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MODEL-FREE OUTPUT FEEDBACK CONTROLLER DESIGN FOR TWIN ROTOR SYSTEMS

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Abstract

In this work, tracking control of twin rotor systems is aimed. The control problem is restricted by the lack of mathematical model of the twin rotor and further complicated by the unavailability of the angular velocity measurements. A model–free controller in conjunction with a high gain observer is designed. Experiments performed on a twin rotor system demonstrates the viability of the controller–observer couple.

Keywords: Twin rotor system, Model-free controller, Observer.

ÇİFT ROTORLU SİSTEMLER İÇİN MODEL GEREKTİRMEYEN ÇIKIŞ GERİ BESLEMELİ DENETÇİ TASARIMI

Özet

Bu çalışmada çift rotorlu sistemlerin yörünge takibi amaçlanmıştır. Çift rotorlu sistemin matematiksel modelinin belirsizlikler içermesi ve açısal hız ölçümlerinin bulunmaması denetim problemini zorlaştırmakta ve karmaşıklaştırmaktadır. Bu sebeple, yüksek kazançlı bir gözlemciyle birleştirilmiş model bilgisi gerektirmeyen bir denetleyici tasarlanmıştır. Çift rotorlu sistem üzerinde yapılan deneyler, denetleyici-gözlemci ikilisinin etkinliğini göstermiştir.

Anahtar Kelimeler: Çift rotorlu sistem, Model gerektirmeyen denetçi, Gözlemci.

Cite

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1. Introduction

Twin rotor system is a laboratory setup resembling a simplified helicopter model that moves on both horizontal and vertical axes. In a twin rotor system, two rotors, named as main rotor and the tail rotor, adjust the angular positions on pitch and yaw axes. The main rotor directly adjusts the movement of the nose of twin rotor system up or down, while the tail rotor causes side to side movement of the nose of the twin rotor system. There are some commercial twin rotor systems developed by companies while there are only a few developed in an academic setting ([1], [2], [3]). Feedback Instruments Limited has produced a twin rotor system for educational purposes and control experiments. Quanser Inc. has also developed experimental helicopter systems.

The twin rotor system is highly nonlinear due to significant amount of cross-coupling between the two motion axes and the system does not have a common dynamic model due to aerodynamic effects being hard to model. Researchers investigated different parts of dynamic modelling with various techniques. [4] presented modelling of one degree of freedom (dof) motion of a twin rotor system by using black box system identification technique. [5] used black box system identification technique to obtain dynamic model of two dof twin rotor systems. [6] utilized both analytical and empirical approaches for modelling of twin rotor system. [7] obtained dynamic model of a twin rotor system with grey box system identification technique. [8] used artificial neural network based modelling to characterize the dynamic behaviour of one dof motion of a twin rotor system about the vertical plane. They utilized multilayered perceptron neural networks by using Levenberg-Morquardt based training algorithm and Elman recurrent neural networks to identify the dynamics of the system. [9] developed nonlinear dynamic model of a twin rotor system based on feed–forward neural networks by using resilient propagation algorithm to obtain a model via using optimum number of neurons. However, as it is apparent from the literature there is no commonly agreed on dynamic model for these systems. When it comes to control design, the parametric uncertainties in the model and the unmodelled effects due to the shortcomings of the modelling approaches mandates the use of model–free controllers. Most of these controllers utilized the signum function in their designs to compensate for uncertainties ([1], [10], [11], [12]) while other robust control techniques were also utilized.

While lack of accurate dynamic models were addressed via robust terms in the control designs, almost all of these past works considered full–state feedback as both angular position and angular velocity measurements were required to implement the controllers ([1], [10], [12], [13], [14], [15], [16]). Only a few results considered output feedback (i.e., only angular position information being available) ([11], [17], [18], [19]).

The lack of accurate dynamic models and angular velocity measurements constituted the main motivation of this work. As a novel departure from the existing control literature on twin rotor systems, a completely model–free controller formulation with only angular position feedback is aimed. To address the lack of angular velocity measurements, a model–free high gain observer is utilized. Then a simple controller is proposed. The controller makes use of observer terms in conjunction with a feed–forward component. The observer term is saturated to ensure boundedness of the control input and to avoid peaking phenomenon ([20]). In the case of partial model knowledge being available, that information can be used as the feed–forward component or artificial intelligence like methods such as neural networks similar to the work of [12] can be utilized.

The rest of the paper is organized in the following manner. Dynamic model for twin rotor system is given in Section 2. Observer and controller formulations are developed in Section 3. Experimental results are given in Section 4, and conclusions are given in Section 5.

