1. *Let G be the benzene ring embedded in p-type surface network. Then we have,*

$$M(G) = 4(m+n)x^2y^2 + 16mnx^2y^3 + 2(8mn-3m-3n)x^3y^3.$$

Proof. The edge set of G can be partitioned as,

$$|E_{\{2,2\}}| = |\{uv \in E(G) : d_u = 2, d_v = 2\}| = 4(m+n) = m_{22}.$$

$$|E_{\{2,3\}}| = |\{uv \in E(G) : d_u = 2, d_v = 3\}| = 16mn = m_{23}.$$

$$|E_{\{3,3\}}| = |\{uv \in E(G) : d_u = 3, d_v = 3\}| = 2(8mn-3m-3n) = m_{33}.$$

From the definition, the M-polynomial of G is obtained bellow.

$$\begin{aligned} M(G) &= \sum_{i\leqslant j} m_{ij}x^iy^j \\ &= m_{22}x^2y^2 + m_{23}x^2y^3 + m_{33}x^3y^3 \\ &= 4(m+n)x^2y^2 + 16mnx^2y^3 + 2(8mn-3m-3n)x^3y^3. \end{aligned}$$

This completes the proof. □

Now using this M-polynomial, we calculate some degree based topological indices of the benzene ring embedded in p-type surface network in the following theorem.

Theorem 2. *Let G be the benzene ring embedded in p-type surface network. Then we have,*

1. $M_1(G) = 4(44mn-5m-5n)$,
2. $M_2(G) = 2(120mn-19m-19n)$,
3. $F(G) = 4(124mn-19m-19n)$,
4. $M_2^m(G) = \frac{1}{3}(\frac{40}{3}mn+m+n)$,
5. $R_\alpha(G) = 2^{2(\alpha+1)}(m+n)+2^{(4+\alpha)}3^\alpha mn+2.3^{2\alpha}(8mn-3m-3n)$,
6. $RR_\alpha(G) = 2^{2(1-\alpha)}(m+n)+2^{(4-\alpha)}3^{-\alpha}mn+2.3^{-2\alpha}(8mn-3m-3n)$,
7. $SDD(G) = \frac{200}{3}mn-4m-4n$,
8. $H(G) = \frac{176}{15}mn$,
9. $I(G) = \frac{216}{5}mn-5m-5n$,
10. $A(G) = \frac{1241}{4}mn-\frac{1163}{32}m-\frac{1163}{32}n$.

Proof. Let $M(G) = f(x,y) = 4(m+n)x^2y^2+16mnx^2y^3+2(8mn-3m-3n)x^3y^3$.
Then we have,

$$\begin{aligned}
(D_x+D_y)(f(x,y)) &= 16(m+n)x^2y^2+80mnx^2y^3+12(8mn-3m-3n)x^3y^3, \\
D_xD_y(f(x,y)) &= 16(m+n)x^2y^2+96mnx^2y^3+18(8mn-3m-3n)x^3y^3, \\
(D_x^2+D_y^2)(f(x,y)) &= 32(m+n)x^2y^2+208mnx^2y^3+36(8mn-3m-3n)x^3y^3, \\
S_xS_y(f(x,y)) &= (m+n)x^2y^2+\frac{8}{3}mnx^2y^3+\frac{2}{9}(8mn-3m-3n)x^3y^3, \\
D_x^\alpha D_y^\alpha(f(x,y)) &= 2^{2(\alpha+1)}(m+n)x^2y^2+2^{4+\alpha}3^\alpha mnx^2y^3+2(3^{2\alpha})(8mn-3m-3n)x^3y^3, \\
S_x^\alpha S_y^\alpha(f(x,y)) &= 2^{2(1-\alpha)}(m+n)x^2y^2+2^{4-\alpha}3^{-\alpha}mnx^2y^3+2(3^{-2\alpha})(8mn-3m-3n)x^3y^3, \\
(D_xS_y+S_xD_y)(f(x,y)) &= 8(m+n)x^2y^2+\frac{104}{3}mnx^2y^3+4(8mn-3m-3n)x^3y^3, \\
S_xJ(f(x,y)) &= (m+n)x^4+\frac{16}{5}mnx^5+\frac{1}{3}(8mn-3m-3n)x^6, \\
S_xJD_xD_y(f(x,y)) &= 4(m+n)x^4+\frac{96}{5}mnx^5+3(8mn-3m-3n)x^6, \\
S_x^3Q_{-2}JD_x^3D_y^3(f(x,y)) &= 32(m+n)x^2+128mnx^3+\frac{729}{32}(8mn-3m-3n)x^4.
\end{aligned}$$

Using table 1, we can easily obtain the required result. □

3.2. Computational aspects of the Tickysim SpiNNaker Model (TSM) sheet.

We compute the M-polynomial of the TSM sheet in the following theorem.

