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Analysis of Philosophy of Mathematics Activities on Students' Attitudes and Beliefs Towards Mathematics[1]

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ABSTRACT

When the strong relationship between mathematics and philosophy, behavioral objectives from the secondary mathematics education curriculum of Ministry of National Education and the goal of the philosophy of mathematics is considered, it is realized that there is no scientific research scrutinizing the philosophy of mathematics activities. From this perspective, it is important to investigate the effects of these activities on the attitudes and beliefs towards mathematics. This study has two aims. One of them is conceptualize the Philosophy of Mathematics Activity and the other one is specify the effects of these activities on 9th grade students' attitudes and beliefs towards mathematics. As the quantitative research methodology pretest-posttest control- experimental design, as the qualitative research methodology, phenomenological study formed the methodology. At the end of the treatment semi-structured interviews were held to collect further data about attitudes and beliefs. At the end of the analyses, it is demonstrated that Philosophy of Mathematics Activities increases students' attitudes and beliefs towards mathematics.

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1. Introduction

When the lives of famous mathematicians who contributed to the world of mathematics is analyzed, it is seen that they did not solely study mathematics. These mathematicians who had an impact on the history, took mathematics and philosophy altogether. Mathematicians like Frege, Russell, Whitehead, Wittgenstein, Quine, Hilbert, Gödel, Leibniz and Bolzano are also important philosophers. In other words, they are mathematicians as well as a philosopher or the other way around a philosopher as well as a mathematician (Koyuncu, 2018). On the other hand, German mathematician and philosopher Gottfried Wilhelm Leibnitz who improved infinitesimal calculus said that "We can't infuse philosophy in depth without mathematics. And without philosophy, we can't infuse in depth into mathematics. Without both, we can't infuse into nothing." (Baki, 2014). However, mathematician and philosopher Gottlob Frege who formed modern mathematics argued that "A philosopher is a half one if he is not interested in geometry, a philosopher who is not interested in philosophy is a half mathematician. Distancing of two disciplines from each other is not beneficial to either of them." (Gür, 2004). When we think of words of Leibniz and Frege, it can be concluded that mathematics, geometry and philosophy should be acknowledged together and that is the only way to understand the reality of the things.

Descartes who is known as the developer of the modern philosophy and analytic geometry argued that geometric methods can be used to explain and demonstrate philosophical doctrines. Descartes

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formed analytic geometry with these thoughts in mind (McKeon, 1930). In addition, Gaarder stated (1994) "Descartes; wanted to use 'mathematical method' in philosophical topics, to prove philosophical truths with mathematical certainty and to consult mind that we use while calculating. Because only mind can give us certainty. It was certain not to put trust in senses. About this, he said that we think that Descartes and Plato who thought that mathematics and numerical relations are more reliable than senses, are in the same line of thought". According to Gür (2004), when the relation between mathematics and philosophy is taken into consideration, philosophy of mathematics as a whole is; studying the nature of mathematics, reality, goals, justifications, scope and content; a branch of philosophy which investigates how to reach knowledge on mathematical objects. Topics of mathematical philosophy can be categorized in two headings. First one is, foundational problems that include mathematical epistemology and ontology, and the second is axiology problems which consist mathematics and human interaction. In the following section, approaches like abilities that philosophy generates in individuals in the scope of the mathematics education before defining Philosophy of Mathematics Activity (PMA) as a concept, goals of using philosophy in mathematics education, relation of the objectives of secondary mathematics education curriculum to philosophy of mathematics, including philosophy of mathematics into mathematics education are taken into consideration. And finally, all these reasons and literature are taken as a reference and so PMA is conceptualized. According to De la Garza (1999) educational setting that is prepared for children by philosophy is an inquiry community. Such a community develops abilities that can be classified into four groups: Logic, inquiry, concept formation and translation abilities.

Activities that are planned to be used in this study are aimed to contribute to students in terms of giving meaning to mathematical concepts and mathematical structures and increasing their motivations in the positive sense. In this regard, it can be said that philosophy will be used as a tool. To put it differently, activities planned to be used in this study include knowledge about epistemology and ontology of the mathematical concepts and try to explain the philosophical perspectives of the mathematics. In this manner, philosophy will be used as an aim in this study. When two of them are read together, it can be concluded that philosophy will be used both as a tool and as an aim in the context of mathematics education. Curriculum emphasizes that due to the rapid change in technology and science, the acts of the individuals' volatile. Hence, individuals need to have such qualities as knowledge builders, users of knowledge operationally in life, problem solvers, critical thinkers, entrepreneurs, decisive, having effective communication abilities, emphatic, contributing to culture and community (MNE, 2018).

