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Applications of Measure of Noncompactness in the Series Spaces of Generalized Absolute Cesàro Means

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Abstract

In this study, we characterize some matrix transformations from the generalized absolute Cesàro series spaces $\left|C_{\lambda,\mu}\right|_p$ $(p \geq 1)$ to the classical sequence spaces ℓ_∞, c and c_0. Besides this, we obtain some identities or estimates for the norms of the bounded linear operators corresponding these matrix transformations. Further, by applying the Hausdorff measure of noncompactness, we give the necessary and sufficient conditions for such operators to be compact.

Keywords: Sequence Spaces, Matrix Operators, BK Spaces, Compact Operators, Hausdorff Measure of Noncompactness.

Genelleştirilmiş Mutlak Cesàro Seri Uzaylarında Nonkompaktlık Ölçüsünün Uygulamaları

Öz

Bu çalışmada, $\left|C_{\lambda,\mu}\right|_p (p \geq 1)$ genelleştirilmiş mutlak Cesàro seri uzaylarından ℓ_∞, c ve c_0 klasik dizi uzaylarına bazı matris dönüşümleri karakterize edilmiştir. Bunun yanı sıra, bu matris dönüşümlerine karşılık gelen sınırlı lineer operatörlerin normları için bazı özdeşlikler veya tahminler verilmiştir. Ayrıca, nonkompaktlık Hausdorff ölçüsünün uygulaması ile bu operatörlerin kompakt olması için gerek ve yeter şartlar elde edilmiştir.

Anahtar Kelimeler: Dizi Uzayları, Matris Operatörleri, BK Uzayları, Kompakt Operatörler, Nonkompaktlık Hausdorff Ölçüsü.

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1. Introduction

One of the research areas in the theory of summability is absolute summability factors and comparison of the methods, which plays an important role in Fourier Analysis and approximation theory and has been widely studied by many authors in the literature (Borwein, 1958; Çanak, 2020; Das, 1970; Flett, 1957; Hazar and Sarıgöl, 2018a, b; Mazhar, 1971; Mehdi, 1960; Mohapatra and Sarıgöl, 2018; Nur and Gunawan, 2019; Sarıgöl, 2015, 2016; Sezer and Çanak, 2015). Recently, independently of these topics, some sequence spaces have been generated and examined by several authors (Altay and Başar, 2007; Altay et al., 2009; Başarır and Kara, 2011a,b, 2012a,b, 2013; Et and Işık, 2012; Hazar, 2020; İlkhan and Kara, 2019; İlkhan, 2020; Kara and İlkhan, 2016; Kara and Başarır, 2011; Karakaya et al., 2011; Sarıgöl, 2016; Zengin Alp and İlkhan, 2019).

The Hausdorff measure of noncompactness was defined by Goldenštein et al. (1957). Using the Hausdorff measure of noncompactness, several authors have characterized some classes of compact operators on certain sequence spaces (Başarır and Kara, 2013; Djolović, 2010; Malkowsky et al., 2002; Malkowsky and Rakočević, 2000; Mursaleen and Noman, 2010, 2011, 2014; Rakočević, 1998).

Moreover, Hazar and Sarıgöl (2018) have introduced the new space $\left|C_{\lambda,\mu}\right|_p$ which is reduced to $|C_\lambda|_p$ (Sarıgöl, 2016) for $\mu = 0$, and proved some theorems related to its topological structures and matrix mappings, where μ and $\lambda + \mu$ are nonnegative integers and $1 \leq p < \infty$.

The aim of this paper is to characterize the classes of infinite matrices $(\left|C_{\lambda,\mu}\right|_p, X)$, where μ and $\lambda + \mu$ are nonnegative integers and $1 \leq p < \infty$, $X = \{\ell_\infty, c, c_0\}$, and also to characterize the classes of compact operators from $\left|C_{\lambda,\mu}\right|_p$ to ℓ_∞, c, c_0 and $\ell_p, 1 \leq p < \infty$ by using the Hausdorff measure of noncompactness.

Let $(X, \|.\|)$ be a normed space. The unit sphere in X is denoted by $S_X = \{x \in X : \|x\| = 1\}$. If X and Y are Banach spaces and $L : X \to Y$ is a linear operator, then, we write $\mathcal{B}(X,Y)$ for the set of all bounded linear operators from X into Y, which is a Banach space with the operator norm given by $\|L\|_{(X,Y)} = sup_{x \in S_X} \|L(x)\|_Y$.

A linear operator $L : X \to Y$ is said to be compact if its domain is all of X and for every bounded sequence $x = (x_n) \in X$, the sequence $\left(L(x_n)\right)$ has a convergent subsequence in Y. We denote the class of such operators by $\mathcal{C}(X,Y)$.

Let w be the space of all complex sequences and ℓ_∞, c, c_0 and ϕ denote the sets of all bounded, convergent, null and finite sequences, respectively.

Further, $\ell_p = \{x \in w : \sum_{v=0}^{\infty} |x_v|^p < \infty\}$ for $1 \leq p < \infty, (\ell_1 = \ell)$. We write $e^{(n)}$ $(n = 0,1,\dots)$ for the sequence with $e_n^{(n)} = 1$, $e_v^{(n)} = 0 (v \neq n)$ for all $n \geq 0$.

A BK- space X is a Banach space with continuous coordinates $P_n : X \to \mathbb{C}$, where $\mathbb{C}$ denotes the complex field and $P_n(x) = x_n$ for all $x \in X$ and $n \geq 0$. Also, a BK- space X containing ϕ is said to have AK if every sequence $x = (x_v) \in X$ has a unique representation $x = \sum_{v=0}^{\infty} x_v e^{(v)}$(Altay et al., 2009). For example, the classical sequence spaces ℓ_∞, c, c_0 and ℓ_p are BK-spaces with their natural norms. Moreover, the spaces c_0 and ℓ_p $(1 \leq p < \infty)$ have AK (Malkowsky and Rakočević, 2000).