Theorem 3. *Let G be the TSM sheet. Then we have,*

$$M(G) = 4x^2y^4+4x^3y^4+2x^3y^6+2(m+n-5)x^4y^4+4(m+n-5)x^4y^6+(3mn-8m-8n+21)x^6y^6.$$

Proof. The edge set of G has partitions as follows,

$$|E_{\{2,4\}}| = |\{uv \in E(L(S(T_{n,k}))) : d_u = 2, d_v = 4\}| = 4 = m_{24}.$$

$$|E_{\{3,4\}}| = |\{uv \in E(L(S(T_{n,k}))) : d_u = 3, d_v = 4\}| = 4 = m_{34}.$$

$$|E_{\{3,6\}}| = |\{uv \in E(L(S(T_{n,k}))) : d_u = 3, d_v = 6\}| = 2 = m_{36}.$$

$$|E_{\{4,4\}}| = |\{uv \in E(L(S(T_{n,k}))) : d_u = 4, d_v = 4\}| = 2(m+n-5) = m_{44}.$$

$$|E_{\{4,6\}}| = |\{uv \in E(L(S(T_{n,k}))) : d_u = 4, d_v = 6\}| = 4(m+n-5) = m_{46}.$$

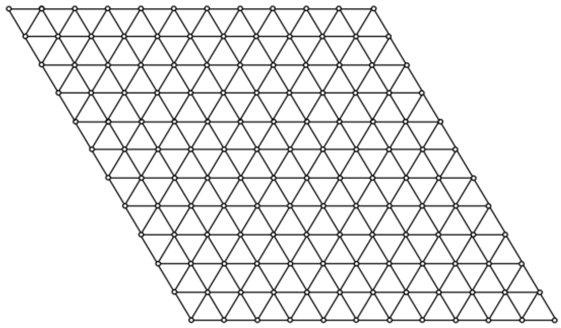
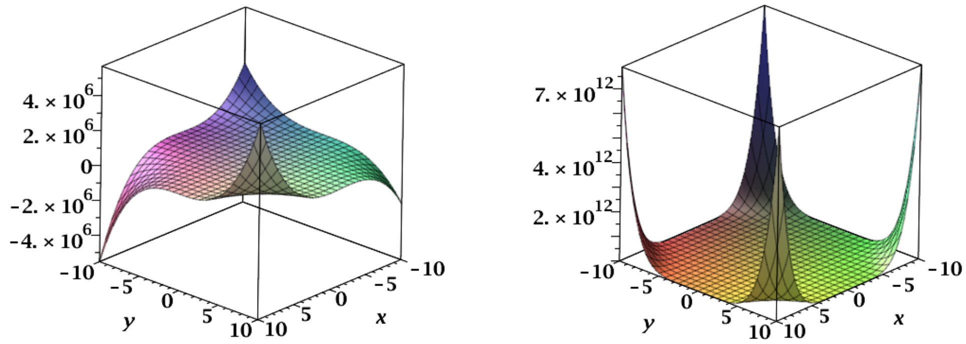


Figure 3.2: The graphical representation of the TSM sheet

.



(a) (b)

Figure 3.3: The M-polynomal of (a) the benzene ring embedded in p-type surface network and (b) the TSM sheet.

$$|E_{\{6,6\}}| = |\{uv \in E(L(S(T_{n,k}))) : d_u = 6, d_v = 6\}| = 3mn - 8m - 8n + 21 = m_{66}.$$