Furthermore, in the definition of the philosophy of mathematics, topics are given in two headings: (1) Foundational problems including epistemology and ontology; (2) Axiological problems covering mathematics and human communication. In addition, Turkish educational system aims to develop verbal and written expression and interpretation of the concepts, opinions, views, sensations, and phenomena. These concepts are the main problems of philosophy of mathematics and solely philosophy as well. So, the position of philosophy of mathematics which includes epistemology and ontology, helps mathematics course curriculum to actualize its objectives. On the other hand, in the view of the mathematics course curriculum, the importance of educating individuals on having values, and knowledge, ability and behaviors united with competencies, is emphasized. Even, it is identified as the main goal of national educational system. This situation is related to the axiological problems that cover mathematics and human communication in the frame of the definition of the philosophy of mathematics. As a result, it can be said that the general goals of the secondary mathematics education curriculum, expectations from individuals, and the expected behaviors from the individuals are all related reciprocally to one another. Hence, the philosophy of mathematics is not included in the curriculum, nor in this study. In other words, none of the mathematical topics was carried philosophically. Just as philosophy of law does not do legislation, history does not draw the boundaries of country, mathematics is carried out by using philosophy of mathematics. The only thing that is aimed could be shedding light upon mathematics and mathematical concepts along with philosophy of mathematics. Moreover, De la Garza (1999) states that intellectual mathematics teachers generally use philosophy of mathematics to motivate and surprise solid students, and x they can even use philosophy for weak students.

Definition of activity in the context of the education and teaching, according to Wallace (2008) is a task or exercise which is decided by teacher, has a goal, is taken over by the learner, and has a learning product. In this context, activities which meet the following criteria, are named as Philosophy of Mathematics Activities (PMA).

a) Having a plan,
b) Having a topic in the context of philosophy of mathematics,
c) Having a name,
d) Having a purpose,
e) The determined purpose be related to the topic, interest or purpose of the philosophy of mathematics,
f) Identifying the approach for incorporating mathematics philosophy into mathematics teaching,
g) Measurability of affective and cognitive changes in students at the end of the activity,
h) Compatibility with the skills, behaviors and competencies that are aimed to be gained in the curricular program,
i) Accordance with the age, interests and levels of individuals,
j) Methods and techniques to be used be clearly identified,
k) Tools, materials, resources, methods and techniques to be used be clearly defined.

When all things are considered, since PMA planned to be used in this study, is aimed to think critically about the source of the knowledge about mathematics, specifically with regard to if it is a game human mind produces, if mathematical knowledge exists in nature, if mathematics is an invention or discovery; it is convenient to say that planned activities fall within epistemological part of the philosophy of mathematics. In addition, since planned activities target to examine if mathematical structures are intellectual concepts and not physical, it can be said that planned activities concern philosophy of mathematics ontologically. On the other hand, since planned activities contribute to listening culture of students one another, it is aimed to develop communication and showing empathy abilities, it can be said to concern philosophy of mathematics axiomatically.

Experimental studies on mathematics held on activity-based (addressing attitudes and beliefs) were generally conducted with secondary school students in our country (Işık & Çağdaşer, 2009; Çağlar et al., 2018; Şen, 2019). From this point of view, this study -which uses experimental design and activities- is important for filling this gap in the literature. When the importance of study and deficiency in literature is concerned, the aim of the study is to identify beliefs and attitudes towards mathematics in the teaching coherent with philosophy of mathematics of secondary mathematics teaching. In the line with the importance and aim of the study, the following questions were addressed:

1. What is the effect of PMA on students' attitudes towards mathematics?
2. What is the effect of PMA on student's beliefs of mathematics?
3. What kind of changes have happened about the thoughts of definition of mathematics?
4. What kind of changes have happened on thinking about what kind of lesson mathematics is?
5. What kind of changes have happened about thoughts that which activities to be done in mathematics class in order to learn mathematics well?
6. If there is a change in the thoughts towards mathematics course, which activities most affected this change and how?

2. Methodology

This study is an experimental study which is investigating the impacts of activities prepared in compliance with philosophy of mathematics on students' attitudes and beliefs towards mathematics.

Qualitative and quantitative research methods used together. Therefore, according to Patton (2001), this study has a mixed research method approach. In addition, convergent parallel design was applied in this research and it is classified by Creswell and Plano-Clark (2017). Study design is given in Figure 1.

It is quantitative because of scalar results on Attitudes Towards Mathematics (ATM) and Mathematics Belief (MB) scales' evaluation; and qualitative because of verbal results on answers of students to interview protocols and in debates to evaluate the interpretation, hence this study has an enriched design. The qualitative part of the study is a "case study", and model of this part is "phenomenological research". Hence, in this study ATM and MB are dependent variables; PMA is an independent variable.