The β-dual of a subset X of w is the set $X^\beta = \{t \in w : \sum_{v=0}^{\infty} t_v x_v \text{ is convergent for all } x \in X\}$.

If $X \supset \phi$ is a BK- space and $t = (t_v) \in w$, then we write

$$\|t\|_X^* = \sup_{x \in S_X} \left|\sum_{v=0}^{\infty} t_v x_v\right| \tag{1}$$

provided the expression on the right is defined and finite which is the case whenever $t \in X^\beta$ (Malkowsky et al., 2002).

Let $T = (t_{nv})$ be an infinite matrix of complex numbers, X and Y be subsets of w. Then, we write $T_n = (t_{nv})_{v=0}^{\infty}$ for the sequence in the n-th row of T. Also, we say that T defines a matrix mapping from X into Y, and we denote it by $T : X \to Y$, if, for all $x = (x_v) \in X$, the sequence $T(x) = \big(T_n(x)\big)$, the T-transform of x, exists and belongs to Y , where

$$T_n(x) = \sum_{v=0}^{\infty} t_{nv} x_v$$

provided the series on the right converges for $n \geq 0$. The notation (X, Y) denotes the class of all matrices T such that $T : X \to Y$. Thus, $T \in (X, Y)$ if and only if $T_n = (t_{nv})_{v=0}^{\infty} \in X^\beta$ for each n and $T(x) \in Y$ for all $x \in X$.

The matrix domain of an infinite matrix T in X is defined by

$$X_T = \{x \in w : T(x) \in X\}. \tag{2}$$

An infinite matrix $T = (t_{nv})$ is called a triangle if $t_{nn} \neq 0$, and $t_{nv} = 0$ for $v > n$, which has a unique inverse (Wilansky, 1984). Throughout paper, q denotes the conjugate of $p > 1$, i.e., $1/p + 1/q = 1$ and $1/q = 0$ for $p = 1$.

The following result is fundamental for our work.

Remark 1.1.

a-) (Malkowsky and Rakočević, 2000). Let $1 < p < \infty$ and $q = p/(p-1)$. Then, we have $\ell_\infty^\beta = c^\beta = c_0^\beta = \ell_1$, $\ell_1^\beta = \ell_\infty$ and $\ell_p^\beta = \ell_q$. Furthermore, let X denote any of the spaces $\ell_\infty, c, c_0, \ell_1$ and ℓ_p. Then, we have $\|t\|_X^* = \|t\|_{X^\beta}$ for all $t \in X^\beta$, where $\|.\|_{X^\beta}$ is the natural norm on the dual space X^β.

b-) (Malkowsky and Rakočević, 2000). Let X and Y be BK spaces. Then, we have $(X, Y) \subset \mathcal{B}(X, Y)$, that is, every matrix $T \in (X, Y)$ defines an operator $L_T \in \mathcal{B}(X, Y)$ by $L_T(x) = T(x)$ for all $x \in X$.

c-) (Djolović, 2010). Let $X \supset \phi$ be a BK space and Y be any of the spaces ℓ_∞, c, c_0. If $T \in (X,Y)$, then $\|L_T\| = \|T\|_{(X,\ell_\infty)} = sup_n \|T_n\|_X^* < \infty$.

Let (X,d) be a complete metric space, $\varepsilon > 0$, and also S and H be subsets of X. Then, S is called an ε-net of H in X, if for every $x \in H$ there exists $s \in S$ such that $d(x,s) < \varepsilon$. Further, if S is finite, then the ε-net S of H is called a finite ε-net of H.

By $\mathcal{M}_X$, we denote the collection of all bounded subsets of a metric space (X,d). If $Q \in \mathcal{M}_X$, then the Hausdorff measure of noncompactness of Q, denoted by $\chi(Q)$, is defined by

$$\chi(Q) = inf\{\varepsilon > 0 : Q \text{ has a finite } \varepsilon - \text{net in } X\}.$$

The function $\chi : \mathcal{M}_X \to [0,\infty)$ is called the Hausdorff measure of noncompactness (Rakočević, 1998).

Lemma 1.2 (Djolović, 2010). Let X and Y be Banach spaces, $L \in \mathcal{B}(X,Y)$. Then, the Hausdorff measure of noncompactness of L, denoted by $\|L\|_\chi$, is defined by

$$\|L\|_\chi = \chi\big(L(S_X)\big),$$

and L is compact iff $\|L\|_\chi = 0$.

Lemma 1.3 (Rakočević, 1998). Let Q be a bounded subset of the normed space X, where $X = \ell_p$ for $1 \le p < \infty$. If $P_r : X \to X$ is the operator defined by $P_r(x) = (x_0, x_1, \ldots, x_r, 0, \ldots)$ for all $x \in X$, then

$$\chi(Q) = \lim_{r\to\infty} \sup_{x\in Q} \|(I - P_r)(x)\|_{\ell_p},$$

where I is the identity operator on X.

2. The Series Spaces of generalized absolute Cesàro methods

Let $\sum x_n$ be an infinite series with partial sums (s_n), and (σ_n^λ) be the nth Cesàro mean (C,λ) of order $\lambda > -1$ of the sequence (s_n), i.e., $\sigma_n^\lambda = \left(E_n^\lambda\right)^{-1} \sum_{k=0}^n E_{n-k}^{\lambda-1} s_k$, where E_n^λ is the binomial coefficient of z^n in the power series expansion of the function $(1-z)^{-\lambda-1}$ in $|z| < 1$. Then, the series $\sum x_n$ is said to be summable $|C,\lambda|_p$ with index $p \ge 1$, if (Flett, 1957)

$$\sum_{n=1}^\infty n^{p-1} \left|\sigma_n^\lambda - \sigma_{n-1}^\lambda\right|^p < \infty. \tag{3}$$