2. Dynamic Model of a Twin Rotor System

The mathematical model for a twin rotor system can be represented by ([21])

$$M(\theta)\ddot{\theta} + N(\theta, \dot{\theta}, t) = \tau(t) \tag{1}$$

where $\theta(t)$, $\dot{\theta}(t)$, $\ddot{\theta}(\mathrm{t}) \in \mathbb{R}^2$ represent the angular position, velocity and acceleration vectors, respectively, $M(\theta) \in \mathbb{R}^{2x2}$ represents the inertia matrix, $N(\theta, \dot{\theta}, t) \in \mathbb{R}^2$ represents Centripetal and Coriolis effects, and $\tau(t) \in \mathbb{R}^2$ represents the control input vector. Twin rotor system moves on both horizontal and vertical axes, therefore, $\theta(t)$ is defined as the combination of yaw and pitch motions in the sense that $\theta = [\theta_h, \theta_v]^T$ with $\theta_h(t)$ and $\theta_v(t)$ denoting horizontal and vertical angular positions, respectively ([12]).

Assumption 1. The inertia matrix is symmetric and positive definite and satisfies the following inequalities ([12], [21])

$$m_1\|\xi\|^2 \leq \xi^T M(\theta)\xi \leq m_2\|\xi\|^2 \tag{2}$$

where $\forall\xi \in \mathbb{R}^2$ and m_1 and m_2 are positive bounding constants.

3. Observer & Controller Design

The main control objective is to control both angular pitch and yaw positions to track desired trajectories. The control problem is restricted by only the angular positions being measurable. This problem will be addressed via the design of an observer. Furthermore, the dynamical terms M and N are considered to include both parametric and unstructured uncertainties and thus cannot be utilized in the design of neither the observer nor the controller. As a result, both observer and controller should be model–free.

To initiate the design of the robust output feedback controller, the tracking error $e(t) \in \mathbb{R}^2$ is defined as

$$e \doteq \theta_d - \theta \tag{3}$$

where $\theta_d(\mathrm{t}) \in \mathbb{R}^2$ is the desired angular position. It is assumed that the desired angular position is sufficiently smooth. An auxiliary filtered error, $r(t) \in \mathbb{R}^2$, is defined as

$$r \doteq \dot{e} + \gamma e \tag{4}$$

where $\gamma \in \mathbb{R}^{2x2}$ is a positive definite diagonal gain matrix. Since only output feedback is available, we will follow an observer based strategy. Firstly, an observed filtered error, denoted by $\dot{\hat{r}} \in \mathbb{R}^2$, is designed as

$$\dot{\hat{r}} \doteq \frac{1}{\epsilon^2}\alpha_2(e - \hat{e}) \tag{5}$$

with $\hat{e} \in \mathbb{R}^2$ being the observed position error that is updated according to

$$\dot{\hat{e}} = \hat{r} - \gamma\hat{e} + \frac{1}{\epsilon}\alpha_1(e - \hat{e}) \tag{6}$$

where $\epsilon \in \mathbb{R}$ is a small positive constant, and $\alpha_1, \alpha_2 \in \mathbb{R}^{2x2}$ are positive definite diagonal observer gain matrices.

The control input is designed as

$$\tau = Sat(K\hat{r}) + \tau_{ff} \tag{7}$$

where $K \in \mathbb{R}^{2x2}$ is positive definite diagonal control gain matrix, $\tau_{ff}(t) \in \mathbb{R}^2$ is the feed–forward component of the control input, and $Sat(\cdot) \in \mathbb{R}^2$ is the vector saturation function. The feed-forward term is used to compensate for some parts of the unknown dynamics and can be set to zero without affecting the stability analysis. This choice of the feed-forward component will cause higher control gains but when this is an issue then feed-forward component could be made use of to compensate for some part of the uncertainties and thus relatively smaller control gains could be used. In the control design, saturation function is introduced to keep the control input bounded and to avoid the possibility of peaking phenomenon. When the dynamic model is partially available it can be utilized as the feed–forward component, and when there is no a priori knowledge of the dynamic model, neural networks can be utilized as the feed-forward component. Finally, it is clear that the control input can be obtained by only the measurements of $\theta(t)$.

Theorem. The controller in (7) with the observer design in (5) and (6) yields a semi global uniformly ultimately bounded tracking result in the sense that

$$\|e(t)\| \leq \epsilon_b \quad (8)$$

here ϵ_b is a known positive bounding constant that can be adjusted arbitrarily small.