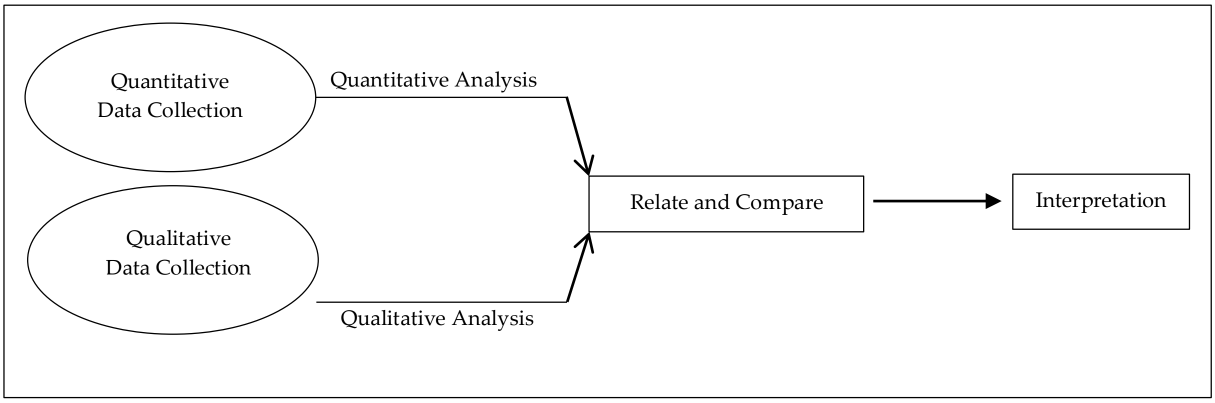

Figure 1. Study design demonstration

2.1. Study Group

In the study, 30 students from two classes of 9th grade sections of a private science high school which is in the Istanbul city center participated using random selection. In the school, there are three ninth grade section and, in each section, there are 15 students. The groups were randomly appointed as experimental and control groups. To demonstrate that there is no statistical difference among their mathematical levels academically of 15 experimental (8 boys, 7 girls) and 15 control (8 boys, 7 girls) students who came after graduating different primary schools to the high school, their TEOG test results from 2016 are taken into account.

2.2. Data Collection Instruments, Data Collection

In the study, quantitative and qualitative data collection instruments were applied altogether. Qualitative data were collected through semi-structured interview forms and quantitative data were collected by Attitudes Towards Mathematics Scale (ATMS) and Mathematics Belief Scale (MBS). To support quantitative methods and to analyze the reasons under the basis of the differences appearing, qualitative data were benefited from. In the study, to measure MB of participants, MBS which was developed by Kandemir (2011) was used. Reliability and validity of the scale was carried under the doctoral thesis of the same author. Reliability measure as Cronbach Alpha of the Scale is calculated as 0,826. This level is sufficient for reliability of a scale (Creswell, 2002). In the study, to measure the ATM of participants, ATMS which was developed by Baykul (1990) was used. This measure was used on 1056 people by Baykul (1990) and its Cronbach alpha reliability constant is 0,96. This level shows adequacy for reliability and validity of the scale (Creswell, 2002). Reliability and validity analysis of the ATMS was carried out again by Kandemir (2011). This time, scale was applied to 340 students who were educated in three different secondary schools. Cronbach alfa reliability constant of ATMS was found to be as 0,976. This level shows adequacy for reliability and validity of the scale. In the qualitative part of the study, interview technique was used which is the most frequently used data collection method in qualitative studies (Patton, 2001).

2.3. Data Analysis

For quantitative analyses, a statistics program is used for t-test analysis. Along the duration of the qualitative data analysis, interviews are taped with the consent of the participants. The video records were transcribed, and then pattern coding was implemented in order to find and interpret the recurring themes in the data.

2.4. Study Process

In the data collection and the implementation stage, with both experimental and control groups students, a traditional teaching program coherent with the 9th grade mathematics curriculum was carried out and to experimental group two PMA course hours are added. With the aim of locating and guarding possible problems that may be encountered beforehand during the study progress, students were presented the activities coherent with philosophy of mathematics for two weeks. During the study, both experimental and control groups were taught by the researcher. Totally, 14 course hours were used for data collection. This includes seven weeks and two hours in a week. Activities coherent with philosophy of mathematics from the book "Philosophy for Children" of White (2001), were applied to experimental group students as worksheets. To provide the relatability of the activities, area experts and advisor views are collected. Throughout the data collection of the study, the researchers provided all applications to be coordinated continuously, but did not affect the groups positively or negatively. With respect to PMA plan developed by experts and advisors, data collection process was started.