Also, this method was extended by Das (1970) to summability $|C,\lambda,\mu|_p$, for $\lambda > -1$, $\lambda + \mu \neq -1, -2, \ldots$, using

$$\sum_{n=1}^\infty n^{p-1} \left|\sigma_n^{\lambda,\mu} - \sigma_{n-1}^{\lambda,\mu}\right|^p < \infty,$$

in place of (3), where $\left(\sigma_n^{\lambda,\mu}\right)$ is the nth Cesàro mean (C,λ,μ) of order (λ,μ) of the sequence (s_n), which was defined by Borwein (1958) as follows:

$$\sigma_n^{\lambda,\mu}=\frac{1}{E_n^{\lambda+\mu}}\sum_{k=0}^{n}E_{n-k}^{\lambda-1}E_k^{\mu}s_k.$$

Now, we denote by $\bar{u}_n^{\lambda,\mu}$ the *n*th Cesàro mean (C,λ,μ) of sequence (nx_n), i.e.,

$$\bar{u}_0^{\lambda,\mu}=x_0,\qquad \bar{u}_n^{\lambda,\mu}=\frac{1}{E_n^{\lambda+\mu}}\sum_{k=1}^{n}E_{n-k}^{\lambda-1}E_k^{\mu}kx_k.$$

By the identity $\bar{u}_n^{\lambda,\mu}=n\left(\sigma_n^{\lambda,\mu}-\sigma_{n-1}^{\lambda,\mu}\right)$ (Das, 1970), the series space $\left|C_{\lambda,\mu}\right|_p$ $\left(\left|C_{\lambda,\mu}\right|_1=\left|C_{\lambda,\mu}\right|\right)$ is stated by (Hazar and Sarıgöl, 2018)

$$\left|C_{\lambda,\mu}\right|_p=\left\{x=(x_v)\in w:U^{(\lambda,\mu,p)}(x)=\left(U_n^{(\lambda,\mu,p)}(x)\right)\in\ell_p\right\},$$

where

$$U_0^{(\lambda,\mu,p)}(x)=x_0,\qquad U_n^{(\lambda,\mu,p)}(x)=\frac{1}{n^{1/p}E_n^{\lambda+\mu}}\sum_{k=1}^{n}E_{n-k}^{\lambda-1}E_k^{\mu}kx_k\,,\qquad n\geq 1.$$

According to (2),

$$\left|C_{\lambda,\mu}\right|_p=\left(\ell_p\right)_{U^{(\lambda,\mu,p)}},$$

(Hazar and Sarıgöl, 2018), where the matrix $U^{(\lambda,\mu,p)}=(u_{nk}^{(\lambda,\mu,p)})$ is defined by $u_{00}^{(\lambda,\mu,p)}=1$ and

$$u_{nk}^{(\lambda,\mu,p)}=\begin{cases}\dfrac{E_{n-k}^{\lambda-1}E_k^{\mu}k}{n^{1/p}E_n^{\lambda+\mu}}, 1\leq k\leq n,\\ 0,\qquad k>n.\end{cases}$$

There exists the inverse matrix $\widetilde{U}^{(\lambda,\mu,p)}=\left(\tilde{u}_{nk}^{(\lambda,\mu,p)}\right)$ of the matrix $U^{(\lambda,\mu,p)}$, which is given by $\tilde{u}_{00}^{(\lambda,\mu,p)}=1$ and, for $\mu,\lambda+\mu\neq-1,-2,\ldots$,

$$\tilde{u}_{nk}^{(\lambda,\mu,p)}=\begin{cases}\frac{E_{n-k}^{-\lambda-1}k^{1/p}E_k^{\lambda+\mu}}{nE_n^{\mu}}, 1\leq k\leq n,\\ 0,\qquad\qquad k>n.\end{cases}\tag{4}$$

It is obvious that $\left|C_{\lambda,\mu}\right|_p$ is the BK-space with the norm (Hazar and Sarıgöl, 2018), for $\mu,\lambda+\mu\neq-1,-2,\ldots$,

$$\|x\|_{\left|C_{\lambda,\mu}\right|_p}=\left\|U^{(\lambda,\mu,p)}(x)\right\|_{\ell_p}.\tag{5}$$

Throughout, for any given sequence $x=(x_n)\in\left|C_{\lambda,\mu}\right|_p$, we define the associated sequence $y=(y_n)$ as the $U^{(\lambda,\mu,p)}$ transform of x, that is, $y=U^{(\lambda,\mu,p)}(x)$, and so

$$y_0=x_0 \text{ and } y_n=\tfrac{1}{n^{1/p}E_n^{\lambda+\mu}}\textstyle\sum_{k=1}^{n}E_{n-k}^{\lambda-1}E_k^{\mu}kx_k,\ n\geq 1.\tag{6}$$

If the sequences x and y are connected by the relation (6), then $x\in\left|C_{\lambda,\mu}\right|_p$ if and only if $y\in\ell_p$, furthermore, if $x\in\left|C_{\lambda,\mu}\right|_p$, then $\|x\|_{\left|C_{\lambda,\mu}\right|_p}=\|y\|_{\ell_p}$. In fact, the linear operator $U^{(\lambda,\mu,p)}:$

$\left|C_{\lambda,\mu}\right|_p \to \ell_p$, which maps every sequence $x \in \left|C_{\lambda,\mu}\right|_p$ to its associated sequence $y \in \ell_p$, is bijective and norm preserving.

Also, we state the notations Λ_c, Λ_∞ and Λ_s as follows:

$$\Lambda_c = \left\{\varepsilon \in w : \lim_m \overline{E}_r^{(m)} \text{ exists for all } r \in \mathbb{N}\right\},$$

$$\Lambda_\infty = \left\{\varepsilon \in w : \sup_{m,r} \left| r E_r^{\lambda+\mu} \overline{E}_r^{(m)} \right| < \infty\right\},$$

$$\Lambda_s = \left\{\varepsilon \in w : \sup_m \sum_{r=1}^{m} \left| r^{1/p} E_r^{\lambda+\mu} \overline{E}_r^{(m)} \right|^q < \infty\right\},$$

where

$$\overline{E}_r^{(m)} = \sum_{k=r}^{m} \frac{E_{k-r}^{-\lambda-1}\varepsilon_k}{kE_k^{\mu}}; m, r \geq 1.$$

Also, we need the following known results for our investigation.