Proof. See Appendix A.

4. Experimental Results

To show the effectiveness of the controller, experimental studies performed on a twin rotor system are given. The twin rotor system which is used in this study and shown in Figure 1 was developed in Control Laboratory of Electrical & Electronics Engineering Department at Izmir Institute of Technology [3]. Labview was used as the software to monitor the twin rotor system and to provide online communication with it through the serial port.

Experimental results of the proposed controller are investigated for set–point control of the twin rotor system. The desired angular positions were chosen as 30 degrees and 20 degrees for pitch and yaw motions, respectively. The control gain matrices were chosen as $K = \text{diag}([10; 50])$, $\gamma = \text{diag}([2; 2])$ and since satisfactory performance was obtained with only the feedback part of the controllers, feed–forward compensation term was not utilized. For the high gain observer, the observer gains were chosen as $\alpha_1 = \alpha_2 = diag([10; 10])$, and $\epsilon = 0.1$.

The results are shown in Figures 2–5. In Figure 2, the angular positions of both vertical (i.e., θ_v) and horizontal (i.e., θ_h) axes are presented. In Figure 3, the position tracking error for both vertical and horizontal axes are given. In Figure 4, difference between actual and position observer error (i.e., $e(t) - \hat{e}(t)$) is presented. In Figure 5, control input voltage for both axes are given. From Figures 2 and 3, it can be seen that vertical angular position is unsmooth while horizontal angular positon is pretty smooth. This is caused by the different characteristics of the dynamics of vertical and horizontal movements. The tracking capability of such systems for vertical axes is usually not as good as the horizontal axes'. This situation also causes the vertical observation more difficult. However, from Figure 3, it is obvious that the control objective was achieved while it is clear that the observer error is driven to the vicinity of zero from Figure 4.

5. Conclusions

In this paper, we presented a new controller for twin rotor systems. In a novel departure from the existing works on twin rotor systems in the literature, the controller does not require neither system dynamics information nor angular velocity measurements. As a result, a simple model-free controller structure that requires only angular position measurements is proposed. The controller ensures uniform ultimate boundedness of the angular position tracking error and the observer errors. The performance of the controller and the high gain observer were verified via experimental studies on the twin rotor system.

As possible research avenues, the proposed controller may be applied to other experimental testbeds. In addition, the assumption which requires the desired position be sufficiently smooth will be tried to be relaxed to constitute a more general control strategy.

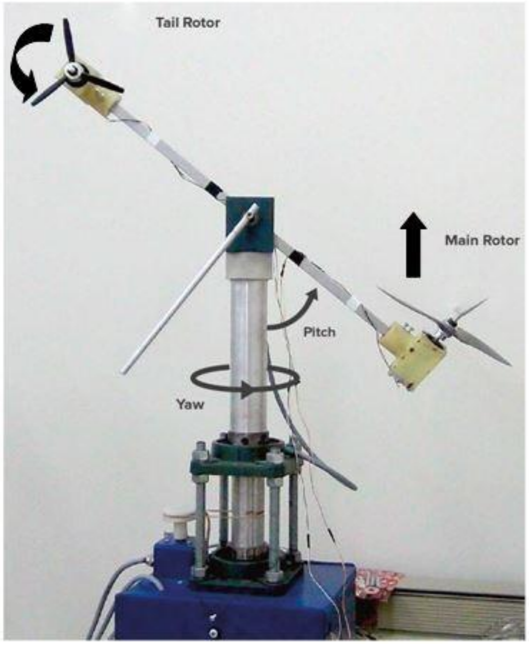

Figure 1: View of twin rotor system [3]

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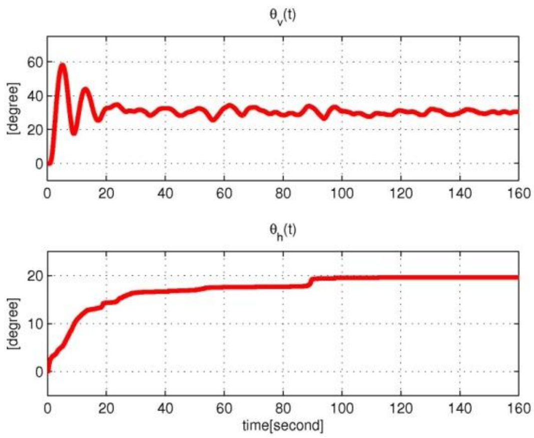