1. In the first week, a visit was made with experimental group's students to Istanbul Bilgi University's Mathematics Department in order to talk with Professor Ali Nesin about the philosophical dimension of mathematics. The students asked the questions they had prepared before. At the end of the activity students were asked what impression this visit had on them.
2. In the second week, to the experimental group students, to examine the concept of "*Reality of Mathematical Objects and Mathematical Certainty*" and to talk about "*Platonism*" from philosophy of mathematics "*Allegory of the Cave*" from Plato's book "*Republic*" was presented visually. Following the video and the presentation, with the experimental group students, a debate was initiated on the reality of concrete objects and mathematical objects under the condition of thinking about the cave allegory. It was discussed how mathematics as an abstract entity and the concrete world comply with one another.
3. In the third week, with the experimental group, a lesson was held on "*Whether mathematics is discover or invented?*" Hersh (1997), topic on which philosophers had long discussions on it. Afterwards, students from experimental group, were divided into two groups as mathematics as a discovery and mathematics as an invention. The, "Discovery-invention" debate was carried out between the groups.
4. In the fourth week, with the experimental group students, role playing was followed up. Theatre piece written by Renyi (1999) and consisted of dialogues between Socrates and Hippocrates from "*Dialogues about Mathematics*" book and is on what mathematics is. In the activity, the quality of mathematical objects and its method, its relation to real world was explained.
5. In the fifth week, Xeno's paradox from the book named as "*Philosophy for Children*" of White (2001) was taught as a lecture to experimental group students, which was followed by a role play performed by the same students as an activity. "*Xeno's Paradox*" was discussed with the experimental group students at the end of these activities.
6. In the sixth week, to visualize some terminology (dimension, direction, etc.) from mathematics, TED's "*4th Dimension Documentary*" from its distance education portal was watched by the experimental group (TEDEd, 2016). To identify the effects of documentary on students, some worksheets on the documentary were distributed as a follow-up handout. After viewing the documentary, the same questions were asked again to the same students. Then, the answers were compared.

All these activities were videotaped by the consent of the students.

3. Results

3.1. Results belonging to the first sub-problem of the study

The first sub-problem of the study is, *"What is the effect of PMA from 9th grade secondary education on ATM?"*. The first result regarding the first sub-problem is on *"If there is a significant difference between pretest results of experimental and control group students on ATM".* To investigate if there is significant difference between experimental and control group students before the study regarding the ATM's mean points, independent groups t-test was carried out. The results can be seen in Table 1.

Table 1. Independent group t-test results of experimental and control group students on ATMS pretest means

Groups	N	X	SS	Sd	t	p
Experiment	15	2,9120	,69531	28	-,174	,863
Control	15	2,9533	,60618			

As displayed in Table 1, there is no statistical significance between group means [*t*=-,174; p>0,05]. In this way, it is concluded that there is no meaningful difference before the study between experimental and control group students' ATM (p=0.05).

Second result from the first sub-problem of the study; *"If there is a significant difference between experimental and control group students on ATM on the posttest results."* To test if there is a statistically significant difference between experimental and control groups ATM mean points, independent groups t-test was performed. The results are in Table 2.

Table 2. Independent group t-test results of experimental and control group students on ATMS posttest means

Groups	N	X	SS	Sd	t	p
Experiment	15	3,6822	,50283	28	3,478	,002
Control	15	2,9947	,57734			

When Table 2 is investigated, statistical difference between group means is meaningful [*t*= 3,478; p<0.05]. Experimental group students' mean point is higher. In this respect, at the end of the study, there is a meaningful difference between experimental and control groups students' mean points. This statistical difference shows that teaching with activities coherent with philosophy of mathematics increases ATM significantly compared to traditional teaching.

Third result related to the first sub-problem *"If there is a significant difference between pre and post test results of experimental group students on ATM."* To test this hypothesis, dependent groups t-test analysis was used between ATM group means. The results are shown in Table 3.

Table 3. Dependent group t-test results of pre and post test results of experimental group students on ATMS

Exp. Group	N	X	SS	Sd	t	p
Pretest	15	2,9120	,69531	14	-4,625	,000
Post test	15	3,6820	,50283			

When Table 3 is analyzed, there is a statistical difference on behalf of posttest results between pre and posttest results of ATM [*t*=-4,625; p<0.05]. This result shows that PMA on experimental group affects students' ATM positively.

Fourth result on the first sub-problem of the study *"If there is a statistical difference between pre and post test results of the control group students on ATM."* To test this hypothesis, dependent groups t-test was carried out between pre and post test results of control group students on ATM group means. Results can be seen in Table 4.

Table 4. Independent group t-test results of pre and post test results of control group students on ATMS

Control Group	N	X	SS	Sd	t	p
Pretest	15	2,9533	,60618	14	-,556	,578
Post test	15	2,9947	,57734			

When Table 4 is analyzed, ATM pre and post test results are found to be insignificant [*t*=-,556; p>0.05]. This result shows that traditional teaching is not effective in changing ATM.

3.2. Results belonging to the second sub-problem of the study

Second sub-problem of the study; *"What is the effect of PMA on the beliefs of the secondary 9th grade students' towards mathematics?"*. First result of the second sub-problem of the study *"If there is a significant difference between experimental and control group students' pretest results on MBS."* To test this hypothesis, independent group t-test was carried out between experimental and control groups MBS group means before the study. Results can be seen in Table 5.

Table 5. Independent group t-test results of experimental and control group students on MB pretest

Groups	N	X	SS	Sd	t	p
Experimental	15	3,4013	,29000	28	-,721	,477
Control	15	3,4920	,39150			

When Table 5 is analyzed, difference between group means is found to be statistically insignificant [*t*=-,721; p>0.05]. In this manner, regarding the MB data before the study, experimental and control group students do not differ regarding the MB at the p= 0.05 level.