From the definition, the M-polynomial of G is obtained bellow,

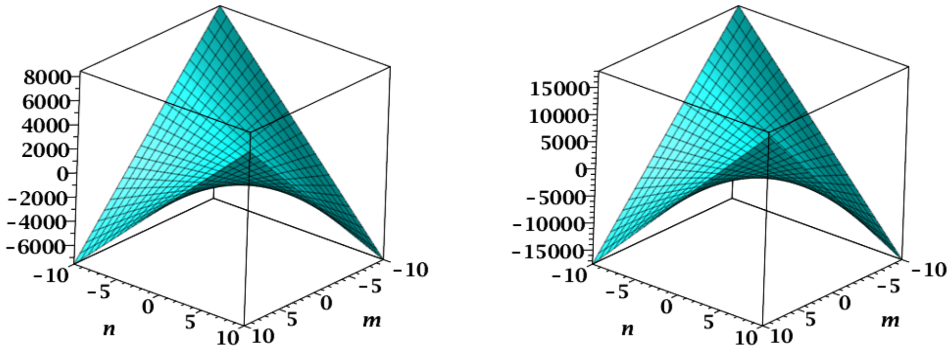
$$\begin{aligned}
M(G) &= \sum_{i \leqslant j} m_{ij} x^i y^j \\
&= m_{24}x^2y^4 + m_{34}x^3y^4 + m_{36}x^3y^6 + m_{44}x^4y^4 + m_{46}x^4y^6 + m_{66}x^6y^6 \\
&= 4x^2y^4 + 4x^3y^4 + 2x^3y^6 + 2(m+n-5)x^4y^4 + 4(m+n-5)x^4y^6 + (3mn-8m-8n+21)x^6y^6.
\end{aligned}$$

This completes the proof. □

Now using this M-polynomial, we calculate some degree based topological index of the TSM sheet in the following theorem.

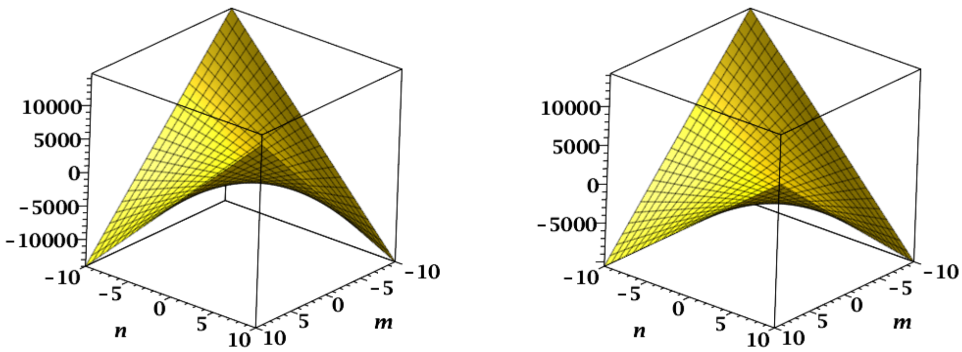
Theorem 4. *Let G be the TSM sheet. Then we have,*

1. $M_1(G) = 2(38mn - 20m - 20n + 21)$,
2. $M_2(G) = 4(27mn - 40m - 40n + 58)$,
3. $F(G) = 2(108mn - 152m - 152n + 211)$
4. $M_2^m(G) = \frac{1}{12}(mn + \frac{5}{6}m + \frac{5}{6}n + \frac{5}{6})$,
5. $R_\alpha(G) = 2^{3\alpha+2} + 3^\alpha 2^{2(\alpha+1)} + 2^{\alpha+1}3^{2\alpha} + 2^{4\alpha+1}(m+n-5) + 2^{3\alpha+2}3^\alpha(m+n-5) + 6^{2\alpha}(3mn - 8m - 8n + 21)$,
6. $RR_\alpha(G) = 2^{2-3\alpha} + 2^{2-2\alpha}3^{-\alpha} + 2^{1-\alpha}3^{-2\alpha} + 2^{1-4\alpha}(m+n-5) + 2^{2-3\alpha}3^{-\alpha}(m+n-5) + 6^{-2\alpha}(3mn - 8m - 8n + 21)$,
7. $SDD(G) = 6mn - \frac{10}{3}m - \frac{10}{3}n + 2$,
8. $H(G) = \frac{1}{2}mn - \frac{1}{30}m - \frac{1}{30}n - \frac{5}{63}$,
9. $I(G) = 9mn - \frac{52}{5}m - \frac{52}{5}n + \frac{235}{21}$,
10. $A(G) = \frac{139968}{1000}mn - \frac{9683696}{2700}m - \frac{9683696}{2700}n + 371.448$.



(a) (b)

Figure 3.4: The first Zagreb index of (a) the benzene ring embedded in p-type surface network and (b) the TSM sheet.