Lemma 2.1 (Sarıgöl, 2015). Let $1 < p < \infty$. Then, $T = (t_{nk}) \in (\ell_p, \ell)$ if and only if

$$\|T\|'_{(\ell_p,\ell)} = \left\{\sum_{k=0}^{\infty} \left(\sum_{n=0}^{\infty} |t_{nk}|\right)^q\right\}^{1/q} < \infty,$$

and there exists $1 \leq \xi \leq 4$ such that $\|T\|'_{(\ell_p,\ell)} = \xi \|T\|_{(\ell_p,\ell)}$.

Lemma 2.2 (Maddox, 1970). Let $1 \leq p < \infty$. Then, $T = (t_{nk}) \in (\ell, \ell_p)$ if and only if

$$\|T\|_{(\ell,\ell_p)} = \sup_k \left\{\sum_{n=0}^{\infty} |t_{nk}|^p\right\}^{1/p} < \infty.$$

Lemma 2.3 (Hazar and Sarıgöl, 2018). If $1 < p < \infty$, λ and $\lambda + \mu$ are nonnegative integers, then, $\left|C_{\lambda,\mu}\right|_p^{\beta} = \Lambda_c \cap \Lambda_s$ and $\left|C_{\lambda,\mu}\right|^{\beta} = \Lambda_c \cap \Lambda_\infty$.

Now, we prove followings.

First, by taking into account the inverse $\widetilde{U}^{(\lambda,\mu,p)}$ of the $U^{(\lambda,\mu,p)}$, we state the following lemma by the previous result.

Lemma 2.4. Let λ and $\lambda + \mu$ be nonnegative integers and $1 \leq p < \infty$. If $t = (t_k) \in \left(\left|C_{\lambda,\mu}\right|_p\right)^{\beta}$, then $\tilde{t}^{(p)} = \left(\tilde{t}_k^{(p)}\right) \in \ell_q$ for $p > 1$ and $\tilde{t}^{(1)} \in \ell_\infty$ for $p = 1$ and the equality

$$\sum_{k=1}^{\infty} t_k x_v = \sum_{k=1}^{\infty} \tilde{t}_k^{(p)} y_k \tag{7}$$

is satisfied for every $x = (x_k) \in \left|C_{\lambda,\mu}\right|_p$, where $y = U^{(\lambda,\mu,p)}(x)$ is the associated sequence as in (6) and $\tilde{t}^{(p)} = \left(\tilde{t}_k^{(p)}\right)$ is defined by

$$\tilde{t}_k^{(p)} = k^{1/p} E_k^{\lambda+\mu} \sum_{r=k}^{\infty} \frac{t_r}{r E_r^{\mu}} E_{r-k}^{-\lambda-1} = \sum_{r=k}^{\infty} t_r \tilde{u}_{rk}^{(\lambda,\mu,p)}.$$

Lemma 2.5. If $1 < p < \infty$, then we have $\|t\|_{|C_{\lambda,\mu}|_p}^* = \|\tilde{t}^{(p)}\|_{\ell_q}$ and if $p = 1$, then we have $\|t\|_{|C_{\lambda,\mu}|}^* = \|\tilde{t}^{(1)}\|_{\ell_\infty}$, for all $t \in \left(|C_{\lambda,\mu}|_p\right)^{\beta}$, where $\tilde{t}^{(p)} = \left(\tilde{t}_k^{(p)}\right)$ is as in Lemma 2.4, λ and $\lambda + \mu$ are nonnegative integers.

Proof. Let $1 < p < \infty$ and $t \in \left(|C_{\lambda,\mu}|_p\right)^{\beta}$. Then, by using Lemma 2.4, we write $\tilde{t}^{(p)} = \left(\tilde{t}_k^{(p)}\right) \in \ell_q$ and the equality (7) holds for all sequence $x \in |C_{\lambda,\mu}|_p$ and $y \in \ell_p$ which are connected by equation (6). Further, it follows from (5) that $x \in S_{|C_{\lambda,\mu}|_p}$ if and only if $y \in S_{\ell_p}$. Therefore, we deduce from (1) and (7) that

$$\|t\|_{|C_{\lambda,\mu}|_p}^* = \sup_{x \in S_{|C_{\lambda,\mu}|_p}} \left| \sum_{k=1}^{\infty} t_k x_k \right| = \sup_{y \in S_{\ell_p}} \left| \sum_{k=1}^{\infty} \tilde{t}_k^{(p)} y_k \right| = \|\tilde{t}^{(p)}\|_{\ell_p}^*$$

This completes the proof.

The proof is elementary and left to the reader for $p = 1$.

Throughout, we denote the associated matrix $\tilde{T}^{(p)} = \left(\tilde{t}_{nk}^{(p)}\right)$ of an infinite matrix $T = (t_{nk})$ by

$$\tilde{t}_{nk}^{(p)} = k^{1/p} E_k^{\lambda+\mu} \sum_{r=k}^{\infty} \frac{t_{nr}}{r E_r^{\mu}} E_{r-k}^{-\lambda-1} = \sum_{r=k}^{\infty} t_{nr} \tilde{u}_{rk}^{(\lambda,\mu,p)} \tag{8}$$

provided the series on the right converges for all $n, k \geq 1$.

Lemma 2.6. Let Z be a sequence space, $T = (t_{nk})$ be an infinite matrix and $1 \leq p < \infty$. If $T \in \left(|C_{\lambda,\mu}|_p, Z\right)$, then $\tilde{T}^{(p)} \in (\ell_p, Z)$ such that $T(x) = \tilde{T}^{(p)}(y)$ for all $x \in |C_{\lambda,\mu}|_p$ and $y \in \ell_p$ which are connected by the equation (6), where $\tilde{T}^{(p)}$ associated matrix is defined by (8), λ, μ and $\lambda + \mu$ are nonnegative integers.

Proof. This can be proved easily by using Lemma 2.4.

Finally, we end this section with the following Lemmas on operator norms.