Second result related to the second sub-problem of the study *"If there is a statistical difference between posttest results of experimental and control group students' MB."* To test this hypothesis, independent groups t-test is carried between the MB group means of experimental and control group means after the treatment. Results are shown in Table 6.

Table 6. Independent group t-test results of experimental and control group students on MB posttest

Groups	N	X	SS	Sd	t	p
Experimental	15	3,6940	,40802	28	2,183	,038
Control	15	3,3900	,35276			

When Table 6 is analyzed, difference between groups means is found to be statistically significant [*t*= 2,183; p<0,05]. This difference is actualized on the behalf of the experimental group students. This result point to the increased MB as a result of the PMA teaching compared to traditional teaching.

Third result related to the second sub-problem of the study *"If there is a statistically significant difference between pre and post test results of the experimental group students on MB."* To test this hypothesis, dependent group t-test was performed between pre and post test results of experimental group students on MB. Results can be seen in Table 7.

Table 7. Dependent group t-test results of experimental group students on MB pre and post test

Experimental Group	N	X	SS	Sd	t	p
Pretest	15	3,4013	,29000	14	-2,449	,028
Post test	15	3,6940	,40802			

When Table 7 is analyzed, there is a statistically significant difference between pre and posttests on the behalf of the posttests of MB [*t*=-2,449; p<0,05]. This result shows that PMA applied to the experimental group is effective on increasing the MB positively.

Fourth result with respect to the second sub-problem of the study" If *there is a significant difference between pre and posttest results of the control group students on MB*." To test this hypothesis, dependent group t-test was carried out between MB results before and after the study of the control group. Results can be seen Table 8.

Table 8. Dependent group t-test results of control group students' MBS pre and posttest mean points

Control Group	N	X	SS	Sd	t	p
Pretest	15	3,4920	,39150	14	,032	,975
Post test	15	3,4873	,40346			

When Table 8 is analyzed, no statistical difference is found between MBS pre and post test results of control group students [*t*= ,032; p>0,05]. This result shows that traditional teaching is not effective in changing the MB of control group students.

3.3. Results belonging to the third sub-problem of the study

The third sub-problem of the study is *"What kind of changes have happened about the thoughts of definition of mathematics?"*. In the scope of the study, as a result of the phenomenological analysis of the experimental students' answers to "*What do you think mathematics is? What are your thoughts about mathematics?*" from the semi-constructed interview protocol, answers of experimental group students on open ended items are listed in Table 9.

Table 9. Expression of experimental group students on the definition of mathematics

Theme	Code	f	%
Definition of Mathematics	Numbers	5	33,3
	Operation	4	26,6
	Problem	4	26,6
	Life	2	13,3

From the data of Table 9, experimental students associate mathematics with school mathematics while defining. Reason for this can be students' seeing mathematics as a school subject while defining. The students want to associate mathematics with life to overcome daily life and some of the problems that it poses because students' comments on life are centered on shopping and calculation and present opinions on these topics and effectiveness of mathematics on these. In brief, students perceive mathematics as a school subject. From the experimental group students, students coded as S1, S2, S3, S4 and S5 code give opinion as mathematics is made up of numbers. Related to this code, an expression of a participant is like this:

"I think that there is no mathematics without numbers. In my opinion, there are numbers in wherever mathematics is. In short, mathematics means numbers and we do this course with numbers." (S1).

3.4. Results belonging to the fourth sub-problem of the study

The fourth sub-problem of the study is "*What kind of changes have happened on thinking about what kind of lesson mathematics is?*". In the scope of the study, as a result of the phenomenological analysis of the experimental students' answers to "*What do you think mathematics course is like? Can you explain it?*" from the semi-constructed interview protocol, answers of experimental group students on open ended items are listed in Table 10.

Table 10. Expressions of experimental group students on what kind of a course mathematics is

Theme	Code	f	%
What type of a course math is	Logical	5	33,3
	Enjoyable	4	26,6
	Easy	4	26,6
	Number	2	13,3

Students who see mathematics as a logical course, from the view of the seeing mathematics that can't go out of the logical rules, possibly state that mathematics is a logical course. Students who express mathematics as an enjoyable course, may view mathematics as a course which does not ask for memorization and they may see themselves as not so good in memorization. In addition, after the activities were carried out, them being aware of the philosophy of mathematics may cause them to enjoy mathematics. Students viewing mathematics as an easy course, on the other hand, by comparison of mathematics with other courses, seeing the certainty in mathematics not existing in other courses may deduce this a convenience, and may see mathematics as an easy course. Students who view mathematics as a quantitative course; may give opinion as mathematics being quantitative from calculations and the secondary computation courses, realizing that mathematics is one of the numerical courses. From the experimental group students; students with codes S1, S2, S3, S4 and S5, state that mathematics is logical course. Related to this code, an expression of a participant is like this:

"For example, if we write 2x2=5, it is wrong. Hence, we correct immediately as 2x2=4. So, we correct something in case it does not suit to our logic. That is why, mathematics is a logical course." (S3).