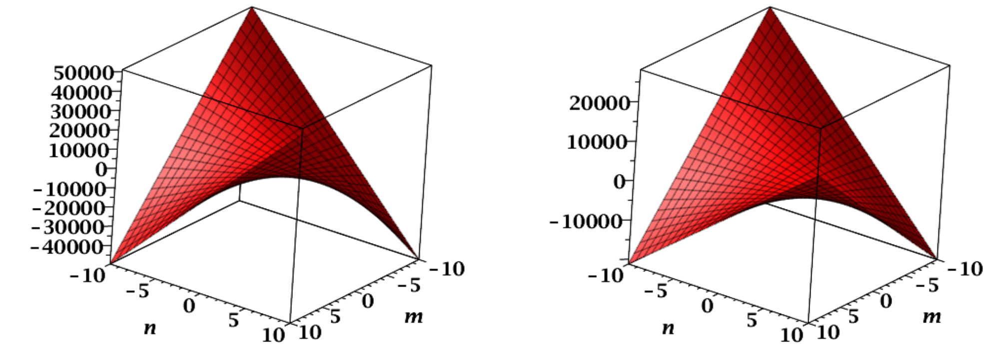
(a) (b)

Figure 3.5: The second Zagreb index of (a) the benzene ring embedded in p-type surface network and (b) the TSM sheet.

Proof. Let $M(G)=f(x,y)=4x^2y^4+4x^3y^4+2x^3y^6+2(m+n-5)x^4y^4+4(m+n-5)x^4y^6+(3mn-8m-8n+21)x^6y^6$.
Then we have,

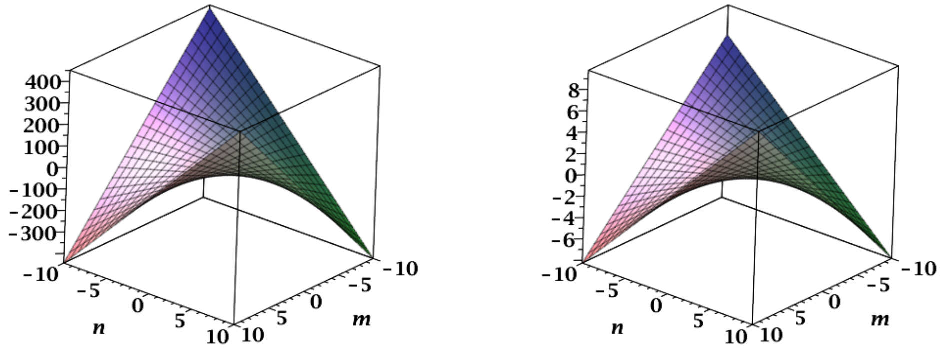
$$
\begin{aligned}
(D_x+D_y)(f(x,y)) &= 24x^2y^4+28x^3y^4+18x^3y^6+16(m+n-5)x^4y^4+40(m+n-5)x^4y^6+12(3mn-8m-8n+21)x^6y^6,\\
D_xD_y(f(x,y)) &= 32x^2y^4+48x^3y^4+36x^3y^6+32(m+n-5)x^4y^4+96(m+n-5)x^4y^6+36(3mn-8m-8n+21)x^6y^6,\\
(D_x^2+D_y^2)(f(x,y)) &= 80x^2y^4+100x^3y^4+90x^3y^6+64(m+n-5)x^4y^4+208(m+n-5)x^4y^6+72(3mn-8m-8n+21)x^6y^6,\\
S_xS_y(f(x,y)) &= \frac{1}{2}x^2y^4+\frac{1}{3}x^3y^4+\frac{1}{9}x^3y^6+\frac{1}{8}(m+n-5)x^4y^4+\frac{1}{6}(m+n-5)x^4y^6+\frac{1}{36}(3mn-8m-8n+21)x^6y^6,\\
D_x^{\alpha}D_y^{\alpha}(f(x,y)) &= 2^{3\alpha+2}x^2y^4+3^{\alpha}4^{\alpha+1}x^3y^4+2^{\alpha+1}3^{2\alpha}x^3y^6+2^{4\alpha+1}(m+n-5)x^4y^4+2^{3\alpha+2}3^{\alpha}(m+n-5)x^4y^6\\
&\quad +6^{2\alpha}(3mn-8m-8n+21)x^6y^6,\\
S_x^{\alpha}S_y^{\alpha}(f(x,y)) &= 2^{2-3\alpha}x^2y^4+2^{2-2\alpha}3^{-\alpha}x^3y^4+2^{1-\alpha}3^{-2\alpha}x^3y^6+2^{1-4\alpha}(m+n-5)x^4y^4+2^{2-3\alpha}3^{-\alpha}(m+n-5)x^4y^6\\
&\quad +6^{-2\alpha}(3mn-8m-8n+21)x^6y^6,\\
(D_xS_y+S_xD_y)(f(x,y)) &= 10x^2y^4+\frac{25}{3}x^3y^4+5x^3y^6+4(m+n-5)x^4y^4+\frac{26}{3}(m+n-5)x^4y^6+2(3mn-8m-8n+21)x^6y^6,\\
S_xJ(f(x,y)) &= \frac{2}{3}x^6+\frac{4}{7}x^7+\frac{2}{9}x^9+\frac{1}{4}(m+n-5)x^8+\frac{2}{5}(m+n-5)x^{10}+\frac{1}{12}(3mn-8m-8n+21)x^{12},\\
S_xJD_xD_y(f(x,y)) &= \frac{16}{3}x^6+\frac{48}{7}x^7+4x^9+4(m+n-5)x^8+\frac{48}{5}(m+n-5)x^{10}+3(3mn-8m-8n+21)x^{12},\\
S_x^3Q_{-2}JD_x^3D_y^3(f(x,y)) &= 32x^4+\frac{6912}{125}x^5+\frac{11664}{343}x^7+\frac{1024}{27}(m+n-5)x^6+108(m+n-5)x^8+\frac{46656}{1000}(3mn-8m-8n+21)x^{10}.
\end{aligned}
$$