Lemma 2.7. Let $T = (t_{nk})$ be an infinite matrix and $\tilde{T}^{(p)}$ associated matrix given by (8). If T is in any of the classes $\left(|C_{\lambda,\mu}|_p, c_0\right)$, $\left(|C_{\lambda,\mu}|_p, c\right)$ and $\left(|C_{\lambda,\mu}|_p, \ell_\infty\right)$, then, we have, for $1 < p < \infty$, λ, μ and $\lambda + \mu$ nonnegative integers,

$$\|L_T\| = \|T\|_{\left(|C_{\lambda,\mu}|_p, \ell_\infty\right)} = \sup_n \left\|\tilde{T}_n^{(p)}\right\|_{\ell_q}$$

and for $p = 1$ and $\lambda, \mu, \lambda + \mu$ nonnegative integers,

$$\|L_T\| = \|T\|_{(|C_{\lambda,\mu}|, \ell_\infty)} = \sup_n \left\|\tilde{T}_n^{(1)}\right\|_{\ell_\infty}.$$

Proof. This can be obtained by combining Remark 1.1 and Lemma 2.5.

Lemma 2.8. Let $T = (t_{nk})$ be an infinite matrix and $\tilde{T}^{(p)} = \left(\tilde{t}_{nk}^{(p)}\right)$ be associated matrix given by (8). Then, we have:

a-) If $T \in \left(\left|C_{\lambda,\mu}\right|, \ell_p\right)$, then, for $p \geq 1$,

$$\|L_T\| = \|T\|_{(|C_{\lambda,\mu}|,\ell_p)} = \left\|\tilde{T}^{(1)}\right\|_{(\ell,\ell_p)}.$$

b-) If $T \in \left(\left|C_{\lambda,\mu}\right|_p, \ell\right)$, then, for $1 < p < \infty$, there exists $1 \leq \xi \leq 4$ such that

$$\|L_T\| = \|T\|_{\left(|C_{\lambda,\mu}|_p,\ell\right)} = \left\|\tilde{T}^{(p)}\right\|_{(\ell_p,\ell)} = \frac{1}{\xi}\left\|\tilde{T}^{(p)}\right\|'_{(\ell_p,\ell)}.$$

Proof. Part of a-) and b-) can be obtained by combining Remark 1.1 with Lemma 2.2 and Lemma 2.1, respectively.

3. Compact matrix operators on $\left|C_{\lambda,\mu}\right|_p$

In this section, we characterize the classes of infinite matrices $(\left|C_{\lambda,\mu}\right|_p, X)$, where $p \geq 1,\ X = \{c_0, c, \ell_\infty\}$. Also, we establish the Hausdorff measures of noncompactness of certain matrix operators on the generalized absolute Cesàro series spaces and using the Hausdorff measure of noncompactness, we give the necessary and sufficient conditions for such operators to be compact.

Now, we are ready to give following results.

Lemma 3.1 (Stieglitz and Tietz, 1977).

a-) $T = (t_{nr}) \in (\ell, c) \Leftrightarrow$ (i) $lim_n t_{nr}$ exists, $r \geq 0$, (ii) $sup_{n,r}|t_{nr}| < \infty$.

b-) $T = (t_{nr}) \in (\ell, \ell_\infty) \Leftrightarrow$ (ii) holds.

c-) Let $1 < p < \infty$. $T = (t_{nr}) \in \left(\ell_p, c\right) \Leftrightarrow$ (i) holds, (iii) $sup_n \sum_{r=0}^{\infty} |t_{nr}|^q < \infty$.

d-) Let $1 < p < \infty$. $T = (t_{nr}) \in \left(\ell_p, \ell_\infty\right) \Leftrightarrow$ (iii) holds.

e-) Let $1 < p < \infty$. $T = (t_{nr}) \in \left(\ell_p, c_0\right) \Leftrightarrow$ (iii) holds, (iv) $lim_n t_{nr} = 0, r \geq 0$.

f-) $T = (t_{nr}) \in (\ell, c_0) \Leftrightarrow$ (ii) and (iv) holds.

Now, we prove our first main result.

Theorem 3.2. Suppose that $T = (t_{nk})$ is an infinite matrix of complex numbers for all $n, k \geq 1$, the associated matrix $\tilde{T}^{(1)} = \left(\tilde{t}_{nk}^{(1)}\right)$ is defined by

$$\tilde{t}_{nk}^{(1)} = kE_k^{\lambda+\mu} \sum_{r=k}^{\infty} \frac{t_{nr}}{rE_r^{\mu}} E_{r-k}^{-\lambda-1}, \tag{9}$$

μ and $\lambda + \mu$ are nonnegative integers. Then

a-) $T \in \left(\left|C_{\lambda,\mu}\right|, \ell_\infty\right)$ if and only if

$$\lim_m \sum_{k=r}^{m} \frac{E_{k-r}^{-\lambda-1} t_{nk}}{kE_k^{\mu}} \text{ exists for } n, r \geq 1, \tag{10}$$

$$\sup_{m,i}\left|iE_i^{\lambda+\mu}\sum_{r=i}^{m}\frac{E_{r-i}^{-\lambda-1}t_{nr}}{rE_r^{\mu}}\right|<\infty, \text{for } n\geq 1, \tag{11}$$

$$\sup_{n,k}\left|\tilde{t}_{nk}^{(1)}\right|<\infty. \tag{12}$$

b-) $T\in\left(\left|C_{\lambda,\mu}\right|,c\right)$ if and only if $(10),(11),(12)$ hold and

$$\lim_{n}\tilde{t}_{nk}^{(1)} \text{exists for each } k.$$

c-) $T\in\left(\left|C_{\lambda,\mu}\right|,c_0\right)$ if and only if $(10),(11),(12)$ hold and