Figure 2: Vertical and horizontal angular positions

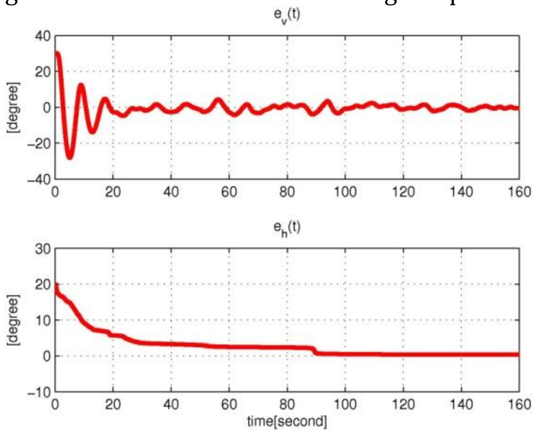

Figure 3: Vertical and horizontal tracking errors

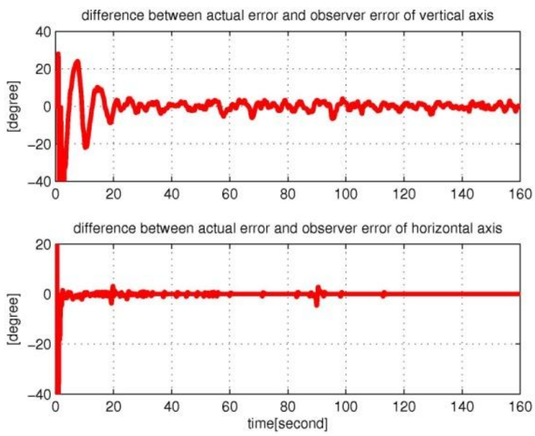

Figure 4: Differences between actual error and observed error for vertical and horizontal axes

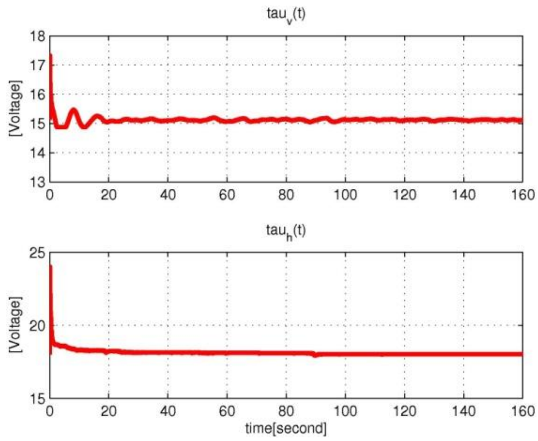

Figure 5: Control input voltages for main and tail rotors

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Appendix A

The full proof of the theorem is lengthy and presenting it in full would the repetition of the knowledge already available in the literature (see [22], [23], [24]), and to mainly focus on the presentation of the experimental verification on the twin rotor system, only the fundamental steps of the proof are provided below.

Proof of Theorem: First, the stability of the system is investigated under the assumption that the full state feedback is available. After that, the stability of the observer dynamics is analyzed. Finally, these two results are combined to obtain the stability of the closed loop system under the proposed output feedback controller.

The proof begins by taking the time derivative of $r(t)$ in (4) and then multiplying with $M(\theta)$ yields

$$M\dot{r} = M\big(\ddot{\theta}_d + \gamma\dot{e}\big) + N\big(\theta,\dot{\theta},t\big) - \tau \tag{A1}$$

where (1) was utilized. After defining auxiliary vectors $f(t), f_d(t), \tilde{f}(t) \in \mathbb{R}^2$ as

$$f \doteq \frac{1}{2}\dot{M}r + e + M\big(\ddot{\theta}_d + \gamma\dot{e}\big) + N \tag{A2}$$

$$f_d \doteq f|_{\theta=\theta_d,\dot{\theta}=\dot{\theta}_d} \tag{A3}$$

$$\tilde{f} = f - f_d \tag{A4}$$

following expression is reached

$$M\dot{r} = \tilde{f} + f_d - e - K\hat{r} - \tau_{ff} - \tfrac{1}{2}\dot{M}r. \tag{A5}$$

The structures of (A3) and (A4) allows the below bounds to be obtained

$$\|f_d\| \le c\,,\ \|\tilde{f}\| \le \rho(\|z\|)\|z\| \tag{A6}$$

where $c \in \mathbb{R}^+$ is bounding constant, $z \doteq [e^T \quad r^T]^T$ is the combined error vector and $\rho(\cdot)$ is a nondecreasing function.