Using table 1, the required result can be obtained easily. □



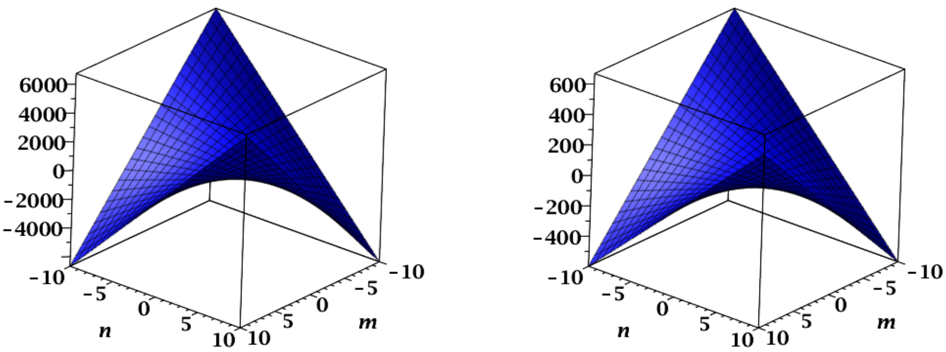
(a) (b)

Figure 3.6: The forgotten topological index of (a) the benzene ring embedded in p-type surface network and (b) the TSM sheet.



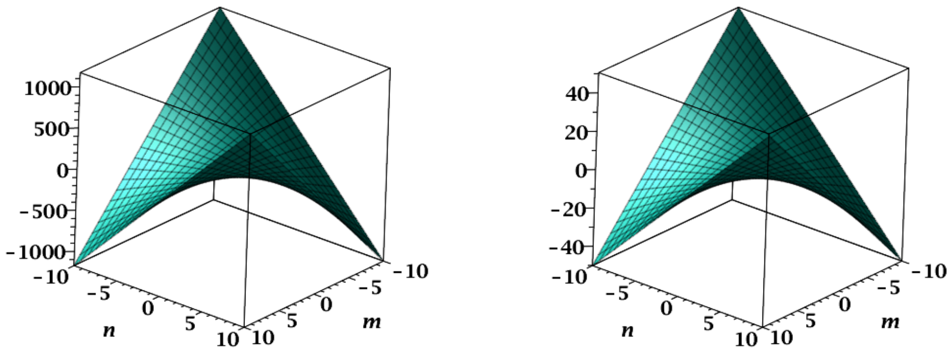
(a) (b)

Figure 3.7: The modified second Zagreb index of (a) the benzene ring embedded in p-type surface network and (b) the TSM sheet.