3.5. Results belonging to the fifth sub-problem of the study

The fifth sub-problem of the study is *"What kind of changes have happened about thoughts that which activities to be done in mathematics class in order to learn mathematics well?"*. In the scope of the study, as a result of the phenomenological analysis of the experimental students' answers to *"What should be done to learn mathematics better?"* from the semi-constructed interview protocol, answers of experimental group students on open ended items are listed in Table 11.

Table 11. Experimental group students' expressions of suggestions for learning mathematics better

Theme	Code	f	%
Suggestions to learn mathematics better	Daily Life	5	33,3
	Problem solving	5	33,3
	Explaining logic	3	20
	Activities	2	13,3

To learn mathematics better, students' ratio who view that mathematics coursework should be integrated with daily life to students' ratio who view that a lot of problems should be solved are equal and can be seen in the top two suggestions as can be seen in Table 11. Regarding this code, a participant statement is:

"If I do not solve enough number of problems, I feel insecure and fearful about that topic. For me, just at the end of the topic, we need to solve a lot of problems." (S7).

To learn mathematics better, when students with codes S1, S2, S3, S4, S5, S6, S7, S8, S9 and S10 expressions are thought together, it can be deduced that real life problems should be solved in mathematics courses and their numbers should be increased. In addition, from students' expressions, PMA may be found as effective to show that mathematics is related to real life. Students think that they carry mathematics learning as in language learning by imitation to a better point because they suppose that solving more problems may make them good mathematics learners. Science learning mathematics better is thought to be in parallel with less memorization, explaining the logic of a topic can be helpful. In a group where making activities is thought to be not so effective, students who gave opinion based on activities, stated things such as going outside the classroom and going on trips. Hoping for activities in mathematics looking like activities in other courses, is another result from their expressions.

3.6. Results belonging to the sixth sub-problem of the study

Sixth problem of the study is *"If there is a change in the thoughts towards mathematics course, which activities most affected this change and how?"*. In the scope of the study, as a result of the phenomenological analysis of the experimental students' answers to fourth question as "*Did activities change your views related to mathematics course? If your answer is yes, which activities or activity were the most effective? Can you explain the reason?"* from the semi-constructed interview protocol, answers of experimental group students on open ended items are listed in Table 12.

Table 12. Expressions of experimental group students on view changes at the end of the activities

Theme	Code	f	%
Thought changes at the end of the activities	Xeno's Paradox	4	26,6
	Discovery-Invention Debate	3	20
	Plato's Cave Allegory	3	20
	Dialog between Socrates and Hippocrates	2	13,3
	A lesson with Prof. Ali Nesin on the philosophy of mathematics	2	13,3
	Documentary	1	6,6

As seen in Table 12, during the study progress, regarding the PMA activities that were applied to the experimental group result in opinion changes under following categories; "Xeno's Paradox, Discovery-Invention Debate, Plato's Cave Allegory, Dialogue between Socrates and Hippocrates, and

A lesson with Prof. Ali Nesin on the philosophy of mathematics". As a result of the PMA applied to experimental group students, as can be understood from the expressions of students, there were some changes in thoughts towards mathematics course. According to the data in Table 12, within the applied PMA, the most effective activity in change of views is "Xeno's Paradox" with 26.6 %. The reason, based on the students' views, could be the fact that it gives an idea on what an indefinite concept like infinity means and makes students worry about this mysterious concept and makes them think about it. A participant expression related to this code is as follows:

"My views towards mathematics course changed incredibly. I started to think that mathematics is not only restricted to the one we know, but also incorporates many philosophical views as well. The most affecting activity was Xeno's paradox, but others were beautiful as much. The mysterious and inexplicable infinity concept from the Xeno's paradox preoccupied me, immensely. As with the other activities, …" (S4).

Accordingly, as can be understood from experimental group students coded as S1, S2, S3 and S4, the most effective factor here can be counted as identification of difficult concepts that are hard to visualize and fictionalize activities that can draw attention of students and transfer to students. On the contrary, with respect to the data in Table 12, in the PMA, activity which caused the least percentage of change in students' views is 'documentary' with 6.6%. The reason for this can be the limited number of documentaries that draw students' attention and have the subject as philosophy of mathematics. In result, it can be deduced that either experts should make documentaries qualified and visually satisfying and having subject as philosophy of mathematics and these videos should be watched by students, or in the scope of PMA less documentary viewing activities should be considered.