$$\lim_{n}\tilde{t}_{nk}^{(1)}=0, \text{for each } k.$$

Proof. a-) $T\in\left(\left|C_{\lambda,\mu}\right|,\ell_\infty\right)$ iff $(t_{nv})_{v=1}^{\infty}\in\left|C_{\lambda,\mu}\right|^{\beta}(n\in\mathbb{N})$ and $T(x)\in\ell_\infty$ for every $x\in\left|C_{\lambda,\mu}\right|$, and also, by Lemma 2.3, $(t_{nv})_{v=1}^{\infty}\in\left|C_{\lambda,\mu}\right|^{\beta}$ iff (10) and (11) hold. Moreover, the series $\sum_v t_{nv}x_v$ converges uniformly in n and so

$$\lim_{n}T_n(x)=\sum_{v=0}^{\infty}\lim_{n}t_{nv}x_v. \tag{13}$$

To prove necessity and sufficiency of condition (12), for every given $x\in\left|C_{\lambda,\mu}\right|$ define the operator $U^{(\lambda,\mu,1)}:\left|C_{\lambda,\mu}\right|\to\ell$ by $U^{(\lambda,\mu,1)}(x)=y$. It is clear that this operator is bijection and the matrix corresponding to this operator is triangle. Further, $U^{(\lambda,\mu,1)}(x)=y\in\ell$ iff $x=\widetilde{U}^{(\lambda,\mu,1)}(y)$, where $\widetilde{U}^{(\lambda,\mu,1)}=\left(\tilde{u}_{nv}^{(\lambda,\mu,1)}\right)$ is the inverse of $U^{(\lambda,\mu,1)}$ and it is defined by (4) with $p=1$. Then it follows that

$$\sum_{k=1}^{m}t_{nk}x_k=\sum_{j=1}^{m}\left(\sum_{k=j}^{m}t_{nk}\tilde{u}_{kj}^{(\lambda,\mu,1)}\right)y_j=\sum_{j=1}^{m}\varphi_{mj}^{(n)}y_j$$

where the matrix $\varphi^{(n)}=\left(\varphi_{mj}^{(n)}\right)$, for $j,m=1,2,,\ldots$, is defined by

$$\varphi_{mj}^{(n)}=\begin{cases}\sum_{k=j}^{m}t_{nk}\tilde{u}_{kj}^{(\lambda,\mu,1)}, & 1\leq j\leq m\\ 0, & j>m.\end{cases}$$

Thus, from (10) and (11), by applying the matrix $\varphi^{(n)}=\left(\varphi_{mj}^{(n)}\right)$ to (13), we get that

$$T_n(x)=\lim_{m}\sum_{j=1}^{m}\varphi_{mj}^{(n)}y_j=\sum_{j=1}^{\infty}\left(\sum_{k=j}^{\infty}t_{nk}\tilde{u}_{kj}^{(\lambda,\mu,1)}\right)y_j=\sum_{j=1}^{\infty}\tilde{t}_{nj}^{(1)}y_j=\tilde{T}_n^{(1)}(y)$$

converges for all $n\geq 1$, where $\tilde{T}^{(1)}=\left(\tilde{t}_{nj}^{(1)}\right)$ is defined by $\tilde{t}_{nj}^{(1)}=\lim_m\varphi_{mj}^{(n)}$ for $j,m=1,2,\ldots$, which is same as in (9).

This shows that the sequence $T(x)=(T_n(x))$ exists. So, we obtain that $T:\left|C_{\lambda,\mu}\right|\to\ell_\infty$ iff $\tilde{T}^{(1)}:\ell\to\ell_\infty$, and also a few calculations reveal that $\tilde{T}^{(1)}=To\widetilde{U}^{(\lambda,\mu,1)}$. Thus, it follows by applying Lemma 3.1 with the matrix $\tilde{T}^{(1)}$ that $\tilde{T}^{(1)}:\ell\to\ell_\infty$ iff (12) holds, and this concludes the proof of the part a-).

Part b-) and c-) can be proved similarly by using Lemma 3.1.

Theorem 3.3. Suppose that $T = (t_{nk})$ is an infinite matrix of complex numbers for all $n, k \geq 1$ and the associated matrix $\tilde{T}^{(p)} = \left(\tilde{t}_{nk}^{(p)}\right)$ is defined by (8), μ and $\lambda + \mu$ are nonnegative integers and $1 < p < \infty$. Then,

a-) $T \in \left(\left|C_{\lambda,\mu}\right|_p, \ell_\infty\right)$ if and only if (10) holds, and

$$\sup_m \sum_{i=1}^m \left| i^{1/p} E_i^{\lambda+\mu} \sum_{r=i}^m \frac{E_{r-i}^{-\lambda-1} t_{vr}}{r E_r^{\mu}} \right|^q < \infty, \; v \geq 1, \tag{14}$$

$$\sup_n \sum_{r=1}^\infty \left| \tilde{t}_{nr}^{(p)} \right|^q < \infty. \tag{15}$$

b-) $T \in \left(\left|C_{\lambda,\mu}\right|_p, c\right)$ if and only if $(10), (14), (15)$ hold and

$$\lim_n \tilde{t}_{nj}^{(p)} \text{exists for each } j.$$

c-) $T \in \left(\left|C_{\lambda,\mu}\right|_p, c_0\right)$ if and only if $(10), (14), (15)$ hold and

$$\lim_n \tilde{t}_{nj}^{(p)} = 0, \text{for each } j.$$

Proof. a-) $T \in \left(\left|C_{\lambda,\mu}\right|_p, \ell_\infty\right)$ iff $(t_{nv})_{v=1}^\infty \in \left|C_{\lambda,\mu}\right|_p^\beta$ $(n \in \mathbb{N})$ and $T(x) \in \ell_\infty$ for every $x \in \left|C_{\lambda,\mu}\right|_p$. Also, using Lemma 2.3, it follows that $(t_{nv})_{v=1}^\infty \in \left|C_{\lambda,\mu}\right|_p^\beta$ iff (10) and (14) hold. Moreover, the series $\sum_v t_{nv} x_v$ converges uniformly in n and so (13) holds.