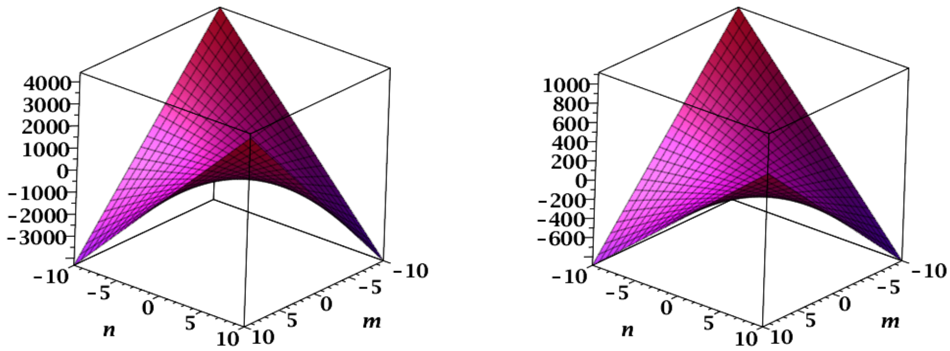
(a) (b)

Figure 3.8: The symmetric division index of (a) the benzene ring embedded in p-type surface network and (b) the TSM sheet.



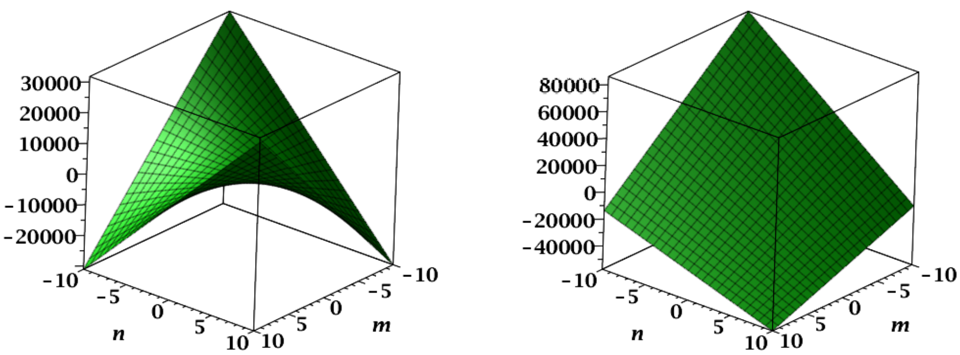
(*a*) (*b*)

Figure 3.9: The harmonic index of (a) the benzene ring embedded in p-type surface network and (b) the TSM sheet.



(*a*) (*b*)

Figure 3.10: The inverse sum index index of (a) the benzene ring embedded in p-type surface network and (b) the TSM sheet.



(*a*) (*b*)

Figure 3.11: The augmented Zagreb index of (a) the benzene ring embedded in p-type surface network and (b) the TSM sheet.

4. Conclusion

We obtained many degree based topological indices for benzene ring embedded in P-type-surface network and Tickysim SpiNNaker Model (TSM) sheet. Firstly, we computed M-polynomial of these graphs and later recovered many degree-based topological indices applying it. Further we have shown the results graphically. These results can play an important role to visualize the topology of the aforesaid networks.

Acknowledgement

The first author is very obliged to the Department of Science and Technology (DST), Government of India for the Inspire Fellowship [IF170148].