3.7. Results obtained from worksheets

It is aimed to identify the effect of PMA on students by asking same questions before and after the activity by using worksheets. To this end, by carrying analysis of worksheets, answers were interpreted. Some student views are given in this part. Instead of students' real names, codes like S1, S2, S3 are used. For example, before the "4th Dimension Documentary" activity, student coded as S1 gives the answer to "What is the direction?" as *"Direction is the movement of an object from right, left, front and back sides." (S1)*. The same student gives answer to the same question as after the activity as "*Direction in reality does not exist, but it is a concept to clarify things in science*" (S1). When the answers are analyzed, it is seen that the student defines the concept of direction with physical laws as change in the situation and place of object. After the activity, student coded as S1 gives a more philosophical and sophisticated answer by ontological analysis. This shows that PMA used in the study gives positive result. Before the activities; drama activity "Dialogue between Socrates and Hippocrates" and "Plato's Cave Allegory Documentary" activity, student coded S5 answers questions; "if there can be two dimensional object or not" or "if there are perfect triangle, square, circle in nature or not ", as "*There is! Lines are two dimensional*." (S5) by thinking lines drawn on paper as two dimensional. In other words, student coded as S5 gives opinion before the activity about that there can be two dimensional objects. Student coded S5 gives the answer as "*There is not. All of the objects in the world have length and thickness*." (S5) after the activity. Student points to the existence of the concept of volume by stressing the dimensions of the objects. When the differences in answers are analyzed, PMA applied in the study results in positive change in the views of the S5.

4. Discussion and Conclusion

In this part, outcomes of the study are discussed as compared with those from the previous studies. In this study, it is seen that PMA increases beliefs and attitudes towards mathematics positively. This situation supports indirectly the study conducted by Jankvist and Iversen (2014). The participants gave positive answers to questions such as "What is the direction?", "What is dimension?", "Is there a two dimensional object?", "Is there a perfect triangle, square, circle etc. in nature?". Namely, superficial and even wrong answers given before activity and interview leave their places into philosophically deeps. In addition, students who used to be close to formalism before the study, got closer to Platonist philosophy after the intervention. Consequently, it can be said that PMA causes positive change in the views towards mathematics, and philosophically deep interpretations. From the results found, it can be said that PMA coherent with philosophy of mathematics, provides an insight with students to define some miscellaneous concepts (like direction, dimension) philosophically and to gain knowledge to be able to carry some debates. Thus, PMA can be said to be helpful in explaining and defining visually difficult concepts existing in students' mathematical world. This result supports Jankvist and Iversen's (2014) study.

PMA can be said to inevitably affect mathematics teaching and learning since it affects ATM positively because Ernest (2004), states that philosophy of mathematics education is not independent from philosophy of mathematics. At the same time, studies of Ma & Kishor (1997) and Anderson (2007) puts forward a relation between teachers' and students' attitudes on mathematics teaching and learning. This result is consistent with the results of this study. In addition, in this study, it is deduced that PMA affects MB positively. Raymond (1997) argues that belief concept as individual value judgement is an importance gaining concept lately. Consequently, PMA is expected to a find a better place in mathematics education. Besides, PMA can shape their views towards mathematics world while identifying MB. Thus, Frank (1988) and Schoenfeld (1992) in their studies, determine MB establishes in schooling. In addition, in the studies, it is concluded that individuals' MB identifies their views towards the world of mathematics.

It can be said that as a result of the education using PMA since their attitudes and beliefs towards mathematics is affected positively, educational activities they would carry after their graduation from teaching can be free of bad perceptions. However, similarly, Moodley, Adendorff and Pather (2016), in their study, preservice teachers express that if they finish the high school with a bad mathematics perception and become a teacher, these bad perceptions towards mathematics may affect their ATM teaching negatively. Furthermore, mathematics teaching with PMA, may give rise to number of students with positive ATM. Hereby, teacher raising institutions, may be provided to have teacher candidates with positive ATM from their previous lives. Similarly, Mutodi, Ngirande (2014) and Pather (2012), argue that to help changing the negative perceptions and ATM of pre-service teachers, teacher educating institutions, primarily it is important to investigate perceptions of preservice teachers that they receive in their past. In their studies, Cobb (1986) and Thompson (1984), designate that MB is important to find out the meaning of what is learned, and this belief has an indirect or direct effect on students' achievement. Results from these studies can be taken in two parts. In the first part, the hypothesis that MB would help to explain meanings of what is learned, and in the second part, ascertain that there is an effect of belief on achievement. The first result supports the results of this study, for the second part it can be said that PMA increases mathematical achievement from the point of its increasing mathematical belief.

Thompson (1984), Carter & Norwood (1997), Hart (2002), Hart (2004) and Swars, Hart, Smith, Smith & Tolar (2007) determine that beliefs related with the nature of mathematics and mathematical education are related to the education given to students. And in this study as well, it is seen that PMA is effective in increasing students' MB positively. Consequently, it may be said that PMA may increase students' MB positively. Even, Kloosterman & Stage (1992), determine that individuals' MB is effective on learning and problem solving. Due to this result, teaching with PMA, may be said to influence students' learning and problem-solving abilities.