Substituting the full state feedback version of the control input in (7) as $\tau = Kr + \tau_{ff}$ into (A5) yields

$$M\dot{r} = \tilde{f} + f_d - e - Kr - \tau_{ff} - \tfrac{1}{2}\dot{M}r. \tag{A7}$$

The stability of the system under full state feedback being available is investigated via

$$V \doteq \frac{1}{2}e^Te + \frac{1}{2}r^TMr \tag{A8}$$

which in view of (2) can be bounded as

$$\tfrac{1}{2}\min\{1, m_1\}\|z\|^2 \le V \le \tfrac{1}{2}\max\{1, m_2\}\|z\|^2. \tag{A9}$$

The time derivative of (A8) is obtained as [22]

$$\dot{V} \le -K_1 V + \epsilon_1 \tag{A10}$$

where K_1 and ϵ_1 are some positive constants. Thus uniform ultimate boundedness of e(t) and r(t) are guaranteed. Having completed the stability analysis of the closed loop system under full state feedback, next the stability of the observer dynamics is analyzed. We will first define auxiliary observer errors denoted as $\eta_1(t)$ and $\eta_2(t)$ for $e(t)$ and $r(t)$, respectively as follows

$$\eta_1 \doteq \frac{1}{\varepsilon}(e - \hat{e}) \tag{A11}$$

$$\eta_2 \doteq r - \hat{r}. \tag{A12}$$

The time derivatives of observer errors are obtained as follows

$$\varepsilon\dot{\eta} = A_0\eta + \varepsilon g \tag{A13}$$

where $\eta = [\eta_1 \quad \eta_2]^T$, $g = [-\gamma\eta_1 \quad \dot{r}]^T$ and

$$A_0 = \begin{bmatrix} -\alpha_1 & I_2 \\ -\alpha_2 & O_{2x2} \end{bmatrix} \tag{A14}$$

The following Lyapunov function is selected to investigate the stability of the unforced observer dynamics (i.e., $\varepsilon\dot{\eta} = A_0\eta$)

$$V_0 \doteq \eta^T P_0 \eta \tag{A15}$$

where $P_0 \in \mathbb{R}^{4x4}$ is selected such as $P_0A_0 + A_0{}^TP_0 = -I_4$ where I_4 is the standard identity matrix. It is easy practice to show that the right hand side of (A15) can be bounded as

$$\lambda_{min}(P_0)\|\eta\|^2 \leq V_0 \leq \lambda_{max}(P_0)\|\eta\|^2. \tag{A16}$$

The time derivative of (A15) can be obtained as

$$\dot{V}_0 = \tfrac{\partial V_0}{\partial \eta}\dot{\eta} \leq -\|\eta\|^2. \tag{A17}$$

For the perturbed system, the time derivative of (A15) can be obtained as

$$\dot{V}_0 = \frac{\partial V_0}{\partial \eta}\frac{1}{\varepsilon}A_0\eta + \frac{\partial V_0}{\partial \eta}g \tag{A18}$$

where

$$\|g\| \leq \sigma_1\|\eta\| + \sigma_2 \tag{A19}$$

where $\sigma_1, \sigma_2 \in \mathbb{R}$ are constants.

By substituting (A17) and (A19), (A18) can be upper bounded as

$$\dot{V}_0 \leq -\left(\tfrac{1}{\varepsilon} - 2\|P_0\|\sigma_1\right)\|\eta\|^2 + 2\|P_0\|\sigma_2\|\eta\|. \tag{A20}$$

From (A20), η is bounded in the sense that $\|\eta\| \leq \delta$, where $\delta \in \mathbb{R}^+$.

The solution of η in (A20) contains terms in the form of decreasing exponential function. So peaking phenomenon may occur [20]. One way to deal with it, is to saturate the control input when using these terms. And, inside the invariant set, the saturation function does not apply. So by using (A12), (4) can be written as

$$M\dot{r} = \tilde{f} + f_d - e - Kr - K\eta_2 - \tau_{ff} - \tfrac{1}{2}\dot{M}r. \tag{A21}$$

From (A21), the time derivative of V in (A8) becomes

$$\dot{V} \leq -K_1V + \epsilon_1 + K\|\eta\|. \tag{A22}$$

Since from the observer analysis, $\|\eta\| \leq \delta$, then from (A22), following expression is obtained

$$\dot{V} \leq -K_1V + \epsilon_2 \tag{A23}$$

where $\epsilon_2 \in \mathbb{R}^+$ is a positive constant. Thus uniform ultimate boundedness of $e(t)$ and $r(t)$ are guaranteed.