References

[1] E. Deutsch and S. Klavzar, M-Polynomial, and degree-based topological indices, Iran. J. Math. Chem., Vol:6,(2015), 93-102.
[2] I. Gutman, Some properties of the Wiener polynomials, Graph Theory Notes N.Y., Vol:125, (1993), 13-18.
[3] V. Alamian ,A. Bahrami and B. Edalatzadeh, PI Polynomial of V-Phenylenic nanotubes and nanotori, Int. J. Mole. Sci., Vol:9, (2008), 229-234. doi: 10.3390/ijms9030229.
[4] M. R. Farahani, Computing theta polynomial, and theta index of V-phenylenic planar, nanotubes and nanotoris, Int. J. Theoretical Chem., Vol:1, No.1, (2013), 01-09.
[5] M. Munir, W. Nazeer, S. Shahzadi and S. M. Kang , Some invariants of circulant graphs, Symmetry, Vol:8, No.11, (2016), 134. doi: 10.3390/sym8110134.
[6] I. Gutman, Degree-based topological indices, Croat. Chem. Acta, Vol:86, (2013), 351-361.
[7] I. Gutman and N. Trinajstic, Graph theory and molecular orbitals total π-electron energy of alternant hydrocarbons, Chem. Phys. Lett., Vol:17,(1972), 535-538.
[8] B. Bollobas and P. Erdös, Graphs of extremal weights, Ars Combin., Vol:50,(1998), 225-233.
[9] D. Amic, D. Beslo, B. Lucic, S. Nikolic and N. Trinajstić, The vertex-connectivity index revisited, J. Chem. Inf. Comput. Sci., Vol:38,(1998), 819-822.
[10] Y. Hu, X. Li,Y. Shi, T. Xu and I. Gutman, On molecular graphs with smallest and greatest zeroth-Corder general Randić index, MATCH Commun. Math. Comput.Chem., Vol:54,(2005), pp. 425-434.
[11] X. Li and I. Gutman, Mathematical aspects of Randić-type molecular structure descriptors, Mathematical Chemistry Monographs, No. 1, Publisher Univ. Kragujevac, Kragujevac, (2006).
[12] G. Caporossi, I. Gutman, P. Hansen and L. Pavlovic, Graphs with maximum connectivity index, Comput. Biol. Chem., Vol:27,(2003), 85-90.
[13] S. Fajtlowicz, On conjectures of Graffiti II, Congr. Numer., Vol:60,(1987), 189-197.
[14] A. T. Balaban, Highly discriminating distance based numerical descriptor, Chem.Phys. Lett., Vol:89,(1982), 399-404.
[15] B. Furtula, A. Graovac and D.Vukićević, Augmented Zagreb index, J. Math. Chem., Vol:48, (2010), 370-380.
[16] M.ÓKeeffe, G. B. Adams, and O. F. Sankey, Predicted new low energy forms of carbon, Phys. Rev. Lett., Vol:68, (1992), 2325-2328.
[17] M. V. Diudea (Ed.), *Nanostructures*, Novel Architecture, NOVA, New York, (2005).
[18] M. V. Diudea, Cs. L. Nagy, Periodic Nanostructures, Springer, Dordrecht, (2007).
[19] I. Stojmenovic, Honeycomb Networks: Topological Properties and Communication Algorithms, IEEE Trans. Parallel Distrib. Syst., Vol:8, (1997), 1036-1042.
[20] A. Ahmad, On the degree based topological indices of benzene ring embedded in P-type-surface in 2D network, Hacettepe Journal of Mathematics and Statistics, Vol:47,(2018), 9-18.
[21] M. Imran, M. K. Siddiqui, A. Ahmad, U. Ali, and N. Hanif, On the Degree-Based Topological Indices of the Tickysim SpiNNaker Model, Axioms, Vol:7,(2018), 73-85.
[22] Y. C. Kwun, M. Munir, W. Nazeer, S. Rafque and S. M. Kang, M-Polynomials and topological indices of V-Phenylenic Nanotubes and Nanotori, Scientific Reports, Vol:7, (2017), doi:10.1038/s41598-017-08309-y.
[23] S. Mondal, N. De, and A. Pal, The M-Polynomial of Line graph of Subdivision graphs, Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat., Vol:68, No.2, (2019), 2104-2116.
[24] M. S. Ahmad, W. Nazeer, S. M. Kang, and C. Y. Jung, M-polynomials and Degree based Topological Indices for the Line Graph of Firecracker Graph, Global Journal of Pure and Applied Mathematics, Vol:13, (2017), 2749-2776.
[25] M. Munir, W. Nazeer, S. Rafique, and S. M. Kang, M-Polynomial and Degree-Based Topological Indices of Polyhex Nanotubes, Symmetry, Vol:8, (2016), doi:10.3390/sym8120149.
[26] M. Munir, W. Nazeer, S. M. Kang, M. I. Qureshi , A. R. Nizami, and Y. C. Kwun, Some Invariants of Jahangir Graphs, Symmetry, Vol:9, (2017), doi:10.3390/sym9010017.
[27] A. Verma, S. Mondal, N. De, and A. Pal, Topological Properties of Bismuth tri-iodide Using Neighborhood M-Polynomial , International Journal of Mathematics Trends and Technology, Vol:65, (2019), 83-90.