According to Mason (2003), unsuccessful students in mathematics course, may be not aware of their negative MB. These negative beliefs may be effective on students' learning and achievement, negatively. As can be seen, in this study as well, learning and mathematical achievement are tied to MB. Starting from this, it can be deduced that by the help of the education with PMA, for students with negative MB, these negative beliefs can be decreased, their awareness about the obstacles before their success are their negative beliefs and it is possible to achieve in mathematics courses.

Kloosterman & Stage (1992) and Mason (2003), in their study point to the MB and problem solving whenever they state, "Students' individual MB and mathematics learning and their ability to solve mathematical problems may be understood by their beliefs about problem solving." In this study, it is determined that teaching with PMA increases attitudes and MB positively. This result may explain the results of Di Martino and Zan (2001)'s study results.

Ernest (2004), indicate that beliefs and attitudes strongly affect students' and teachers' way of learning and belief studies in mathematics education, focus on how teachers take the nature of mathematics, teaching and teaching in general. From this point on, it can be said that PMA may bring positive effects on teachers and students on what is mathematics, how it can be learned, and in general, it can guide students and teachers on "teaching". Since teaching with PMA affects students' attitudes and beliefs, these activities guide actions within the and effect students' and teachers' change process, which is result is parallel to Pajares (1992).

Reynolds, Schoenfeld, Tchoshanov; cited in Moodley, Adendorff & Pather (2016), in their study they posit that students' beliefs, attitudes, understanding, and experiences in mathematics learning, is shaped by exposure to rule-based teaching in primary and secondary level for a long time. In this respect, regarding the result that teaching coherent with PMA increases positively students' MB and ATM, students' perceptions related to mathematics may be increased. De la Garza (1999) points that mathematics teachers usually use philosophical questions to motivate successful students and to surprise them. In addition, he adds to his study results that it is a possibility to be benefited from philosophy even for academically poor students. For this reason, in the learning environments with PMA, students' motivation and activation of students' astonishment grow.

Nale (1969) while defining ATM, uses the term as "Trend to participate in mathematical activities or avoidance from them." For this reason, students' eagerness to participate in PMA may increase their positive ATM. In addition, from this definition, participation of students in PMA not only in this but in other courses and their desire for other courses to be taught by the activities may increase students' attitudes positively. Furthermore, according to Bloom (2012) in case of students' having positive ATM, one of the biggest obstacles in mathematical success may be removed. In this study, since used PMA increases students' ATM positively, activities done may be helpful in removing the obstacles in front of the students' success in mathematics.

Students intuit that theoretically each number can be divided into two, may continue to divide in two up to infinity. Hence, they mention that under the atomic dimensions as well, they can reach to a small number. However, physical space (real world that we live in) cannot be divided into two continuously, since after a while, continuous division into two may be precluded by nature. Consequently, students realize that physical space cannot be divided into two continuously, but mathematical space that we visualize can be divided into two continuously. This activates the idea in students that mathematical world and the world that we live by may not be coherent with one another. In addition, students realize that due to the existence of the concept of infinity and attention of mathematical discipline with this concept, mathematics' only problem is not the physical world's problems that can be understood by our five senses. On the other hand, students see that the thing with infinity concept that it is an important concept itself but also important to understand mathematics as well. In short, understanding of "$n=\infty$" of the concept of infinity leaves its place into notational "$n \to \infty$" which is important for mathematics and has a place in mathematical philosophy. These results support Crilly (2013) and Nesin's (2007) studies.

After the "Plato's Cave Allegory" activity, it was discussed if it would be possible to have a relation between real world with no metaphysics experienced with senses and realities that can be perceived only by thoughts. Moreover, a brainstorm was carried out on the concepts of "reality" and "mathematical reality". Before everything, it is observed that students are so eager and excited while there are talks about topics they have not heard before. Furthermore, these students started to think about reality of the knowledge, mental world and mathematical objects at the end of this activity, and they became motivated from the point of the learning inside story of the mathematics and geometry –essence of the knowledge-.

5. Suggestions

In this part, there will be some suggestions for researchers and practitioners with respect to the results about students' attitudes and beliefs towards mathematics and beliefs on using PMA in learning environments.

In this study, the effects of PMA on students' attitudes and beliefs towards mathematics were scrutinized. In the following studies, other sensual and cognitive variables of PMA can be investigated; In the future research, the number of PMA can be increased. It can be furnished with richer content; In the learning environments supported with PMA the duration of the experimental studies on attitudes and beliefs towards mathematics can be increased; a book designed with respect to age and class level can be prepared for PMA; the number of mathematics and philosophy documentaries that are filmed by area expert investigators professionally and with visually rich content can be increased; for teachers to carry PMA to their classroom effectively, and since the educational programs should be suitable for this, people who prepare educational programs may give some place to PMA in the activities part; implementations rich with verbal and visual animations can be prepared and can be publicized through the internet. Future researches may include studies planned in accordance with PMA education at different grade levels. Additionally, certain effects of PMA implementations such as creativity and communication skills may research in future studies.

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