To get the condition (15), as in the proof of Theorem 3.2, for every given $x \in \left|C_{\lambda,\mu}\right|_p$ define the operator $U^{(\lambda,\mu,p)} : \left|C_{\lambda,\mu}\right|_p \to \ell_p$ by $U^{(\lambda,\mu,p)}(x) = y$. Also, the inverse matrix $\widetilde{U}^{(\lambda,\mu,p)} = \left(\tilde{u}_{nv}^{(\lambda,\mu,p)}\right)$ of $U^{(\lambda,\mu,p)}$ is defined by (4). So we obtain that

$$\sum_{v=1}^m t_{nv} x_v = \sum_{j=1}^m \left(\sum_{v=j}^m t_{nv} \tilde{u}_{vj}^{(\lambda,\mu,p)} \right) y_j = \sum_{j=1}^m b_{mj}^{(n)} y_j$$

where for $j, m = 1,2, \ldots$, the matrix $B^{(n)} = \left(b_{mj}^{(n)}\right)$ is defined by

$$b_{mj}^{(n)} = \begin{cases} \sum_{v=j}^m t_{nv} \tilde{u}_{vj}^{(\lambda,\mu,p)}, & 1 \leq j \leq m \\ 0, & j > m. \end{cases}$$

Thus, from (10) and (14), by applying the matrix $B^{(n)} = \left(b_{mj}^{(n)}\right)$ to (13), it can be written that

$$T_n(x) = \lim_m \sum_{j=1}^m b_{mj}^{(n)} y_j = \sum_{j=1}^\infty \left(\sum_{v=j}^\infty t_{nv} \tilde{u}_{vj}^{(\lambda,\mu,p)} \right) y_j = \sum_{j=1}^\infty \tilde{t}_{nj}^{(p)} y_j = \tilde{T}_n^{(p)}(y)$$

converges for all $n \geq 1$, where $\tilde{T}^{(p)} = \left(\tilde{t}_{nj}^{(p)}\right)$ is defined by $\tilde{t}_{nj}^{(p)} = \lim_m b_{mj}^{(n)}$ for $j, m = 1,2, \ldots$, as in (8). This leads us that $T : \left|C_{\lambda,\mu}\right|_p \to \ell_\infty$ if and only if $\tilde{T}^{(p)} : \ell_p \to \ell_\infty$. Further, it can be easily calculated that $\tilde{T}^{(p)} = To\widetilde{U}^{(\lambda,\mu,p)}$. Hence, by Lemma 3.1, $\tilde{T}^{(p)} : \ell_p \to \ell_\infty$ iff (15) holds, and this proves the part of a-).

Part b-) and c-) can be proved similarly by using Lemma 3.1.

We may state the following lemma on the Hausdorff measures of noncompactness.

Lemma 3.4. (Mursaleen and Noman, 2010; Theorem 3.7) Let $X \supset \phi$ be a BK space. Then, we have:

a-) If $T \in (X, \ell_\infty)$, then

$$0 \leq \|L_T\|_\chi \leq \limsup_{n\to\infty} \|T_n\|_X^*.$$

b-) If $T \in (X, c_0)$, then

$$\|L_T\|_\chi = \limsup_{n\to\infty} \|T_n\|_X^*.$$

c-) If X has AK or $X = \ell_\infty$ and $T \in (X, c)$, then

$$\frac{1}{2}\limsup_{n\to\infty} \|T_n - \gamma\|_X^* \leq \|L_T\|_\chi \leq \limsup_{n\to\infty} \|T_n - \gamma\|_X^*,$$

where $\gamma = (\gamma_v)$ with $\gamma_v = lim_{n\to\infty} t_{nv}$ for all $v \in \mathbb{N}$.

Now, let $T = (t_{nk})$ be an infinite matrix and $\tilde{T}^{(p)} = \left(\tilde{t}_{nk}^{(p)}\right)$ the associated matrix defined by (8). Then connected with Lemma 3.4, we can prove next result using Lemma 2.5 and Lemma 2.6.

Theorem 3.5. Let μ and $\lambda + \mu$ be nonnegative integers and $p \geq 1$. Then, we have:

a-) If $T \in \left(\left|C_{\lambda,\mu}\right|_p, \ell_\infty\right)$, then

$$0 \leq \|L_T\|_\chi \leq \limsup_{n\to\infty} \left\|\tilde{T}_n^{(p)}\right\|_{\ell_p}^* \tag{16}$$

and

$$L_T \text{ is compact if } \lim_{n\to\infty} \left\|\tilde{T}_n^{(p)}\right\|_{\ell_p}^* = 0. \tag{17}$$

b-) If $T \in \left(\left|C_{\lambda,\mu}\right|_p, c_0\right)$, then

$$\|L_T\|_\chi = \limsup_{n\to\infty} \left\|\tilde{T}_n^{(p)}\right\|_{\ell_p}^* \tag{18}$$

and

$$L_T \text{ is compact if and only if } \lim_{n\to\infty} \left\|\tilde{T}_n^{(p)}\right\|_{\ell_p}^* = 0. \tag{19}$$

c-) If $T \in \left(\left|C_{\lambda,\mu}\right|_p, c\right)$, then

$$\frac{1}{2}\limsup_{n\to\infty} \left\|\tilde{T}_n^{(p)} - \tilde{\gamma}\right\|_{\ell_p}^* \leq \|L_T\|_\chi \leq \limsup_{n\to\infty} \left\|\tilde{T}_n^{(p)} - \tilde{\gamma}\right\|_{\ell_p}^*, \tag{20}$$

and

$$L_T \text{ is compact if and only if } \lim_{n\to\infty} \left\|\tilde{T}_n^{(p)} - \tilde{\gamma}\right\|_{\ell_p}^* = 0, \tag{21}$$

where $\tilde{\gamma} = (\tilde{\gamma}_v)$ with $\tilde{\gamma}_v = lim_{n\to\infty}\tilde{t}_{nv}^{(p)}$ for all $v \in \mathbb{N}$.

Proof. Considering Lemma 1.2, we derive the conditions (17),(19) and (21) from the conditions (16), (18) and (20), respectively. So, we may prove (16), (18) and (20).

Since $\left|C_{\lambda,\mu}\right|_p, p \geq 1$ is a *BK*-space, by combining parts a-) and b-) of Lemma 3.4 with Lemma 2.5, we get the conditions (16) and (18), respectively.

Now, we show that the condition (20) holds. Let $T \in \left(\left|C_{\lambda,\mu}\right|_p, c\right)$ be given, then it follows from Lemma 2.6 that $\tilde{T}^{(p)} \in (\ell_p, c)$, where $\tilde{T}^{(p)}$ is defined by (8). Also, if we take $X = \ell_p$, which has AK, in part c-) of Lemma 3.4, then $\tilde{T}^{(p)} \in (\ell_p, c)$ implies that

$$\frac{1}{2}\limsup_{n\to\infty}\left\|\tilde{T}_n^{(p)} - \tilde{\gamma}\right\|_{\ell_p}^{*} \leq \left\|L_{\tilde{T}^{(p)}}\right\|_{\chi} \leq \limsup_{n\to\infty}\left\|\tilde{T}_n^{(p)} - \tilde{\gamma}\right\|_{\ell_p}^{*}, \tag{22}$$

where $\tilde{\gamma} = (\tilde{\gamma}_v)$ with $\tilde{\gamma}_v = lim_{n\to\infty}\tilde{t}_{nv}^{(p)}$ for all $v \in \mathbb{N}$.

On the other hand, let $S_{\left|C_{\lambda,\mu}\right|_p}$ be the unit sphere in $\left|C_{\lambda,\mu}\right|_p$. Then, we can write that $x \in S_{\left|C_{\lambda,\mu}\right|_p}$ if and only if $y \in S_{\ell_p}$, where S_{ℓ_p} denotes the unit sphere in ℓ_p, $x \in \left|C_{\lambda,\mu}\right|_p$ and $y \in \ell_p$, since $T(x) = \tilde{T}^{(p)}(y)$ by Lemma 2.6. For brevity, we use the notation $S_{\left|C_{\lambda,\mu}\right|_p} = S$ and $S_{\ell_p} = \bar{S}$. So, this leads us by Remark 1.1, Lemma 1.2, and Lemma 2.6 to the consequence that

$$\|L_T\|_{\chi} = \chi(TS) = \chi\left(\tilde{T}^{(p)}\bar{S}\right) = \left\|L_{\tilde{T}^{(p)}}\right\|_{\chi}. \tag{23}$$

This completes the proof by (22) and (23) .

Concerning the compactness characterizations of $\left(\left|C_{\lambda,\mu}\right|, \ell_p\right)$ for $1 \leq p < \infty$ and $\left(\left|C_{\lambda,\mu}\right|_p, \ell\right)$ for $p > 1$, we have next result.

Theorem 3.6 Let μ and $\lambda + \mu$ be nonnegative integers, $T = (t_{nk})$ be an infinite matrix and $\tilde{T}^{(p)} = \left(\tilde{t}_{nk}^{(p)}\right)$ the associated matrix defined by (8).

a-) If $T \in \left(\left|C_{\lambda,\mu}\right|, \ell_p\right)$, then, for $1 \leq p < \infty$,

$$\|L_T\|_{\chi} = \lim_{r\to\infty}\sup_{v}\left(\sum_{n=r+1}^{\infty}\left|\tilde{t}_{nv}^{(1)}\right|^p\right)^{1/p} \tag{24}$$

and

$$L_T \text{ is compact iff } \lim_{r\to\infty}\sup_{v}\sum_{n=r+1}^{\infty}\left|\tilde{t}_{nv}^{(1)}\right|^p = 0. \tag{25}$$

b-) If $T \in \left(\left|C_{\lambda,\mu}\right|_p, \ell\right)$, then, for $p > 1$, there exists $1 \leq \xi \leq 4$ such that

$$\|L_T\|_{\chi} = \frac{1}{\xi}\lim_{r\to\infty}\left(\sum_{v=1}^{\infty}\left(\sum_{n=r+1}^{\infty}\left|\tilde{t}_{nv}^{(p)}\right|\right)^q\right)^{1/q}$$

and

$$L_T \text{ is compact iff } \lim_{r\to\infty} \sum_{v=1}^{\infty} \left(\sum_{n=r+1}^{\infty} \left| \tilde{t}_{nv}^{(p)} \right| \right)^q = 0.$$

Proof. a-) Let $S_{|C_{\lambda,\mu}|}$ be the unit sphere in $|C_{\lambda,\mu}|$, that is, $S_{|C_{\lambda,\mu}|} = \{x \in |C_{\lambda,\mu}| : \|x\| = 1\}$. Then, from (5), we know that $x \in S_{|C_{\lambda,\mu}|}$ if and only if $y \in S_\ell$, where S_ℓ denotes the unit sphere in ℓ, $x \in |C_{\lambda,\mu}|$ and $y \in \ell$ are connected by the equation (6). For brevity, we write $S_{|C_{\lambda,\mu}|} = S$ and $S_\ell = \bar{S}$. So, using Remark 1.1, Lemma 1.2 and Lemma 1.3, we obtain

$$\begin{aligned} \|L_T\|_\chi &= \chi(TS) = \chi(\tilde{T}^{(1)}\bar{S}) \\ &= \lim_{r\to\infty} \sup_{y\in\bar{S}} \left\| (I-P_r)\tilde{T}^{(1)}(y) \right\|_{\ell_p} \\ &= \lim_{r\to\infty} \sup_{v} \left(\sum_{n=r+1}^{\infty} \left| \tilde{t}_{nv}^{(1)} \right|^p \right)^{1/p}, \end{aligned}$$

where $P_r : \ell_p \to \ell_p$ is defined by $P_r(y) = (y_0, y_1, \ldots, y_r, 0, \ldots)$, which completes the asserted by Lemma 2.2.

Also, we get the condition (25) from the condition (24) using Lemma 1.2.

Since one can easily prove part b-) as in part a-) using Lemma 2.1 instead of Lemma 2.2, so we omit the detail.

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